MATH 602 HOMEWORK 2

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Exercise. (Problem 1) We will assume that the problem meant to say "su with $s \in S \setminus \{0\}$" because it would be trivial otherwise. Choose $a_{n-1}, \cdots, a_0 \in R$ such that $u^n + a_{n-1}u^{n-1} + \cdots + a_1u^1 + a_0 = 0$. If $a_0 = 0$, then $u(u^{n-1} + a_{n-1}u^{n-2} + \cdots + a_1) = 0$. Since we are dealing with integral domains, this implies $u^{n-1} + a_{n-1}u^{n-2} + \cdots + a_1 = 0$. By repeating this process, we obtain a monic polynomial with coefficients in R and a nonzero constant term that u satisfies.

Therefore, we may assume $a_0 \neq 0$. Then $u(a_1 + a_2u + \cdots + a_{n-1}u^{n-2} + u^{n-1}) = -a_0 \in R$. Since $a_0 \neq 0$, $a_1 + a_2u + \cdots + a_{n-1}u^{n-2} + u^{n-1}$ is a nonzero element in S. Hence, we showed that some multiple of u lives in R.

Exercise. (Problem 2) Let $R = \mathbb{Z}$ and $S = 2\mathbb{Z}$. $R \setminus S$ is not even an ideal because $0 \notin R \setminus S$. Thus $R \setminus S$ is not a prime ideal.