MATH 633 (HOMEWORK 11)

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Theorem 0.1. *If a function f is holomorphic in an open set that contains a simple closed piecewise-smooth curve Γ and its interior, then*

$$\int_{\Gamma} f = 0.$$

Proof. Let Ω denote the interior of Γ. Let $\mathcal{O}$ denote an open set on which f is holomorphic and which contains Γ and Ω. Such an open set must exist as it is given in the problem statement.

Choose $\epsilon > 0$ such that $N(x, \epsilon) \subset \mathcal{O}$ for each $x \in \Gamma$. This is possible because Γ is a compact subset of an open set $\mathcal{O}$.

Next, let $P_1, \cdots, P_n$ denote the consecutive points where smooth parts of Γ join. We may pick $\delta < \epsilon/10$ such that each circle C_j centered at a point P_j and of radius δ intersects Γ in precisely two distinct points. This is possible by Lemma 2.10 because for each $P_j = \Gamma(t_j)$, C_j intersects $\Gamma([0, t_j])$ and $\Gamma([t_j, 1])$ once each.

These two points on C_j determine two arcs of circles and one is entirely contained in Ω and the other one does not intersect Ω. TODO

$\square$