ALGEBRAIC TOPOLOGY EXAMPLES 1

HIDENORI SHINOHARA

1. Covering spaces

Examples 1.1. S^1 can be seen as a 2-sheeted covering space of S^1.

- Define a 2-sheeted covering map $p : S^1 \to S^1$.
- Let $X = S^1, \tilde{X} = S^1$. Compute $\pi_1(X)$ and $\pi_1(\tilde{X})$.
- Compute the deck transformation group of $\tilde{X}$.
- Is $\tilde{X}$ normal?
- Is $\tilde{X}$ abelian?
- Confirm that the groups calculated above are indeed correct by using Proposition 1.39(b).
- Is there a group G such that p can be expressed as $S^1 \mapsto S^1/G$? Is the action properly discontinuous?

Examples 1.2. Given a covering map $p : (\tilde{X}, \tilde{x_0}) \to (X, x_0)$ with $\tilde{X}$ path connected, any deck transformation τ is uniquely determined by $\tau(\tilde{x_0})$. (This is from 10/2)

Check that this is indeed true by looking at the universal covering of $S^1 \vee S^1$.

Find $(X, x_0), (\tilde{X}, \tilde{x_0})$ such that

- $\tilde{X}$ is a covering space of X.
- $\tilde{X}$ is not path connected.
- A deck transformation τ cannot be uniquely determined by $\tau(\tilde{x_0})$.

Examples 1.3. Consider the following covering spaces:

- $p : S^1 \vee S^1 \vee S^1 \vee S^1 \vee S^1 \to S^1 \vee S^1$.
- $p : S^1 \vee S^1 \vee S^1 \vee S^1 \to S^1 \vee S^1$.
- $p : S^1 \vee S^1 \vee S^1 \to S^1 \vee S^1$.
- $p : S^1 \vee S^1 \to S^1 \vee S^1$.

For each of the above,

- Determine if it is normal by using the definition, and come up with a proof by picture.
- Calculate the deck transformation group.
- Calculate $H = p_*(\pi_1(\tilde{X})) \subset \pi_1(X)$. Is there any connection between the deck transformation group and H?
- Apply Proposition 1.39(a) to confirm that we get the same answer.