MATH 633(HOMEWORK 10)

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Exercise. (Exercise 1(b)) When $t = 0$, $\gamma(t) = \gamma_\epsilon(t) = 0$, so γ and γ_ϵ intersect for any $\epsilon > 0$.

Exercise. (Exercise 2) By Theorem 2.1 (P.351), $\mathbb{C} \setminus \gamma$ is connected. Suppose $\mathbb{C} \setminus C$ is not connected. Then $\mathbb{C} \setminus C = A_1 \cup A_2$ where A_1, A_2 are nonempty, disjoint closed sets. This implies that $\mathbb{C} \setminus \gamma = (A_1 \cup D \setminus \{1\}) \cup A_2$. Since $A_1 \cup D_1$ and A_2 are disjoint and closed in $\mathbb{C} \setminus \gamma$, at least one of them has to be nonempty. Clearly, $D_1 \neq \emptyset$, $A_2 = \emptyset$. However, this is a contradiction because A_2 is supposed to be nonempty.