

# MYTITLE

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### 1. MODULES

**Exercise.** (Problem 6) Take four  $4 \times 4$  matrices with integer entries and check if the abelian group presented by the matrix is cyclic.

*Proof.*

$$\begin{aligned} & \begin{bmatrix} -166 & -74 & 254 & 347 \\ 140 & -93 & 246 & 425 \\ -196 & 57 & -363 & 202 \\ 325 & 257 & 314 & -389 \end{bmatrix} \rightarrow [18444530375 \quad 1 \quad 1 \quad 1] \\ & \begin{bmatrix} 237 & -81 & 332 & -132 \\ 95 & 268 & 229 & 498 \\ 387 & 213 & 46 & 55 \\ 88 & -126 & -380 & -447 \end{bmatrix} \rightarrow [2610768268 \quad 1 \quad 1 \quad 1] \\ & \begin{bmatrix} -275 & -22 & -207 & -276 \\ -469 & -342 & 240 & -101 \\ -41 & 455 & 51 & -151 \\ 267 & -450 & 98 & -40 \end{bmatrix} \rightarrow [33644517767 \quad 1 \quad 1 \quad 1] \\ & \begin{bmatrix} 48 & 29 & 22 & -481 \\ 388 & -468 & -137 & -491 \\ 84 & -352 & 85 & -384 \\ -226 & -486 & 102 & -156 \end{bmatrix} = [13267264454 \quad 1 \quad 1 \quad 1] \end{aligned}$$

Each of the groups contains 4 generators, so none of them are cyclic. □

### 2. GALOIS THEORY

**Exercise.** (Problem 1) Let  $F = \mathbb{Q}$ . Let  $L = \mathbb{Q}(\sqrt{7}, \sqrt{-11})$ . To what familiar group is  $\text{Aut}(L/F)$  is isomorphic?

*Proof.*  $[K : \mathbb{Q}(\sqrt{7})] = [K : \mathbb{Q}(\sqrt{-11})] = 2$ . Since the characteristic of  $K$  is not 2, by the argument presented on P.3 of the Galois Theory handout,  $\text{Aut}(K/\mathbb{Q}(\sqrt{7}))$  and  $\text{Aut}(K/\mathbb{Q}(\sqrt{-11}))$  have 2 elements. For instance,  $\alpha = \sqrt{7}$  and the minimal monic polynomial is  $x^2 - 7$ . This gives  $D = 28$  and two automorphisms in  $\text{Aut}(K/\mathbb{Q}(\sqrt{7}))$ , the identity map, and  $\sigma : \sqrt{D} \mapsto -\sqrt{D}$  as discussed in the handout. Similarly,  $\text{Aut}(K/\mathbb{Q}(\sqrt{-11}))$  contains the identity map and  $\sigma : \sqrt{D} \mapsto -\sqrt{D}$  where  $D = -44$ .

Finish this proof.

□

**Exercise.** (Problem 2) Let  $F \subset K$  be a field extension.

- (1) Prove in at most two sentences that each  $\sigma \in \text{Aut}(K/F)$  is an  $F$ -linear transformation of the  $F$ -vector space,  $K$ .
- (2) Does the same condition hold in general for  $\sigma \in \text{Aut}(K)$ ? Prove or give a counterexample.

*Proof.*

- (1) For any  $a \in F$  and  $v, w \in K$ ,  $\sigma(av + w) = \sigma(a)\sigma(v) + \sigma(w) = a\sigma(v) + \sigma(w)$ , so  $\sigma$  is indeed an  $F$ -linear transformation.
- (2) Let  $F = \mathbb{Q}(\sqrt{7})$  and  $K = \mathbb{Q}(\sqrt{7}, \sqrt{-11})$ . Let  $\sigma \in \text{Aut}(K/\mathbb{Q})$  such that  $\sigma(\sqrt{7}) = -\sqrt{7}$ ,  $\sigma(\sqrt{-11}) = -\sqrt{-11}$ . The existence of such an automorphism is shown in the solution to Problem 1.  $K$  is an  $F$ -vector space. However,  $\sigma(\sqrt{7} \cdot 1) = -\sqrt{7} \neq \sqrt{7} = \sqrt{7}(\sigma(1))$ , so  $\sigma$  is not an  $F$ -linear transformation.

□

**Exercise.** (Problem 3) Let  $\zeta = \exp(2\pi i/3) \in \mathbb{C}$ . Consider the following subfields of  $\mathbb{C}$ . Let  $F = \mathbb{Q}(\zeta)$ . For  $i \in \{0, 1, 2\}$ , let  $K_i = \mathbb{Q}(\zeta^i 7^{1/3})$ . Let  $L = \mathbb{Q}(7^{1/3}, \zeta 7^{1/3}, \zeta^2 7^{1/3})$ .

*Proof.*

- (1)  $[F : \mathbb{Q}] = 3$ .

□