QUALIFYING EXAM PREP

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ABSTRACT. In order to prepare for the qualifying exam, I decided to solve problems from Hatcher and Dummit and Foote.

CONTENTS

1. ALGEBRA

1.1. **Groups.** The topics to cover: Elementary concepts (homomorphism, subgroup, coset, normal subgroup), solvable groups, commutator subgroup, Sylow theorems, structure of finitely generated Abelian groups. Symmetric, alternating, dihedral, and general linear groups.

1.2. **Rings.** The topics to cover: Commutative rings and ideals (principal, prime, maximal). Integral domains, Euclidean domains, principal ideal domains, polynomial rings, Eisenstein's irreducibility criterion, Chinese remainder theorem. Structure of finitely generated modules over a principal ideal domain.

1.3. **Fields Extensions.** Finite, algebraic, separable, inseparable, transcendental, splitting field of a polynomial, primitive element theorem, algebraic closure. Finite fields.

1.4. **Galois Theory.** Finite Galois extensions and the Galois correspondence between subgroups of the Galois group and sub-extensions. Solvable extensions and solving equations by radicals.

2. ALGEBRAIC TOPOLOGY

2.1. **Fundamental group.** Computation of the fundamental group, van Kampen's theorem, covering spaces.

2.2. **Homology.** Singular chains, chain complexes, homotopy invariance. Relationship between the first homology and the fundamental group, relative homology. The long exact sequence of relative homology. The Mayer-Vietoris sequence.