

## MATH 602 HOMEWORK 2

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**Exercise.** (Problem 1) We will assume that the problem meant to say “ $su$  with  $s \in S \setminus \{0\}$ ” because it would be trivial otherwise. Choose  $a_{n-1}, \dots, a_0 \in R$  such that  $u^n + a_{n-1}u^{n-1} + \dots + a_1u + a_0 = 0$ . If  $a_0 = 0$ , then  $u(u^{n-1} + a_{n-1}u^{n-2} + \dots + a_1) = 0$ . Since we are dealing with integral domains, this implies  $u^{n-1} + a_{n-1}u^{n-2} + \dots + a_1 = 0$ . By repeating this process, we obtain a monic polynomial with coefficients in  $R$  and a nonzero constant term that  $u$  satisfies.

Therefore, we may assume  $a_0 \neq 0$ . Then  $u(a_1 + a_2u + \dots + a_{n-1}u^{n-2} + u^{n-1}) = -a_0 \in R$ . Since  $a_0 \neq 0$ ,  $a_1 + a_2u + \dots + a_{n-1}u^{n-2} + u^{n-1}$  is a nonzero element in  $S$ . Hence, we showed that some multiple of  $u$  lives in  $R$ .

**Exercise.** (Problem 2) Let  $R = \mathbb{Z}$  and  $S = 5\mathbb{Z}$ .  $R \setminus S$  is not even an ideal because  $0 \notin R \setminus S$ . Thus  $R \setminus S$  is not a prime ideal.