MATH 611 (DUE 10/23)

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1. Simplicial and Singular Homology

Exercise. (Problem 2) Show that the Δ-complex obtained from Δ^3 by performing the edge identifications $[v_0, v_1] \sim [v_1, v_3]$ and $[v_0, v_2] \sim [v_2, v_3]$ deformation retracts onto a Klein bottle. Find other pairs of identifications of edges that produce Δ-complexes deformation retracting onto a torus, a 2-sphere, and $\mathbb{R}\mathbf{P}^2$.

Proof. Maybe something like this? Either way, I noticed that it looks like it contains 2 $\mathbb{R}\mathbf{P}^2$.

□

Exercise. (Problem 4) Compute the simplicial homology groups of the triangular parachute obtained from Δ^2 by identifying its three vertices to a single point.

Proof. Let v_0 denote the only vertex, e_1, e_2, e_3 denote the three edges of the parachute, and σ denote the face of the parachute. $C_k = 0$ for $k \geq 3$ because Δ^2 with the vertices identified does not contain any k-dimensional simplicies. $C_2 = \langle \sigma \rangle, C_1 = \langle e_1, e_2, e_3 \rangle, C_0 = \langle v_0 \rangle$. Let $n \in \mathbb{N}$. ∂_n is defined such that $\partial_n(\sigma_\alpha) = \sum_i (-1)^i \sigma_\alpha \mid [v_0, \cdots, \hat{v}_i, \cdots, v_n]$. Since there is only one vertex, ∂_n is the zero map. Therefore, $H_n = \ker(\partial_n)/\operatorname{Im}(\partial_{n+1}) = C_n/\langle 0 \rangle = C_n$. Thus

$$H_n = \begin{cases} \{0\} & (n \geq 3) \\ \langle \sigma \rangle \cong \mathbb{Z} & (n = 2) \\ \langle e_1, e_2, e_3 \rangle \cong \mathbb{Z}^3 & (n = 1) \\ \langle v_0 \rangle \cong \mathbb{Z} & (n = 0). \end{cases}$$

I'm not sure if this is correct.

□

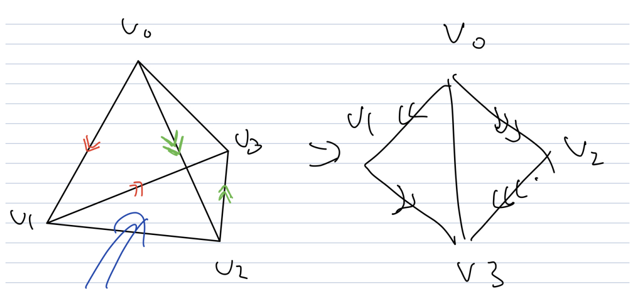

Figure 1. mycaption

Exercise. (Problem 5) Compute the simplicial homology groups of the Klein bottle using the Δ-complex structure described at the beginning of this section.

Proof. $\square$