MATH 611 HOMEWORK (DUE 9/18)

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Exercise. (Problem 12, Chapter 1.2) The Klein bottle is usually pictured as a subspace of $\mathbb{R}^3$ like the subspace $X \subset \mathbb{R}^3$ shown in the first figure at the right. If one wanted a model that could actually function as a bottle, one would delete the open disk bounded by the circle of self-intersection of X, producing a subspace $Y \subset X$. Show that $\pi_1(X) \approx \mathbb{Z} * \mathbb{Z}$ and that $\pi_1(Y)$ has the presentation $\langle a, b, c \mid aba^{-1}b^{-1}cb^{\epsilon}c^{-1} \rangle$ for $\epsilon = \pm 1$. Show also that $\pi_1(Y)$ is isomorphic to $\pi_1(\mathbb{R}^3 \setminus Z)$ for Z the graph shown in the figure.

Proof. Tried for 15 minutes. I'm having a hard time figuring out the fundamental group of the Klein bottle using cell complexes. See Figure 1.

□

Exercise. (Problem 14, Chapter 1.2) Consider the quotient space of a cube I^3 obtained by identifying each square face with the opposite square face via the right-handed screw motion consisting of a translation by one unit in the direction perpendicular to the face combined with a one-quarter twist of the face about its center point. Show this quotient space X is a cell complex with two 0 cells, four 1 cells, three 2 cells, and one 3 cell. Using this structure, show that $\pi_1(X)$ is the quaternion group $\{\pm 1, \pm i, \pm j, \pm k\}$ of order eight.

Proof. Tried for 20 minutes. I was able to identity 0-cells, 1-cells, 3-cells, but I can't tell what would be 2-cells. I think it'll be even harder to figure out the fundamental group.

□

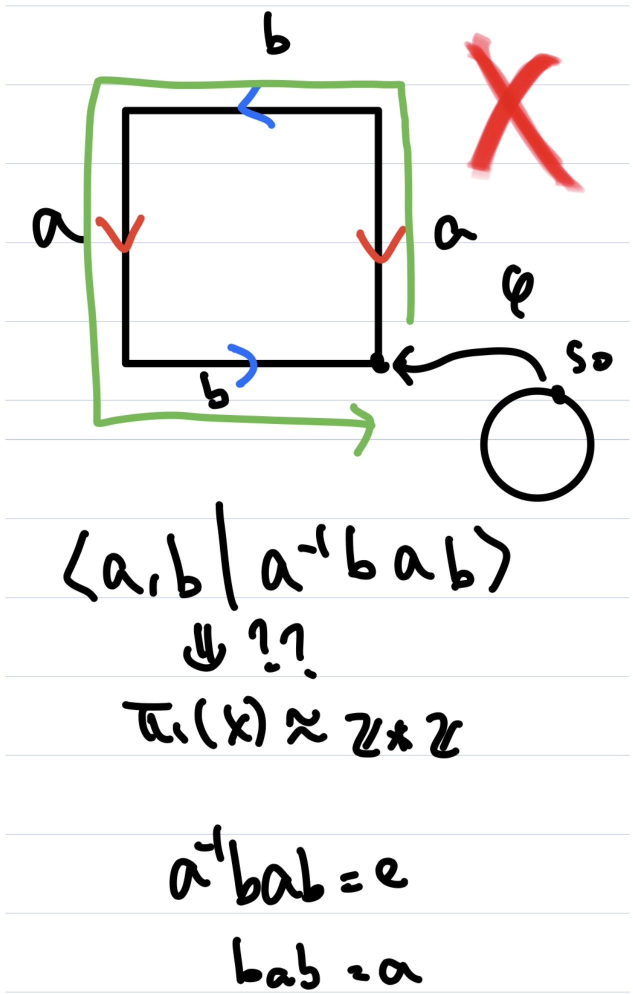

Figure 1. Attempt

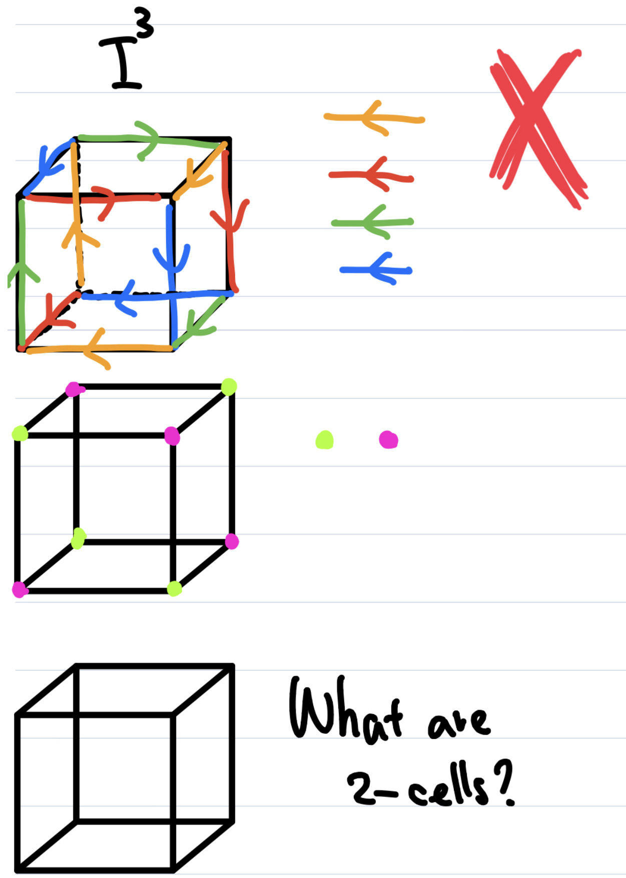

Figure 2. Quotient