## INTRODUCTION TO SMOOTH MANIFOLDS

## HIDENORI SHINOHARA

ABSTRACT. I am not good at smooth manifolds, so I decided to read and solve problems from Introduction to Smooth Manifolds.

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**Exercise 1.1.** Show that equivalent definitions of manifolds are obtained if instead of allowing U to be homeomorphic to any open subset of  $\mathbb{R}^n$ , we require it to be homeomorphic to an open ball in  $\mathbb{R}^n$ , or to  $\mathbb{R}^n$  itself.

*Proof.* It is clear that a "manifold" satisfying the open-ball or  $\mathbb{R}^n$  definition satisfies the open-subset definition. Let M be a manifold satisfying the open-subset definition. Let  $x \in M$  be given and let  $U, \hat{U}, \phi$  be given according to the definition. Since  $\hat{U}$  is open, there exists an open ball B such that  $\phi(x) \in B \subset \hat{U}$ . Restrict  $\phi$  to  $\phi^{-1}(B)$ . Then  $\phi^{-1}(B)$  is an open subset of M containing x, and  $\phi \mid_{\phi^{-1}(B)}$  is a homeomorphism between  $\phi^{-1}(B)$  and B. Thus M satisfies the open-ball definition.

 $B(x,r) \subset \mathbb{R}^n$  is homeomorphic to  $\mathbb{R}^n$  by the map  $(x_1 + a_1, \dots, x_n + a_n) \mapsto (\frac{a_1}{r - a_1}, \dots, \frac{a_n}{r - a_n})$  where  $x = (x_1, \dots, x_n)$  is the center of B(x,r) and r is the radius. Since the composition of two homeomorphisms gives a homeomorphism, M also satisfies the  $\mathbb{R}^n$  definition as well.