

MATH 611 HOMEWORK (DUE 9/18)

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Exercise. (Problem 12, Chapter 1.2) The Klein bottle is usually pictured as a subspace of \mathbb{R}^3 like the subspace $X \subset \mathbb{R}^3$ shown in the first figure at the right. If one wanted a model that could actually function as a bottle, one would delete the open disk bounded by the circle of self-intersection of X , producing a subspace $Y \subset X$. Show that $\pi_1(X) \approx \mathbb{Z} * \mathbb{Z}$ and that $\pi_1(Y)$ has the presentation $\langle a, b, c \mid aba^{-1}b^{-1}cb^\epsilon c^{-1} \rangle$ for $\epsilon = \pm 1$. Show also that $\pi_1(Y)$ is isomorphic to $\pi_1(\mathbb{R}^3 \setminus Z)$ for Z the graph shown in the figure.

Proof.

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