MATH 633

HIDENORI SHINOHARA

1. Homework 4

Exercise. (Problem 1) Show that if f is entire and $\mathrm{Re}(f)$ is bounded above then f is constant.

Proof. $|\exp(f)| = \exp(\mathrm{Re}(f))$. Since $\mathrm{Re}(f)$ is bounded above, $\exp(f)$ is bounded. By Liouville's theorem, $\exp(f)$ is constant. Thus f is constant because f is continuous and $\exp(z) = \exp(w)$ if and only if $z - w = 2k\pi i$ for some $k \in \mathbb{Z}$. $\square$