MATH 633(HOMEWORK 2)

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Exercise. (Problem 3)

$$\begin{aligned}\int_a^b |z'(t)|dt &= \int_c^d |z'(t(s))|t'(s)ds \\ &= \int_c^d |z'(t(s))t'(s)|ds \\ &= \int_c^d |\tilde{z}'(s)|ds\end{aligned}$$

where $\tilde{z}(s) : [c, d] \to \mathbb{C}$ is a reparametrization of $z(t) : [a, b] \to \mathbb{C}$.

Exercise. (Problem 4) If $t^\star \in \Omega_1$, then there exists an open neighborhood U of $z(t^\star)$ contained in Ω_1. Then $z^{-1}(U)$ is a neighborhood of $t^\star$ in $[0, 1]$ because z is continuous. Since $z(1) \in \Omega_2$, $t^\star \neq 1$. However, this implies the existence of $\epsilon > 0$ such that $t^\star + \epsilon < 1$ and $z(t^\star + \epsilon) \in \Omega_1$. This is a contradiction.

If $t^\star \in \Omega_2$, then there exists an open neighborhood U of $z(t^\star)$ contained in Ω_2. Since U is open, $z^{-1}(U)$ is a neighborhood of $t^\star$ in $[0, 1]$, so $\exists \epsilon > 0$ such that $z(t^\star - \epsilon) \in \Omega_2$.

In each case, we reached a contradiction, so Ω is not disconnected.