MATH 633 HOMEWORK 6

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Exercise. (1) Define the map $f : H \to \Omega_1$ such that $f(z) = \exp(\log(z)/\alpha)$ where log denotes the principal branch of the complex logarithm function. This is well defined because H does not contain the real line. Moreover, this is holomorphic because it is the composition of holomorphic functions. Finally, $f'(z) = \exp(\log(z)/\alpha)/z \neq 0$ on H. Thus f is conformal.

Exercise. (2) $z \mapsto az + b$ and $z \mapsto cz + d$ are clearly entire. If $c = 0$, then $\phi : z \mapsto (az + b)/(cz + d)$ is entire. If $c \neq 0$, then ϕ is holomorphic everywhere except for $-d/c$ and at $-d/c$, ϕ has a pole because $\phi(-d/c) = \infty$. In other words, it is meromorphic.

Let $\phi : z \mapsto (az + b)/(cz + d)$ and $\psi : z \mapsto (-dz + b)/(cz - a)$. Then $\phi(\psi(z)) = z$ and $\psi(\phi(z)) = z$, and $(-d)(-a) - bc = ad - bc \neq 0$.

Finish the last part.

Exercise. (3)