

# MATH 601 (DUE 10/23)

HIDENORI SHINOHARA

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### 1. FIELD EXTENSION

**Exercise.** (Problem 1) Let  $p$  be a prime number. Let  $K = \mathbb{Z}/p\mathbb{Z}(t)$  be the fraction field of  $\mathbb{Z}/p\mathbb{Z}[t]$ .

- (i) What is the characteristic of  $K$ ?
- (ii) What is the characteristic of any extension field of  $K$ ?
- (iii) Show that the Frobenius endomorphism,  $F : K \rightarrow K$  is not a ring isomorphism.
- (iv) Let  $f(x) = x^p - t \in K[x]$ . Prove that  $f(x)$  is irreducible.
- (v) Prove that  $f(x)$  is not a separable polynomial.
- (vi) Construct an explicit field extension  $K \subset L$  such that  $f(x) \in L[x]$  has a factor of positive degree  $\nmid p$ .
- (vii) With  $f$  and  $L$  above find all the roots of  $f(x)$  in  $L$  and determine their multiplicities.