MATH 633 (HOMEWORK 5)

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Exercise. (Problem 1)

Exercise. (Problem 2)

Exercise. (Problem 3) $p(z) = az + b$ with $a \neq 0$ are the only bijective polynomials.

By the fundamental theorem of algebra, every polynomial $p(z)$ with coefficients in $\mathbb{C}$ is of the form $a \prod_{i=1}^{n}(z - a_i)$ for $a \neq 0, a_1, \cdots, a_n \in \mathbb{C}$. If $a_i \neq a_j$ for some i, j, then p cannot be injective. Thus any bijective polynomials must be of the form $a(z - b)^n$ for some $a \neq 0$ and $b \in \mathbb{C}$. If $n \geq 2$, then $p(\omega + b) = a\omega^n = a$ where $\omega = e^{2\pi i j/n}$ where $j = 0, \cdots, n - 1$. Thus $n = 1$ if the polynomial is injective. In other words, any bijective polynomial must be linear.

On the other hand, it is clear that any non-constant linear function is bijective.