

MATH 602 HOMEWORK 2

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Exercise. (Problem 1) We will assume that the problem meant to say “ su with $s \in S \setminus \{0\}$ ” because it would be trivial otherwise. Choose $a_{n-1}, \dots, a_0 \in R$ such that $u^n + a_{n-1}u^{n-1} + \dots + a_1u + a_0 = 0$. If $a_0 = 0$, then $u(u^{n-1} + a_{n-1}u^{n-2} + \dots + a_1) = 0$. Since we are dealing with integral domains, this implies $u^{n-1} + a_{n-1}u^{n-2} + \dots + a_1 = 0$. By repeating this process, we obtain a monic polynomial with coefficients in R and a nonzero constant term that u satisfies.

Therefore, we may assume $a_0 \neq 0$. Then $u(a_1 + a_2u + \dots + a_{n-1}u^{n-2} + u^{n-1}) = -a_0 \in R$. Since $a_0 \neq 0$, $a_1 + a_2u + \dots + a_{n-1}u^{n-2} + u^{n-1}$ is a nonzero element in S . Hence, we showed that some multiple of u lives in R .