MATH 611 (DUE 10/2)

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Exercise. (Problem 10, Chapter 1.3) Find all the connected 2-sheeted and 3-sheeted covering spaces of $S^1 \vee S^1$, up to isomorphisms of covering spaces without base points.

Proof.

For the first part, I ended up with the two graphs in Figure 1.

- There have to be exactly two points in a covering space with 4 edges.
- Every other point has a neighborhood such that the point has only two edges.
- A covering space has to be path connected.

Based on these three things, it's not hard to get to the following two possibilities. However, I'm not sure if this is rigorous enough. Also, I don't know how this can be applied to the case of 3. There are many ways to connect vertices and it doesn't seem doable, which suggests that there might be better ways to solve this.
I tried to use the universal covering and consider a subset. I feel that this would make sure that I'm covering all the cases, although I don't know how to show it rigorously.

□

Exercise. (Problem 11, Chapter 1.3) Construct finite graphs X_1 and X_2 having a common finite-sheeted covering space $\tilde{X}_1 = \tilde{X}_2$, but such that there is no space having both X_1 and X_2 as covering spaces.

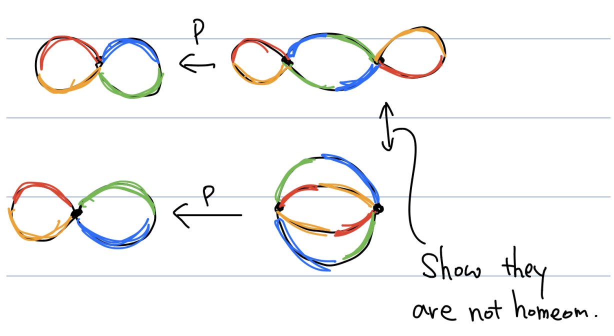

FIGURE 1. Problem 10 Idea

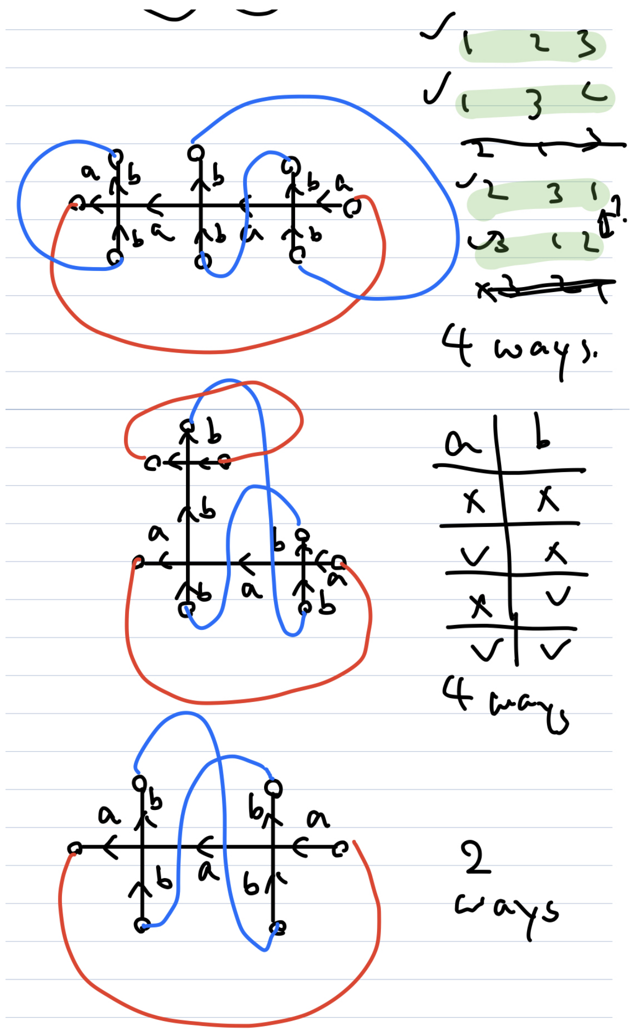

FIGURE 2. Problem 10 Idea 2

Proof. Since they are finite graphs, we can analyze the degree of each vertex. The degree of each vertex is preserved when lifted, so that's something I'm trying to make use of. So, something like Figure 3 is what I'm trying to do. The idea is that X_1, X_2 would have 2 and 3 vertices with degree 4, so X must have exactly one vertex with degree 4. However, in this case, $S \vee S$ has both X_1 and X_2 as covering spaces. I've also tried graphs with vertices of different degrees like 3. I think this is the right approach. I just need some more time.

I think Figure 4 is a correct solution, but I'm not certain. There exists no graph consisting of one vertex of degree 3, so if this example doesn't work, there has to be a covering map between them. But I don't think they are homeomophic to each other, so there should be a way to show that they are like pretty different or something.

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Exercise. (Problem 14, Chapter 1.3) Find all the connected covering spaces of $\mathbb{R}P^2 \vee \mathbb{R}P^2$.

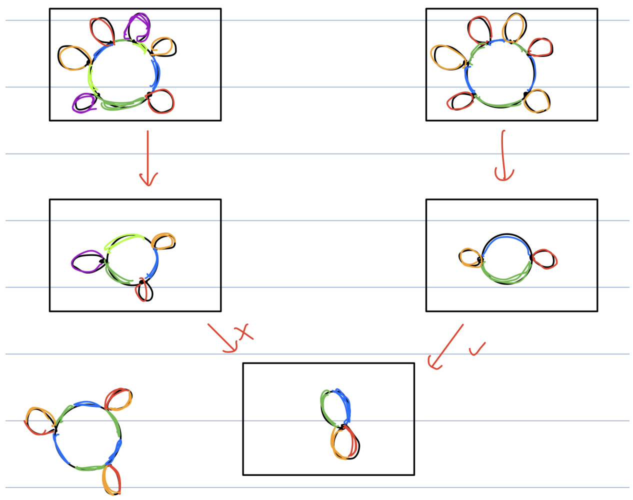

FIGURE 3. Problem 11 Idea

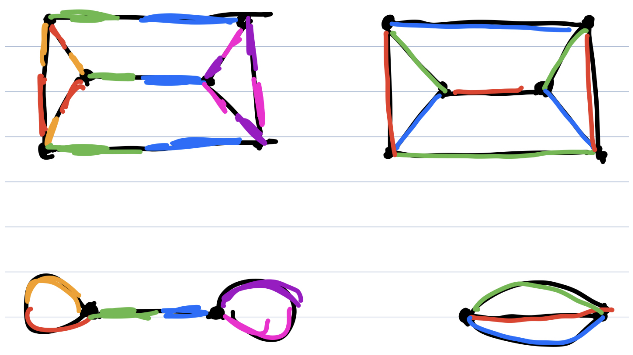

FIGURE 4. Problem 11 Idea 2(Probably correct?)

Proof. I think Figure 5 is the universal covering of $\mathbb{P}_2 \wedge \mathbb{P}_2$, but I'm not certain. $\square$

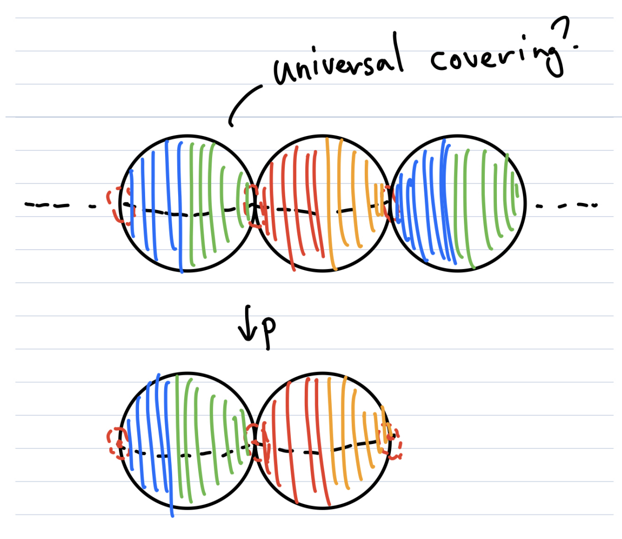

Figure 5. Problem 14 Idea 2