MATH 601 (DUE 11/22)

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Contents

1. THE THEOREM ON SYMMETRIC POLYNOMIALS

Exercise. (Problem 1) By substituting $u_4 = 0$, we get $u_1^2u_2u_3 + u_1u_2^2u_3 + u_1u_2u_3^2 = s_3s_1$. s_3s_1 with 4 variables expands to $u_1^2u_2u_3 + u_1^2u_2u_4 + u_1^2u_3u_4 + u_1u_2^2u_3 + u_1u_2^2u_4 + u_1u_2u_3^2 + 4u_1u_2u_3u_4 + u_1u_2u_4^2 + u_1u_3^2u_4 + u_1u_3u_4^2 + u_2^2u_3u_4 + u_2u_3^2u_4 + u_2u_3u_4^2$. Then $s_3s_1 - f$ where f is the original polynomial gives us $4u_1u_2u_3u_4 = 4s_4$. Therefore, $f = s_3s_1 - 4s_4$.

Exercise. (Problem 2) We are given that $|M - xI| = x^3 - ax^2 + bx - c$. This implies that $|M - (-x)I| = -x^3 - ax^2 - bx - c$. Since the determinant function preserves multiplication, $|M - xI||M - (-x)I| = |M^2 - x^2I|$. This implies $|M^2 - x^2I| = -x^6 + (a^2 - 2b)x^4 + (b^2 + 2ac)x^2 + c^2$. Therefore, the characteristic polynomial of M is $-x^3 + (a^2 - 2b)x^2 + (b^2 + 2ac)x + c^2$.