MATH 612 (HOMEWORK 1)

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Exercise. (Exercise 1(a)) The case of $G = \mathbb{Z}$ is discussed in Example 2.42.

$$H_k(\mathbb{R}\mathrm{P}^n; \mathbb{Z}) = \begin{cases} \mathbb{Z} & \text{for } k = 0 \text{ and for } k = n \text{ odd} \\ \mathbb{Z}_2 & \text{for } k \text{ odd, } 0 < k < n \\ 0 & \text{otherwise.} \end{cases}$$

Suppose n is even. For any field F, we obtain the cellular chain complex

$$0 \to F \xrightarrow{2} F \xrightarrow{0} \cdots \xrightarrow{2} F \xrightarrow{0} F \to 0.$$

If the characteristic is 2, then all maps are 0. Therefore, $H_k(\mathbb{R}\mathrm{P}^n; F) = F$ if $k \leq n$ and $H_k(\mathbb{R}\mathrm{P}^n; F) = 0$ otherwise. If the characteristic is not 2, then $H_0(\mathbb{R}\mathrm{P}^n; F) = F$ and all other homology groups are 0. If n is odd, we obtain

$$0 \to F \xrightarrow{0} F \xrightarrow{2} \cdots \xrightarrow{2} F \xrightarrow{0} F \to 0.$$

If the characteristic is 2, $H_k(\mathbb{R}\mathrm{P}^n; F) = F$ if $k \leq n$ and $H_k(\mathbb{R}\mathrm{P}^n; F) = 0$ otherwise. Otherwise, $H_0(\mathbb{R}\mathrm{P}^n; F) = H_n(\mathbb{R}\mathrm{P}^n; F) = F$ and all other homology groups are 0.