MATH 602 HOMEWORK 4

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Exercise. (1) Let $a/s \in S^{-1}\sqrt{I}$. Then $a^n \in I$ and $s \in S$ for some $n \in \mathbb{N}$. This implies $(a/s)^n \in S^{-1}I$, so $a/s \in \sqrt{S^{-1}I}$.

Let $a/s \in \sqrt{S^{-1}I}$. Then $a^n/s^n \in S^{-1}I$ for some $n \in \mathbb{N}$. Then $a^n \in I$, so $a \in \sqrt{I}$. Since $s \in S$, $a/s \in S^{-1}\sqrt{I}$.

Exercise. (6a) $(M : N)$ is nonempty. For any $a, b \in (M : N)$, $(a - b)N = aN + (-b)N = aN + bN \subset M$, so $a - b \in (M : N)$. Finally, for any $a \in (M : N), x \in R$, $(xa)N = a(xN) \subset aN \subset M$, $ax \in (M : N)$.