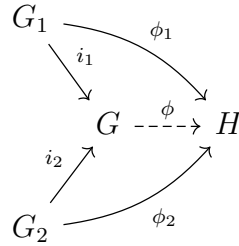


# MYTITLE

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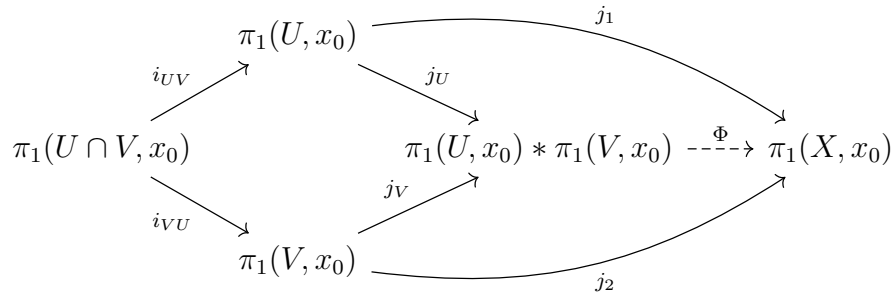
**Theorem 0.1** (Universal property of free products). *Let  $G = G_1 * G_2$  and  $i_1 : G_1 \rightarrow G, i_2 : G_2 \rightarrow G$  be given. For any group  $H$  and homomorphisms  $\phi_1 : G_1 \rightarrow H, \phi_2 : G_2 \rightarrow H$ , there exists a unique homomorphism  $\phi : G \rightarrow H$  such that  $\phi \circ i_1 = \phi_1$  and  $\phi \circ i_2 = \phi_2$ .*



**Theorem 0.2.** *Let  $U, V \subset X$  be open. Suppose:*

- $X = U \cup V$ .
- $U, V, U \cap V$  are all path connected.
- $x_0 \in U \cap V$ .

*By the universal property, there exists a homomorphism  $\Phi : \pi_1(U, x_0) \times \pi_1(V, x_0) \rightarrow \pi_1(X, x_0)$ .*



*Then  $\Phi$  is surjective and  $\ker \Phi$  is the normal subgroup generated by  $\{i_{UV}(g)i_{VU}(g)^{-1} \mid g \in \pi_1(U \cap V, x_0)\}$ .*

*Therefore, we can calculate  $(\pi_1(U, x_0) * \pi_1(V, x_0)) / \ker \Phi$  to find a group isomorphic to  $\pi_1(X, x_0)$ .*