

## MATH 611 (DUE 11/6)

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### 1. SIMPLICIAL AND SINGULAR HOMOLOGY

**Exercise.** (Problem 14) Determine whether there exists a short exact sequence  $0 \rightarrow \mathbb{Z}_4 \rightarrow \mathbb{Z}_8 \oplus \mathbb{Z}_2 \rightarrow \mathbb{Z}_4 \rightarrow 0$ . More generally, determine which abelian groups  $A$  fit into a short exact sequence  $0 \rightarrow \mathbb{Z}_{p^m} \rightarrow A \rightarrow \mathbb{Z}_{p^n} \rightarrow 0$  with  $p$  prime. What about the case of short exact sequences  $0 \rightarrow A \rightarrow \mathbb{Z}_n \rightarrow 0$ ?

*Proof.* Let  $\phi_1 : \mathbb{Z}_4 \rightarrow \mathbb{Z}_8 \oplus \mathbb{Z}_2, \phi_2 : \mathbb{Z}_8 \oplus \mathbb{Z}_2 \rightarrow \mathbb{Z}_4$  be defined such that  $\phi_1(a) = (2a, a)$  and  $\phi_2(a, b) = 2b - a$ .

Solve this!

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