

# MATH 601 HOMEWORK (DUE 10/16)

HIDENORI SHINOHARA

## CONTENTS

1. Jordan Canonical Form	1
--------------------------	---

### 1. JORDAN CANONICAL FORM

Let  $k$  be a field,  $V$  a finite dimensional  $k$ -vector space, and  $T \in \text{End}_k(V)$  a linear transformation.

**Exercise.** (Problem 1) Show that the set  $\{p(x) \in k[x] \mid p(T) = 0 \in \text{End}_k(V)\}$  is an ideal,  $I \subset k[x]$ . Also, show that  $I \neq 0$ .

*Proof.*

- Claim 1:  $I$  is nonempty.

Use Cayley-Hamilton to find a non-trivial element. I'm still trying to understand C-H. One thing I learned today is that the determinant function is independent of the choice of the basis.

- Claim 2:  $I$  is closed under subtraction. Let  $p(x), q(x) \in I$ . Then  $p(x) - q(x) \in I$  because  $p(T) - q(T) = 0 - 0 = 0$ .
- Claim 3:  $I$  is closed under multiplication by elements in  $k[x]$ . Let  $p(x) \in I, r(x) \in k[x]$ . Then  $p(T)r(T) = 0r(T) = 0$ , so  $r(x)p(x) \in I$ .

By Claim 1 and 2,  $I$  is a subgroup of  $k[x]$  under addition. Then Claim 3 implies that  $I$  is an ideal. By Claim 1,  $I \neq 0$ .  $\square$