MATH 611 (DUE 10/2)

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Exercise. (Problem 10, Chapter 1.3) Find all the connected 2-sheeted and 3-sheeted covering spaces of $S^1 \vee S^1$, up to isomorphisms of covering spaces without base points.

For the first part, I ended up with the two graphs in Figure 1. Based on these three things, it's not hard to get to the following two possibilities. However, I'm not sure if this is rigorous enough. Also, I don't know how this can be applied to the case of 3. There are many ways to connect vertices and it doesn't seem doable, which suggests that there might be better ways to solve this.
I tried to use the universal covering and consider a subset. I feel that this would make sure that I'm covering all the cases, although I don't know how to show it rigorously.

Proof.

- There have to be exactly two points in a covering space with 4 edges.
- Every other point has a neighborhood such that the point has only two edges.
- A covering space has to be path connected.

□

Exercise. (Problem 11, Chapter 1.3) Construct finite graphs X_1 and X_2 having a common finite-sheeted covering space $\tilde{X}_1 = \tilde{X}_2$, but such that there is no space having both X_1 and X_2 as covering spaces.

Figure 3 seems very promising. It won't be too rigorous, but I can show that there is no space that has these two spaces as covering spaces.

Proof.

- The degree of each vertex must be 3.
- The number of vertices must be at most 2.
- The number of vertices must be 2.
- Consider a path from one point to the other. You can't lift all of them given that the map must be locally homeomorphic around each vertex.

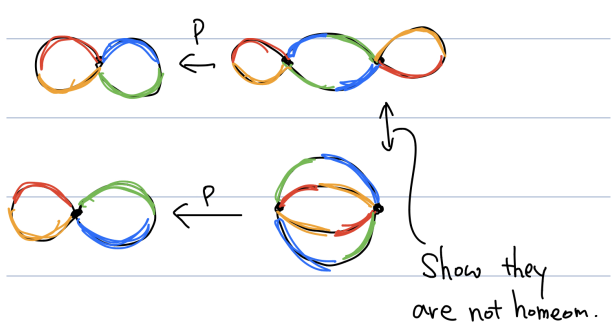

FIGURE 1. Problem 10 Idea

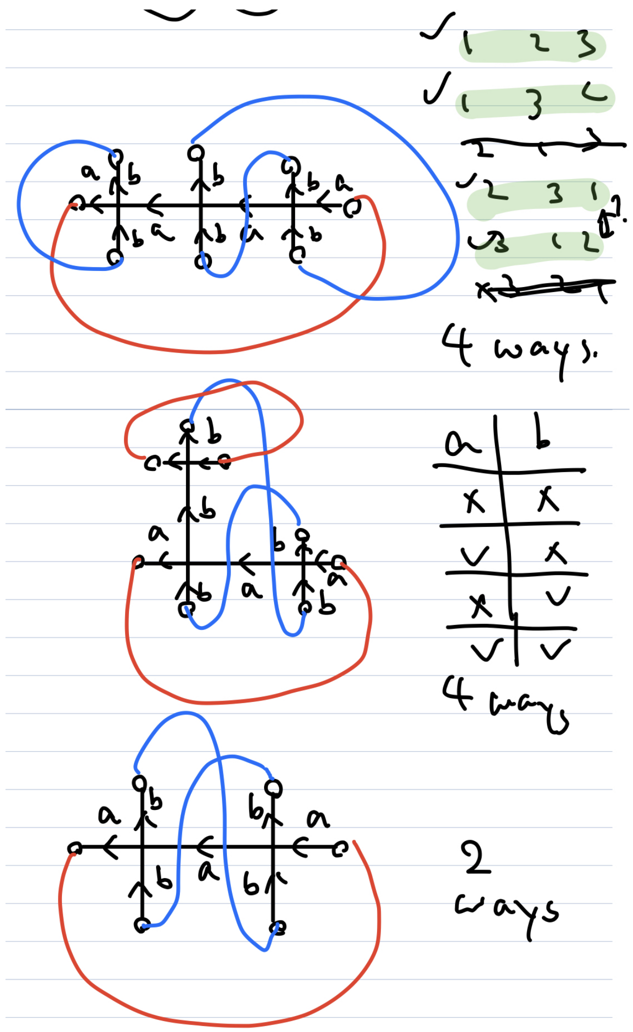

FIGURE 2. Problem 10 Idea 2

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Exercise. (Problem 14, Chapter 1.3) Find all the connected covering spaces of $\mathbb{R}P^2 \vee \mathbb{R}P^2$.

Proof. I think Figure 4 is the universal covering of $\mathbb{P}_2 \wedge \mathbb{P}_2$, but I'm not certain.

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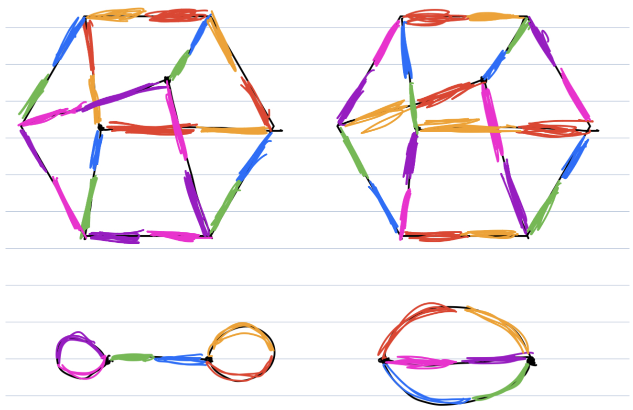

FIGURE 3. Problem 11 Idea 3

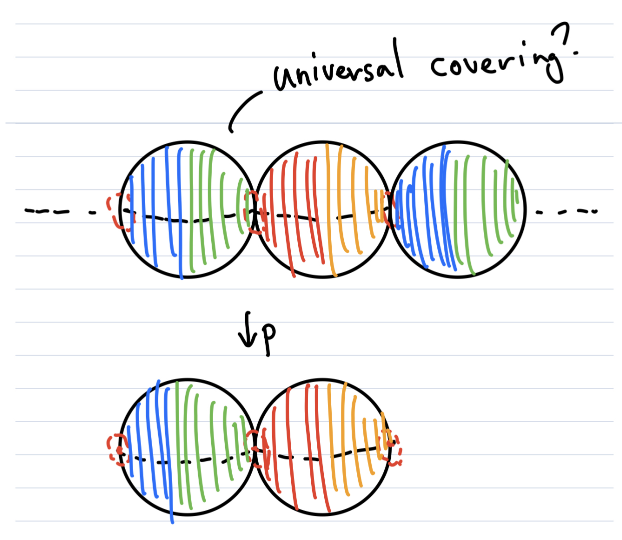

FIGURE 4. Problem 14 Idea 2