# MATH 602(HOMEWORK 3)

HIDENORI SHINOHARA

## 1. Exercises

**Exercise.** (Exercise 1) The ideal generated by the three polynomials contains $-yz^4+yz^2+y=(xy^2-xz+y)-y(xy-z^2)+z(x-yz^4)$. However, its leading term $-yz^4$ is not in the ideal generated by the leading terms of the three polynomials.

**Exercise.** (Exercise 2)

Solve this.

**Exercise.** (Exercise 3)

Solve this.

**Exercise.** (Exercise 4) $0 \in \sqrt{0}$, $a, b \in \sqrt{0} \implies (a+b)^{m+n-1} = \sum_{i=0}^{m+n-1} \binom{m+n-1}{i} a^i b^{m+n-1-i} = 0$, and $\forall a \in \sqrt{0}, \forall x \in R, (ax)^n = a^n x^n = 0$, so $\sqrt{0}$ is an ideal.

**Exercise.** (Exercise 5)

Solve this.

**Exercise.** (Exercise 6) Tensoring an exact sequence with $M \otimes_A N$ is the same as tensoring it with $M$ first and tensoring the resulting sequence with $N$ later.

**Exercise.** (Exercise 7)

Solve this.

**Exercise.** (Exercise 8) Let $pa+qb=1$ for some $p, q \in \mathbb{Z}$. Then $1 \otimes 1 = (pa+qb) \otimes (pa+qb) = pa \otimes pa + pa \otimes qb + qb \otimes pa + qb \otimes qb = 0+0+0+0 = 0$.