CSE276C - Linear Systems of Equations

Henrik I. Christensen

Computer Science and Engineering
University of California, San Diego
http://cri.ucsd.edu

October 2020

Outline

- Linear Systems of Equations
- Solution Techniques - Gauss Jordan
- Matrix Decomposition
- Matrix Factorization
- Singular Value Decomposition
- Rank and sensitivity

Linear Systems of Equations

- One of the most basic tasks is solve for a set of unknowns

$$
\begin{array}{rcl}
a_{00}x_0 + a_{01}x_1 + a_{02}x_2 + \ldots + a_{0n-1}x_{n-1} & = & b_0 \\
a_{10}x_0 + a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n-1}x_{n-1} & = & b_1 \\
\vdots & & \\
a_{m-10}x_0 + a_{m-11}x_1 + a_{m-12}x_2 + \ldots + a_{m-1n-1}x_{n-1} & = & b_{m-1}
\end{array}
$$

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a_{m-10}x_0 + a_{m-11}x_1 + a_{m-12}x_2 + \ldots + a_{m-1n-1}x_{n-1} & = & b_{m-1}
\end{array}
$$

- which we can rewrite

$$A\vec{x} = \vec{b}$$

where

$$
A = \begin{pmatrix}
a_{00} & a_{01} & a_{01} & \cdots & a_{0n-1} \\
a_{10} & a_{11} & a_{11} & \cdots & a_{1n-1} \\
 & & \vdots & & \\
a_{m-10} & a_{m-11} & a_{m-11} & \cdots & a_{m-1n-1}
\end{pmatrix}
\vec{b} = \begin{pmatrix}
b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_{m-1}
\end{pmatrix}
$$

Matrix Properties

- Given an $m \times n$ matrix A we define
  - Column space - Linear combination of columns
  - Row space - Linear combination of row
- We can consider A a mapping:

$$A : R^n \rightarrow R^m$$

$$\begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{pmatrix} \rightarrow \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{m-1} \end{pmatrix} = \mathrm{A} \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{pmatrix}$$

- Column space of A is vector subspace of R^m that image vectors under A

Null Space

- We define the null-space: set of vectors $x \in R^n$ where

$$Ax = 0$$

- The row space and the null space are complementary

$$n = dim(row\ space) + dim(null\ space)$$

Questions

Gauss-Jordan Elimination

- How can we solve the equation system?
- The standard form

$$A\vec{x} = \vec{b} \;\rightarrow\; U\vec{x}' = \vec{b}'$$