MYTITLE

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Theorem 0.1 (Universal property of free products)**.** *Let $G = G_1 * G_2$ and $i_1 : G_1 \to G, i_2 : G_2 \to G$ be given. For any group H and homomorphisms $\phi_1 : G_1 \to H, \phi_2 : G_2 \to H$, there exists a unique homomorphism $\phi : G \to H$ such that $\phi \circ i_1 = \phi_1$ and $\phi \circ i_2 = \phi_2$.*

$$\begin{tikzcd}
G_1 \arrow[dr, "i_1"] \arrow[drr, bend left, "\phi_1"] & & \\
& G \arrow[r, dashed, "\phi"] & H \\
G_2 \arrow[ur, "i_2"] \arrow[urr, bend right, "\phi_2"'] & &
\end{tikzcd}$$

Theorem 0.2. *Let $U, V \subset X$ be open. Suppose:*

- $X = U \cup V$.
- *$U, V, U \cap V$ are all path connected.*
- $x_0 \in U \cap V$.

By the universal property, there exists a homomorphism $\Phi : \pi_1(U, x_0) * \pi_1(V, x_0) \to \pi_1(X, x_0)$.

$$\begin{tikzcd}
& \pi_1(U, x_0) \arrow[dr, "j_U"] \arrow[drr, bend left, "j_1"] & & \\
\pi_1(U \cap V, x_0) \arrow[ur, "i_{UV}"] \arrow[dr, "i_{VU}"'] & & \pi_1(U, x_0) * \pi_1(V, x_0) \arrow[r, dashed, "\Phi"] & \pi_1(X, x_0) \\
& \pi_1(V, x_0) \arrow[ur, "j_V"] \arrow[urr, bend right, "j_2"'] & &
\end{tikzcd}$$

Then Φ is surjective and $\ker \Phi$ is the normal subgroup generated by $\{i_{UV}(g) i_{VU}(g)^{-1} \mid g \in \pi_1(U \cap V, x_0)\}$.

Therefore, we can calculate $(\pi_1(U, x_0) * \pi_1(V, x_0)) / \ker \Phi$ to find a group isomorphic to $\pi_1(X, x_0)$.