

# CSE276C - Linear Systems of Equations

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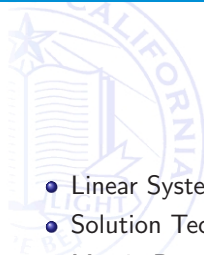


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# Outline

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- Linear Systems of Equations
  - Solution Techniques - Gauss Jordan
  - Matrix Decomposition
  - Matrix Factorization
  - Singular Value Decomposition
  - Rank and sensitivity

# Linear Systems of Equations

- One of the most basic tasks is solve for a set of unknowns

$$a_{00}x_0 + a_{01}x_1 + a_{02}x_2 + \dots + a_{0n-1}x_{n-1} = b_0$$

$$a_{10}x_0 + a_{11}x_1 + a_{12}x_2 + \dots + a_{1n-1}x_{n-1} = b_1$$

$$\vdots$$

$$a_{m-10}x_0 + a_{m-11}x_1 + a_{m-12}x_2 + \dots + a_{m-1n-1}x_{n-1} = b_{m-1}$$

# Linear Systems of Equations

- One of the most basic tasks is solve for a set of unknowns

$$\begin{aligned}a_{00}x_0 + a_{01}x_1 + a_{02}x_2 + \dots + a_{0n-1}x_{n-1} &= b_0 \\a_{10}x_0 + a_{11}x_1 + a_{12}x_2 + \dots + a_{1n-1}x_{n-1} &= b_1 \\&\vdots \\a_{m-10}x_0 + a_{m-11}x_1 + a_{m-12}x_2 + \dots + a_{m-1n-1}x_{n-1} &= b_{m-1}\end{aligned}$$

- which we can rewrite

$$A\vec{x} = \vec{b}$$

where

$$A = \begin{pmatrix} a_{00} & a_{01} & a_{01} & \cdots & a_{0n-1} \\ a_{10} & a_{11} & a_{11} & \cdots & a_{1n-1} \\ & & \vdots & & \\ a_{m-10} & a_{m-11} & a_{m-11} & \cdots & a_{m-1n-1} \end{pmatrix} \quad \vec{b} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_{m-1} \end{pmatrix}$$

# Matrix Properties

- Given an  $m \times n$  matrix  $A$  we define
  - Column space - Linear combination of columns
  - Row space - Linear combination of row
- We can consider  $A$  a mapping:

$$A : R^n \rightarrow R^m$$

$$\begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{pmatrix} \rightarrow \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{m-1} \end{pmatrix} = A \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{pmatrix}$$

- Column space of  $A$  is vector subspace of  $R^m$  that image vectors under  $A$

# Null Space

- We define the null-space: set of vectors  $x \in \mathbb{R}^n$  where

$$Ax = 0$$

- The row space and the null space are complementary

$$n = \dim(\text{row space}) + \dim(\text{null space})$$



# Questions

# Gauss-Jordan Elimination



- How can we solve the equation system?
- The standard form

$$A\vec{x} = \vec{b} \rightarrow U\vec{x}' = \vec{b}'$$