MATH 611 HOMEWORK (DUE 10/16)

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Exercise. (Problem 18) For a path-connected, locally path-connected, and semilocally simply-connected space X, call a path-connected covering space $X \to X$ abelian if it is normal and has abelian deck transformation group. Show that X has an abelian covering space that is a covering space of every other abelian covering space of X, and that such a 'universal' abelian covering space is unique up to isomorphism. Describe this covering space explicitly for $X = S^1 \vee S^1$ and $X = S^1 \vee S^1 \vee S^1$.

Proof.

- What is the hypothesis?
- What is the conclusion?
- Is it possible to satisfy the condition?
- Draw a figure.
- Separate the various parts of the hypothesis.
- Find the connection between the hypothesis and the conclusion.
- Have you seen it before?
- Look at the conclusion! And try to think of a familiar theorem having the same or a similar conclusion.
- Keep only a part of the hypothesis, drop the other part; is the conclusion still valid?
- Could you derive something useful from the hypothesis?
- Could you think of another hypothesis from which you could easily derive the conclusion?
- Could you change the hypothesis, or the conclusion, or both if necessary, so that the new hypothesis and the new conclusion are nearer to each other?
- Did you use the whole hypothesis?

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