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by

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Bayesian methods for interpretation and control in multi-agent vision systems

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Abstract

Interpretation of images is a context dependent activity and therefore saturated with uncertainty.

It is outlined how causal probabilistic networks (CPNs) together with strict and efficient Bayesian methods can be used for modeling contexts and for interpretation of findings.

For illustration purposes a 2-agent system consisting of an interpreter using a CPN and a findings catcher using an image processor is designed.

It is argued that the architecture should be a system of agents with instincts, each of them acting to improve their own situation.

Going through an interpretation session it is shown how the Bayesian paradigm very neatly supports the agents-with-instincts control paradigm such that the system through private benefit maximizing in an efficient way reaches its goal.

1 Introduction

In the ESPRIT basic research project "Vision As Process" (VAP) [Crowley *et al.*, 1989] we are aiming at constructing a full vision system which by use of a number of different modules performs interpretations of a dynamically changing scene consisting of man-made objects.

Interpretation of images will usually consist of a segmentation of the image (mainly data driven) and an interpretation of the segments (mainly expectation driven). The interpretation of the segments is heavily context dependent, and therefore conclusions as well as expectations will be defected by uncertainty. This uncertainty however, is not an unavoidable nuisance due to imperfect data: it is the driving force in the interpretation, and since uncertainty is inherent in the interpretation task, any interpretation must incorporate ways of handling it.

We therefore have started an investigation of how a system may be constructed if it uses *causal probabilistic networks* (CPNs) as models for the interpretation module. For domains with inherent uncertainty, CPNs is a very promising modeling framework [Pearl, 1988], and it offers a unified method for efficient deductive and abductive reasoning following the laws of probability [Lauritzen and Spiegelhalter, 1988; Jensen *et al.*, 1990a].

Our first considerations were reported at the ESPRIT vision workshop at Crete, september 1990 [Jensen *et al.*, 1990b], and the present paper is a clarification and further development of the points made here.

Several other groups are working with the use of Bayesian approaches to computer vision. In particular, the group from the Robotics Laboratory at Stanford University is working along the same lines as we are [Levitt *et al.*, 1989; Binford *et al.*, 1989]. However, while they are attempting to build a vision system from top to bottom as an *influence diagram* [Shachter, 1988], we are considering the CPN as part of one agent in a multi-agent system, and our principal interest is how such a 'Bayesian' agent may interface to other agents.

2 Interacting Vision Agents

In the VAP system the interaction between agents (knowledge sources, procedural data generators etc) is modeled as a local feedback loop. The basic principle is shown in figure 1.

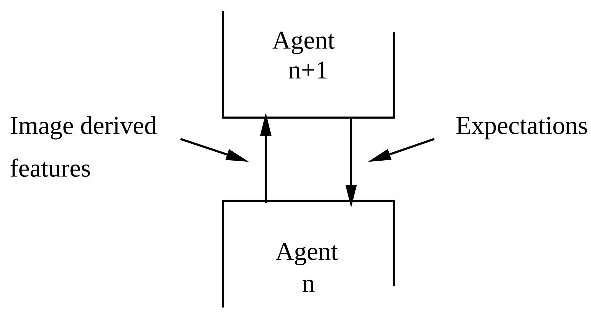

Figure 1: The basic interaction between agents

A "lower" level agent provides data derived features to another agent. This agent then in turn provides expectations (based on contextual knowledge) which may guide processing. The "lower" level agent has encoded knowledge about cost; i.e. the amount of resources (computational and storage) needed for extraction of each of the features which the agent may produce.

In such a system the total amount of information necessary for optimal performance tend to grow intractably, and any centralized control mechanism will therefore in itself require so many resources that the cost of it far exceeds the savings by determining optimal rather than sub-optimal processing (an experience which politically is getting more and more acknowledged).

Hierarchical decision procedures like data driven control (bottom up) or expectation driven control (top down) lack flexibility and are not able to pursue that expectations may give hints on what to do next as well as the chance that evidence may pop up from an unexpected corner.

It is often stated, that the control mechanism in interpreting vision systems is a hypothezise and test cycle [Tsotsos, 1990]. In itself such a statement only says that sometimes expectations should drive the system and sometimes data is driving. However, nothing operational is said about how the control actually is shifting, nothing is said about when the cycle should stop, and nothing is said about the conditions under which the cycle will improve the situation and perhaps converge towards a stop situation.

We have taken the approach to control that since the information on costs is resident in the agents themselves, we will also provide them with a (private) concept of benefit together with a set of 'instincts' which make them act in order to improve there own situation. The task for the system designer is then to create instincts and interaction procedures so that when all agents act according to their instincts, then the system will efficiently reach its goals.

The designer is a creator of species and at the same time a legislator in a society without a police force.

We have chosen to try out a **free market model** [1]: The agent who needs information which holds the information of expected benefit and he will request information by offering to pay a certain amount for the processing. To leave room for profit (reserves against future losses) the requesting agent may offer e.g. $\frac{3}{4}$ of the expected benefit. The information provider, which holds the information of costs of collecting it will among its various offers decide to perform the collecting which maximizes the difference between offer and cost. This includes doing nothing.

3 Causal probabilistic networks

As we have used causal probabilistic networks for the interpretation agent we shortly review some of the notions used here.

A causal probabilistic network (CPN) is constructed over a Universe U consisting of a set of variables, each variable having a finite set of states. The universe is organized as a directed acyclic graph, i.e. the graph has no directed cycles. To each variable A is attached the conditional probability $P(A|pa(A))$, where $pa(A)$ denotes the set of parents for A.

The network figures shown in the rest of this paper are causal probabilistic networks taken from the HUGIN[2] shell. HUGIN is a shell for constructing systems based on CPN models [Andersen *et al.*, 1989]. In this shell tools for creating, altering, and use of CPNs are provided.

The steps necessary to create a CPN are the following:

- Decide on the direct cause-effect relations in the domain in question and represent them as a directed acyclic graph. The nodes in the graph are called variables.

- Decide the set of possible states for each node in the graph, e.g. in the network presented later the set of states for **Cutlery** are *Fork*, *Knife*, *Spoon*, and *No*. This means that the variable **Cutlery** is in exactly one of these states. Which one may be unknown to us.

[1] This does not mean that the authors subscribe to political liberalism and abandon any kind of socialism.

[2] HUGIN is a trademark of Hugin Expert, Ltd.

- Decide on the conditional probabilities for all states in a node conditioned on the states in the parent nodes. That is, the strength of the cause-effect relations are modeled as conditional probabilities.

The probabilities can be compiled in many different ways - ranging from statistical information to subjective assessment (we have done the latter). This does not affect the calculations, but naturally one can not expect responses of higher quality than the information put into the model.

Information is entered to the network in the form of evidence on the states of certain variables. Here we will restrict ourselves to the type of evidence which we call findings: A statement that certain states of specific variables are impossible (typically a finding points out one particular state to be the only possible).

It would be nice to have an updating scheme performing local evidence passing through the network. However, this only works if the network is a tree (without cycles), and the networks here do contain cycles. Therefore, HUGIN constructs another structure, a junction tree, for the evidence updating. For more information on this subject the reader is referred to the literature on the methods used in HUGIN [Jensen *et al.*, 1989; Jensen *et al.*, 1990a].

Since there is no difference in the model between variables for entering information and the other variables it is very easy to do hypothesizing if the hypothesis can be expressed as a statement that a specific variable has a set of impossible states.

The model shown in figure 2 is made for discrimination between a british and a continental breakfast table[3]. It consists of two plate settings, a pot and a jar. Each plate setting may include a plate, a cup, a knife, and a fork. The pot is composed of a body, a neck, and a handle. The meaning of the states of the variables should be obvious. In this network it is shown how both the plate setting and the nature of the pot influences e.g. the probability for finding various kinds of cups.

4 Modeling a domain

In order to simplify the exposition, we restrict ourselves to a **plate setting** only, and we imagine that the task for the system is to decide whether it is *British* or *Continental.*

A plate setting consists of a **plate** which may be *Big* or *Small*, possibly a **cup** and at most two pieces of **cutlery** (which may be a *knife*, a *fork* or a *spoon*). The cup may be a *tea* cup or a *coffee* cup, and a coffee cup need not have a *handle*. *British* plate settings are more likely to have big plates and tea cups then continental plate settings, and forks are more likely to appear on British plate settings (for bacon and eggs) than on continental ones (see figure 3).

4.1 Undirected constraints

There is on extra constraint to the plate setting, namely what kinds of cutlery may be found together. They never come in identical pairs, and you will never find a spoon together with a fork.

[3]To non-Europeans: British breakfasts usually are composed of tea, bacon and eggs and toast with marmalade, while continental breakfasts consist of coffee, boiled eggs and roles with jam.

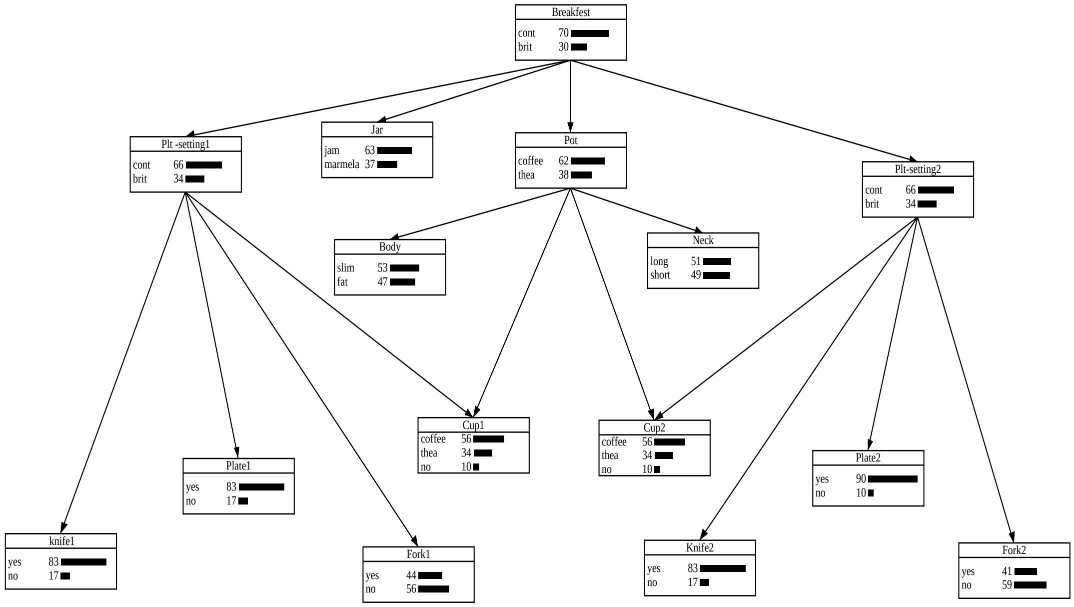

Figure 2: A CPN model for distinguishing between a continental and a british breakfast table

This constraint is not a directed impact from one variable to another, so it can not be modeled directly through a CPN. There is a way though:

A consistency variable **consist** is introduced with states *Yes* and *No* and with **cutlery1** and **cutlery2** as parents. The conditional probability table is a logical table giving *Yes* when the constraint is satisfied and *No* otherwise. The constraint is entered to the model by entering the finding *Yes* to **consist**. This is shown in figure 4.

5 Expected benefit

The task of the CPN module is to discriminate between *British* and *Continental* for **Plate setting**. A perfect determination of the type of the plate setting has a certain value, B.

If the discrimination is not 100%, a value less then B should be given, and to this end a value function $V(p)$ is attached (p the probability of *British*). We use the function

$$V(p) = 3 \times (100(p - \frac{1}{2}))^2 \tag{1}$$

The function V reflects a symmetry between *Continental* and *British*, and furthermore it is convex: The closer to the end points the steeper it is.

In order to reach to a conclusion, the module may request findings from some other module. A simple request may be of the form 'Please, tell me the state of the variable **X**'. Before the request is posed, it may be worthwhile to find out what to expect from it. Suppose that the outcome of the request is 'variable **X** is in state x'. The value of this outcome together with previously obtained evidence, e, is

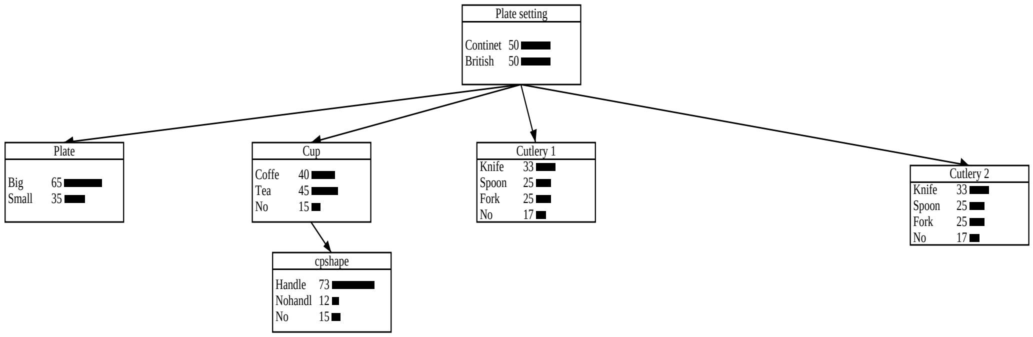

Figure 3: A CPN model of the plate setting, with initial probabilities

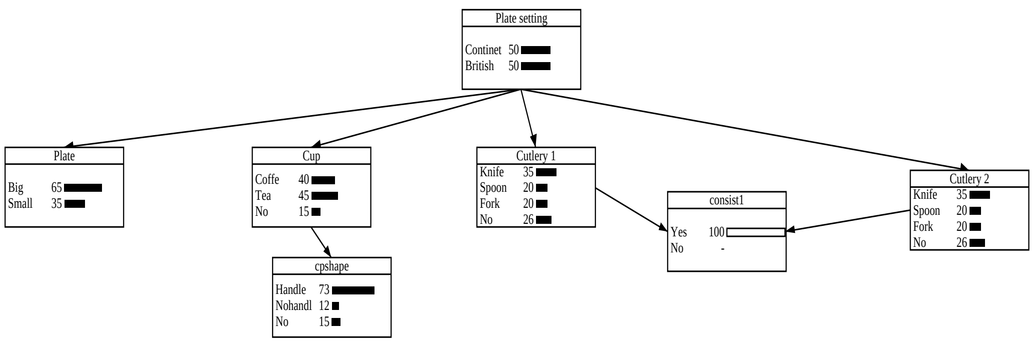

Figure 4: The CPN for plate setting with the constraints modeled through a **consistency** variable

$$V(P(British|x,e)) \tag{2}$$

$P(British|x,e)$ can be obtained through simulating x as a finding in the CPN. The expected contribution to the value is obtained by multiplying with $P(x|e)$, which is available in the CPN. The total value is the sum of the contributions from each possible outcome. The **expected benefit** from the request is the difference between the expected value and the present value, $V(P(British|e))$.

Since the expected value from getting the state of a variable is a sum of contributions from each possible outcome, it may seem that a simulation is needed for each state. However, Bayes' theorem yields a method which only requires one simulation.

$$P(British|x,e) = \frac{P(x|British,e)P(British|e)}{P(x|e)} \tag{3}$$

By entering *British* as a finding and propagating we get all the probabilities $P(x|British,e)$

not only for the variable **X**, but for all variables in the model, and the formula above will give us the required probabilities.

It can be proved (by Jensen's inequality) that since V is convex, the expected benefit is never negative.

6 The 2-agent system

The two agents in our system are the **interpreter** (using a CPN model) and the **findings catcher** (using an image processor).

The tasks for the findings catcher is to use the image processor such that information about the picture can be communicated to the interpreter in the form of statements on the state of certain variables.

The task for the interpreter is to take the findings and use them in the CPN in order to come up with a judgment on the type of the plate setting.

Comparing the kind of output from the image processor with the variables in figure 4, it is obvious that a bridge is needed. The findings catcher must perform a further processing of the segmentation done by the processor, and the interpreter must expand the CPN down to a description suitable for the findings provided by the findings catcher. An overview of the system is shown is figure 5.

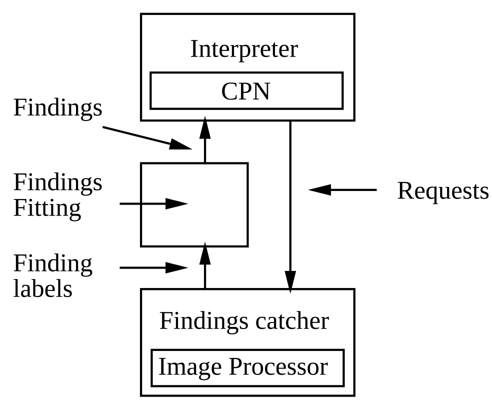

Figure 5: The 2-agent system

7 The sensor: A 2D image processor

The input to the system is a 512×512 image from a camera. The image is acquired just above the tabletop. In the image processor a gray-scale image pyramid with 6 resolution levels is constructed using binomial filters. This process is entirely pre-attentive. The grey-level pyramid is shown in figure 6.

The images (in the pyramid) are subsequently segmented into homogeneous regions with a constant aspect ratio (width/height). The image processor does not necessarily process the entire image, it may perform one of three possible actions.

1. segment the entire image

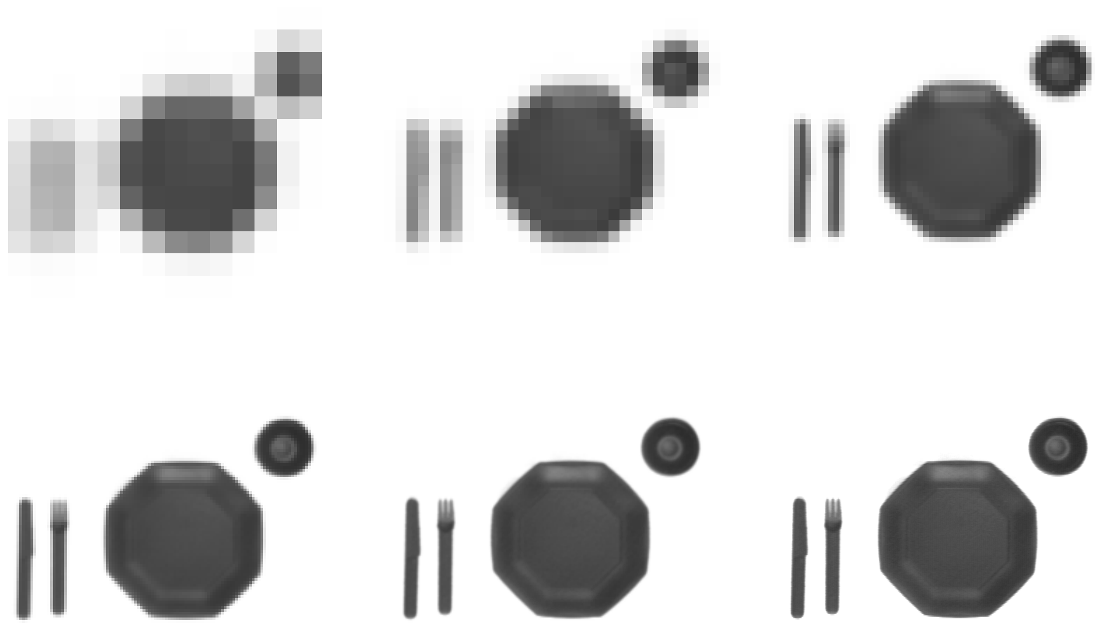

Figure 6: A plate setting as it appears on the 6 resolution levels of the image processor

2. verify existence of a particular segment
3. process region (circular) surrounding a segment

Each of these actions has an associated cost which is calculated according to the formula below.

$$Cost(a, l) = a3^{6-l} + 3 \tag{4}$$

Where l is the resolution level, $l = 6$ denotes the coarsest resolution while $l = 1$ denotes the finest resolution (the original image).

The parameter a denotes expected area of a segment. The following values have been used for a.

item	area
Entire image	300
Circular region	30
The Plate	12
The Cup	5
A piece of cutlery	2

Each of the segments extracted by the image processor is approximated by an ellipse and described by position, area and eccentricity.

The actual actions performed by the image processor depend on expectations received from the interpretation/findings catcher module.

8 Modeling the sensor

The sensor is modeled by indicating its expected behavior under known conditions. Take e.g. plates for level 1. If there is a big plate at the scene, then the sensor will at level 1 detect a large segment. The area and the eccentricity of this segment is estimated through various experiments, and the result is distributions for the two parameters.

Plate For plates, the distributions for the area for *Big* and *Small* at level 1 are disjoint, so it is modeled by introducing a variable **pts1** with parent **plate** and with the same states. The conditional probability table is a trivial logical connection *Big* to *Big* and *Small* to *Small*.

For level 2, the situation is exactly the same, and a variable **pts2** reflects that. At level 3 it appears that the two area distributions overlap. Therefore a new area interval is determined. It is called *Medium*. In the CPN a variable **pts3** is introduced. It has states *Big*, *Medium* and *Small* and it has **plate** as its only parent. The conditional probabilities reflect the probability for a *Big* **plate** and a *Small* **plate** to be detected at level 3 with *Big*, *Medium* and *Small* areas respectively.

At level 4, a variable **pts4** is used with the same three states as above. It has parents **plate** and **pts3**, and the probabilities reflect the various effects on level 4 for *Big* and *Small* plates given how they are detected at level 3. The same is done at level 5 and level 6 (see figure 7).

Cup The sensor cannot distinguish the body of a tea cup from the body of a coffee cup and only the detection of a handle may help in our task. Therefore we introduce a variable **cpshape** with states *Handle*, *Nohandle* and *No*, and for each level, i, a variable **cpsi** is introduced. The states of these variables are the possible numbers of segments which may be detected in a picture of a cup.

The CPN is shown in figure 7. Note that from level 3 on and down there is a possibility that a cup may not be detected at all. The states of the variable **cps4** reflect that at level 4, a cup with a handle may be detected as an elliptic segment or it may be detected as two segments (if it is detected at all). At levels 5 and 6 only one segment may be detected.

Cutlery The considerations for modeling the sensor's behavior on cutlery are similar to those for cups (see figure 7). The states at level 4 model that at this level any kind of cutlery is detected as one segment (if detected), but knifes will be detected with a larger eccentricity than spoons and forks.

As described above, the sensor can perform a processing of a circular area at a given position. This is used in our experiment as an option of processing an area surrounding a segment already detected. This option creates findings, and it may also be modeled in a CPN. However, for the considerations to come, this would be a complicating factor, and we have postponed it to a forthcoming study.

In this experiment we give the processor the option of processing an area of fixed size surrounding a plate segment and a cup segment. We have decided that the probability for a cup to be in the surroundings of the plate is 0.5, for a piece of cutlery the probability of being close to the plate it is also 0.5, and it is 0.4 of being close to the cup (if it is present).

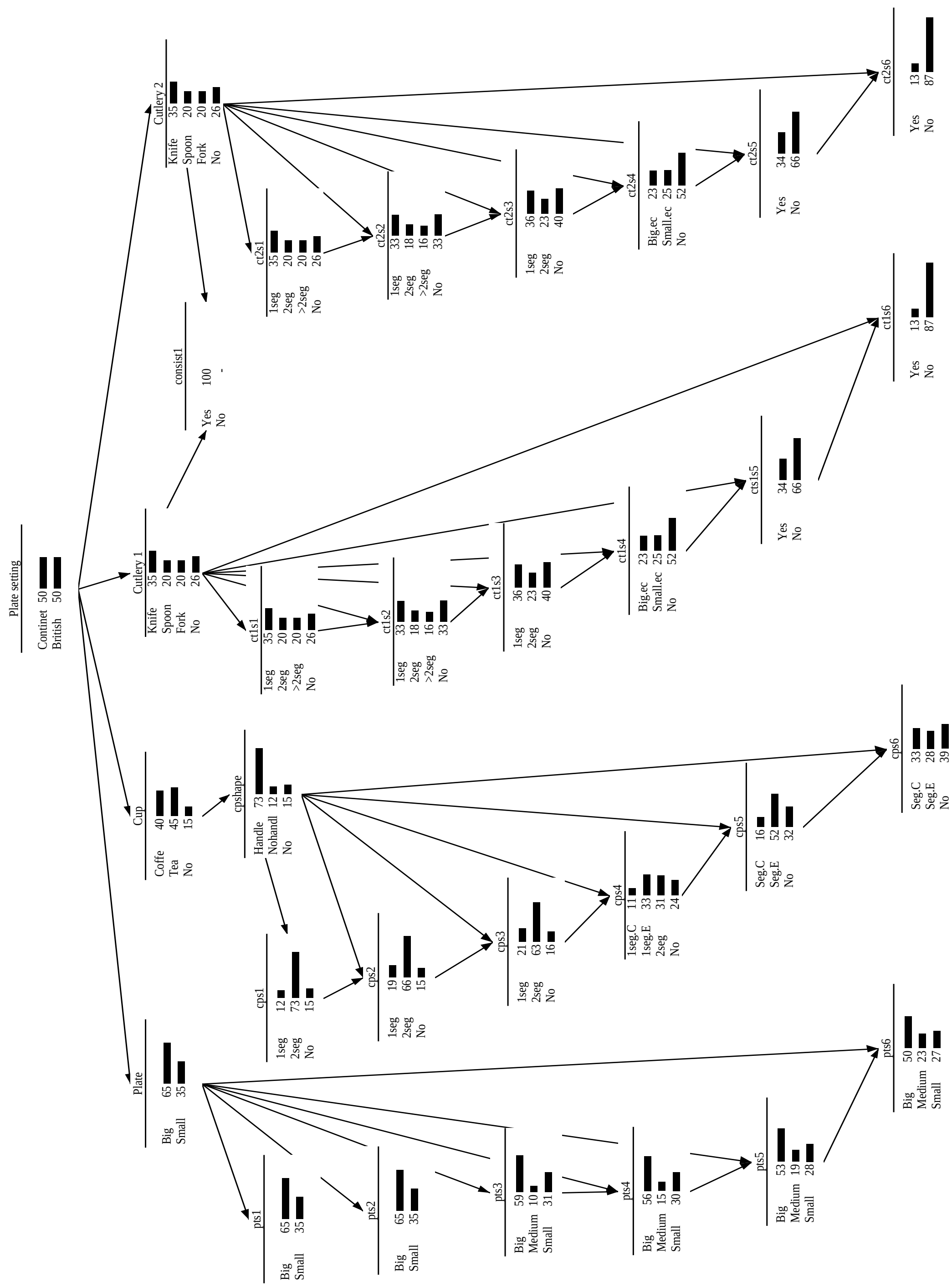

Figure 7: The model used by the interpreter

9 Production of findings

When the image has been segmented into regions which are described in terms of ellipses it is necessary to associate labels with each of the ellipses in order to facilitate matching into the CPN network. For the assignment of labels the area and eccentricity is used.

Based on a number of experiments a simple pattern recognizer has been constructed. It is here assumed that the measurement process may provide inaccurate results due to poor segmentation or image noise. To enable handling of this uncertainty both the area and the eccentricity parameters have associated confidence regions. For each of the possible labels two different matchings are possible: a *correct* match and a *possible* match. For the *correct* match it is required that both area and eccentricity is within a 10% limit of the "ideal" values. If one or both parameters differs more than 10% but less that 30% from the "ideal" values a *possible* match is indicated. For each of the image regions a match to the "ideal" values is performed and all possible matches are reported in a list which is ranked according to quality of match; i.e., the label which has the best fit with measured data is at the top while *possible* matches are at the bottom of the list.

10 Findings fitting

In findings fitting the segment labels are entered into the CPN as findings. This fitting process is not a trivial one-to-one matching task, as the image processor may provide several possible labels for a given segment and a given label may be entered at more than one node in the network.

An optimal fitting of findings into a CPN requires use of an iterative procedure where the entering of a new finding is associated with a re-evaluation of all previous findings, in order to facilitate back-tracking so that findings may have been entered incorrectly can be re-entered at new locations. This is a highly complicated process.

For these initial experiments a simple strategy has been chosen. The label which obtained the best fit to the prior defined "ideal" values is selected as the optimal label. If this label may fit into the CPN at multiple nodes the nodes which corresponds to the highest model probability receives the finding.

11 A session with a British plate setting

To illustrate how the ideas may work out, we give a session with a *British* plate setting (the one from figure 6).

The CPN module can request information in three different ways. It can at any level of resolution ask for the processing of

- an identified segment
- a circular region centered around an identified segment
- the entire picture

We shall go through a session where the levels are taken one by one.

Level 6 To start up, a processing of the entire picture at level 6 is performed (an introductory offer from the image processor). The result is two segments lying close to each other. One of them fits to *Medium* in **pts6** and the other fits to **Seg.C** in **cps6**. The resulting probabilities are shown in figure 8.

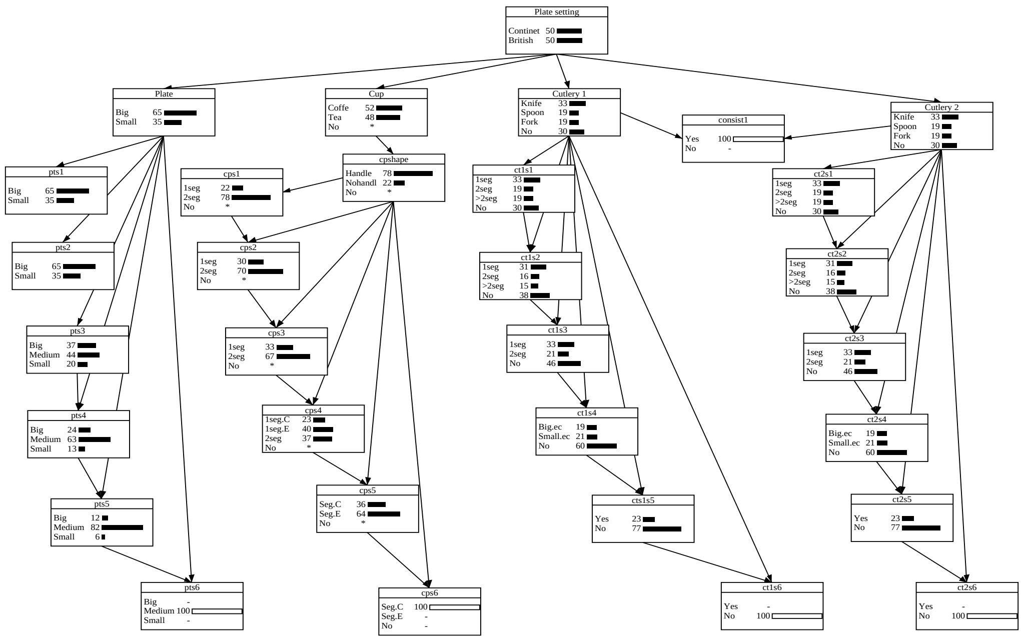

Figure 8: The probabilities after the first set of findings

Level 5 There are the following options for processing

- the plate segment
- the cup segment
- an area centered at the plate segment
- an area centered at the cup segment
- the entire picture
- nothing

To calculate the expected benefit from processing the plate segment we need $P(British|x, e)$ for each state of **pts5** (e is the evidence entered already. From now on we will skip the 'e'). These are obtained as described in section5 through a propagation of the simulated finding *British*. The result is shown in figure 9. The expected benefit is 63.

The expected benefit from processing the cup segment is calculated in the same way, and it is 201.

The calculation of the expected benefit from processing an area centered at one of the segments detected causes a couple of new challenges.

First of all, there is a chance that one or two cutlery segments may be discovered. However, as it is seen from the simulation of *British*, findings on the cutlery segments at level 5 have no impact on the hypothesis (see figures 8 and 9) and we need not address that problem here.

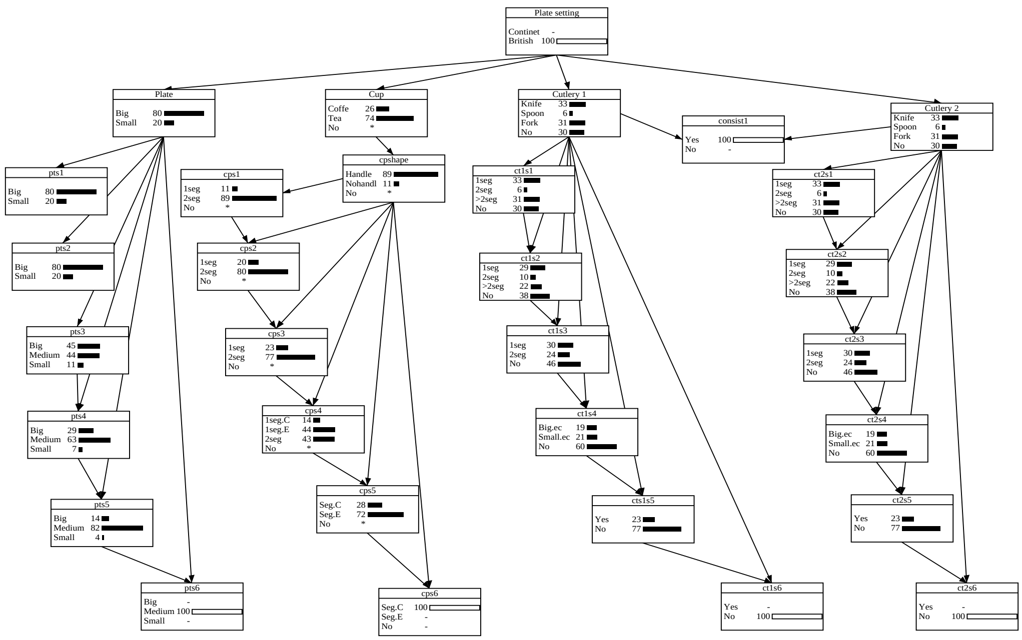

Figure 9: Probabilities after the initial findings and hypothesizing of *British*

Secondly, the processing will yield findings on two variables, **pts5** and **cps5**. In order to calculate the expected values, we need the probabilities $P(British|x,y)$. Bayes' theorem transforms it to a need for the joint probabilities $P(x,y|British)$ and $P(x,y)$.

Since

$$P(x,y) = P(x,y|Continental)P(Continental) + P(x,y|British)P(British) \quad (5)$$

the two conditional probabilities above are sufficient. There are ways in HUGIN to achieve them, but as yet they require too many propagations. In the present situation there is an easy way:

The plate variables and the cup variables are conditional independent given *British* and given *Continental*. So

$$\begin{aligned} P(x,y|British) &= P(x|British)P(y|British) && (6) \\ P(x,y|Continental) &= P(x|Continental)P(y|Continental) && (7) \end{aligned}$$

Using the formula

$$P(x) = P(x|Continental)P(Continental) + P(x|British)P(British) \quad (8)$$

we can obtain all required probabilities through a propagation of *British* only.

The expected benefit from processing the surroundings of any of the two segments is calculated to 315.

The interpreter offers $\frac{3}{4}$ of the expected benefit to the findings catcher. It is 48 for the plate segment, 150 for the cup segment and 237 for any surrounding.

For the image processor the costs are 45 for processing the plate segment, it is 24 for processing the cup segment, and it is 99 for the surroundings. The findings capturer decides to do a surrounding processing of the plate segment (The interpreter was not too clever here. It might have got a processing of both segments a bit cheaper).

The result is *Big* for **pts5**, *Seg.C* for **cps5** and *Yes* for **cts5**. The new probability are for *British* is 51, and it seems that the interpreter has wasted its money.

Level 4 The only variable which may have an impact on the hypothesis is **cps4**. The expected benefit is 519, the offer is 490, and the cost is 42. The image processor performs a processing of **cps4**, and the result is *1seg.E* (see figure 10.

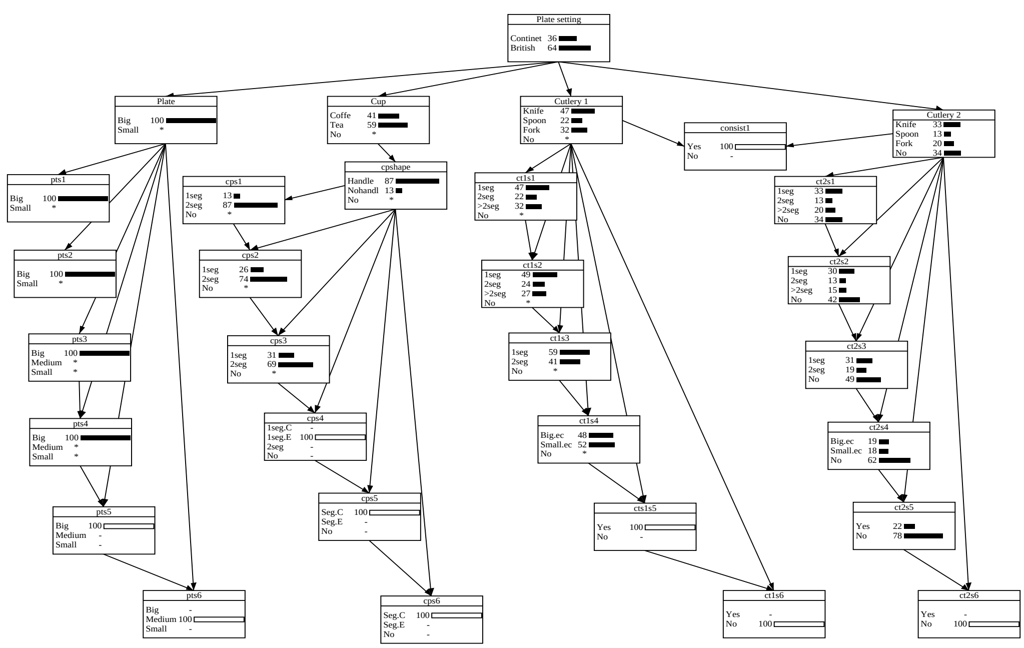

Figure 10: Probabilities after findings from level 4

Level 3 The expected benefit from processing **cps3** is 174 and for **cts3** it is 69 (see figure 11). Costs exceed the offers, so nothing is done at level 3.

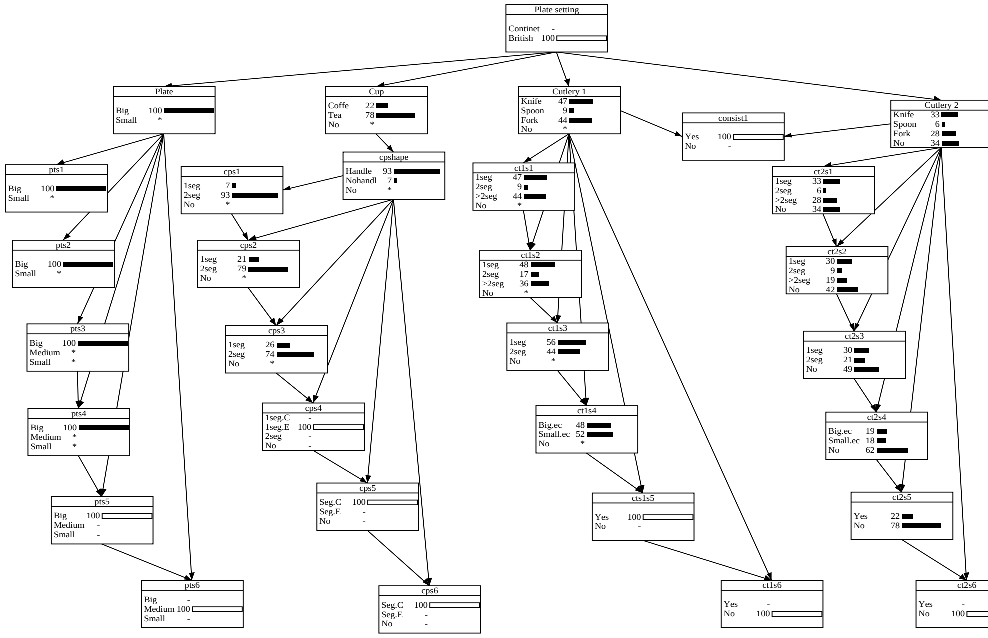

Figure 11: Probabilities after findings from level 4 and hypothesizing *British*

Level 2 The expected benefit from processing **cps2** is 135 and for **ct1s2** it is 753 (see figures 10 and 11). Costs are 414 and 171 respectively, but before we let the system decide, we should also consider a processing of the surroundings of the plate segment. Apart from a processing of **cps2** and **ct1s2** this may also discover a second cutlery segment.

The cup variables and the **cutlery1** variables are conditional independent given *British* and *Continental*, but this does not hold for the two sets of cutlery variables: If one piece of cutlery turns up to be a knife then the other must be a spoon or a fork and vice versa.

This means that only in the case of *1seg* for **ct1s2** will a surroundings processing give hope for a higher value then the one obtained by processing **ct1s2** and **cps2** separately. It is possible to calculate the appropriate probabilities. This will be based on an analysis of the concrete situation, though, and it can hardly be used as an offset for general automatic methods.

Instead we perform a calculation considering all variables to be conditional independent. The probabilities for the non-No states of **ct2s2** are multiplied by the probability of finding it close to the cup (0.4) and the probability for *No* is the complement hereof. The expected benefit is calculated to 879, and the cost is 2439.

Now, the rest is plain sailing. First **ct1s2** is processed yielding the finding *2seg*; next **cps2** gives an expected benefit of 10, so it is dropped. The expected benefit from processing the cutlery segment at level 1 is 1032 (offer 774), and the costs are 495. Processing of the cutlery segment at level 1 yields *2seg*, which results in $P(British) = 0.90$. A further analysis shows that it might be possible to achieve a probability of 0.92, but it does not pay.

Note that apart from finding the best sources of information at each level, the paradigm also ensures that it has not been necessary to predefine any stop values for the process. It stops by itself at a moment which is most beneficial for the particular situation.

12 Conclusion

The work presented goes in two directions. It is a study on how causal probabilistic networks and the new Bayesian computational methods can be used for modeling and knowledge updating in interpretation tasks, and it is a first attempt at operationalizing the control paradigm 'agents with instincts' in full vision systems.

We have shown that CPNs are very good for modeling scenes consisting of parts whose interpretation is context dependent, and this context dependence is fairly easy to represent as conditional probabilities. Having a good model, the updating of probabilities and the production of expectations for the various knowledge sources cause no problem.

We have also shown that precisely the ability to handle uncertainty in a coherent and efficient manner makes it possible to use expectations and simulated inputs from knowledge sources in planning of data requests, so that it is possible to estimate an expected benefit from consulting the possible knowledge sources. These estimates are easy to communicate to other agents (e.g. in the form of offers of payment for information) and therefore it fits neatly to the agents-with-instincts paradigm.

For both directions of the work we feel that we can conclude, that what we have discovered so far looks very promising, but still very much work and many new ideas are necessary before we have a proper, full and efficient system. To name a few major problems:

- We still need to find out how spatial relations are best modeled. Should they be modeled directly in the CPNs or is it rather a task for a findings fitter ?
- We have not touched upon temporal reasoning (e.g. changing scenes).
- Findings fitting (which really is the indexing problem) has not found a good solution.
- We need efficient ways of calculating the expected benefit from knowledge sources with multiple variables as well as methods for planning of sequences of data requests.
- We require all the dependencies between knowledge sources to be modeled in the CPN. What if they are not ?

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