

MATH 601 (DUE 11/22)

HIDENORI SHINOHARA

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1. THE THEOREM ON SYMMETRIC POLYNOMIALS

Exercise. (Problem 1) By substituting $u_4 = 0$, we get $u_1^2 u_2 u_3 + u_1 u_2^2 u_3 + u_1 u_2 u_3^2 = s_3 s_1$. $s_3 s_1$ with 4 variables expands to $u_1^2 u_2 u_3 + u_1^2 u_2 u_4 + u_1^2 u_3 u_4 + u_1 u_2^2 u_3 + u_1 u_2^2 u_4 + u_1 u_2 u_3^2 + 4u_1 u_2 u_3 u_4 + u_1 u_2 u_4^2 + u_1 u_3^2 u_4 + u_1 u_3 u_4^2 + u_2^2 u_3 u_4 + u_2 u_3^2 u_4 + u_2 u_3 u_4^2$. Then $s_3 s_1 - f$ where f is the original polynomial gives us $4u_1 u_2 u_3 u_4 = 4s_4$. Therefore, $f = s_3 s_1 - 4s_4$.

Exercise. (Problem 2) We are given that $|M - xI| = x^3 - ax^2 + bx - c$. This implies that $|M - (-x)I| = -x^3 - ax^2 - bx - c$. Since the determinant function preserves multiplication, $|M - xI||M - (-x)I| = |M^2 - x^2I|$. This implies $|M^2 - x^2I| = -x^6 + (a^2 - 2b)x^4 + (b^2 + 2ac)x^2 + c^2$. Therefore, the characteristic polynomial of M is $-x^3 + (a^2 - 2b)x^2 + (b^2 + 2ac)x + c^2$.

2. GALOIS THEORY VI

Exercise. (Problem 3)

- (a) $\{(123), (132), e\}$ is clearly a subgroup of the stabilizer group S_v of v . Since $(12) \notin S_v$, $3 \leq |S_v| \leq 5$. By Lagrange's Theorem, $S_v = \langle (123) \rangle$.
- (b) By (i), $S_3 v$ contains only $[S_3 : S_v] = 2$ elements. Thus $v' = (12) \cdot v = u_2 u_1^2 + u_1 u_3^2 + u_3 u_2^2$.