

## MATH 612 (HOMEWORK 3)

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**Exercise.** (3.1.11) Using the cellular homology, we obtain

$$\begin{aligned}\tilde{H}_i(X) &= \begin{cases} \mathbb{Z}/m\mathbb{Z} & (i = n) \\ 0 & (i \neq n). \end{cases} \\ \tilde{H}^i(X) &= \begin{cases} \mathbb{Z}/m\mathbb{Z} & (i = n + 1) \\ 0 & (i \neq n + 1). \end{cases}\end{aligned}$$

From previous homework,

$$\tilde{H}_i(X/S^n) = \tilde{H}_i(S^{n+1}) = \begin{cases} \mathbb{Z} & (i = n + 1) \\ 0 & (i \neq n + 1). \end{cases}$$

Thus the map on  $\tilde{H}_i(-; \mathbb{Z})$  is the zero map for each  $i$ . On the other hand, the long exact sequence of a pair gives us  $\tilde{H}^{n+1}(X, S^n; \mathbb{Z}) \xrightarrow{q^*} \tilde{H}^{n+1}(X; \mathbb{Z}) \rightarrow \tilde{H}^{n+1}(S^n; \mathbb{Z})$  where  $\tilde{H}^{n+1}(S^n; \mathbb{Z}) = 0$ , so  $q^*$  is surjective. Therefore, it is nontrivial because  $\tilde{H}^{n+1}(X; \mathbb{Z}) \neq 0$ .

Natural?

The long exact sequence of a pair gives us  $\tilde{H}_n(S^n; \mathbb{Z}) \rightarrow \tilde{H}_n(X; \mathbb{Z}) \rightarrow \tilde{H}_n(X, S^n; \mathbb{Z}) = \tilde{H}_n(S^{n+1}; \mathbb{Z}) = 0$  which implies the surjectivity of the induced map. Since  $\tilde{H}_n(X; \mathbb{Z}) \neq 0$ , the induced map is nonzero.

The map induced on  $\tilde{H}^i(-; \mathbb{Z})$  is the zero map for any  $i$  because at least one of  $\tilde{H}^i(S^n; \mathbb{Z})$  or  $\tilde{H}^i(X; \mathbb{Z})$  is 0 for each  $i$ .

**Exercise.** (3.1.13)

**Exercise.** (3.2.1)

**Exercise.** (3.2.2)

**Exercise.** (3.2.3)

**Exercise.** (3.2.6)

**Exercise.** (3.2.7)