MATH 612 FINAL PROJECT

HIDENORI SHINOHARA

Abstract. This is based on the book *4-Manifolds and Kirby Calculus* by Robert E. Gompf and Andras I. Stipsicz.

Contents

1. Manifolds

Definition 1.1. A topological manifold is a separable Hausdorff space such that every point has a neighborhood which is homeomorphic to an open subset of $\mathbb{R}^n_+$. Each pair (U_α, ϕ_α) containing a neighborhood and a homeomorphism is called a chart, and a collection of charts covering the manifold is called an atlas of the manifold.

Definition 1.2. A topological manifold is called a C^r-manifold if, for every pair of charts in the given atlas, the transition function $\phi_\alpha \circ \phi_\beta^{-1}$ is C^r.

This definition makes sense because $\phi_\alpha \circ \phi_\beta^{-1}$ maps U_β into U_α, both of which are open subsets of $\mathbb{R}^n_+$, thus the usual calculus definition of C^r is applied. More rigorously, a C^r-manifold is $(X, \mathcal{T}, \mathcal{A})$ where X is the set, $\mathcal{T}$ is the set of open subsets of X, and $\mathcal{A}$ is the atlas of X. However, just like we normally say a topological space X instead of $(X, \mathcal{T})$, we normally just say a C^r-manifold X without specifying the atlas.

Definition 1.3. Let X, X' be C^r-manifolds. Then a map $f : X \to X'$ is called a C^r-map if $\phi_\alpha \circ f \circ \phi_\beta^{-1}$ is C^r for α, β. Moreover, f is called a C^r-diffeomorphism if f is bijective and both f and f^{-1} are C^r-maps.

Again, in this definition, the usual calculus definition of C^r is used for $\phi_\alpha \circ f \circ \phi_\beta^{-1}$.

Definition 1.4. Let X be a topological manifold. Let A, A' be two atlases of X such that (X, A) and (X, A') are both C^r manifolds. The two structures are called isotopic if the "identity" map $(X, A) \mapsto (X, A')$ is isotopic to a C^r-diffeomorphism between (X, A) and (X, A').

We will usually consider structures up to isotopy.

Example 1.5. TODO Examples of isotopic structures.

Definition 1.6. A compact manifold with no boundary is said to be closed.

Definition 1.7. A space X is said to be a singular manifold if the complement of a finite subset of X is a smooth manifold.

Definition 1.8. TODO Tangent bundle