

MATH 633 HOMEWORK 3

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Exercise. (Problem 1) A simply connected space is clearly piecewise smooth simply connected. Let Ω be piecewise smooth simply connected and $\gamma_1, \gamma_2 : [0, 1] \rightarrow \Omega$ be two continuous curves with the same end points. Since Ω is open, $\gamma_1(t)$ has an open ball around it that is contained in Ω for each $t \in [0, 1]$. Since $[0, 1]$ is compact and γ_1 is continuous, $\gamma_1([0, 1])$ is compact. Hence, there is a finite partition $0 = t_0 < t_1 < \cdots < t_{n-1} < t_n = 1$ such that $\gamma_1([t_i, t_{i+1}])$ is contained in an open ball $\subset \Omega$ for each i . Then γ_1 is homotopic to the curve $\gamma_{1'}$ that consists of n straight lines, i th of which is the line between $\gamma_1(t_i)$ and $\gamma_1(t_{i-1})$ where $i = 1, \dots, n$. This can be shown by the “straight-line” homotopy because $\gamma_1([t_{i-1}, t_i])$ and the i th straight line are in an open ball contained in Ω .

A similar argument can be applied to show that γ_2 is homotopic to a curve $\gamma_{2'}$ that consists of finitely many straight lines. A curve consisting of finitely many straight lines is clearly piecewise smooth.

Therefore, $\gamma_1 \sim \gamma_{1'} \sim \gamma_{2'} \sim \gamma_2$. Thus Ω is simply connected.