MATH 612(HOMEWORK 4)

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Exercise. (8) By using cellular cohomology, we obtain

$$H^i(X;\mathbb{Z}) = H^i(Y;\mathbb{Z}) = \begin{cases} \mathbb{Z} & (i = 0, 4), \\ \mathbb{Z}_p & (i = 3), \end{cases}$$

$$H^i(X;\mathbb{Z}_p) = H^i(Y;\mathbb{Z}_p) = \begin{cases} \mathbb{Z}_p & (i = 0, 2, 3, 4), \end{cases}$$

Therefore, we cannot distinguish X from Y by looking at the cohomology groups. When using the coefficient $\mathbb{Z}$, cup products are simply 0 because nontrivial cohomology groups are of order 3 and 4. Thus we cannot distinguish X from Y by looking at the cohomology rings of X and Y. Since $H^i(Y;\mathbb{Z}_p) = H^i(S^4;\mathbb{Z}_p) \oplus H^i(M(\mathbb{Z}_p, 2);\mathbb{Z}_p)$ and the cup product of elements from different "components" in a wedge sum is 0, cup products in $H^*(Y;\mathbb{Z}_p)$ are all 0. On the other hand, the cup product $\alpha \smile \alpha$ where α is a generator of $H^2(\mathbb{C}P^2;\mathbb{Z}_p)$ is nontrivial because $\alpha \smile \alpha$ is a generator of $H^4(\mathbb{C}P^2;\mathbb{Z}_p)$.