

MATH 601 HOMEWORK (DUE 9/18)

HIDENORI SHINOHARA

Exercise. Let R be a commutative ring with one. Explain why there is a unique ring homomorphism, $\mathbb{Z} \rightarrow R$.

Proof. The existence of a ring homomorphism is clear since $\phi(n) = 1_R + \cdots + 1_R$ and $\phi(-n) = -\phi(n)$ define a homomorphism.

We will show the uniqueness of a ring homomorphism. Let $\phi_1, \phi_2 : \mathbb{Z} \rightarrow R$ be ring homomorphisms.

We claim that $\phi_1(n) = \phi_2(n)$ for each $n \in \mathbb{N}$.

- By definition, $\phi_1(1) = \phi_2(1) = 1_R$.
- Suppose $\phi_1(n) = \phi_2(n)$ for some $n \in \mathbb{N}$. Then $\phi_1(n+1) = \phi_1(n) + \phi_1(1) = \phi_2(n) + \phi_2(1) = \phi_2(n+1)$.

By mathematical induction, $\phi_1(n) = \phi_2(n)$ for each $n \in \mathbb{N}$.

For every $n \in \mathbb{N}$, $\phi_1(-n) = -\phi_1(n) = -\phi_2(n) = \phi_2(-n)$. Finally, $\phi_1(0) = \phi_1(0+0) = \phi_1(0) + \phi_1(0)$, so $\phi_1(0) = 0_R$. Similarly, $\phi_2(0) = 0_R$. Thus $\phi_1(0) = \phi_2(0)$.

Hence, we have shown that $\phi_1 = \phi_2$. □

Exercise. (Problem 2) Let $I \subset R$ be an ideal in a commutative ring. Describe a bijective correspondence between ideals in R/I and certain ideals in R .

Proof.

Tried this for about 10 minutes. I think this must be related to some special ideals, so I checked the annihilator, but that doesn't really work. I suspect that this problem is fairly simple once I notice what it is, but it'll take time until I notice it.

□