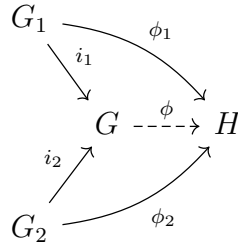


MYTITLE

HIDENORI SHINOHARA

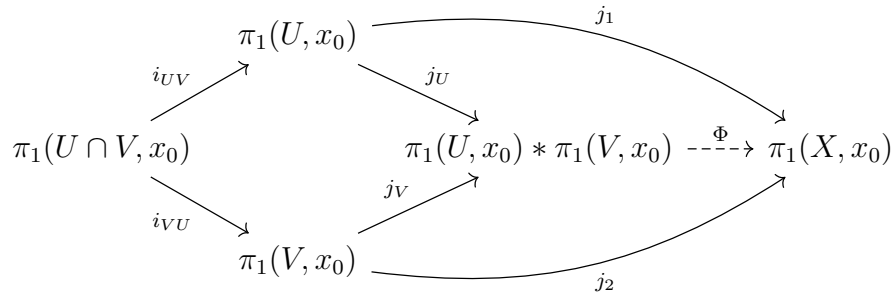
Theorem 0.1 (Universal property of free products). *Let $G = G_1 * G_2$ and $i_1 : G_1 \rightarrow G, i_2 : G_2 \rightarrow G$ be given. For any group H and homomorphisms $\phi_1 : G_1 \rightarrow H, \phi_2 : G_2 \rightarrow H$, there exists a unique homomorphism $\phi : G \rightarrow H$ such that $\phi \circ i_1 = \phi_1$ and $\phi \circ i_2 = \phi_2$.*



Theorem 0.2. *Let $U, V \subset X$ be open. Suppose:*

- $X = U \cup V$.
- $U, V, U \cap V$ are all path connected.
- $x_0 \in U \cap V$.

*By the universal property, there exists a homomorphism $\Phi : \pi_1(U, x_0) * \pi_1(V, x_0) \rightarrow \pi_1(X, x_0)$.*



Then Φ is surjective and $\ker \Phi$ is the normal subgroup generated by $\{i_{UV}(g)i_{VU}(g)^{-1} \mid g \in \pi_1(U \cap V, x_0)\}$.

*Therefore, we can calculate $(\pi_1(U, x_0) * \pi_1(V, x_0)) / \ker \Phi$ to find a group isomorphic to $\pi_1(X, x_0)$.*