ALGEBRAIC TOPOLOGY II LECTURE NOTES

HIDENORI SHINOHARA

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1. Poincare Duality

Since $H^*(T^n) \equiv \wedge_{\mathbb{Z}} M$ where $M = \langle v_1, \cdots, v_n \rangle$, we have

k	0	1	2	$\cdots$	$n-1$	n
$\operatorname{rank} H^k(T^n;\mathbb{Z})$	$\binom{n}{0}$	$\binom{n}{1}$	$\binom{n}{2}$	$\cdots$	$\binom{n}{n-1}$	$\binom{n}{n}$

This symmetry phenomenon is true in general and very useful. Another example: The cellular complex for $\mathbb{CP}^n$ is $\mathbb{Z} \to 0 \to \mathbb{Z} \to 0 \to \cdots \to 0 \to \mathbb{Z}$. Thus

k	0	1	2	$\cdots$	$2n-1$	$2n$
$\operatorname{rank} H^k(\mathbb{CP}^n;\mathbb{Z})$	1	0	1	$\cdots$	0	1