MATH 633(HOMEWORK 7)

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Exercise. (1) Suppose f is locally bijective. For every $p \in U$, f is locally injective. Therefore, f' is nonzero in the neighborhood around p. In other words, f' is nonzero on U.

The other direction

Exercise. (12(a)) Let $a \neq b$ be two fixed points. Let $\sigma(z) = (z-a)/(1-\overline{a}z)$. Then σ sends a to 0 and maps D to D bijectively. Let $g = \sigma \circ f \circ \sigma^{-1}$. g has two fixed points, 0 and $\sigma(b)$. By applying Lemma 2.1, g is a rotation. However, g fixes $\sigma(b) \neq 0$, so g must be the identity map. Then f must be the identity.

Exercise. (12(b)) The map $\sigma : z \mapsto (z-i)/(z+i)$ maps the upper half-plane to the unit disk bijectively. Then $\sigma \circ f \circ \sigma^{-1}$ where $f(z) = z+1$ is a holomorphic bijection on f that has no fixed point because f has no fixed point.