

MATH 633 MIDTERM

HIDENORI SHINOHARA

Proposition 0.1 (Goursat's Theorem). *If Ω is an open set in \mathbb{C} , and $T \subset \Omega$ is a triangle whose interior is also contained in Ω , then*

$$\int_{\gamma} f(z)dz = 0$$

whenever f is holomorphic in Ω .

Proof.

- Let $T^0 = T$. Having created T^i , create 4 triangles from T^i as shown in the textbook with the natural orientation. Then one of the 4 triangles, denoted by T^{i+1} , must satisfy $|\int_{T^i} f(z)dz| \leq 4|\int_{T^{i+1}} f(z)dz|$. Since $\{T_i\}$ is a sequence of nonempty compact sets whose diameter diminishes, there must exist a unique point z_0 that belongs to all T^i .
- Since f is holomorphic at z_0 , $f(z) = f(z_0) + f'(z_0)(z - z_0) + \psi(z)(z - z_0)$ where $\psi(z) \rightarrow 0$ as $z \rightarrow z_0$.
- Since $f(z_0) + f'(z_0)(z - z_0)$ has a primitive, $\int_{T^n} f(z)dz = \int_{T^n} \psi(z)(z - z_0)dz$ for any n . $|\int_{T^n} \psi(z)(z - z_0)dz| \leq \epsilon_n dp/4^n$ where $\epsilon_n = \sup_{z \in T^n} |\psi(z)|$, d the diameter of T , and p the perimeter of T . $\epsilon_n \rightarrow 0$ as $n \rightarrow \infty$, so $|\int_T f(z)dz| \leq \epsilon_n dp = 0$ as $n \rightarrow \infty$.

□