INTRODUCTION TO MATROID AND ITS REPRESENTABILITY

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ABSTRACT. This paper introduces a mathematical structure called Matroid which abstracts the concept of linear independence. The goal of this paper is to discuss the representability of matroid, which will be defined later in this paper, and introduce some examples, sketches of proofs about some of the important theorems and conjectures about representability and others.

Contents

1. Introduction to Matroid

Introduction the similarity of linearly independence and cycles

Definition 1.1. A definition of matroid

2. Some basic definitions/results

This chapter will introduce some of the basic results, definition which will be necessary to start discussing the representability.

Theorem 2.1. All maximal independent sets have the same size.

Theorem 2.2. Basis and circuits

Most results/definitions in here will come from Oxley's lecture note and Wikipedia. Since this is not an introduction to matroid, I will omit those propositions/definitions that are not necessary. I will add more results that I personally found useful and rephrase some of the definitions so it's easier to use.

3. Introduction to representability

This chapter will introduce definitions and results that are necessary to discuss the representability.

More specifically, this chapter will introduce:

- Why we care about representability
 - If all matroids are representable, we are just giving a vector space another name.
 - Therefore, it is important that not all matroids are representable.
 - Show some examples of unrepresentable matroids
 - Now that we know the existence, the question is, which one?
- The mathematical definition of "representability"

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Definition 3.1. contraction

Definition 3.2. deletion

- 3.1. What do contraction and deletion mean in graphs and vector spaces?
- 3.2. Why do these matter? Because of a matroid is representable over some field

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, its minor is always representable over

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- 3.3. The introduction of Rota's conjecture.
 - 4. More discussion on matroid representability

This is going to be the bread and butter of the paper. I am planning to write several chapters about matroid representability. Some ideas I have at this point include:

- (Sketch of) proofs of some small cases of Rota's conjecture
- List of interesting matroids that are not representable