

## MATH 601 HOMEWORK (DUE 9/4)

HIDENORI SHINOHARA

**Exercise 0.1.** Exercise 2.2 Let  $\ast : G \times S \rightarrow S$  be a left action of  $G$ . Show that  $s \star g = g^{-1} \ast s$  defines a right action of  $G$  on  $S$ .

*Proof.* Let  $s \in S, g, h \in G$  be given.

$$\begin{aligned}(s \star g) \star h &= h^{-1} \ast (s \star g) \\ &= h^{-1} \ast (g^{-1} \ast s) \\ &= (h^{-1}g^{-1}) \ast s \\ &= (gh)^{-1} \ast s \\ &= s \star (gh).\end{aligned}$$

Let  $e \in G$  denote the identity element and let  $s \in S$  be given.

$$\begin{aligned}s \star e &= e^{-1} \ast s \\ &= e \ast s \\ &= s.\end{aligned}$$

Therefore,  $\star$  is indeed a right action of  $G$  on  $S$ . □