University of Calgary

TEAM NOTEBOOK

ACM International Collegiate Programming Contest
2005 World Finals

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Our Magic Incantations List

When Choosing a Problem

- Find out which balloons are the popular ones!
- Pick one with a nice, ***clean solution*** that you are totally convinced ***will work*** to do first.

Before Designing Your Solution

- Highlight the important information on the problem statement – input bounds, special rules, formatting, etc.
- Look for code in this notebook that you can use!
- Convince yourself that your algorithm will run with time to spare on the biggest input.
- Create several ***test cases*** that you will use, especially for ***special or boundary cases***.

Prior to Submitting

- Check ***maximum*** input, ***zero*** input, and other ***degenerate*** test cases.
- Cross check with team mates' supplementary test cases.
- Read the problem ***output specification*** one more time – your program's output behaviour is fresh in your mind.
- Does your program work with ***negative*** numbers?
- Make sure that your program is reading from an appropriate ***input file***.
- Check all ***variable initialisation***, ***array bounds***, and ***loop variables*** (`i` vs `j`, `m` vs `n`, etc.).
- Finally, run a `diff` on the provided sample output and your program's output.
- And don't forget to submit your solution under the ***correct problem number!***

After Submitting

- Immediately ***print a copy*** of your source.
- Staple the solution to the problem statement and keep them safe. Do not lose them!

If It Doesn't Work…

- Remember that a ***run-time error*** can be ***division by zero***.
- If the solution is not complex, allow a team mate to start the problem afresh.
- Don't waste a lot of time – it's not shameful to ***simply give up!!!***

Remember to ***HAVE FUN!!!***

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Graph Algorithms Summary

Undirected Graphs

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Directed Graphs

Cycle Detection	DFS Back Edges	Linear
Transitive Closure	Warshall's Algorithm	V^3
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DAG Transitive Closure	DFS with DP, 'or' the rows	Linear
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Mail Carrier	Mincost Maxflow	?

Hard Problems

Hamiltonian Path/Tour	Exhaustive Search	V!
General Shortest Path	Exhaustive Search	V!
Even Cycle Detection	No Nice Algorithm Known	?
Graph Isomorphism	?	?

```cpp
//---------------------------------------------------------------------------
// GRAPH ALGORITHMS ON ADJACENCY MATRIX GRAPH
//
// This file contains a graph class that uses an adjacency matrix
// representation, and implementations of the following algorithms:
//    - Breadth First Search for counting connected components
//    - Prim's algorithm for minimum spanning trees
//    - Dijkstra's algorithm for shortest paths
//    - Floyd's algorithm for all pairs shortest paths
//
// Author:  Sonny Chan
// Date:    November 10, 2003
//---------------------------------------------------------------------------

#include <iostream>
#include <iomanip>
#include <vector>
#include <queue>
#include <algorithm>
#include <iterator>

using namespace std;

//---------------------------------------------------------------------------
// graph class with adjacency matrix representation
//    - template parameter T is edge type (eg. int, double, or edge struct)

template <class T>
struct graph {

   int vertices;                 // number of vertices in the graph

   vector< vector<T> > d;        // distance matrix
   vector< vector<bool> > c;     // connectivity matrix (true if edge exists)

   vector<bool> present;         // indicates which vertices actually exist

   vector< vector<T> > spd;      // shortest paths adjacency matrix
   vector< vector<int> > spp;    // shortest paths tree for all vertices
   vector< vector<bool> > tcc;   // transitive closure connectivity matrix

   // constructor takes size as (maximum) number of vertices
   graph(int size = 1, bool allv = true) : vertices(size)
   {
      // initialise vertex existence vector
      present = vector<bool>(size, allv);

      // create connectivity and distance matrices
      d = vector< vector<T> >(size, vector<T>(size));
      c = vector< vector<bool> >(size, vector<bool>(size, false));

      // create shortest path and transitive closure matrices
      spd = d;
      tcc = c;
      spp = vector< vector<int> >(size, vector<int>(size, -1));
   }

   // adds a directed edge between vertices a and b with weight w
   void add(int a, int b, const T &w)
   {
      present[a] = present[b] = true;
      c[a][b] = true;
      d[a][b] = w;
   }

   // adds an undirected edge between vertices a and b with weight w
   void addu(int a, int b, const T &w)
   {
      add(a, b, w);
      add(b, a, w);
   }

   // computes and returns the degree of vertex v
   //    (V complexity)
   int degree(int v)
   {
      return count(c[v].begin(), c[v].end(), true);
   }

   // counts the number of connected components in the graph
   //    (V^2 complexity, worst case)
   int components()
   {
      // component numbering of the vertices
      vector<int> cn(vertices, -1);
      int ct = 0;

      for (int v = 0; v < vertices; ++v) {

         // check if vertex v is not present or has been processed already
         if (!present[v] || cn[v] != -1)
            continue;

         // do a breadth first search on this component
         queue<int> q;
         q.push(v);
         cn[v] = ct;
         while (!q.empty()) {
            int x = q.front(); q.pop();
            for (int w = 0; w < vertices; ++w)
               if (cn[w] == -1 && c[x][w]) {
                  q.push(w);
                  cn[w] = ct;
               }
         }

         // increment the component number
         ++ct;
      }

      // return number of components found
      return ct;
   }

   // computes the minimum spanning tree using Prim's algorithm
   // assumes the graph is connected, and will return V-1 edges of the tree
   //    (V^2 complexity)
   vector< pair<int, int> > prim()
   {
      vector< pair<int, int> > edges;

      int s = -1;
      for (int i = 0; i < vertices && s == -1; ++i)
         if (present[i]) s = i;
      if (s == -1) return edges;

      vector<bool> seen(vertices, false); // seen[v] true if weight[v] is valid
      vector<bool> used(vertices, false); // used[v] true if v added to mst
```

```
   vector<int> mst(vertices, -1);      // the minimum spanning tree
   vector<T> weight(vertices, T());    // fringe vector

   // set algorithm to start on vertex s
   seen[s] = true;

   // loop on all connected vertices
   for (;;) {

      // find closest connected vertex to add to mst
      int v = -1;
      T lo;
      for (int i = 0; i < vertices; ++i) {
         if (seen[i] && !used[i]) {
            if (v == -1 || weight[i] < lo) {
               v = i;
               lo = weight[i];
            }
         }
      }
      if (v == -1) break;

      used[v] = true;

      // update the fringe vertices
      for (int i = 0; i < vertices; ++i) {
         if (c[v][i] && !used[i]) {
            T cost = d[v][i];
            if (!seen[i] || cost < weight[i]) {
               seen[i] = true;
               weight[i] = cost;
               mst[i] = v;
            }
         }
      }
   }

   // extract the edges from the mst
   for (int i = 0; i < vertices; ++i)
      if (mst[i] != -1)
         edges.push_back(pair<int,int>(i, mst[i]));

   return edges;
}

// computes the shortest paths vector from source s using Dijkstra's
// algorithm, then returns the length of the shortest path from s to t
//    (V^2 complexity)
T dijkstra(int s, int t)
{
   vector<bool> seen(vertices, false); // seen[v] true if weight[v] is valid
   vector<bool> used(vertices, false); // used[v] true if relaxed on v
   vector<int> path(vertices, -1);     // the shortest path tree for s
   vector<T> weight(vertices, T());    // shortest distance vector for s

   // set algorithm to start on vertex s
   seen[s] = true;
   path[s] = s;

   // loop on all connected vertices for relaxing
   for (;;) {

      // find closest connected vertex to relax on
      int v = -1;
      T lo;
      for (int i = 0; i < vertices; ++i) {
         if (seen[i] && !used[i]) {
            if (v == -1 || weight[i] < lo) {
               v = i;
               lo = weight[i];
            }
         }
      }
      if (v == -1) break;

      used[v] = true;

      // relax on vertex v
      for (int i = 0; i < vertices; ++i) {
         if (c[v][i]) {
            T cost = weight[v] + d[v][i];
            if (!seen[i] || cost < weight[i]) {
               seen[i] = true;
               weight[i] = cost;
               path[i] = v;
            }
         }
      }
   }

   // update the transitive closure and shortest pathes matrices for s
   tcc[s] = seen;
   spd[s] = weight;
   spp[s] = path;

   // return the shortest distance from s to t
   if (!seen[t]) return -1;      // untested
   return weight[t];
}

// traverses the shortest paths tree for s to find the shortest path to t
vector<int> shortestpath(int s, int t)
{
   vector<int> p;
   if (spp[s][t] != -1) {
      for (int v = t; v != s; v = spp[s][v])
         p.push_back(v);
      p.push_back(s);
   }
   reverse(p.begin(), p.end());
   return p;
}

// computes all pairs shortest paths using Floyd's algorithm
//    (V^3 complexity)
void floyd() {

   // set the transitive closure and shortest path distance matrices
   tcc = c;
   spd = d;

   for (int i = 0; i < vertices; ++i)
      for (int s = 0; s < vertices; ++s)
         if (tcc[s][i])
            for (int t = 0; t < vertices; ++t)
               if (tcc[i][t]) {

                  // if there's not an existing path from s to t,
```

```
                    // create a new one
                    if (!tcc[s][t]) {
                       tcc[s][t] = true;
                       spd[s][t] = spd[s][i] + spd[i][t];
                    }
                    // otherwise update the cost
                    else
                       spd[s][t] = min(spd[s][t], spd[s][i] + spd[i][t]);

                 }
   }
};

//----------------------------------------------------------------------------
// small test bed that does nothing but generate a random graph

int main()
{
   const int v = 10;

   graph<int> g(v, false);

   for (int i = 0; i < 12; ++i)
      g.addu(rand()%v, rand()%v, rand()%10);

   cout << "Number of connected components:" << endl;
   cout << g.components() << endl << endl;

   // test out Dijkstra's shortest path alrogithm
   int s = rand()%v;
   int t = rand()%v;
   cout << "Shortest path between " << s << " and " << t << ':' << endl;
   int length = g.dijkstra(s, t);
   vector<int> path = g.shortestpath(s, t);
   copy(path.begin(), path.end(), ostream_iterator<int>(cout, " "));
   if (g.tcc[s][t])
      cout << endl << "Length: " << length << endl;
   else
      cout << endl << "Unreachable!" << endl;
   cout << endl;

   // test out Floyd's algorithm for transitive closure
   cout << "The graph's transitive closure:" << endl;
   g.floyd();
   for (int i = 0; i < g.vertices; ++i) {
      for (int j = 0; j < g.vertices; ++j)
         if (g.tcc[i][j]) cout << setw(4) << g.spd[i][j];
         else cout << setw(4) << '-';
      cout << endl;
   }
   cout << endl;

   return 0;
}
//----------------------------------------------------------------------------
```

```
//----------------------------------------------------------------------------
// UNION-FIND ALGORITHM FOR CONNECTIVITY
//
// This file mainly features the union-find algorithm for testing for
// connectivity/reachability and keeping track of connected components.
// A graph structure with adjacency list representation is also present to
// demonstrate the use of the union-find algorithm in finding a minimum
// spanning tree using Kruskal's algorithm.
//
// Author:  Sonny Chan
// Date:    March 12, 2004
//----------------------------------------------------------------------------

#include <iostream>
#include <vector>
#include <map>
#include <algorithm>
#include <iterator>
#include <cstdlib>
#include <ctime>

using namespace std;

//----------------------------------------------------------------------------
// union find data structure for keeping track of connectivity
struct ufind {

   int n;                    // number of vertices or elements
   int components;           // number of connected components
   vector<int> c;            // array representation of parent-link tree

   // constructor takes size of structure as argument
   ufind(int size) : n(size), components(size)
   {
      c = vector<int>(n);
      for (int i = 0; i < n; ++i) c[i] = i;
   }

   // finds the root of element a
   int root(int a)
   {
      int r;
      for (r = a; r != c[r]; r = c[r]) c[r] = c[c[r]];
      return r;
   }

   // joins elements a and b
   int join(int a, int b)
   {
      int p = root(a);
      int q = root(b);

      if (p == q) return p;

      --components;
      int r = min(p, q);
      int s = max(p, q);
      c[s] = r;
      return r;
   }

   // answers a query as to whether elements a and b are connected
   bool connected(int a, int b)
   {
```

```
      return root(a) == root(b);
   }

};

//-------------------------------------------------------------------------
// adjacency lists undirected graph structure for minimum spanning tree

struct graph {

   // edge structure with
   struct edge {
      int s, t, cost;
      edge(int a = -1, int b = -1, int c = 0) : s(a), t(b), cost(c) {}
   };

   int n;                        // number of vertices
   vector< map<int, int> > g;    // sparse upper triangle of connectivity

   // constructor takes the number of vertices in the graph, numbered 0 to n
   graph(int vertices = 1) : n(vertices)
   {
      g = vector< map<int, int> >(n);
   }

   // adds an undirected edge to the graph
   void add(int a, int b, int c = 1)
   {
      g[min(a, b)][max(a, b)] = c;
   }

   // computes a minimum spanning tree of the graph using Kruskal's algorithm
   //    (E log E complexity)
   vector<edge> kruskal()
   {
      vector<edge> e;
      for (int i = 0; i < n; ++i)
         for (map<int, int>::iterator jt = g[i].begin(); jt != g[i].end(); ++jt)
            e.push_back(edge(i, jt->first, jt->second));

      sort(e.begin(), e.end());

      ufind uf(n);
      vector<edge> mst;
      for (vector<edge>::iterator it = e.begin(); it != e.end(); ++it) {
         if (!uf.connected(it->s, it->t)) {
            mst.push_back(*it);
            uf.join(it->s, it->t);
            if (uf.components == 1) break;
         }
      }

      return mst;
   }
};

// comparison operator to sort edges according to cost
bool operator<(const graph::edge &a, const graph::edge &b)
{
   return a.cost < b.cost;
}

// insertion operator to output edges
ostream &operator<<(ostream &stream, const graph::edge &e)
{
   stream << '(' << e.s << '-' << e.t << ", " << e.cost << ')';
   return stream;
}

//-------------------------------------------------------------------------
// small test bed that uses union-find and the MST graph

int main()
{
   // try it out on Sedgewick's MST example graph
   cout << "Finding minimum spanning tree for Sedgewick's graph..." << endl;
   graph sedgewick(8);
   sedgewick.add(0, 1, 32);
   sedgewick.add(0, 2, 29);
   sedgewick.add(0, 5, 60);
   sedgewick.add(0, 6, 51);
   sedgewick.add(0, 7, 31);
   sedgewick.add(1, 7, 21);
   sedgewick.add(3, 4, 34);
   sedgewick.add(3, 5, 18);
   sedgewick.add(4, 5, 40);
   sedgewick.add(4, 6, 51);
   sedgewick.add(4, 7, 46);
   sedgewick.add(6, 7, 25);
   cout << "\tMinimum spanning tree: ";
   vector<graph::edge> v = sedgewick.kruskal();
   copy(v.begin(), v.end(), ostream_iterator<graph::edge>(cout, " "));
   cout << endl << endl;

   // try it out on a random graph of size 20
   cout << "Finding MST for random graph with 20 vertices..." << endl;
   srand(time(0));
   graph gr20(20);
   ufind uf20(20);
   for (int i = 0; i < 20; ++i) {
      int n = rand()%5;
      for (int j = 0; j < n; ++j) {
         int v = rand()%20;
         gr20.add(i, v, rand()%100);
         uf20.join(i, v);
      }
   }
   cout << "\tGraph has " << uf20.components << " connected components." <<
endl;
   cout << "\tMinimum spanning tree: ";
   vector<graph::edge> w = gr20.kruskal();
   copy(w.begin(), w.end(), ostream_iterator<graph::edge>(cout, " "));
   cout << endl << endl;

   return 0;
}
//-------------------------------------------------------------------------
```

```cpp
//--------------------------------------------------------------------------
// FORD-FULKERSON ALGORITHM FOR MAXIMUM FLOW
//
// This file contains a network structure which implements the Ford-Fulkerson
// method of augmenting paths to find a maxflow.  The augmenting path search
// method is simply a depth-first search in this implementation -- a better
// search priority may be needed if you're doing a heavy flow application.
//
// Author:  Sonny Chan
// Date:    March 19, 2004
//--------------------------------------------------------------------------

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

//--------------------------------------------------------------------------
// graph structure to implement Ford-Fulkerson for maximum flow

struct graph {

   int n;                         // number of vertices in the graph
   int hicap;                     // highest capacity edge
   vector< vector<int> > g;       // capacity matrix
   vector< bool > seen;           // for marking verices in search

   // constructs a graph of size vertices
   graph(int size = 1) : n(size), hicap(0)
   {
      g = vector< vector<int> >(n, vector<int>(n, 0));
      seen = vector<bool>(n, false);
   }

   // adds a directed edge to the graph
   void add(int s, int t, int c = 1)
   {
      g[s][t] = c;
      hicap = max(hicap, c);
   }

   // use a depth-first search to find an augmenting path from s to sink,
   // and augment along the path on the backward recursion
   int augment(int s, int sink, int cap)
   {
      if (s == sink) return cap;

      seen[s] = true;
      vector<int> &adj = g[s];

      for (int t = 0; t < n; ++t) {
         if (!seen[t] && adj[t] > 0) {
            if (int c = augment(t, sink, min(cap, adj[t]))) {
               g[s][t] -= c;
               g[t][s] += c;
               return c;
            }
         }
      }

      return 0;
   }

   // finds a maximum flow using the Ford-Fulkerson algorithm
   int maxflow(int source, int sink)
   {
      int total = 0;
      fill(seen.begin(), seen.end(), false);
      while (int flow = augment(source, sink, hicap)) {
         total += flow;
         fill(seen.begin(), seen.end(), false);
      }

      // At this point, the maxflow is found, and g is the residual network.
      // To recover the flow, take the original network and subtract g.

      return total;
   }

};

//--------------------------------------------------------------------------
// small test bed to test out the implementation

int main()
{
   // test it out on the small sedgewick graph
   graph sedgewick(6);
   sedgewick.add(0, 1, 2);
   sedgewick.add(0, 2, 3);
   sedgewick.add(1, 3, 3);
   sedgewick.add(1, 4, 1);
   sedgewick.add(2, 3, 1);
   sedgewick.add(2, 4, 1);
   sedgewick.add(3, 5, 2);
   sedgewick.add(4, 5, 3);
   cout << "Maxflow on Sedgewick graph: " << sedgewick.maxflow(0, 5) << endl;

   return 0;
}
//--------------------------------------------------------------------------
```

```
//-------------------------------------------------------------------------
// NETWORK SIMPLEX ALGORITHM FOR MINCOST MAXFLOW
//
// This file contains a network structure which implements a version of the
// network simplex algorithm for finding a mincost maxflow.  Beware that
// this code is not fully tested nor optimised to my liking!
//
// Author:  Sonny Chan
// Date:    March 12, 2004
//-------------------------------------------------------------------------

#include <iostream>
#include <iomanip>
#include <vector>
#include <list>
#include <algorithm>

using namespace std;

// sentries for the maximum cost and flow for the whole network
const double maxcost = 1e15;
const double maxcap = 1e15;

//-------------------------------------------------------------------------
// edge structure to represent a flow edge in the network

struct edge {
   int v, w;
   double cost;
   double capacity;
   double flow;

   edge(int a = 0, int b = 0, double c = 0.0, double k = 0.0)
      : v(a), w(b), cost(c), capacity(k), flow(0.0) {}

   int other(int a)
      { return v == a ? w : v; }

   double costto(int a)
      { return v == a ? -cost : cost; }

   double capacityto(int a)
      { return v == a ? flow : capacity - flow; }

   void addflowto(int a, double f)
      { flow += (v == a ? -f : f); }
};

ostream &operator<<(ostream &stream, const edge &e)
{
   stream << e.v << '-' << e.w;
   return stream;
}

//-------------------------------------------------------------------------
// network structure to implement network simplex for mincost maxflow

struct network {

   typedef list<edge>::iterator edgep;
   list<edge> elist;             // list of all edges in the network

   vector< vector<edgep> > g;    // network connectivity as adjacency list
   int vertices;                 // number for vertices in the network
   int source, sink;             // the source and sink vertices

   vector<edgep> stree;          // spanning tree for network simplex
   vector<int> mark;             // marking to help traverse spanning tree
   int valid;                    // value of next valid mark

   vector<double> phi;           // vertex potentials

   bool backwards;               // tracks whether we're pushing flow backward
   int augmentations;            // counts the total number of augmentations

   // constructor takes size of network as argument
   network(int size = 1) : vertices(size), valid(0)
   {
      g = vector< vector<edgep> >(size);

      stree = vector<edgep>(size);
      mark = vector<int>(size, 0);

      phi = vector<double>(size, 0.0);
   }

   // adds a directed edge from v to w with cost c and capacity k
   void add(int v, int w, double c, double k)
   {
      elist.push_back(edge(v, w, c, k));
      g[v].push_back(--elist.end());
      g[w].push_back(--elist.end());
   }

   // adds an undirected edge between v and w with cost c and capacity k
   void addu(int v, int w, double c, double k)
   {
      add(v, w, c, k);
      add(w, v, c, k);
   }

   int st(int v)
   {
      const edgep &ep = stree[v];
      return v == ep->v ? ep->w : ep->v;
   }

   // recursively builds a spanning tree
   void buildst(int v)
   {
      mark[v] = valid;
      for (vector<edgep>::iterator it = g[v].begin(); it != g[v].end(); ++it) {
         int w = (*it)->w;
         if (mark[w] != valid) {
            stree[w] = *it;
            buildst(w);
         }
      }
   }

   // recursively calculates vertex potentials
   double potential(int v)
   {
      if (mark[v] == valid) return phi[v];
      phi[v] = potential(st(v)) - stree[v]->costto(v);
      return phi[v];
   }
```

```
// calculates potentials for the vertices
void calculatep()
{
   ++valid;
   mark[sink] = valid;
   for (int v = 0; v < vertices; ++v)
      if (mark[v] != valid)
         potential(v);
}

// computes the lowest common ancestor of two vertices v and w
int lca(int v, int w)
{
   ++valid;
   mark[v] = mark[w] = valid;
   while (v != w) {
      if (v != sink) v = st(v);
      if (v != sink && mark[v] == valid) return v;
      mark[v] = valid;
      if (w != sink) w = st(w);
      if (w != sink && mark[w] == valid) return w;
      mark[w] = valid;
   }
   return v;
}

// augments along a negative cycle by adding edge x to the spanning tree
// returns an empty/full edge to remove from the tree
edgep augment(const edgep &x)
{
   int v = x->v, w = x->w;
   if (backwards) swap(v, w);

   int r = lca(v, w);
   double d = x->capacityto(w);

   for (int u = w; u != r; u = st(u))
      d = min(d, stree[u]->capacityto(st(u)));
   for (int u = v; u != r; u = st(u))
      d = min(d, stree[u]->capacityto(u));

   x->addflowto(w, d);
   edgep e = x;

   for (int u = w; u != r; u = st(u)) {
      stree[u]->addflowto(st(u), d);
      if (stree[u]->capacityto(st(u)) == 0.0)
         e = stree[u];
   }
   for (int u = v; u != r; u = st(u)) {
      stree[u]->addflowto(u, d);
      if (stree[u]->capacityto(u) == 0.0)
         e = stree[u];
   }

   return e;
}

// tests if vertex b is on the path from a to c in the spanning tree
bool onpath(int a, int b, int c)
{
   for (int i = a; i != c; i = st(i))
      if (i == b) return true;
   return false;
}

// reverses all the links in the spanning tree from vertex u to x
void reverse(int u, int x)
{
   if (u == x) return;
   edgep e = stree[u];
   int j;
   for (int i = st(u); ; i = j) {
      edgep y = stree[i];
      j = st(i);
      stree[i] = e;
      e = y;
      if (i == x) break;
   }
}

// updates the spanning tree by removing edge w and adding edge y
void update(const edgep &w, const edgep &y)
{
   if (w == y) return;

   int u = y->w, v = y->v, x = w->w;
   if (stree[x] != w) x = w->v;

   int r = lca(u, v);

   if (onpath(u, x, r)) {
      reverse(u, x);
      stree[u] = y;
      return;
   }
   if (onpath(v, x, r)) {
      reverse(v, x);
      stree[v] = y;
      return;
   }
}

// calculates the reduced cost of an edge e from vertex v
double reduced(const edgep &e, int v)
{
   double r = e->cost + phi[e->w] - phi[e->v];
   return (e->v == v ? r : -r);
}

// finds the best eligible edge for augmenting
edgep besteligible()
{
   edgep x;
   double small = maxcost;
   for (int v = 0; v < vertices; ++v) {
      for (vector<edgep>::iterator it = g[v].begin();
         it != g[v].end(); ++it)
      {
         const edgep &e = *it;
         if (e->capacityto(e->other(v)) > 0.0)
            if (e->capacityto(v) == 0.0)
               if (reduced(e, v) < small) {
                  x = e;
                  small = reduced(e, v);
                  backwards = e->v != v;
               }
      }
```

```
      }
      return x;
   }

   // calculates the total cost of the flow
   double totalcost()
   {
      double sum = 0.0;
      for (list<edge>::iterator it = elist.begin(); it != elist.end(); ++it)
         if (it->flow > 0.0) {
            sum += it->cost * it->flow;
         }
      return sum;
   }

   // calculates a mincost maxflow from source s to sink t
   void mincost_maxflow(int s, int t)
   {
      source = s;
      sink = t;

      // add a dummy edge from source to sink
      add(s, t, maxcost, maxcap);
      edgep dummy = --elist.end();
      dummy->addflowto(t, maxcap);

      // build initial spanning tree using recursive DFS
      elist.push_back(edge(t, t));

      ++valid;
      stree[t] = --elist.end();
      mark[t] = valid;
      stree[s] = dummy;
      buildst(s);

      augmentations = 0;
      for ( ; ; ++valid) {
         // calculate vertex potentials
         calculatep();

         // find best eligible edge
         edgep e = besteligible();

         // check for no more eligible edges
         double rcost = reduced(e, (backwards ? e->w : e->v));
         if (rcost == 0.0) break;

         // augment on e
         ++augmentations;
         edgep r = augment(e);

         update(r, e);
      }

      // remove the dummy edges
      elist.pop_back();
      elist.pop_back();

   }

   void printst()
   {
      cout << "Spanning tree:" << endl;
      for (int i = 0; i < vertices; ++i) {
         cout << i << ": " << i;
         for (int j = st(i); ; j = st(j)) {
            cout << '-' << j;
            if (j == st(j)) break;
         }
         cout << endl;
      }
   }

   void printflow()
   {
      cout << "Network flow is:" << endl;
      for (list<edge>::iterator it = elist.begin(); it != elist.end(); ++it)
         if (it->flow > 0.0) {
            cout << '\t' << *it << ' ' << it->cost << ' ' << it->flow
                 << ' ' << it->cost * it->flow << endl;
         }
   }
};

//--------------------------------------------------------------------------
// a somewhat inadequate test bed to try to test this crazy algorithm

int main()
{
   // try it on Sedgewick's sample weighted network
   cout << "Finding mincost maxflow on Sedgewick's sample network..." << endl;
   network rs(6);
   rs.add(0, 1, 3, 3);
   rs.add(0, 2, 1, 3);
   rs.add(1, 3, 1, 2);
   rs.add(1, 4, 1, 2);
   rs.add(2, 3, 4, 1);
   rs.add(2, 4, 2, 2);
   rs.add(3, 5, 2, 2);
   rs.add(4, 5, 1, 2);
   rs.mincost_maxflow(0, 5);
   rs.printflow();
   cout << "Total cost: " << rs.totalcost() << endl;
   cout << "Total augmentations: " << rs.augmentations << endl;
   cout << endl;

   // try it on a weighted bipartite match problem
   cout << "Finding best weighted bipartite match for 7-7 matching..." << endl;
   int iq[14] = {  70, 80, 90,100,105,110,115,110, 85,115,105, 80, 75,120 };
   int ht[14] = { 145,155,165,175,156,158,160,175,170,170,149,155,179,168 };
   network match(16);
   for (int i = 0; i < 7; ++i) {
      match.add(14, i, 0, 1);
      match.add(i+7, 15, 0, 1);
      for (int j = 7; j < 14; ++j)
         match.add(i, j, abs(iq[i]-iq[j]) + abs(ht[i]-ht[j]), 1);
   }
   match.mincost_maxflow(14, 15);
   match.printflow();
   cout << "Total cost: " << match.totalcost() << endl;
   cout << "Total augmentations: " << match.augmentations << endl;
   cout << endl;

   return 0;
}
//--------------------------------------------------------------------------
```

```
//---------------------------------------------------------------------------
// MAXIMUM CLIQUE AND GRAPH COLOURING ALGORITHMS
//
// This file contains optimised implementations of maximum clique and graph
// colouring algorithms for a graph structure with adjacency matrix
// representation.
//
// Author:  Sonny Chan
// Date:    March 12, 2004
//---------------------------------------------------------------------------

#include <iostream>
#include <vector>
#include <set>
#include <map>
#include <algorithm>
#include <iterator>

using namespace std;

//---------------------------------------------------------------------------
// graph class with adjacency matrix for maximum clique and graph colouring

struct graph {

   // vertex structure with vertex number and degree
   struct vertex {
      int v, d;
      vertex(int vn = 0, int deg = 0) : v(vn), d(deg) {}
   };

   int n;                        // number of vertices
   vector< vector<bool> > c;     // connectivity matrix
   vector< set<int> > g;         // sparse upper triangle of above

   vector< vertex > vinfo;       // maps a new vertex number to original
   map< int, int > vmap;         // maps original vertex number to sorted

   vector< int > mark;           // marking for each vertex for search
   vector< int > cv;             // contains the vertices for our clique
   int bestk;                    // max clique or best colouring found so far

   vector< vector<int> > forbidden; // marks forbidden colours for each vertex
   vector< int > colouring;         // contains a colouring of the graph

   int evaluations;                 // number of function evaluations

   // constructor takes the number of vertices in the graph, numbered 0 to n
   graph(int vertices = 1) : n(vertices)
   {
      c = vector< vector<bool> >(n, vector<bool>(n, false));
      g = vector< set<int> >(n);
   }

   // adds an undirected edge to the graph
   void add(int a, int b)
   {
      c[a][b] = true;
      c[b][a] = true;
   }

   // sorts the vertices in descending order of degree
   void sortv()
   {
      vinfo.clear();
      for (int v = 0; v < n; ++v) {
         int degree = count(c[v].begin(), c[v].end(), true);
         vinfo.push_back(vertex(v, degree));
      }
      sort(vinfo.begin(), vinfo.end());

      vmap.clear();
      for (int i = 0; i < n; ++i) {
         vmap[vinfo[i].v] = i;
         g[i].clear();
      }

      // create the upper triangle of connectivity using adjacency lists
      for (int i = 0; i < n; ++i) {
         for (int j = i+1; j < n; ++j)
            if (c[i][j]) {
               int a = vmap[i];
               int b = vmap[j];
               g[min(a, b)].insert(max(a, b));
            }
      }
   }

   // finds the largest clique with vertex v (start with m=1)
   void clique(int v, int m, vector<int> &st)
   {
      ++evaluations;

      st.push_back(v);

      if (m > bestk) { bestk = m; cv = st; }

      set<int> &adj = g[v];

      int tally = 0;
      for (set<int>::iterator it = adj.begin(); it != adj.end(); ++it)
         if (mark[*it] == m-1 && vinfo[*it].d >= bestk-1) {
            ++mark[*it];
            ++tally;
         }

      for (set<int>::iterator it = adj.begin(); it != adj.end(); ++it) {
         if (mark[*it] == m) {
            if (tally >= bestk-m) clique(*it, m+1, st);
            --mark[*it];
            --tally;
         }
      }

      st.pop_back();
   }

   // find a maximum clique in the graph
   int maxclique()
   {
      sortv();

      evaluations = 0;
      bestk = 0;
      mark = vector<int>(n, 0);
      vector<int> st;
      for (int v = 0; v < n; ++v) {
         if (v + bestk >= n) break;
```

```cpp
         if (vinfo[v+bestk].d < bestk) break;
         clique(v, 1, st);
      }

      // transform the clique vertices to their originals
      for (vector<int>::iterator it = cv.begin(); it != cv.end(); ++it)
         *it = vinfo[*it].v;

      return bestk;
   }

   bool colour(int v, int k)
   {
      ++evaluations;

      if (v == n) return true;

      for (int c = 0; c < min(k, v+1); ++c) {
         if (forbidden[v][c] == 0) {
            set<int> &adj = g[v];

            for (set<int>::iterator it = adj.begin(); it != adj.end(); ++it)
               ++forbidden[*it][c];

            colouring[v] = c;
            if (colour(v+1, k)) return true;

            for (set<int>::iterator it = adj.begin(); it != adj.end(); ++it)
               --forbidden[*it][c];
         }
      }

      return false;
   }

   // find a minimum colouring of the graph
   int mincolouring()
   {
      sortv();

      evaluations = 0;
      bestk = 0;
      colouring = vector<int>(n, -1);

      // linear search on k... we assume it's small
      for (int k = 1; k <= n; ++k) {
         forbidden = vector< vector<int> >(n, vector<int>(k, 0));
         if (colour(0, k)) {
            bestk = k;
            break;
         }
      }

      // transform the colouring to their original (unsorted) vertex indices
      vector<int> tc(n);
      for (int i = 0; i < n; ++i)
         tc[vinfo[i].v] = colouring[i];
      colouring.swap(tc);

      return bestk;
   }

};
```

```cpp
// comparator to sort vertices according to degree
bool operator<(const graph::vertex &a, const graph::vertex &b)
{
   return a.d > b.d;
}

//---------------------------------------------------------------------------
// test bed with fixed and random graphs

int main()
{
   // try it out on a 10-clique graph with 12 vertices
   cout << "Finding maximum clique for graph with k10..." << endl;
   graph gk10(12);
   for (int i = 1; i <= 10; ++i)
      for (int j = i+1; j <= 10; ++j)
         gk10.add(i, j);
   gk10.add(0, 9);
   gk10.add(11, 5);
   cout << "\tClique size:       " << gk10.maxclique() << endl;
   cout << "\tTotal evaluations: " << gk10.evaluations << endl;
   cout << "\tClique vertices:   ";
   copy(gk10.cv.begin(), gk10.cv.end(), ostream_iterator<int>(cout, " "));
   cout << endl << endl;
   cout << "Finding best colouring for graph with k10..." << endl;
   cout << "\tColours used:      " << gk10.mincolouring() << endl;
   cout << "\tTotal evaluations: " << gk10.evaluations << endl;
   cout << "\tVertex colouring:  ";
   copy(gk10.colouring.begin(), gk10.colouring.end(),
      ostream_iterator<int>(cout, " "));

   // try it out on the Coxeter graph
   cout << "Finding maximum clique in the complement Coxeter graph..." << endl;
   graph coxeter(28);
   for (int i = 0; i < 7; ++i) {
      coxeter.add(i, i+7);
      coxeter.add(i, i+14);
      coxeter.add(i, i+21);
      coxeter.add(i+7,  (i+1)%7 + 7);
      coxeter.add(i+14, (i+2)%7 + 14);
      coxeter.add(i+21, (i+3)%7 + 21);
   }
   // complement the graph
   for (int i = 0; i < 28; ++i)
      for (int j = 0; j < 28; ++j)
         if (i != j) coxeter.c[i][j] = !coxeter.c[i][j];
   cout << "\tClique size:       " << coxeter.maxclique() << endl;
   cout << "\tTotal evaluations: " << coxeter.evaluations << endl;
   cout << "\tClique vertices:   ";
   copy(coxeter.cv.begin(), coxeter.cv.end(), ostream_iterator<int>(cout, " "));
   cout << endl << endl;

   return 0;
}
//---------------------------------------------------------------------------
```

```
//----------------------------------------------------------------------------
// POWERS OF TWO AND COMBINATIONS
//
// This file includes nifty bit manipulation algorithms to calculate:
//    - power of 2 floor and ceiling for integer
//    - determining whether or not an integer is a power of 2
//    - next integer with same amout of 1 bits (snoob)
//
// These algorithms are courtesy of "Hacker's Delight" by Henry S. Warren Jr.
// (Addison-Wesley 2003 ISBN 0-201-91465-4)
//
// Author:  Sonny Chan
// Date:    November 12, 2003
//----------------------------------------------------------------------------

#include <iostream>

using namespace std;

//----------------------------------------------------------------------------
// integer power of 2 floor function
unsigned int p2floor(unsigned int x)
{
   x = x | (x >> 1);
   x = x | (x >> 2);
   x = x | (x >> 4);
   x = x | (x >> 8);
   x = x | (x >> 16);
   return x - (x >> 1);
}

// integer power of 2 ceiling function
unsigned int p2ceiling(unsigned int x)
{
   x -= 1;
   x = x | (x >> 1);
   x = x | (x >> 2);
   x = x | (x >> 4);
   x = x | (x >> 8);
   x = x | (x >> 16);
   return x + 1;
}

// determines whether an integer is a power of 2
bool ispower2(unsigned int x)
{
   return (!(x & (x-1)) && x);
}

//----------------------------------------------------------------------------
// calculates the next integer with the same number of 1-bits
unsigned int snoob(unsigned int x)
{
   unsigned int smallest, ripple, ones;
                                    // x = xxx0 1111 0000
   smallest = x & -x;               //     0000 0001 0000
   ripple = x + smallest;           //     xxx1 0000 0000
   ones = x ^ ripple;               //     0001 1111 0000
   ones = (ones >> 2) / smallest;   //     0000 0000 0111
   return ripple | ones;            //     xxx1 0000 0111
}
//----------------------------------------------------------------------------
```

```
/****************************************************************************
BINARY SEARCH
Submitted March 21, 2004 by Kelly Poon
Original source courtesy of The University of Alberta
****************************************************************************/

/* Returns non-zero if x is found, and zero otherwise.  If x is found, then
A[index] = x.  If not, then index is the place x should be inserted into A. */
int bin_search(int *A, int n, int x, int *index){
   int low, up, mid;

   if (n <= 0 || x < A[0]) { *index = 0; return 0; }
   if (A[n-1] < x) { *index = n; return 0; }
   if (x == A[n-1]) { *index = n-1; return 1; }
   for(low = 0, up = n-1; low + 1 < up;){
     mid = (low+up)/2;
     if (A[mid] <= x) low = mid;
     else up = mid;
   }
   if (A[low] == x) { *index = low; return 1; }
   else { *index = up; return 0; }
}

/****************************************************************************
GEOMETRY ROUTINES
Submitted March 21, 2004 by Alex Fink
Original source courtesy of The University of Alberta
****************************************************************************/

#define EPS 1E-8
#define SQR(x)  ((x)*(x))
#define SGN(x)  ((x)<0?-1:1)

typedef struct {
  double x, y;
} Point;

typedef struct {
  Point o;
  double r;
} Circle;

/* distance squared */
double dist2(Point a, Point b) {
  return SQR(a.x-b.x) + SQR(a.y-b.y);
}

/* distance */
double dist_2d(Point a, Point b){
  return sqrt(SQR(a.x-b.x)+SQR(a.y-b.y));
}

/* which side of a line */
enum {LEFT, RIGHT, CL};

int whichside(Point a, Point b, Point p)
{
  double res;

  res = (p.x - a.x)*(b.y - a.y) -
        (p.y - a.y)*(b.x - a.x);

  if (fabs(res) < EPSILON)
    return CL;
```

```
  else if (res > 0.0)
    return RIGHT;
  return LEFT;
}

/* angle */
double angle2d(Point a, Point b, Point c){
  double dx1 = a.x - b.x, dy1 = a.y - b.y;
  double dx2 = c.x - b.x, dy2 = c.y - b.y;
  double dot = dx1 * dx2 + dy1 * dy2;
  double l1 = sqrt(SQR(dx1)+SQR(dy1));
  double l2 = sqrt(SQR(dx2)+SQR(dy2));

  return acos(dot / (l1*l2));
}

/* ax+by=c equation of line */
typedef struct{
  double a, b, c;
} Line;

Line pt2line(Point a, Point b){
  double dx = a.x-b.x, dy = a.y-b.y;
  double len = sqrt(SQR(dx)+SQR(dy));
  Line res;

  if(dy < 0){
    dy *= -1;
    dx *= -1;
  }
  res.a = dy/len;
  res.b = -dx/len;
  res.c = res.a*a.x + res.b*a.y;
  return res;
}

/* closest point to c on line ab */
Point closest_pt_iline(Point a, Point b, Point c) {
  Point p;
  double dp;

  b.x -= a.x;
  b.y -= a.y;
  dp = (b.x*(c.x-a.x) + b.y*(c.y-a.y)) / (SQR(b.x)+SQR(b.y));
  p.x = b.x*dp + a.x;
  p.y = b.y*dp + a.y;
  return p;
}

/* closest point to c on segment ab */
Point closest_pt_lineseg(Point a, Point b, Point c) {
  Point p;
  double dp;

  b.x -= a.x;
  b.y -= a.y;
  if (fabs(b.x) < EPS && fabs(b.y) < EPS) return a;
  dp = (b.x*(c.x-a.x) + b.y*(c.y-a.y))/(SQR(b.x)+SQR(b.y));
  if (dp > 1) dp = 1;
  if (dp < 0) dp = 0;
  p.x = b.x*dp + a.x;
  p.y = b.y*dp + a.y;
  return p;
}

/* distance from p to line ab */
double dist_iline(Point a, Point b, Point p){
  return fabs(((a.y-p.y)*(b.x-a.x)-
               (a.x-p.x)*(b.y-a.y))
              /dist_2d(a,b));
}

/* reflection of c across ab */
Point reflect(Point a, Point b, Point c) {
  Point d, p;

  d = closest_pt_iline(a,b,c);
  p.x = 2.0*d.x - c.x;
  p.y = 2.0*d.y - c.y;
  return p;
}

/* rotation of p around o */
Point rotate_2d(Point p, Point o, double theta){
  double m[2][2];
  Point r;

  m[0][0] = m[1][1] = cos(theta);
  m[0][1] = -sin(theta);
  m[1][0] = -m[0][1];
  p.x -= o.x;
  p.y -= o.y;
  r.x = m[0][0] * p.x + m[0][1] * p.y + o.x;
  r.y = m[1][0] * p.x + m[1][1] * p.y + o.y;
  if(fabs(r.x) < EPS) r.x = 0;
  if(fabs(r.y) < EPS) r.y = 0;
  return r;
}

/* intersection of lines */
int isect_iline(Point a, Point b, Point c, Point d, Point *p){
  double r, denom, num1;

  num1  = (a.y - c.y) * (d.x - c.x) - (a.x - c.x) * (d.y - c.y);
  denom = (b.x - a.x) * (d.y - c.y) - (b.y - a.y) * (d.x - c.x);

  if (fabs(denom) >= EPS) {
    r = num1 / denom;
    p->x = a.x + r*(b.x - a.x);
    p->y = a.y + r*(b.y - a.y);
    return 1;
  }
  if (fabs(num1) >= EPS) return 0;
  return -1;
}

/* intersection of segments */
int intersect_line(Point a, Point b, Point c, Point d, Point *p){
  Point t; double r, s, denom, num1, num2;

  num1  = (a.y - c.y)*(d.x - c.x) - (a.x - c.x)*(d.y - c.y);
  num2  = (a.y - c.y)*(b.x - a.x) - (a.x - c.x)*(b.y - a.y);
  denom = (b.x - a.x)*(d.y - c.y) - (b.y - a.y)*(d.x - c.x);

  if (fabs(denom) >= EPS) {
    r = num1 / denom;
    s = num2 / denom;
    if (0-EPS <= r && r <= 1+EPS &&
```

```
        0-EPS <= s && s <= 1+EPS) {
      p->x = a.x + r*(b.x - a.x);
      p->y = a.y + r*(b.y - a.y);
      return 1;
    }
    return 0;
  }
  if (fabs(num1) >= EPS) return 0;
  if (a.x > b.x || (a.x == b.x && a.y > b.y)) { t = a; a = b; b = t; }
  if (c.x > d.x || (c.x == d.x && c.y > d.y)) { t = c; c = d; d = t; }
  if (a.x == b.x) {
    if (b.y == c.y) {
      *p = b; return 1;
    } else if (a.y == d.y) {
      *p = a; return 1;
    } else if (b.y < c.y || d.y < a.y)
      return 0;
  } else {
    if (b.x == c.x) {
      *p = b; return 1;
    } else if (a.x == d.x) {
      *p = a; return 1;
    } else if (b.x < c.x || d.x < a.x)
      return 0;
  }
  return -1;
}

/* triangle area */
double area_tri(Point a, Point b, Point c){
  double area;

  area = (b.x-a.x) * (c.y-a.y)
        -(b.y-a.y) * (c.x-a.x);
  return (fabs(area))/2;
}

/* triangle area -- Heron's formula */
double area_heron(double a, double b, double c){
  double s = (a+b+c)/2.0;
  if(a > s || b > s || c > s) return -1;
  return sqrt(s*(s-a)*(s-b)*(s-c));
}

/* signed polygon area (counterclockwise positive) */
double area_poly(Point *p, int n){
  double sum = 0;
  int i, j;

  for(i = n-1, j = 0; j < n; i = j++)
    sum += p[i].x * p[j].y -
           p[i].y * p[j].x;
  return sum/2.0;
}

/* point in polygon */
#define BOUNDARY 1 // what to return for boundary points

int pt_in_poly(Point *p, int n, Point a) {
  int i, j, c = 0;

  for (i = 0, j = n-1; i < n; j = i++) {
    if (dist_2d(p[i],a)+dist_2d(p[j],a)-dist_2d(p[i],p[j]) < EPS)
      return BOUNDARY;
    if ((((p[i].y<=a.y) && (a.y<p[j].y)) ||
         ((p[j].y<=a.y) && (a.y<p[i].y))) &&
        (a.x < (p[j].x-p[i].x) * (a.y - p[i].y)
               / (p[j].y-p[i].y) + p[i].x)) c = !c;
  }
  return c;
}

/* centroid */
Point centroid(Point *p, int n) {
  double area, sum;
  Point c;
  int i, j;

  c.x = c.y = sum = 0.0;
  for(i = n-1, j = 0; j < n; i = j++) {
    sum += area = p[i].x * p[j].y - p[i].y * p[j].x;
    c.x += (p[i].x + p[j].x)*area;
    c.y += (p[i].y + p[j].y)*area;
  }
  sum *= 3.0;
  c.x /= sum;
  c.y /= sum;
  return c;
}

/* Pick's theorem */
void lat_poly_pick(Point *p, int n, long long *I, long long *B){
  int i, j, dx, dy;
  double A = fabs(area_poly(p, n));

  *B = 0;
  for(i = n-1, j = 0; j < n; i = j++){
    dx = abs(p[i].x - p[j].x);
    dy = abs(p[i].y - p[j].y);
    *B += gcd(dx,dy);
  }
  *I = A+1-*B/2.0;
}

/* convex hull (counterclockwise, minimum size) */
Point *P0;

enum {CCW, CW, CL};

int cross_prod(Point *p1, Point *p2, Point *p0)
{
  double res, x1, x2, y1, y2;

  x1 = p1->x - p0->x;
  x2 = p2->x - p0->x;
  y1 = p1->y - p0->y;
  y2 = p2->y - p0->y;

  res = x1*y2 - x2*y1;

  if (fabs(res) < EPSILON)
    return CL;
  else if (res > 0.0)
    return CW;
  else
    return CCW;
}
```

```c
int polar_cmp(Point *p1, Point *p2)
{
  int res;
  double d, x1, x2, y1, y2;

  res = cross_prod(p1, p2, P0);

  if (res == CW)
    return -1;
  else if (res == CCW)
    return 1;
  else {
    x1 = p1->x - P0->x;
    x2 = p2->x - P0->x;
    y1 = p1->y - P0->y;
    y2 = p2->y - P0->y;

    d = ((x1*x1) + (y1*y1)) - ((x2*x2) + (y2*y2));

    if (fabs(d) < EPSILON)
      return 0;
    else if (d < 0.0)
      return -1;
    else
      return 1;
  }
}

int convex_hull(Point *poly, int n, Point *hull)
{
  int i, min, h;

  if (n < 1)
    return 0;

  min = 0;
  P0 = &hull[0];
  *P0 = poly[0];

  for (i = 1; i < n; i++) {
    if ((poly[i].y < P0->y) ||
        ((poly[i].y == P0->y) && (poly[i].x < P0->x))) {
      min = i;
      *P0 = poly[i];
    }
  }

  poly[min] = poly[0];
  poly[0] = *P0;
  h = 1;

  if (n == 1)
    return h;

  qsort(poly+1, n-1, sizeof(poly[1]),
        (int (*)(const void *, const void *))polar_cmp);

  for (i = 1; i < n; i++) {
    if ((fabs(poly[i].x - hull[0].x) > EPSILON) ||
        (fabs(poly[i].y - hull[0].y) > EPSILON))  {
      break;
    }
  }

  if (i == n)
    return h;

  hull[h++] = poly[i++];

  for ( ; i < n; i++) {
    while ((h > 1) &&
           (cross_prod(&poly[i], &hull[h-1], &hull[h-2]) != CCW)) {
      h--;
    }

    hull[h++] = poly[i];
  }

  return h;
}
/* end of convex hull */

/* circle through 3 points */
int circle(Point p1, Point p2, Point p3, Point *center, double *r)
{
  double a,b,c,d,e,f,g;

  a = p2.x - p1.x;  b = p2.y - p1.y;
  c = p3.x - p1.x;  d = p3.y - p1.y;
  e = a*(p1.x + p2.x) + b*(p1.y + p2.y);
  f = c*(p1.x + p3.x) + d*(p1.y + p3.y);
  g = 2.0*(a*(p3.y - p2.y) - b*(p3.x - p2.x));
  if (fabs(g) < EPS) return 0;
  center->x = (d*e - b*f) / g;
  center->y = (a*f - c*e) / g;
  *r = sqrt((p1.x-center->x)*(p1.x-center->x) +
            (p1.y-center->y)*(p1.y-center->y));
  return 1;
}

/* tangents from point p to circle c, r */
void circ_tangents(Point c, double r, Point p, Point *a, Point *b) {
  double perp, para, tmp = dist2(p,c);

  para = r*r/tmp;
  perp = r*sqrt(tmp-r*r)/tmp;

  a->x = c.x + (p.x-c.x)*para - (p.y-c.y)*perp;
  a->y = c.y + (p.y-c.y)*para + (p.x-c.x)*perp;
  b->x = c.x + (p.x-c.x)*para + (p.y-c.y)*perp;
  b->y = c.y + (p.y-c.y)*para - (p.x-c.x)*perp;
}

/* intersection of circle and line */
int circ_iline_isect(Circle c, Point a, Point b,
                     Point *r1, Point *r2){
  double dx = b.x-a.x, dy = b.y-a.y;
  double sdr = SQR(dx)+SQR(dy), dr = sqrt(sdr);
  double D,disc,x,y;

  a.x -= c.o.x; a.y -= c.o.y;
  b.x -= c.o.x; b.y -= c.o.y;
  D = a.x*b.y - b.x*a.y;
  disc = SQR(c.r*dr)-SQR(D);

  if(disc < 0) return 0;
  x = SGN(dy)*dx*sqrt(disc);
  y = fabs(dy)*sqrt(disc);
```

```
  r1->x = (D*dy + x)/sdr + c.o.x;
  r2->x = (D*dy - x)/sdr + c.o.x;
  r1->y = (-D*dx + y)/sdr + c.o.y;
  r2->y = (-D*dx - y)/sdr + c.o.y;
  return disc == 0 ? 1 : 2;
}

/* intersection of circle and segment */
int circ_lineseg_isect(Circle c, Point a, Point b,
                        Point *r1, Point *r2){
  double d = dist_2d(a,b);
  int res = circ_iline_isect(c,a,b,r1,r2);

  if(res == 2 && dist_2d(a,*r2)+dist_2d(*r2,b) != d) res--;
  if(res >= 1 && dist_2d(a,*r1)+dist_2d(*r1,b) != d){
    *r1 = *r2;
    res--;
  }
  return res;
}

/* intersection of circles */
enum int_t {NONE=0, ONE, TWO, AEQUALSB, AINB, BINA,
            AINB_TANGENT, BINA_TANGENT};

Point rotate_2d(Point p, Point o, double theta){
  double m[2][2];
  Point r;

  m[0][0] = m[1][1] = cos(theta);
  m[0][1] = -sin(theta);
  m[1][0] = -m[0][1];
  p.x -= o.x;
  p.y -= o.y;
  r.x = m[0][0] * p.x + m[0][1] * p.y + o.x;
  r.y = m[1][0] * p.x + m[1][1] * p.y + o.y;
  if(fabs(r.x) < EPS) r.x = 0;
  if(fabs(r.y) < EPS) r.y = 0;
  return r;
}

int CIType (Circle A, Circle B) {
  double distance, dx = A.o.x - B.o.x, dy = A.o.y - B.o.y;

  distance = sqrt(dx*dx + dy*dy);

  if (distance < EPS && fabs(A.r-B.r) < EPS) return AEQUALSB;
  if (fabs(distance - (A.r + B.r)) < EPS) return ONE;
  if (distance > A.r + B.r) return NONE;
  if (distance + A.r <= B.r) {
    if (B.r - (distance+A.r) < EPS) return AINB_TANGENT;
    return AINB;
  }
  if (distance + B.r <= A.r) {
    if (A.r - (distance+B.r) < EPS) return BINA_TANGENT;
    return BINA;
  }
  return TWO;
}

int CIPoints (Circle A, Circle B, Point *s, Point *t) {
  double dx = B.o.x-A.o.x, dy = B.o.y-A.o.y;
  double dA, d, c, a;
  int type;

  type = CIType(A, B);

  d = sqrt(dx*dx + dy*dy);

  switch (type) {
  case AINB_TANGENT:
    s->x = B.o.x + (B.r/d)*-dx;
    s->y = B.o.y + (B.r/d)*-dy;
    return 1;

  case BINA_TANGENT: case ONE:
    s->x = A.o.x + (A.r/d)*dx;
    s->y = A.o.y + (A.r/d)*dy;
    return 1;

  case TWO:
    c = atan2(dy, dx);

    a = sqrt(4*SQR(d)*SQR(A.r) - SQR(SQR(d)-SQR(B.r)+SQR(A.r))) /d;

    dA = (SQR(d) - SQR(B.r) + SQR(A.r)) / (2*d);

    t->x = s->x = dA + A.o.x;
    s->y = a/2 + A.o.y;
    t->y = -a/2 + A.o.y;

    /* Rotate these points */
    *s = rotate_2d(*s, A.o, c);
    *t = rotate_2d(*t, A.o, c);

    return 2;

  default:
    return type;
  }
}
/* end of intersection of circles */
```

```
/***************************************************************************
3D GEOMETRY ROUTINES
Submitted March 21, 2004 by Kelly Poon
Original source courtesy of The University of Alberta
***************************************************************************/

#include <math.h>
#define EPS 1E-8
#define pt(a) &(a.x), &(a.y), &(a.z)

struct Point{
  double x, y, z;
  Point(){};
  Point(double xi, double yi, double zi){x = xi; y = yi; z = zi;}
};

Point operator + (const Point& a, const Point& b) {
  return Point(a.x + b.x, a.y + b.y, a.z + b.z);
}

Point operator * (double k, const Point& a) {
  return Point(k*a.x, k*a.y, k*a.z);
}

Point operator - (const Point& a, const Point& b) {
  return Point(a.x - b.x, a.y - b.y, a.z - b.z);
}

Point operator * (Point a, double k) {
  return (k*a);
}

Point operator / (Point a, double k) {
  return (1.0/k)*a;
}

double dot(const Point& a, const Point& b) {
  return a.x*b.x + a.y*b.y + a.z*b.z;
}

Point cross(const Point& a, const Point& b) {
  return Point(a.y*b.z-b.y*a.z, b.x*a.z-a.x*b.z, a.x*b.y-b.x*a.y);
}

double length2(const Point& a) {
  return dot(a,a);
}

double length(const Point& a) {
  return sqrt(dot(a,a));
}

Point closest_pt_iline(const Point& a, const Point& b, const Point& p) {
  double along = dot(b-a,p-a)/length2(b-a);
  return (b-a)*along + a;
}

Point closest_pt_seg(const Point& a, const Point& b, const Point& p) {
  double along;

  if (length2(b-a) < EPS) return a;
  along = dot(b-a,p-a)/length2(b-a);
  if (along < 0) along = 0;
  if (along > 1) along = 1;
  return (b-a)*along + a;
}

/* plane represented by a normal and a point on plane */
Point closest_pt_plane(const Point& norm, const Point& a, const Point& p) {
  Point res = cross(cross(norm,p-a),norm);
  if (length2(res) < EPS) return a;
  return res*dot(res,p-a)/length2(res);
}

/* plane represented by three points */
Point closest_pt_plane(const Point& a, const Point& b, const Point& c, const
Point& p) {
  Point norm;

  norm = cross(b-a,c-a);
  /*assert(length2(norm) > EPS);*/  // collinearity
  return closest_pt_plane(norm,a,p);
}

/* returns number of intersections and the intersections*/
int sphere_iline_isect(const Point& c, double r, const Point& a, const Point& b,
                      Point *p, Point *q) {
  Point vec, mid = closest_pt_iline(a,b,c);

  if (length2(c-mid) > r*r) return 0;
  vec = (a-b)*sqrt((r*r - length2(c-mid))/length2(a-b));
  *p = mid + vec;
  *q = mid - vec;
  return ((length2(vec) > EPS) ? 2 : 1);
}

/* project point p to the plane defined by a, b and c */
Point to_plane(const Point& a, const Point& b, const Point& c, const Point& p) {
  Point norm, ydir, xdir, res;

  norm = cross(b-a,c-a);
  /*assert(length2(norm) > EPS);*/  // collinearity
  xdir = (b-a)/length(b-a); // create orthonormal vectors
  ydir = cross(norm,xdir);
  ydir = ydir/length(ydir);
  res.x = dot(p-a,xdir);
  res.y = dot(p-a,ydir);
  res.z = 0;
  return res;
}

/* given two lines in 3D space, find distance of closest approach */
double line_line_dist(const Point& a, const Point& b, const Point& c, const
Point& d) {
  Point perp = cross(b-a,d-c);

  if (length2(perp) < EPS)  /* parallel */
    perp = cross(b-a,cross(b-a,c-a));
  if (length2(perp) < EPS) return 0; /* coincident */

  return fabs(dot(a-c,perp))/length(perp);
}

/* same as line_line_dist, but returns the points of closest approach */
double closest_approach(const Point& a, const Point& b, const Point& c, const
Point& d,
                                                  Point *p, Point *q) {
  double s = dot(d-c,b-a), t = dot(a-c,d-c);
```

```
  double num, den, tmp;

  den = length2(b-a)*length2(d-c) - s*s;
  num = t*s - dot(a-c,b-a)*length2(d-c);
  if (fabs(den) < EPS) { /* parallel */
    *p = a;
    *q = (d-c)*t/length2(d-c) + c;
    if (fabs(s) < EPS) *q = a;  /* coincident */
  } else {  /* skew */
    tmp = num/den;
    *p = a + (b-a)*tmp;
    *q = c + (d-c)*(t + s*tmp)/length2(d-c);
  }
  return length(*p-*q);
}

/* is the point p on the infinite line ab? */
int on_iline(const Point& a, const Point& b, const Point& p) {
  return (length2(p-closest_pt_iline(a,b,p)) < EPS);
}

/* is the point p on the segment ab? */
int on_seg(const Point& a, const Point& b, const Point& p) {
  return (length(a-p) + length(p-b) - length(a-b) < EPS);
}

/* Given a plane and a line ab, determine if the two intersect,
   and if so, find the single point of intersection */
int plane_iline_isect(const Point& norm, const Point& ori, const Point& a, const
Point& b, Point *p) {
  double along, den = dot(norm,b-a);

  if (fabs(den) < EPS) { /* parallel */
    if (length2(cross(ori-a,b-a)) < EPS) return -1; /* coincident */
    return 0;  /* non-intersecting */
  }
  along = dot(norm,ori-a)/den;

  /* if you want to intersect a plane with a finite segment,
     check that (along <= 1 && along => 0) */
  *p = a + along*(b-a);
  return 1;
}
```

```
/* triangulate.h  - triangulates a polygon in O(n^2) time */
/*                 (note: fails on degenerate case of 3 collinear points) */
#include <list>
#include <vector>

using namespace std;

#define EPS 1e-8
#define ORDER 1 /* 1: cw, -1: ccw */

struct Point {
  double x, y;
};
struct Triangle {
  Point p[3];
};

/* classifies p as either being -1 left of, 1 right of or 0 on the line ab. */
int leftRight(Point &a, Point &b, Point &p){
  double res = ((b.x - a.x)*(p.y - a.y) - (p.x - a.x)*(b.y - a.y));
  if (res > EPS) return -1;
  else if (res < -EPS) return 1;
  return 0;
}

/* returns non-0 if b in the sequence a->b->c is concave, 0 for convex. */
int isConcave(Point &a, Point &b, Point &c){
  return (ORDER*leftRight(a, b, c) <= 0);
}

/* returns non-zero if point p is located on or inside the triangle <a b c>. */
int isInsideTriangle(Point &a, Point &b, Point &c, Point &p){
  int r1 = leftRight(a, b, p);
  int r2 = leftRight(b, c, p);
  int r3 = leftRight(c, a, p);
  return ((ORDER*r1 >= 0) && (ORDER*r2 >= 0) && (ORDER*r3 >= 0));
}

/* P - n cw-ordered points of a polygon (n>=3, P modified during function
   T - n-2 triangles, returns the triangulation of P */
void triangulate(list<Point> &P, vector<Triangle> &T){
  list<Point>::iterator a, b, c, q;
  Triangle t;

  T.clear();
  if (P.size() < 3) return;

  for (a=b=P.begin(), c=++b, ++c; c != P.end(); a=b, c=++b, ++c) {
    if (!isConcave(*a, *b, *c)) {
      for (q = P.begin(); q != P.end(); q++) {
        if (q == a) { ++q; ++q; continue; }
        if (isInsideTriangle(*a, *b, *c, *q)) break;
      }
      if (q == P.end()) {
        t.p[0] = *a; t.p[1] = *b; t.p[2] = *c;
        T.push_back(t);
        P.erase(b);
        b = a;
        if (b != P.begin()) b--;
      }
    }
  }
}
```

```
/**************************************************************************
LONGEST INCREASING SUBSEQUENCE
Submitted March 21, 2004 by Kelly Poon
Original source courtesy of University of Alberta
**************************************************************************/

#include <stdlib.h>

/* Given an array of size n, asc_seq returns the length
   of the longest ascending subsequence, as well as one
   of the subsequences in S.*/
int asc_seq(int *A, int n, int *S){
  int *m, *seq, i, k, low, up, mid, start;

  m   = (int*)malloc((n+1) * sizeof(int));
  seq = (int*)malloc(n * sizeof(int));
  /* assert(m && seq); */

  for (i = 0; i < n; i++) seq[i] = -1;
  m[1] = start = 0;
  for (k = i = 1; i < n; i++) {
    if (A[i] >= A[m[k]]) {
      seq[i] = m[k++];
      start = m[k] = i;
    } else if (A[i] < A[m[1]]) {
      m[1] = i;
    } else {
      /* assert(A[m[1]] <= A[c] && A[c] < A[m[k]]); */
      low = 1;
      up = k;
      while (low != up-1) {
         mid = (low+up)/2;
         if(A[m[mid]] <= A[i]) low = mid;
         else up = mid;
      }
      seq[i] = m[low];
      m[up] = i;
    }
  }
  for (i = k-1; i >= 0; i--) {
    S[i] = A[start];
    start = seq[start];
  }
  free(m); free(seq);
  return k;
}
```

```
/**************************************************************************
LONGEST COMMON SUBSEQUENCE AND EDIT DISTANCE
Submitted March 23, 2004 by Kelly Poon
Original source courtesy of University of Alberta
**************************************************************************/

#include <stdlib.h>

#define MAXN 20
#define Atype int
#define max(x,y) (((x)>(y))?(x):(y))

int LCS(Atype *A, int n, Atype *B, int m, Atype *s)
{
  int L[MAXN+1][MAXN+1];
  int i, j, k;

  for(i = n; i >= 0; i--) for(j = m; j >= 0; j--){
    if(i == n || j == m){
      L[i][j] = 0;
    } else if(A[i] == B[j]){
      L[i][j] = 1 + L[i+1][j+1];
    } else {
      L[i][j] = max(L[i+1][j], L[i][j+1]);
    }
  }

  /* The following is not needed if you are not interested in
     returning a longest common subsequence */
  k = 0;
  i = j = 0;
  while(i < n && j < m){
    if(A[i] == B[j]){
      s[k++] = A[i++];
      j++;
    } else if(L[i+1][j] > L[i][j+1]){
      i++;
    } else if(L[i+1][j] < L[i][j+1]){
      j++;
    } else {
      /* tie breaking conditions here*/
      j++;
    }
  }
  return L[0][0];
}


/*** EDIT DISTANCE CODE ***/

#include <string.h>

#define MAXN 90

char move[MAXN][MAXN];  /* Type of command used */
int  g[MAXN][MAXN];      /* Cost of changes */

int editDistance(char *src, char *dst, int replace, int insert, int delete){
  int i, j, l1, l2;

  l1 = strlen(src);
  l2 = strlen(dst);
```

```
  for(j = 0; j <= l1; j++){
    g[0][j] = j;
    move[0][j] = 'D';
  }

  for(i = 1; i <= l2; i++){
    g[i][0] = i;
    move[i][0] = 'I';

    for(j = 1; j <= l1; j++){
      g[i][j] = g[i-1][j-1]+replace;
      move[i][j] = 'R';

      if(g[i-1][j]+insert < g[i][j]){
        g[i][j] = g[i-1][j]+insert;
        move[i][j] = 'I';
      }

      if(g[i][j-1]+delete < g[i][j]){
        g[i][j] = g[i][j-1]+delete;
        move[i][j] = 'D';
      }

      if(src[j-1] == dst[i-1] && g[i-1][j-1] < g[i][j]){
        g[i][j] = g[i-1][j-1];
        move[i][j] = 'N';
      }
    }
  }
  return g[l2][l1];
}

int counter;
void PathRecovery(int x, int y, int *delta, char *src, char *dst){
  int ndelta;

  if(x == 0 && y == 0){
    *delta = 0;
    return;
  }
  else {
    switch(move[x][y]){
    case 'R':
      PathRecovery(x-1,y-1,&ndelta,src,dst);
      *delta = ndelta;
      printf("%d Replace %d,%c\n", counter++, y+ndelta, dst[x-1]);
      break;
    case 'I':
      PathRecovery(x-1,y,&ndelta,src,dst);
      *delta = ndelta+1;
      printf("%d Insert %d,%c\n", counter++, y+ndelta+1, dst[x-1]);
      break;
    case 'D':
      PathRecovery(x,y-1,&ndelta,src,dst);
      *delta = ndelta-1;
      printf("%d Delete %d\n", counter++, y+ndelta);
      break;
    case 'N':
      PathRecovery(x-1,y-1,&ndelta,src,dst);
      *delta = ndelta;
      break;
    }
  }
}
```

```
/* matrix.h - contains matrix and vector maths
   NOTE: be careful using homogenous coords */
#include <vector>
#include <math.h>

using namespace std;

#ifndef MATRIX_H
#define MATRIX_H

#define Matrix vector < vector<double> >
#define Vector vector<double>

bool ludcmp(Matrix& a, vector<int>& indx, double& d);
void lubksb(const Matrix& a, const vector<int>& indx, Vector& b);

Vector operator+(const Vector& v1, const Vector& v2){
    Vector ret(v1.size());
    for(int i = 0; i < v1.size(); i++) ret[i] = v1[i] + v2[i];
    return ret;
}
Vector operator-(const Vector& v1, const Vector& v2){
    Vector ret(v1.size());
    for(int i = 0; i < v1.size(); i++) ret[i] = v1[i] - v2[i];
    return ret;
}
Vector operator*(const double d, const Vector& v){
    Vector ret(v.size());
    for(int i = 0; i < v.size(); i++) ret[i] = d*v[i];
    return ret;
}
double dot(const Vector& v1, const Vector& v2){
    double ret = 0.0;
    for(int i = 0; i < v1.size(); i++) ret += v1[i]*v2[i];
    return ret;
}
double length(const Vector& v){
    return sqrt(dot(v,v));
}
Matrix operator+(const Matrix& m1, const Matrix& m2){
    Matrix ret(m1.size(), Vector(m1[0].size()));
    for(int i = 0; i < m1.size(); i++)
        for(int j = 0; j < m1[0].size(); j++)
            ret[i][j] = m1[i][j] + m2[i][j];
    return ret;
}
Matrix operator-(const Matrix& m1, const Matrix& m2){
    Matrix ret(m1.size(), Vector(m1[0].size()));
    for(int i = 0; i < m1.size(); i++)
        for(int j = 0; j < m1[0].size(); j++)
            ret[i][j] = m1[i][j] - m2[i][j];
    return ret;
}
Matrix operator*(const double s, const Matrix& m){
    Matrix ret(m.size(), Vector(m[0].size()));
    for(int i = 0; i < m.size(); i++)
        for(int j = 0; j < m[0].size(); j++)
            ret[i][j] = s*m[i][j];
    return ret;
}
Matrix operator*(const Matrix& m1, const Matrix& m2){
    Matrix ret(m1.size(), Vector(m2[0].size(), 0.0));
    for(int r = 0; r < m1.size(); r++)
        for(int c = 0; c < m2[0].size(); c++)
```

```
            for(int i = 0; i < m2.size(); i++)
                ret[r][c] += m1[r][i]*m2[i][c];
    return ret;
}
Vector operator*(const Matrix& m, const Vector& v){
    Vector ret(m.size(), 0.0);
    for(int r = 0; r < m.size(); r++)
        for(int c = 0; c < m[0].size(); c++)
            ret[r] += m[r][c]*v[c];
    return ret;
}
#include <iostream>
Matrix id(int N){
    Matrix ret(N, Vector(N, 0.0));
    for(int i = 0; i < N; i++)
        ret[i][i] = 1.0;
    return ret;
}

/* inverts m (assumes m is square) */
Matrix inverse(const Matrix& m){
    int N = m.size();
    Matrix mT = m, inv(N, Vector(N, 0.0));
    double d;
    vector<int> indx(N);

    ludcmp(mT, indx, d);
    for(int j = 0; j < N; j++){
        Vector col(N, 0.0); col[j] = 1.0;
        lubksb(mT, indx, col);
        for(int i = 0; i < N; i++) inv[i][j] = col[i];
    }

    return inv;
}

/* returns the determinant of m, an NxN matrix in O(N^3) */
double determinant(const Matrix& m){
    int N = m.size();
    Matrix mT = m;
    double d;
    vector<int> indx(N);

    ludcmp(mT, indx, d);
    for(int j = 0; j < N; j++) d*= mT[j][j];

    return d;
}

/* return the solution, x, to Ax = b (assumes A is NxN and b is N) */
Vector solve(const Matrix& A, const Vector& b){
    int N = A.size();
    Matrix aT = A;
    Vector x = b;
    double d;
    vector<int> indx(N);

    ludcmp(aT, indx, d);
    lubksb(aT, indx, x);

    return x;
}

#endif
```

```
/* lu.h - LU Decomposition */

#include "matrix.h"
#define TINY 1.0e-20

/* replaces A with the LU decomposition of A rowwise permutation of A
   indx records the row permutation effected by pivoting
   d is 1.0 if n interchanges is even, else -1 */
bool ludcmp(Matrix& A, vector<int>& indx, double& d)
{
    int i, j, k, imax = 0, n = A.size();
    double big, dum, sum, temp;
    vector<double> vv(n+1);

    d = 1.0;
    for(i = 0; i < n; i++){
        big = 0.0;
        for(j = 0;j < n; j++)
            if((temp = fabs(A[i][j])) > big) big = temp;
        if(big == 0.0) return false; /* singular matrix */
        vv[i] = 1.0/big;
    }
    for(j = 0; j < n; j++){
        for(i = 0; i < j; i++){
            sum = A[i][j];
            for(k = 0; k < i; k++) sum -= A[i][k]*A[k][j];
            A[i][j] = sum;
        }
        big = 0.0;
        for(i = j; i < n; i++){
            sum = A[i][j];
            for(k = 0; k < j; k++) sum -=A[i][k]*A[k][j];
            A[i][j] = sum;
            if((dum = vv[i]*fabs(sum)) >= big){
                big = dum;
                imax = i;
            }
        }
        if(j != imax){
            for(k = 0; k < n; k++){
                dum = A[imax][k];
                A[imax][k] = A[j][k];
                A[j][k] = dum;
            }
            d = -d;
            vv[imax] = vv[j];
        }
        indx[j] = imax;
        if(A[j][j] == 0.0) A[j][j]=TINY;
        if(j != n-1){
            dum = 1.0/(A[j][j]);
            for(i = j+1; i < n; i++) A[i][j] *= dum;
        }
    }

    return true;
}

/* solves Ax=b (returns x in b) */
void lubksb(const Matrix& A, const vector<int>& indx, Vector& b)
{
    int i, ip, j, ii=0, n = A.size();
    double sum;
```

```
    for(i = 0; i < n; i++){
        ip = indx[i];
        sum = b[ip];
        b[ip] = b[i];
        if(ii) for(j = ii-1; j < i; j++) sum -= A[i][j]*b[j];
        else if(sum) ii = i+1;
        b[i] = sum;
    }
    for(i = n-1; i >= 0; i--){
        sum = b[i];
        for(j = i+1; j < n; j++) sum -= A[i][j]*b[j];
        b[i] = sum/A[i][i];
    }
}
```

Here is a potentially useful excerpt from *3D Game Engine Design* by David H. Eberly (Morgan Kaufmann, 2001):

Angle-Axis to Rotation Matrix

Any standard computer graphics text discusses the relationship between an angle and axis of rotation and the rotation matrix, although the constructions can be varied. A useful one is given here. If θ is the angle of rotation and $\vec{U}$ is the unit-length axis of rotation, then the corresponding rotation matrix is

$$R = I + (\sin\theta)S + (1-\cos\theta)S^2,$$

where I is the identity matrix and

$$S = \begin{bmatrix} 0 & -u_2 & u_1 \\ u_2 & 0 & -u_0 \\ -u_1 & u_0 & 0 \end{bmatrix}$$

a skew-symmetric matrix. For $\theta > 0$, the rotation represent a counterclockwise rotation about the axis. The sense of clockwise or counterclockwise is based on looking at the plane with normal $\vec{U}$ from the side of the plane to which the normal points. Note that $S\vec{V} = \vec{U}\times\vec{V}$ and

$$R\vec{V} = \vec{V} + (\sin\theta)\vec{U}\times\vec{V} + (1-\cos\theta)\vec{U}\times(\vec{U}\times\vec{V}).$$

```
/* rotations.h - makes rotation matrices */
#include "matrix.h"
#include <math.h>

/* rotations about main axes */
Matrix rotX(double angle){
    double cosa = cos(angle), sina = sin(angle);
    Matrix ret = id(4);
    ret[1][1] = cosa; ret[1][2] = -sina;
    ret[2][1] = sina; ret[2][2] = cosa;
    return ret;
}
Matrix rotY(double angle){
    double cosa = cos(angle), sina = sin(angle);
    Matrix ret = id(4);
    ret[0][0] = cosa; ret[0][2] = sina;
    ret[2][0] = -sina; ret[2][2] = cosa;
    return ret;
}
Matrix rotZ(double angle){
    double cosa = cos(angle), sina = sin(angle);
    Matrix ret = id(4);
    ret[0][0] = cosa; ret[0][1] = -sina;
    ret[1][0] = sina; ret[1][1] = cosa;
    return ret;
}

/* rotation about arbitrary axis (flattens to z) */
Matrix rot(double angle, Vector axis){
    double u = axis[0], v = axis[1], w = axis[2];
    double u2 = u*u, v2 = v*v, w2 = w*w, len2 = u2 + v2 + w2, len = sqrt(len2);
    double cosa = cos(angle), sina = sin(angle);

    Matrix ret = id(4);
    ret[0][0] = (u2+(v2+w2)*cosa)/len2;
    ret[0][1] = (u*v*(1-cosa)-w*len*sina)/len2;
    ret[0][3] = (u*w*(1-cosa)+v*len*sina)/len2;
    ret[1][0] = (u*v*(1-cosa)+w*len*sina)/len2;
    ret[1][1] = (v2+(u2+w2)*cosa)/len2;
    ret[1][2] = (v*w*(1-cosa)-u*len*sina)/len2;
    ret[2][0] = (u*w*(1-cosa) - v*len*sina)/len2;
    ret[2][1] = (v*w*(1-cosa)+ u*len*sina)/len2;
    ret[2][2] = (w2+(u2+v2)*cosa)/len2;

    return ret;
}

/* rotation about the axis parallel to axis that goes through point */
Matrix rot(double angle, Vector axis, Vector point){
    double u = axis[0], v = axis[1], w = axis[2];
    double a = point[0], b = point[1], c = point[2];
    double u2 = u*u, v2 = v*v, w2 = w*w, len2 = u2 + v2 + w2, len = sqrt(len2);
    double cosa = cos(angle), sina = sin(angle);

    Matrix ret = rot(angle, axis);
    ret[0][3] = (a*(v2+w2)-u*(b*v-c*w)+(u*(b*v+c*w)-a*(v2+w2))*cosa+(b*w-
c*v)*len*sina)/len2;
    ret[1][3] = (b*(u2+w2)-v*(a*u+c*w)+(v*(a*u+c*w)-b*(u2+w2))*cosa+(c*u-
a*w)*len*sina)/len2;
    ret[2][3] = (c*(u2+v2)-w*(a*u+b*v)+(w*(a*u+b*v)-c*(u2+v2))*cosa+(a*v-
b*u)*len*sina)/len2;

    return ret;
}
```

```c
/******************************************************************************
TABLE OF PRIMES, EXTENDED EUCLIDEAN ALGORITHM, CHINESE REMAINDER THEOREM
    AND CUBIC EQUATION SOLVER
Author:  Alex Fink, with some original source from Howard Cheng
Date:    March 22, 2004
******************************************************************************/

int primes[6543];

void generate_primes(int max) {
  int i, j, k = 1;
  primes[0] = 2;
  for(i = 3; i < max; i += 2) {
    for(j = 0; j < k; j++)
      if (!(i%primes[j]))
        break;
    if (j == k)
      primes[k++] = i;
  }
  primes[k++] = 0;
}

void print_table() {
  int i;
  printf("{");
  for(i = 0; primes[i]; i++)
    printf("%d,%s", primes[i], 12==i%13?"\n":"");
  printf("0}\n");
}

int least_factor(int n) {
  int i;
  for(i = 0; primes[i]; i++)
    if (!(n%primes[i]))
      return primes[i];
  return n;
}

int extended_euclid(int a, int b, int *x, int *y) {
  int d, t;
  if (!b) {
    *x = 1;
    *y = 0;
    return a;
  }
  d = extended_euclid(b, a%b, &t, x);
  *y = t - *x*(a/b);
  return d;
}

/* Chinese remainder theorem for x % m[i] = a[i] */
int cra(int n, int *m, int *a){
  int x, i, k, prod, temp;
  int *gamma, *v;

  gamma = (int *)malloc(n*sizeof(int));
  v     = (int *)malloc(n*sizeof(int));

  /* compute inverses */
  for (k = 1; k < n; k++) {
    prod = m[0] % m[k];
    for (i = 1; i < k; i++) {
      prod = (prod * m[i]) % m[k];
    }
    extended_euclid(prod, m[k], gamma+k, &temp);
    gamma[k] %= m[k];
    if (gamma[k] < 0) {
      gamma[k] += m[k];
    }
  }

  /* compute coefficients */
  v[0] = a[0];
  for (k = 1; k < n; k++) {
    temp = v[k-1];
    for (i = k-2; i >= 0; i--) {
      temp = (temp * m[i] + v[i]) % m[k];
      if (temp < 0) {
        temp += m[k];
      }
    }
    v[k] = ((a[k] - temp) * gamma[k]) % m[k];
    if (v[k] < 0) {
      v[k] += m[k];
    }
  }

  /* convert from mixed-radix representation */
  x = v[n-1];
  for (k = n-2; k >= 0; k--) {
    x = x * m[k] + v[k];
  }
  free(gamma);
  free(v);
  return x;
}

/* solve a cubic equation */
typedef struct{
  int n;        /* Number of solutions */
  double x[3];  /* Solutions */
} Result;

double PI; // PI = acos(-1);

Result solve_cubic(double a, double b, double c, double d){
  Result s;
  long double a1 = b/a, a2 = c/a, a3 = d/a;
  long double q = (a1*a1 - 3*a2)/9.0, sq = -2*sqrt(q);
  long double r = (2*a1*a1*a1 - 9*a1*a2 + 27*a3)/54.0;
  double z = r*r-q*q*q;
  double theta;

  if(z <= 0){
    s.n = 3;
    theta = acos(r/sqrt(q*q*q));
    s.x[0] = sq*cos(theta/3.0) - a1/3.0;
    s.x[1] = sq*cos((theta+2.0*PI)/3.0) - a1/3.0;
    s.x[2] = sq*cos((theta+4.0*PI)/3.0) - a1/3.0;
  } else {
    s.n = 1;
    s.x[0] = pow(sqrt(z)+fabs(r),1/3.0);
    s.x[0] += q/s.x[0];
    s.x[0] *= (r < 0) ? 1 : -1;
    s.x[0] -= a1/3.0;
  }
  return s;
}
```

```
/***************************************************************************
THE ZERO-ONE KNAPSACK PROBLEM
Author:  John Zhang, adapted from Methods to Solve
Date:    March 7, 2005
***************************************************************************/

#include <stdio.h>

#define MAXITEMS 100      /* number of items */
#define MAXWEIGHT 100     /* maximum total weight */

int weight[MAXITEMS];
int value[MAXITEMS];

/* returns max value if there are N items with max total weight of MW */
int knapsack(int N, int MW) {
  int i, w;
  int C[MAXITEMS][MAXWEIGHT];

  /* zero value when no items or max weight is zero */
  for (i=0; i<=N; i++) C[i][0] = 0;
  for (w=0; w<=MW; w++) C[0][w] = 0;

  for (i=1;i<=N;i++) {
    for (w=1;w<=MW;w++) {
      if (Wi[i] > w)
        C[i][w] = C[i-1][w];
      else
        C[i][w] = max(C[i-1][w], C[i-1][w-Wi[i]]+Vi[i]);
    }
  }

  return C[N][MW];
}

/***************************************************************************
SEGMENT INTERSECTION
Author:  Alex Fink
Date:    March 26, 2005
***************************************************************************/

#include <stdio.h>

#define EPS 1e-8

typedef struct {
  double x, y;
} Point;

/* The values here are important!  */
enum {LEFT = -1, RIGHT = 1, CL = 0};

int whichside(Point a, Point b, Point p) {
  double res;

  res = (p.x - a.x)*(b.y - a.y) - (p.y - a.y)*(b.x - a.x);

  if(fabs(res) < EPS)
    return CL;
  else if (res > 0.0)
    return RIGHT;
  return LEFT;
}

int isect_iline(Point a, Point b, Point c, Point d, Point *p) {
  double r, denom, num1;

  num1  = (a.y - c.y) * (d.x - c.x) - (a.x - c.x) * (d.y - c.y);
  denom = (b.x - a.x) * (d.y - c.y) - (b.y - a.y) * (d.x - c.x);

  if (fabs(denom) >= EPS) {
    r = num1 / denom;
    p->x = a.x + r*(b.x - a.x);
    p->y = a.y + r*(b.y - a.y);
    return 1;
  }
  if (fabs(num1) >= EPS) return 0;
  return -1;
}

/* intersection of segments */
int isect_seg(Point a, Point b, Point c, Point d, Point *p) {
  int s1, s2, i = isect_iline(a, b, c, d, p);
  Point  t;

  if (i == 1) {
    if ((whichside(a, b, c)*whichside(a, b, d) == 1) ||
        (whichside(c, d, a)*whichside(c, d, b) == 1))
      return 0;
    return 1;
  }
  else if (i == -1) {
    if (a.x > b.x || (a.x == b.x && a.y > b.y)) {t=a; a=b; b=t;}
    if (c.x > d.x || (c.x == d.x && c.y > d.y)) {t=c; c=d; d=t;}
    if (a.x == b.x) {
      if (b.y == c.y) {
        *p = b; return 1;
      } else if (a.y == d.y) {
        *p = a; return 1;
      } else if (b.y < c.y || d.y < a.y)
        return 0;
    } else {
      if (b.x == c.x) {
        *p = b; return 1;
      } else if (a.x == d.x) {
        *p = a; return 1;
      } else if (b.x < c.x || d.x < a.x)
        return 0;
    }
    return -1;
  }
  else
    return 0;
}

main() {
  Point a, b, c, d, p;
  int i;

  while(scanf("%lf%lf%lf%lf%lf%lf%lf%lf",
              &a.x, &a.y, &b.x, &b.y, &c.x, &c.y, &d.x, &d.y)) {
    if(feof(stdin))
      return -1;
    if((i = isect_seg(a, b, c, d, &p)) == 1)
      printf("%lf %lf ", p.x, p.y);
    printf("(%d)\n",i);
  }
}
```