MATH 601 (DUE 10/9)

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Contents

1. Modules

Exercise. (Problem 1) For each of the $\mathbb{Z}$-modules listed in the handout, answer the questions in the handout.

Proof.

(a) $M = \mathbb{Z}^3 \times \mathbb{Z}/86\mathbb{Z}$.

Solve this problem!

(b) $M = \prod_{n\geq 1} \mathbb{Z}/n\mathbb{Z}$.

Solve this problem!

(c) $M = \mathbb{Z}[1/p] \subset \mathbb{Q}$.

Solve this problem!

(d) $M = \mathbb{Q}/\mathbb{Z}_{(p)}$.

Solve this problem!

□

2. Rings of Fractions

Exercise. (Problem 3) Let $T \subset R$ be the subset consisting of all nonzero divisors.

- Show that T is a multiplicative set.
- Let $s \in T$ and let $S = \{1, s, s^2, s^3, \cdots\} \subset T$. Show that the following rings are isomorphic: $S^{-1}R$, the subring $R[1/s] \subset T^{-1}R$, and the quotient ring $R[x]/(sx-1)$.

Proof.

- Prove this!
- Prove this!

□