

MATH 633 (HOMEWORK 8)

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Exercise. (Problem 13) For any $w \in D$, $\phi_w(z) = (z - w)/(1 - \bar{w}z)$ is an automorphism on D that maps w to 0. Fix $w \in D$. Then $\phi_{f(w)} \circ f \circ \phi_w^{-1}$ is an automorphism that maps 0 to 0. By Lemma 2.1, $|(\phi_{f(w)} \circ f \circ \phi_w^{-1})(z)| \leq |z|$ for all $z \in D$. Fix $z \in D$. Then $\phi_w(z) \in D$, so $|(\phi_{f(w)} \circ f \circ \phi_w^{-1})(\phi_w(z))| \leq |\phi_w(z)|$. This equals to

$$\begin{aligned} |(\phi_{f(w)} \circ f \circ \phi_w^{-1})(\phi_w(z))| &\leq \left| \frac{z - w}{1 - \bar{w}z} \right| \implies |\phi_{f(w)}(f(z))| \leq \left| \frac{z - w}{1 - \bar{w}z} \right| \\ &\implies \left| \frac{f(z) - f(w)}{1 - \overline{f(w)}f(z)} \right| \leq \left| \frac{z - w}{1 - \bar{w}z} \right| \\ &\implies \rho(f(z), f(w)) \leq \rho(z, w). \end{aligned}$$