ADAPT-Pricing: A Dynamic And Predictive Technique for Pricing to Maximize Revenue in Ridesharing Platforms

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ABSTRACT

Ridesharing platforms use dynamic pricing as a means to control the network's supply and demand at different locations and times (e.g., Lyft's Prime Time and Uber's Surge Pricing) to increase revenue. These algorithms only consider the network's *current* supply and demand only at a ride's *origin* to adjust the price of the ride. In this work, we show how we can increase the platform's revenue while lowering the prices as compared to state-of-the-art algorithms, by considering the network's *future* demand. Furthermore, we show if rather than setting the price of a ride only based on the supply and demand at its origin, we use predictive supply and demand at both the ride's *origin and destination*, we can further increase the platform's overall revenue. Using a real-world data set from New York City, we show our pricing method can increase the revenue by up to 15% while reducing the price of the rides by an average of 5%. Furthermore, we show that our methods are resilient to up to 25% error in future demand prediction.

CCS CONCEPTS

• **Information systems** → **Electronic commerce**; *Incentive schemes*;
• **Applied computing** → **Transportation**;

KEYWORDS

Ridesharing, Dynamic Pricing, Revenue Maximization, Price Optimization

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1 INTRODUCTION

The popularity of ridesharing platforms such as Lyft, Uber and Didi has grown them into multi-million dollar markets. Platform providers do not own the resources (i.e., cars) in these markets and hence, cannot directly control the supply (i.e., drivers). They can only encourage drivers to participate in the market by setting appropriate prices. However, the set prices not only impact the availability of the drivers, they also impact the willingness of the passengers to use the platform. Consequently, finding the optimal price to balance the supply and demand has a crucial impact on the performance of the market.

While ridesharing companies can simultaneously have multiple objectives, it goes without saying that revenue maximization is always among the primary goals. Due to its economic value, ridesharing companies do not publish the details of their pricing approaches. However, we know that they use a real-time dynamic algorithm that modifies the prices in different regions depending on the current supply and demand in that region [18, 22]. The set prices control the supply and demand which in turn, impact the total number of serviced trips at each time. Furthermore, the future geographical distribution of the drivers depends on the destination of the trips they are currently assigned to service. Consequently, how the platform provider sets prices in different regions impacts the spatial distribution of the drivers in the future. As a result, locally optimizing the prices at the current time does not necessarily yield to an overall higher revenue.

Recently, many studies have focused on dynamic pricing based on the market's future demand (i.e., predictive pricing) [4, 7, 14]. In the context of ridesharing markets, the problem is studied under certain demand patterns [4]. However, we know from real-world data that the demand pattern of the network can vary during a day [20]. For example, a region with a high demand in the morning rush hour might not necessarily have a high demand in the afternoon. Furthermore, demand prediction is a complex problem and can be erroneous [16, 17, 24]. Consequently, any pricing method which depends on the future demand of the network, must be resilient to inaccuracies in demand prediction.

We present A Dynamic And Predictive Technique for pricing (ADAPT-Pricing) where in addition to the network's current supply and demand, we also consider the predicted future demand. Passengers are more encouraged to use ridesharing platforms over other means of transportation when the prices are low. Therefore, in regions with an abundance of supply, lowering the prices can result in higher demand which in turn increases the number of trips. With predictive pricing (P-Pricing) the goal is to increase the revenue of the platform by increasing the number of trips originating and ending in lower and higher demand regions, respectively.

Changing the prices of every ride based only on the rides' origins, will affect all the trips originating from that location regardless of their destinations. In other words, not all the trips resulted from P-Pricing, would end up in high demand regions in the future. Consequently, the platform must set prices not just based on the origin of

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the ride, but based on the origin-destination (OD) pair for each ride. However, if rides in the same region have different prices, drivers are more attracted to service higher priced rides. Consequently, it is important that the platform providers prevent shortage of supply for rides with lower prices. Thus, we improve the performance of P-Pricing with a predictive origin-destination based pricing (POD-Pricing) method. The goal is to further increase the revenue by better controlling which trips get serviced and where the drivers end up in future.

We evaluate ADAPT-Pricing using a real-world dataset from New York City [20] and compare it with a baseline approach that optimizes the prices at each location only based on the current supply and demand at that location (origin). We show with P-Pricing and POD-Pricing, the generated revenue within each time period increases up to 5% and 15%, respectively. In addition, we show that the ride prices set by our approaches are on average 5% cheaper, i.e., the revenue growth is due to the increase in the number of rides, not higher prices. Furthermore, we show that our pricing methods outperform the baseline approach even with up to 25% error in demand prediction, showing the resilience of our ADAPT-Pricing techniques.

The remainder of the paper is organized as follows. In Section 2, we review the related literature. We formally model a ridesharing market and define the revenue maximization problem in Section 3. In Section 4, we introduce ADAPT-Pricing and present three different pricing methods; A baseline approach (Section 4.1), P-Pricing (Section 4.2) and POD-Pricing (Section 4.3). We present our experimental results in Section 5 and conclude the paper in Section 6.

2 RELATED WORK

There is a rich literature on the behavior of users in commercial sharing platforms. Recent studies have shown that the expected income largely impacts the participation of resource providers (i.e., drivers) [13, 15] and that cost is the most influential factor in their participation [12]. On the other hand, an empirical study on passengers shows that suddenly increasing prices (e.g., Uber's surge pricing) greatly decrease the network's demand [8]. We use the findings of these studies to define our model in Section 3.

Most studies on dynamic pricing in ridesharing markets focus on surge pricing as a means to increase participation of drivers in locations with high demand. Without surge pricing, drivers can end up driving longer distances to pick up passengers as there are no available drivers close to the passenger which results in inefficiency in the market [6]. On the other hand, it is shown that when surge pricing is in effect, it is less likely that the drivers will leave the platform and end their services [9]. Furthermore, with regard to revenue maximization, surge pricing outperforms fixed pricing [3, 5]. These studies only consider the implications of surge pricing at a single region at a particular time. They do not consider the effects on the drivers' movement in the network which determines the future supply in different regions. While [4] does consider the spatial structure of the network and future supply in setting prices, they do not account for fluctuations in future demand.

Other lines of research exist on revenue maximization in ridesharing platforms. Rather than focusing on dynamic pricing, they either optimize the process of matching passengers to drivers [2, 21] or provide pricing mechanisms for splitting the fares between drivers and the platform provider [1, 10]. In particular, in two of our own prior works [1, 2], we assumed the original price of each ride is given and didn't discuss how the original price is computed. The current paper complements our prior work by introducing techniques to find the optimal original price for the rides (which was assumed to be given in our previous studies). Consequently, these studies are orthogonal to our work as the price of a ride is known in advance and other aspects of the problem are optimized. Furthermore, many studies in recent years have focused on demand prediction in ridesharing platform [11, 22, 23]. While our work utilizes the predicted future demand to dynamically optimize prices, we do not focus on demand prediction itself.

3 PROBLEM DEFINITION

3.1 Model

We assume the entire space consists of n different equidistant regions. Also, we consider an infinite time horizon discretized into equisized time periods. Within each time period t, we assume for each region i there are a potential of $\mathcal{R}_i^t$ ride requests.

Every request r has an origin and a destination region (the origin and destination region can be the same). We assume there exists a function $f^r(p)$ which gives the probability of passengers willing to pay price p for their ride. We define the network's *demand* at location i at time t as:

$$D_i^t(p) = \mathcal{R}_i^t \cdot \left(1 - F^r(p)\right) \tag{1}$$

where $F^r(.)$ is the cumulative distribution function (cdf) of $f^r(.)$.

Similarly, during each time period t, there are a potential of $\mathcal{V}_i^t$ drivers available at location i. The probability of drivers participating in the platform for a certain price p is shown with $f^w(p)$. We define the network's *supply* at location i at time t as:

$$S_i^t(p) = \mathcal{V}_i^t \cdot F^w(p) \tag{2}$$

Intuitively, increasing the price of rides increases (/decreases) the chance of drivers (/passengers) participating in the platform. More formally we assume:

ASSUMPTION 1. *$F^r(.)$ and $F^w(.)$ are continuous and strictly increasing. In addition, similar to [13], we assume $F^r(.)$ is strictly convex.*

ASSUMPTION 2. *$F^r(0) = 1$ and $F^w(0) = 0$ and there exists a finite price p_{max} such that $F^r(p_{max}) = 0$ and $F^w(p_{max}) = 1$.*

PROPOSITION 1. *Assuming $\mathcal{R}_i^t > 0$ and $\mathcal{V}_i^t > 0$, there exists a price p^c at which $D_i^t(p^c) = S_i^t(p^c)$.*

We call p^c the *market clearing price*.[1]

PROOF. Proof of all Theorems, Lemmas and Propositions are presented in the Appendix. □

The total number of trips at location i at time t can be computed as:

$$T_i^t(p) = \min\{D_i^t(p), S_i^t(p)\} \tag{3}$$

[1] Hereafter, unless mentioned otherwise, we assume $\mathcal{R}_i^t > 0$ and $\mathcal{V}_i^t > 0$.

PROPOSITION 2. *If for location i at time t there exists a market clearing price, then* $p^c = \arg\max_p T_i^t(p)$.

We assume all trips starting at time period t will end at time period $t+1$. Therefore, the serviced trips during each time period, affect the supply of the network at different locations in the following time period. We assume for each time period there exists a *transition matrix* Θ^t such that the *ij*-th entry of the matrix, θ_{ij}^t, gives the fraction of ride requests at location i with destination location j at time t, we compute the number of potential drivers at each location at time $t+1$ as:

$$\mathcal{V}_j^{t+1} = \left(\sum_i \theta_{ij}^t \cdot T_i^t(p_i^t)\right) + \left(\mathcal{V}_j^t - T_j^t(p_j^t)\right) + \gamma_j^{t+1} \qquad (4)$$

The first term in Eq. (4) gives the drivers that serviced a request in time period t and ended up at location j in $t+1$. The second term refers to those drivers who were already at location j at t and did not service any request and remained at location j at $t+1$. Finally, in each time period, a number of drivers can enter/leave the system. γ_j^{t+1} is the number of drivers that logged in to the platform minus those who left.

3.2 The Revenue Maximization Problem

In our model, we assume passengers pay a fare equal to the price set by the platform provider (p). The provider keeps a portion (λ) of the fare as its own service fee and pays the remainder to the driver. Consequently, for a ride with price p, the share of the platform provider will be λp. The revenue generated at time t for location i is:

$$Rev_i^t(p_i^t) = T_i^t(p_i^t) \cdot \lambda \cdot p_i^t \qquad (5)$$

where p_i^t is the price set by the platform for rides originating at location i at time t.

Furthermore, the platform's total revenue would be:

$$TotalRev = \sum_t \sum_i Rev_i^t(p_i^t) \qquad (6)$$

The *Revenue Maximization Problem* is to determine optimal p_i^t's in order to maximize the generated revenue (Eq. (6)).

Earlier we mentioned that all n regions in the geographical space have the same pairwise distance to each other. We end this section with a discussion on the practicality of this assumption. The assumption of equidistant regions has two consequences that are relevant to our problem. Firstly, when every pair of regions have the same distance to each other, all the trips will have the same travel time and hence, every trip starting at time t will end at time $t+1$. Secondly, all trips originating from the same region will have the same price (unless we utilize POD-Pricing and consider the destination of the ride when setting the prices). Following we discuss the impact of these outcomes on the practicality of our pricing model.

As explained earlier, the basis of ADAPT-Pricing is setting prices for different trips such that in the following time periods there is enough supply to support the demand in different regions. While demand patterns in a location do change, the changes do not happen rapidly (similar to rush hour traffic that does not rapidly change, and thus most navigations, e.g., Waze, use the traffic pattern at the time of departure). Suppose our model assumes a certain location is in high demand in the next time period and adjusts the prices so more drivers are available in that location in the next time period. Even if some drivers take more than one time period to reach their destination, by the time they arrive, most possibly the demand is still high. In a real-world setting, the assumption of all trips having the same travel time affects the solution only on the edge cases of the demand pattern where the relative demand at a certain location makes a sudden shift. Furthermore, as we discuss in Sections 4.2 and 4.3, assuming all trips have similar travel times reduces the complexity of the optimization problems in ADAPT-Pricing by orders of magnitude. Finally, a main goal of this paper is to compare the results of our pricing model with existing studies (e.g., [4]) which use the similar assumption. However, to confirm the practicality of our model, we performed some experiments based on real-world travel times between different regions, and the difference with the results with similar travel times is only 1% (more details are presented in Section 5).

With regard to rides having similar prices, if trips have different distances and travel times, rather than computing the price of the entire trip, our model can use the same logic to compute the "price per unit of time/length" which makes trips with different lengths having different fares. In fact, in the experiments we performed on real-world travel times this is how we set prices for different rides.

4 ADAPT-PRICING

In this section, we present A Dynamic And Predictive Technique for pricing (ADAPT-Pricing). First, we show a baseline approach for dynamic pricing in which, the platform only considers the *current* supply and demand at the origin. Following, we discuss the predictive aspect of ADAPT-Pricing where both the current and future demand of the network are accounted for. We end the section with further optimizing ADAPT-Pricing by setting prices not only based on a ride's origin, but rather considering the origin-destination pair of each ride.

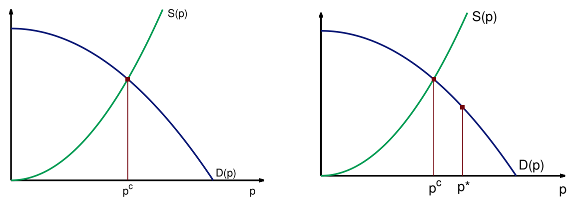
4.1 Baseline: Local Optimization

Ride-sharing applications (e.g., Uber, Lyft, etc.) use a real-time dynamic algorithm to determine the price of rides based on the network's supply and demand. This algorithm considers the *current* supply and demand in the network and when demand is higher than the available drivers, the prices are increased to encourage more drivers to participate in the platform [18, 22]. In this section, we discuss how the optimal price for location i at time t (${p^*}_i^t$) can be computed only considering $\mathcal{D}_i^t(.)$ and $\mathcal{S}_i^t(.)$.

Figure 1(a) shows how the network's supply and demand (y-axis) changes as a function of the price (x-axis) based on the model in Section 3. As depicted, at $p = 0$ we have $\mathcal{S}(p) = 0$ and $\mathcal{D}(p) > 0$ (Assumption 2). As p increases, $\mathcal{S}(p)$ and $\mathcal{D}(p)$ keep increasing and decreasing, respectively (Assumption 1). Finally, where $p = p_c$ the network's supply and demand are equal (Proposition 1)

During each time period, for each of the locations, the network's supply and demand can be modeled similar to Fig. 1(a). Since the platform only considers the supply and demand at the origin location of the ride, the revenue maximization problem reduces to *locally* maximizing $Rev_i^t()$ from Eq. (5) for every location i at time period t.[2]

[2]Hereafter, for simplicity, when the region and time period can be inferred from the context, indices i and t are dropped.



(a) (b)

Figure 1: Supply and Demand at Origin

THEOREM 1. *The optimal price p^* which maximizes $Rev(p)$ from Eq.* (5) *is always greater or equal to p^c.*

For price $p \geq p^c$, we have $\mathcal{S}(p) \geq \mathcal{D}(p)$ and hence, the network's demand becomes the dominant factor in deciding the number of serviced trips (Eq. (3)). Therefore:

$$\forall p \geq p^c \quad Rev(p) = \mathcal{D}(p) \cdot \lambda \cdot p \tag{7}$$

Assuming p^d is the price that maximizes $\mathcal{D}(p) \cdot p$, Theorem 2 gives the optimal price for maximizing $Rev(p)$ in Eq. (5):

THEOREM 2. *The optimal price p^* for maximizing the revenue at each location at each time is:*

(i) $p^* = p^c$ *if* $p^d \leq p^c$
(ii) $p^* = p^d$ *if* $p^d > p^c$

When only considering the current demand and supply of the network, the platform provider sets the price at location i at time t to the optimal price ${p^*}_i^t$. Consequently, the total revenue generated for the platform is:

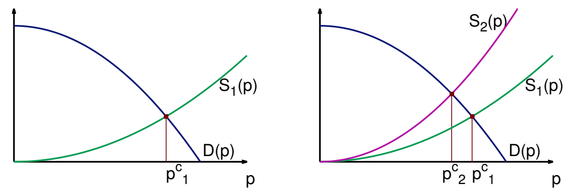
$$TotalRev = \sum_t \sum_i Rev_i^t({p^*}_i^t) \tag{8}$$

4.2 P-Pricing: Predicting Demand at Origin

In recent years, there has been an increasing effort to predict the demand of ride-sharing networks both from academia [16] and the industry [17, 24]. In this section, we focus on utilizing the network's future demand in order to maximize its overall revenue.

With the baseline approach Section 4.1, any location where $p^c < p^*$, for any price p where $p^c \leq p < p^*$, we have $T(p) > T(p^*)$ (Fig. 1(b)). That is, even though setting the price to p will lower the generated revenue (Theorem 2), it will increase the number of serviced trips. Furthermore, we know from Eq. (4) that the number of serviced trips at the current time affects how the drivers are distributed among different locations in the following time period. The basic idea of *predictive pricing* (P-Pricing) is to increase the overall revenue by setting the price at some locations lower than the optimal price in order to increase the number of serviced trips and thus, potentially have more drivers at locations with high demand in the future. Before we formalize the optimization problem for P-Pricing, we need to discuss how the increased number of serviced trips in the current time period, affects the network's supply in the following time period.

Figure 2 shows how supply changes at the destination regions. We assume $\mathcal{S}_1(p)$ gives the network's supply if the price of rides at every location are set to p^* in the previous time period (Fig. 2(a)). In Fig. 2(b) we consider a second scenario where the number of serviced trips in the previous time period are increased at certain locations (by lowering the prices) and as a result, in the current time period we end up with more potential drivers at the destination of those added trips. $\mathcal{S}_2(p)$ in Fig. 2(b) shows the supply at the destination locations in the second scenario.



(a) (b)

Figure 2: Supply and Demand at Destination

LEMMA 1. *If p_1^c and p_2^c are the market clearing prices for $\mathcal{S}_1(.)$ and $\mathcal{S}_2(.)$, respectively and*

$$p^d = \arg\max_p \{\mathcal{D}(p) \cdot p\}$$

the generated revenue from $\mathcal{S}_2(.)$ is larger than that of $\mathcal{S}_1(.)$ if $p_2^c < p_1^c \wedge p^d < p_1^c \wedge p^d \leq p_2^c$.

LEMMA 2. *Assuming $p_2^c = p^d$, adding more potential drivers does not increase the generated revenue.*

Compared to setting all the prices to p^*, lowering the prices at time t reduces the generated revenue at time t. However, it also increases the number of trips at certain locations which can add more supply to high demand regions in the future and thus, increase the generated revenue at $t + 1$. The question that needs to be answered is: *"How much and at which locations should we sacrifice revenue in order to increase the generated revenue in the following time period?"*. To answer this question, first we have to compute the revenue decrease in the current time period as a function of the added trips (due to lowering the prices) and the corresponding revenue increase in the following time period as a function of the added potential drivers.

We start with computing the revenue decrease in the current time period:

$$\begin{aligned} RevDec(\delta T) &= T(p^*) \cdot p^* - \big(T(p^*) + \delta T\big) \cdot \big(p^* - \delta p\big) \\ &= T(p^*) \cdot \delta p + \delta T \cdot \delta p - \delta T \cdot p^* \end{aligned} \tag{9}$$

where,

$$\delta p = p^* - \mathcal{D}^{-1}(T(p^*) + \delta T) \tag{10}$$

$$\delta p < p^* - p^d \tag{11}$$

Similarly, we can compute the revenue increase as a function of the number of added potential drivers as:

$$RevInc = T(p_2^c) \cdot p_2^c - T(p_1^c) \cdot p_1^c \tag{12}$$

Since p_1^c and p_2^c are market clearing prices for $\mathcal{S}_1(.)$ and $\mathcal{S}_2(.)$ respectively, we can re-write Eq. (12) as:

$$RevInc = \mathcal{S}_2(p_2^c) \cdot p_2^c - \mathcal{S}_1(p_1^c) \cdot p_1^c \tag{13}$$

Assuming $\mathcal{S}_1(p) = \mathcal{V} \cdot F^w(p)$, we can compute the revenue increase as a function of the *added* potential vehicles ($\delta\mathcal{V}$) as:

$$RevInc(\delta\mathcal{V}) = (\mathcal{V} + \delta\mathcal{V}) \cdot F^w(p_2^c) \cdot p_2^c - \mathcal{V} \cdot F^w(p_1^c) \cdot p_1^c \tag{14}$$

where $p_2^c \geq p^d$.

We can find the optimal number of trip increases in the current time period by solving the following optimization problem:

$$\begin{aligned} \textbf{maximize} & \sum_j RevInc(\delta\mathcal{V}_j^{t+1}) - \sum_i RevDec(\delta T_i^t) \\ \textbf{subject to} & \quad \Delta T^t \cdot \Theta^t = \Delta\mathcal{V}^{t+1} \\ & \quad \delta T_i^t \geq 0 \qquad \forall i \\ & \quad \delta\mathcal{V}_j^t \geq 0 \qquad \forall j \end{aligned} \tag{15}$$

where,

- Θ^t is the transition matrix at time period t.
- $\Delta T^t = \langle \delta T_1^t, \delta T_2^t, \cdots, \delta T_n^t \rangle$
- $\Delta\mathcal{V}^{t+1} = \langle \delta\mathcal{V}_1^{t+1}, \delta\mathcal{V}_2^{t+1}, \cdots, \delta\mathcal{V}_n^{t+1} \rangle$

Algorithm 1 summarizes the procedure to compute the optimal prices for every region i at time t.

Algorithm 1 P-Pricing($\mathcal{R}_i^t, \mathcal{V}_i^t, \mathcal{R}_j^{t+1}$)

Input: $\mathcal{R}_i^t, \mathcal{V}_i^t$ as the number of requests and available drivers at time t and $\mathcal{R}_j^{t+1}$ as the predicted demand at time $t+1$.

Output: ${p^*}_i^t$ as the optimal prices at time t

1: **for** $i = 1 : n$ **do**
2: $\quad D_i^t(p) = \mathcal{R}_i^t \cdot (1 - F^r(p))$
3: $\quad S_i^t(p) = \mathcal{V}_i^t \cdot F^w(p)$
4: $\quad p_i^t =$ **LocalOptimal**$(D_i^t(p), S_i^t(p))$
5: $\quad$ Compute $RevDec(\delta T_i^t)$ using Eq. (9)
6: **end for**
7: **for** $j = 1 : n$ **do**
8: $\quad D_j^{t+1}(p) = \mathcal{R}_j^{t+1} \cdot (1 - F^r(p))$
9: $\quad$ Compute $\mathcal{V}_j^{t+1}(p_i^t)$ using Eq. (4)
10: $\quad S_j^{t+1}(p) = \mathcal{V}_j^{t+1} \cdot F^w(p)$
11: $\quad p_j^{t+1} =$ **LocalOptimal**$(D_j^{t+1}(p), S_j^{t+1}(p))$
12: $\quad$ Compute $RevInc(\delta V_j^{t+1})$ using Eq. (14)
13: **end for**
14: Solve Eq. (15) and find optimal ΔT^t
15: **for** i 1:n **do**
16: $\quad \delta p_i^t = p_i^t - D_i^{t\,Inv}(D_i^t(p_i^t) - \delta T_i^t)$
17: $\quad {p^*}_i^t = p_i^t - \delta p_i^t$
18: **end for**
19: **return** ${p^*}_i^t$

Algorithm 1 starts with computing the demand and supply of every region i at time t and their local optimal prices using the current demand and supply (lines 1-6). The **LocalOptimal** method on line 4 (and later on line 11) refers to the local optimization method explained in Section 4.1 which uses the current demand and supply in a single region to compute the locally optimal price. Using the locally optimal prices at time t, we can compute the available drivers in each region at time $t+1$ (line 9). Using the available drivers and the predicted demand in each region at time $t+1$, we compute the supply (line 10) and demand (line 8) for each region at time $t+1$. Consequently, we can compute locally optimal prices for each region at time $t+1$ (line 11). Having all this information we compute $RevDec(\delta T_i^t)$ and $RevInc(\delta V_j^{t+1})$ and solve the optimization problem (line 14). Having computed the optimal δT_i^t's from the optimization problem, we can compute the optimal prices for each region at time t (lines 15-18). The method $D_i^{t\,Inv}(.)$ on line 16 is the inverse of $D_i^t(.)$.

With P-Pricing, at time t, even though we consider the *potential* revenue increase at time $t+1$, we only set the prices for time t. Prices for rides at $t+1$ are set at $t+1$ when P-Pricing can partially sacrifice the potential revenues increase computed before, in order to gain even more revenue in the future. Ideally, one would extend the optimization problem in Eq. (15) to perform a global optimization over all time periods. However, many taxi demand prediction studies [11, 23] use markov models where only the demand of one time period ahead is predicted every time. As a result, in addition to the complexity of a global optimization, the current practices in predicting future demand only allows us to perform a one period ahead optimization.

With regard to the complexity of solving the optimization problem in Eq. (15), based on Assumption 1, *RevDec* is always convex and *RevInc* is concave. Consequently, the optimization function becomes convex and can be solved efficiently (proportional to n^3, where n is the number of regions).

4.3 POD-Pricing: Predicting Demand at Origin & Destination

P-Pricing sets a similar price for all rides originating from the same location, regardless of their destination. As a result, not all the drivers servicing the increased trips will end up at a high demand location in the following time period.

Figure 3 shows a simple scenario with three locations. Suppose we know in the following time period l_2 will have a high demand while the demands at l_1 and l_3 will be low. Based on P-Pricing, we decide to lower the prices at l_1 so 10 more trips will be serviced hoping that this will add more potential drivers at l_2 in the following time period. However, this will also lower the prices for rides going from l_1 to l_3. Consequently, out of the 10 added trips, on average 5 of those will end up at l_2 ($\theta_{12} = 0.5$) and the other 5 will end up at l_3. Ideally, we should only lower the prices for those rides going to l_2 so all the 10 drivers servicing the added trips will end up at l_2 in the following time period.

In this section we show how we can set prices for rides not only based on their origin location, but based on their origin-destination pair. With predictive origin-destination pricing (POD-Pricing), once the platform sets different prices for rides originating from the same location, all the potential drivers at that location will try to service rides with higher prices. However, as shown in the example from

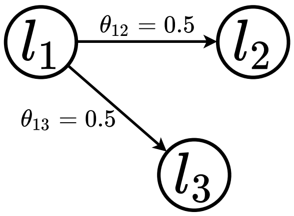


Figure 3: Example of Network's Transition

Fig. 3, trips to high demand locations will have lower prices. To overcome this problem, for a ride request r at location i, rather than notifying every driver at i, we only notify a subset of drivers based on the destination of r. In other words, we divide the available drivers at each location into disjoint sets and use the drivers of each subset to service rides for a specific destination.

We assume β_{ij} gives the fraction of drivers at location i that the platform decides to allocate to service the rides with destination j such that $\sum_j \beta_{ij} = 1$. Then we can re-write Eq. (3) and Eq. (5) as:

$$\begin{aligned} T^t_{ij}(\beta^t_{ij}, p^t_{ij}) &= \min\{\mathcal{D}^t_{ij}(p^t_{ij}), \mathcal{S}^t_{ij}(p^t_{ij})\} \\ &= \min\{\beta^t_{ij}\mathcal{V}^t_i F^w(p^t_{ij}), \theta^t_{ij}\mathcal{R}^t_i F^r(p^t_{ij})\} \end{aligned} \tag{16}$$

$$Rev^t_i(\mathbf{B}^t_i, \mathbf{P}^t_i) = \sum_j T^t_{ij}(\beta^t_{ij}, p^t_{ij}) \cdot \lambda \cdot p^t_{ij} \tag{17}$$

where,

- $\mathbf{B}^t_i = \langle \beta^t_{i1}, \beta^t_{i2}, \cdots, \beta^t_{in} \rangle$.
- $\mathbf{P}^t_i = \langle p^t_{i1}, p^t_{i2}, \cdots, p^t_{in} \rangle$.

PROPOSITION 3. *The generated revenue from Eq.* (17) *is at least as much as that of Eq.* (5)*.*

Similar to P-Pricing, at each point in time we optimize the prices only for the current time period. In addition to the prices, we also need to find optimal β_{ij}'s. Consequently, the optimization problem can be written as:

$$\begin{aligned} \textbf{maximize} \sum_j RevInc(\delta\mathcal{V}^{t+1}_j) - \sum_i \sum_j RevDec(\beta^t_{ij}, \delta T^t_{ij}) \\ \textbf{subject to} \quad \sum_i \delta T^t_{ij} = \delta\mathcal{V}^{t+1}_j \quad & \forall j \\ \sum_j \beta^t_{ij} = 1 \quad & \forall i \\ \delta T^t_{ij} \geq 0 \quad & \forall i, j \\ \delta\mathcal{V}^t_j \geq 0 \quad & \forall j \\ \delta\beta^t_{ij} \geq 0 \quad & \forall i, j \end{aligned} \tag{18}$$

Using Eq. (18) and an algorithm similar to Algorithm 1, we can compute the optimal prices for every origin-destination pair at time t (${p^*}^t_{ij}$).

5 EXPERIMENTS

5.1 Dataset

We evaluate our methods using one month (May, 2013) of New York City's Taxi and Limousine Commission (TLC) dataset [20], which contains 39,437 drivers and around 500,000 trips per day. Each trip in the dataset has a pick-up latitude/longitude, a drop-off latitude/longitude and request time. We consider 1 hour long time periods in our experiments and map each ride to a time period based on its request time. Furthermore, we consider each precinct in the city [19] as a unique location and mapped each ride's pick-up and drop-off points to an origin and destination location, respectively.

5.2 Experimental Methodology

We measure the overall revenue generated by each ADAPT-Pricing method (Section 4). Additionally, we compare the average price set for the rides by each approach. We assume the total number of potential drivers follows the same pattern as the total number of ride requests during different time periods. In other works, the ratio of the total number of potential drivers to the total number of ride requests, showed by ρ, is approximately the same during a day. Furthermore, the drivers are uniformly distributed in different locations in the first time period. For the following time periods, the distribution of the drivers can be computed using Eq. (4).

As discussed in Section 4, with both P-Pricing and POD-Pricing, the optimization problem requires the future demand of the network at different locations. In our experiments, we measure the sensitivity of ADAPT-Pricing to the accuracy of the predicted future demand. We know the exact number of potential ride requests at each region i at any time t ($\mathcal{R}^t_i$), from our dataset. An accuracy of 100% means that the exact $\mathcal{R}^t_i$'s are used to compute the demand (Eq. (1)) which in turn are used in the optimization problems in Eqs. (15) and (18). Furthermore, in the experiments when the accuracy is set to α, assuming that $\mathcal{R}^t_i$ is the exact number of ride request at region i at time t, we compute the predicted number of ride request at region i at time t, $\hat{\mathcal{R}}^t_i$, as:

$$\begin{aligned} \epsilon^t_i &= (1 - \alpha) \cdot \mathcal{R}^t_i \\ \hat{\mathcal{R}}^t_i &= Random(\mathcal{R}^t_i - \epsilon^t_i, \mathcal{R}^t_i + \epsilon^t_i) \end{aligned}$$

where $Random(a, b)$ chooses a random number in the range $[a, b]$.

Table 1 shows the different values we used for the parameters in our experiments.

Parameter	Max-Min (increment)	default
Prediction Accuracy	100%-50% (5%)	100%
ρ	5.0-0.5 (0.5)	2.5

Table 1: Parameters for Pricing Method Comparison

In addition, p_{max} (from Assumption 2), $F^r(.)$ and $F^w(.)$ are set as:

$$p_{max} = \$10 \qquad F^r(p) = F^w(p) = \frac{p^2}{100}$$

We run our experiments for every day in May 2013 and report the average values over all days.

The experiments were run on a single machine equipped with 4th Generation Intel Core i7 processor quad-core [4.0GHz, 8MB Shared Cache] and 12GB DDR3-1600 DIMM RAM on Windows 10 and the methods were implemented using Java 8.

5.3 Pricing Method Comparison

Our first set of results compare the generated revenue and the average price of the rides for all three pricing methods (Table 2). The revenue increase and price discounts in Table 2 are in comparison to the baseline approach. As shown, with P-Pricing and POD-Pricing, we observe a 2.8% and 10.3% increase in revenue, respectively. Furthermore, the prices are lowered which confirms that the revenue increase is the result of increased number of trips and not higher prices.

	Baseline	P-Pricing	POD-Pricing
Revenue	2013341	2069606	2220335
Rev. Increase		2.8%	10.3%
Avg. Price	6.38	6.05	6.08
Price Discount		5.26%	4.6%

Table 2: Overall Results of Method Comparison

The main idea of ADAPT-Pricing is to partially sacrifice revenue in earlier time periods if it results in higher gains in future. Fig. 4 compares the revenue increase of P-Pricing and POD-Pricing as compared to the baseline approach in each time period. As depicted, in the first few time periods (until 6:00am), neither of the pricing methods do much better than the baseline and in fact in some cases they can be slightly worse than the baseline. However, in later time periods both methods outperform the baseline approach.

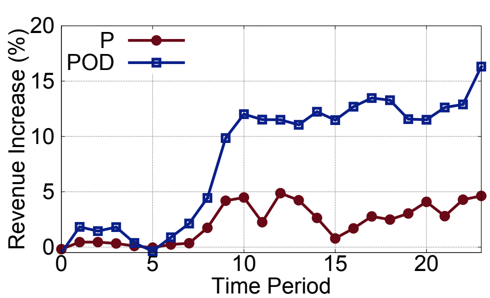


Figure 4: Revenue Increase

Fig. 5 depicts the average price discounts and the percentage of affected regions that encountered a discount in different time periods. As observed, more than 80% of regions, on average, received a 4% discount. While Fig. 5 shows the average price discount across all regions, Fig. 6 shows the price discounts in different origin locations (x-axis) for two time periods. Table 3 summarizes those results as the percentage of regions with less than 5%, 5% − 10% and more than 10% price discount, respectively. As observed in Table 3, in both time periods, more than 10% of regions observe a discount that is higher than 10%.

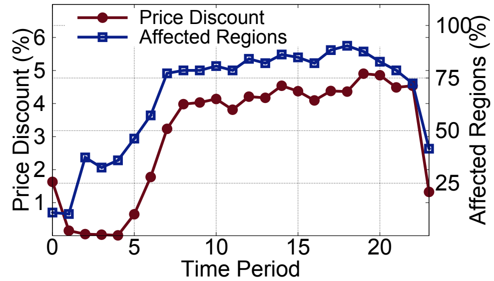


Figure 5: Price Discount

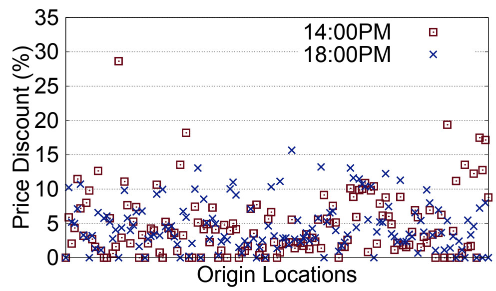


Figure 6: Price Discount per Region

	< 5%	5% − 10%	> 10%
14:00PM	62.76%	24.83%	12.41%
18:00PM	62.07%	25.52%	12.41%

Table 3: Price Discount per Region

Our pricing methods depend on predicting the network's demand in the near future. Accurately predicting the demand is a complex task by itself. In the next set of experiments, we analyzed the dependency of our pricing methods on the accuracy of the demand prediction (Fig. 7). As depicted, with POD-Pricing, even with an 80% accuracy, we still observe of 5% increase in revenue. One interesting observation is that P-Pricing is more resilient to prediction errors as compared to POD-Pricing. When we make a bad decision based on erroneous demand prediction, some of the negative effects are masked by the fact that with P-Pricing the drivers do not always take those rides that we want them to take (as explained in Section 4.3).

We also we evaluated the performance of our pricing methods for various supply/demand ratios (referred to as ρ)(Fig. 8). The parameter ρ is based on the total number of potential drivers and the total number of ride requests in the entire network. The supply to demand ratio in each location is not necessarily the same. For smaller values of ρ, due to limited supply, most locations will have a relatively higher market clearing price. According to Theorem 2, it is likely that the optimal price in the baseline approach is very close to the market clearing price. As a result, lowering the price would no longer increase the number of trips and hence, predictive pricing will not have a huge advantage. On the other hand, for larger values of ρ, due to the abundance of drivers, most high-demand

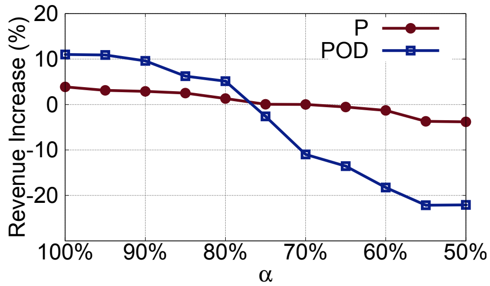


Figure 7: Prediction Accuracy

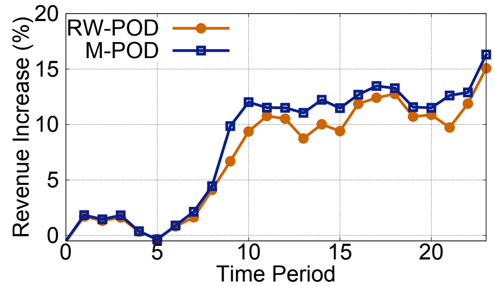


Figure 9: Revenue Increase with Real-world Travel Times

locations will always have enough potential drivers regardless of whether it is predicted or not. Consequently, after the optimal point, the advantage of predictive pricing reduces as ρ increases.

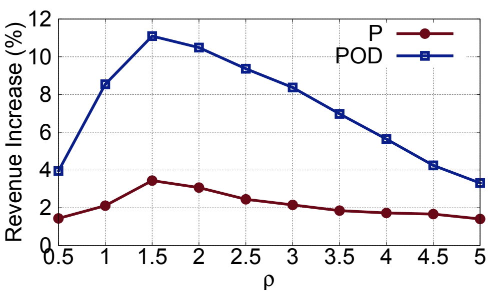


Figure 8: Supply/Demand Ratio

In our model, we assume all regions have the same distance to each other. At the end of Section 3 we provide a supporting argument for this assumption from both a practical and an analytical point of view. To further support the arguments in Section 3, in our last set of experiments, we show that the effect of this assumption is negligible in real-world scenarios. We compute the price for each origin/destination pair using the same method as before. However, for the real-world scenario, we no longer assume that all the trips are completed within one time period and consider the real travel time for each trip. Furthermore, in the real-world scenario, we consider the computed price as the *price per unit of time* for each trip. Consequently, the actual fare for each ride will be the computed price multiplied by the travel time of the trip. As observed in Table 4, even when considering real-worlds travel times, POD-Pricing increases the generated revenue by 9%.

	Baseline	POD-Pricing
Revenue	6043028	6591735
Rev. Increase		9.08%

Table 4: Overall Results with Real-world Travel Times

Furthermore, Fig. 9 depicts the revenue increase within each time period for both real-world travel times (RW-POD) and one time period travel times (M-POD) when using POD-Pricing. As depicted, in most time periods the revenue increase remains the same when considering real-world travel times. As discussed in Section 3, the only case where our model does not perform optimally with real-world travel times is when there is a sudden change in the demand patterns of the regions. It can be observed in Fig. 9 that in the early afternoon and late night when the potential ride requests increase and decrease respectively, the revenue increase with RW-POD decreases by a few points.

6 CONCLUSION & FUTURE WORK

In this paper, we presented ADAPT-Pricing as a dynamic pricing method in ridesharing markets which (1) utilizes the network's predicted future demand and (2) rather than setting prices based on the rides' origins, it sets prices based on the origin-destination pairs of the rides. We introduce P-Pricing and POD-Pricing as two aspects of ADAPT-Pricing and show that compared to a baseline approach they increase the generated revenue in each time period by up to 5% and 15%, respectively, while reducing the price of the rides by 5%. In the multi-million dollar ridesharing market, even a 5% increase accounts for hundreds of thousands of dollars of revenue per day.

Our models were designed based on the fact that completing a task involves workers moving from a starting point to an ending point. However, there are many other applications of spatial crowdsourcing with *instantaneous* tasks that do not require moving between and origin and destination to complete a task. For example, a requester might ask available workers for a picture of a famous statue. Even though our models are guaranteed to not perform worse than alternative approaches, it is not clear if we observe the same level of improvements. As more data from spatial crowdsourcing platforms with instantaneous tasks become publicly available, our models can be further tested and if required improved based on behaviors that are specific to these types of platforms.

7 ACKNOWLEDGEMENT

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REFERENCES

[1] Mohammad Asghari and Cyrus Shahabi. An on-line truthful and individually rational pricing mechanism for ride-sharing. In *Proceedings of the 25th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems*, SIGSPATIAL'17, pages 7:1–7:10, 2017.

[2] Mohammad Asghari, Dingxiong Deng, Cyrus Shahabi, Ugur Demiryurek, and Yaguang Li. Price-aware real-time ride-sharing at scale: An auction-based approach. In *Proceedings of the 24th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems*, GIS '16, pages 3:1–3:10, 2016.

[3] Siddhartha Banerjee, Ramesh Johari, and Carlos Riquelme. Pricing in ride-sharing platforms: A queueing-theoretic approach. In *Proceedings of the Sixteenth ACM Conference on Economics and Computation*, EC '15, pages 639–639, 2015.

[4] Kostas Bimpikis, Ozan Candogan, and Daniela Saban. Spatial pricing in ride-sharing networks. https://papers.ssrn.com/abstract=2868080, 2016.

[5] Gérard P. Cachon, Kaitlin M. Daniels, and Ruben Lobel. The role of surge pricing on a service platform with self-scheduling capacity. *Manufacturing & Service Operations Management*, 19(3):368–384, 2017.

[6] Juan Castillo, Daniel T. Knoepfle, and E. Glen Weyl. Surge pricing solves the wild goose chase. https://papers.ssrn.com/abstract=2890666, 2017.

[7] Yiwei Chen and Ming Hu. Pricing and matching with forward-looking buyers and sellers. https://papers.ssrn.com/abstract=2859864, 2017.

[8] Le Chen, Alan Mislove, and Christo Wilson. Peeking beneath the hood of uber. In *Proceedings of the 2015 Internet Measurement Conference*, IMC '15, pages 495–508, 2015.

[9] M. Keith Chen. Dynamic pricing in a labor market: Surge pricing and flexible work on the uber platform. In *Proceedings of the 2016 ACM Conference on Economics and Computation*, EC '16, pages 455–455, 2016.

[10] Shih-Fen Cheng, Duc Thien Nguyen, and Hoong Chuin Lau. Mechanisms for arranging ride sharing and fare splitting for last-mile travel demands. In *Proceedings of the 2014 International Conference on Autonomous Agents and Multi-agent Systems*, AAMAS '14, pages 1505–1506, 2014.

[11] Meng-Fen Chiang, Tuan-Anh Hoang, and Ee-Peng Lim. Where are the passengers?: A grid-based gaussian mixture model for taxi bookings. In *Proceedings of the 23rd SIGSPATIAL International Conference on Advances in Geographic Information Systems*, SIGSPATIAL '15, pages 32:1–32:10, 2015.

[12] Liran Einav, Chiara Farronato, and Jonathan Levin. Peer-to-peer markets. Technical report, National Bureau of Economic Research, August 2015.

[13] Zhixuan Fang, Longbo Huang, and Adam Wierman. Prices and subsidies in the sharing economy. In *Proceedings of the 26th International Conference on World Wide Web*, WWW '17, pages 53–62, 2017.

[14] Harish Guda and Upender Subramanian. Strategic surge pricing on on-demand service platforms. https://papers.ssrn.com/abstract=2895227, 2017.

[15] Juho Hamari, Mimmi Sjöklint, and Antti Ukkonen. The sharing economy: Why people participate in collaborative consumption. *J. Assoc. Inf. Sci. Technol.*, 67(9):2047–2059, September 2016.

[16] Ramón Iglesias, Federico Rossi, Kevin Wang, David Hallac, Jure Leskovec, and Marco Pavone. Data-driven model predictive control of autonomous mobility-on-demand systems. In *Proceedings of 2018 IEEE International Conference on Robotics and Automation*, ICRA '18, 2017.

[17] Nikolay Laptev, Jason Yosinski, Li Erran Li, and Slawek Smyl. Time-series extreme event forecasting with neural networks at uber. In *Proceedings of the 34th International Conference on Machine Learning*, ICML '17, 2017.

[18] Prime time for drivers. https://help.lyft.com/hc/en-us/articles/115012926467. Accessed: 2018-01-15.

[19] New york city neighborhoods. http://www1.nyc.gov/site/planning/data-maps/open-data.page#other. Accessed: 2017-11-30.

[20] New york city taxi trips. http://www.nyc.gov/html/tlc/html/about/trip_record_data.shtml. Accessed: 2017-03-30.

[21] Erhun Ozkan and Amy Ward. Dynamic matching for real-time ridesharing. 2017. https://papers.ssrn.com/abstract=2844451.

[22] Engineering extreme event forecasting at uber with recurrent neural networks. https://eng.uber.com/neural-networks/. Accessed: 2018-02-21.

[23] K. Zhao, D. Khryashchev, J. Freire, C. Silva, and H. Vo. Predicting taxi demand at high spatial resolution: Approaching the limit of predictability. In *2016 IEEE International Conference on Big Data*, BigData '16, pages 833–842, Dec 2016.

[24] Lingxue Zhu and Nikolay Laptev. Deep and confident prediction for time series at uber. In *2017 IEEE International Conference on Data Mining Workshops*, ICDMW '17, pages 103–110, 2017.

APPENDICES

.1 Proof of Theorems, Propositions & Lemmas

PROPOSITION 1. *Assuming $\mathcal{R}_i^t > 0$ and $\mathcal{V}_i^t > 0$, there exists a price p^c at which $D_i^t(p^c) = S_i^t(p^c)$.*

PROOF. We know from Assumption 2 that $\mathcal{S}(0) < \mathcal{D}(0)$ and $\mathcal{S}(p_{max}) > \mathcal{D}(p_{max})$. Furthermore, both $\mathcal{S}(p)$ and $\mathcal{D}(p)$ are continuous (Assumption 1). Consequently, at some point p^c in the range $(0, p_{max})$ we have $\mathcal{S}(p^c) = \mathcal{D}(p_c)$. □

PROPOSITION 2. *If for location i at time t there exists a market clearing price, then $p^c = \arg\max_p T_i^t(p)$.*

PROOF. According to Assumption 1 $F^r(.)$ is continuous and strictly decreasing with regard to p. Since $R_i^t \geq 0$ for any i and t, we can imply that $\mathcal{D}_i^t(.)$ is continuous and strictly decreasing. Therefore:

$$\forall p < p^c \qquad \mathcal{D}(p) > \mathcal{D}(p^c) \tag{19}$$

$$\forall p > p^c \qquad \mathcal{D}(p) < \mathcal{D}(p^c) \tag{20}$$

Similarly, we can show $\mathcal{S}_i^t(.)$ is continuous and strictly increasing and hence:

$$\forall p < p^c \qquad \mathcal{S}(p) < \mathcal{S}(p^c) \tag{21}$$

$$\forall p > p^c \qquad \mathcal{S}(p) > \mathcal{S}(p^c) \tag{22}$$

Considering that $\mathcal{D}(p^c) = \mathcal{S}(p^c)$ we can say:

$$\forall p < p^c \qquad \mathcal{S}(p) < \mathcal{D}(p) \tag{23}$$

$$\forall p > p^c \qquad \mathcal{S}(p) > \mathcal{D}(p) \tag{24}$$

Based on Eq. (3) we have:

$$\forall p < p^c \qquad T(p) = \mathcal{S}(p) \tag{25}$$

$$\forall p > p^c \qquad T(p) = \mathcal{D}(p) \tag{26}$$

$$if\, p = p^c \qquad T(p) = \mathcal{D}(p) = \mathcal{S}(p) \tag{27}$$

Therefor for $p < p^c$, $T(p)$ will be strictly increasing and for $p > p^c$, $T(p)$ will be strictly decreasing.

Therefore:

$$p^c = \arg\max_p \{T(p)\} \tag{28}$$

□

THEOREM 1. *The optimal price p^* which maximizes $Rev(p)$ from Eq. (5) is always greater or equal to p^c.*

PROOF. $T(p)$ is continuous and strictly increasing for $p < p^c$ and continuous and strictly decreasing for $p > p^c$ (refer to proof of Proposition 2). We also know from Assumption 2 that $T(0) = \mathcal{S}(0) = 0$ and $T(p_{max}) = \mathcal{D}(p_{max}) = 0$. Consequently, for every $p_1 < p^c$, there exists a $p_2 > p^c$ such that $T(p_1) = T(p_2)$. Since, $p_1 > p_2$ we can imply that $T(p_1) \cdot p_1 < T(p_2) \cdot p_2$. Therefore:

$$p^c \leq \arg\max_p \{Rev(p)\}$$

□

THEOREM 2. *The optimal price p^* for maximizing the revenue at each location at each time is:*

(i) $p^* = p^c$ *if* $p^d \leq p^c$

(ii) $p^* = p^d$ *if* $p^d > p^c$

PROOF.
case(i): For all $p \geq p^c$:

$$T(p) = \mathcal{D}(p)$$
$$Rev(p) = \mathcal{D}(p) \cdot \lambda \cdot p$$

Based on Assumption 1, we can say $\mathcal{D}(p)$ is strictly concave in the range $[0, p_{max}]$. Furthermore, $p^d \leq p^c$ and hence, for all $p \geq p^c$:

$$Rev(p^c) \geq Rev(p) \tag{29}$$

On the other hand, if $p < p^c$ we know from Theorem 1 that there exists $p' > p^c$ such that:

$$Rev(p) < Rev(p') \tag{30}$$

Since $p' > p^c$, combining Eq. (29) and Eq. (30) we get:

$$Rev(p) < Rev(p') \leq Rev(p^c)$$

Therefore, if $p^d \leq p^c$ then for all $p \in [0, p_{max}]$ we have:

$$Rev(p) \leq Rev(p^c)$$

case(ii): We know from Theorem 1 that $p^* > p^c$. Also, for $p > p^c$ we have $T(p) = \mathcal{D}(p)$. Furthermore, $\mathcal{D}(p)$ is strictly concave which makes $\mathcal{D}(p) \cdot p$ concave. Consequently, since $p^d = \arg\max_p \{\mathcal{D}(p) \cdot p\}$, it is safe to say $p^* = p^d$. □

LEMMA 1. *If p_1^c and p_2^c are the market clearing prices for $\mathcal{S}_1(.)$ and $\mathcal{S}_2(.)$, respectively and*

$$p^d = \arg\max_p \{\mathcal{D}(p) \cdot p\}$$

the generated revenue from $\mathcal{S}_2(.)$ is larger than that of $\mathcal{S}_1(.)$ if $p_2^c < p_1^c \wedge p^d < p_1^c \wedge p^d \leq p_2^c$.

PROOF. Since $p^d < p_1^c$, we know from Theorem 2 that $p_1^* = p_1^c$, where p_1^* is the optimal price that maximizes the revenue for $\mathcal{S}_1(p)$ and $\mathcal{D}(p)$. Similarly, we can say $p_2^* = p_2^c$, where p_2^* maximizes the revenue for $\mathcal{S}_2(p)$ and $\mathcal{D}(p)$.

On the other hand we know $\mathcal{D}(p) \cdot p$ is concave in the range $[0, p_{max}]$ and since, $p^d < p_2^c < p_1^c$:

$$\mathcal{D}(p_2^c) \cdot p_2^c > \mathcal{D}(p_1^c) \cdot p_1^c$$
$$T(p_2^c) \cdot p_2^c > T(p_1^c) \cdot p_1^c$$
$$Rev(p_2^c) > Rev(p_1^c)$$

Therefore, the generated revenue from $\mathcal{S}_2(.)$ is greater than that of $\mathcal{S}_1(.)$ □

LEMMA 2. *Assuming $p_2^c = p^d$, adding more potential drivers does not increase the generated revenue.*

PROOF. We know from Theorem 2 that if $p_2^c = p^d$, then $p_2^* = p^d$, where p_2* is the optimal price that maximizes the revenue for $\mathcal{S}_2(p)$ and $\mathcal{D}(p)$.

Assuming we show the new supply after adding more potential drivers with $\mathcal{S}_3(p)$, we can find the new market clearing price, p_3^c, where $p_3^c < p_2^c$. Consequently, $p_3^c < p^d$. We know from Theorem 2 that if $p_3^c < p^d$ then $p_3^* = p^d$, where p_3^* is the price that maximizes the revenue for $\mathcal{S}_3(p)$ and $\mathcal{D}(p)$.

Therefore, by adding more potential drivers the optimal price remains at p^d and thus, there will be no increase in the revenue. □

PROPOSITION 3. *The generated revenue from Eq.* (17) *is at least as much as that of Eq.* (5).

PROOF. Using Eq. (17), the total revenue from rides originating at location i can be computed as:

$$\begin{aligned} Rev_i^t(\mathbf{B}_i^t, \mathbf{P}_i^t) &= \sum_j T_{ij}^t(\beta_{ij}^t, p_{ij}^t) \cdot \lambda \cdot p_{ij}^t \\ &= \sum_j \min\{\beta_{ij}^t \mathcal{V}_i^t F^w(p_{ij}^t), \theta_{ij}^t \mathcal{R}_i^t F^r(p_{ij}^t)\} \cdot \lambda \cdot p_{ij}^t \end{aligned}$$

Assuming regardless of its destination, every ride has the same price p_i and we set $\beta_{ij}^t = \theta_{ij}^t$ we get:

$$\begin{aligned} Rev_i^t(\mathbf{B}_i^t, \mathbf{P}_i^t) &= \sum_j \min\{\theta_{ij}^t \mathcal{V}_i^t F^w(p_i^t), \theta_{ij}^t \mathcal{R}_i^t F^r(p_i^t)\} \cdot \lambda \cdot p_i^t \\ &= \sum_j \theta_{ij}^t \cdot \min\{\mathcal{V}_i^t F^w(p_i^t), \mathcal{R}_i^t F^r(p_i^t)\} \cdot \lambda \cdot p_i^t \\ &= \min\{\mathcal{V}_i^t F^w(p_i^t), \mathcal{R}_i^t F^r(p_i^t)\} \cdot \lambda \cdot p_i^t \times \sum_j \theta_{ij}^t \\ &= \min\{\mathcal{S}_i^t(p_i^t), \mathcal{D}_i^t(p_i^t)\} \cdot \lambda \cdot p_i^t \\ &= T_i^t(p_i^t) \cdot \lambda \cdot p_i^t \\ &= Rev_i^t(p_i^t) \end{aligned}$$

Therefore, the generated revenue from Eq. (17) is at least as much as that of Eq. (5). □