

# STELLAR CONSENSUS PROTOCOL

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ABSTRACT. This is my personal notes on the Stellar consensus protocol. This roughly follows the structure of the white paper on <https://www.stellar.org/>.

## 1. BASIC PROPERTIES OF QUORUMS

**Definition 1.1.** Let  $V$  be a set and  $Q : V \rightarrow 2^V \setminus \{\emptyset\}$  be a function such that  $\forall v \in V, \forall q \in Q(v), v \in q$ . Then we call the pair  $\langle V, Q \rangle$  a federated Byzantine agreement system, or FBAS for short.

**Definition 1.2.** Let  $\langle V, Q \rangle$  be an FBAS.  $U \subset V$  is called a quorum if and only if  $\forall v \in U, \exists q \in Q(v), q \subset U$ .

**Theorem 1.3.** In an FBAS  $\langle V, Q \rangle$ , the union of two quorums is a quorum.

*Proof.* Let  $U_1, U_2$  be two quorums. Let  $v \in U_1 \cup U_2$ . Then  $v \in U_i$  for  $i = 1$  or  $i = 2$ . Then  $q \subset U_i$  for some  $q \in Q(v)$ . Therefore,  $q \subset U_1 \cup U_2$ , so  $U_1 \cup U_2$  is indeed a quorum.  $\square$

**Theorem 1.4.** In an FBAS  $\langle V, Q \rangle$ ,  $V$  is a quorum.

*Proof.* For any  $v \in V$ , for any  $q \in Q(v)$ ,  $q \subset V$ . Therefore,  $V$  is indeed a quorum.  $\square$

**Example 1.5.** One might imagine that the intersection of quorums is always a quorum. However, this is not true in general.

Let  $V = \{v_1, \dots, v_4\}$  and

- $Q(v_1) = \{\{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}\},$
- $\vdots$
- $Q(v_4) = \{\{v_1, v_2, v_4\}, \{v_1, v_3, v_4\}, \{v_2, v_3, v_4\}\}.$

In other words,  $Q(v_i) = \{U \mid U \in 2^V, v_i \in U\}$ .

Then  $U_1 = \{v_1, v_2, v_3\}$  is a quorum, and  $U_2 = \{v_2, v_3, v_4\}$  is a quorum. However,  $U_1 \cap U_2 = \{v_2, v_3\}$  is not a quorum because the size of any quorum slice is 3.

**Definition 1.6.** Let  $\langle V, Q \rangle$  be an FBAS. We say  $\langle V, Q \rangle$  enjoys quorum intersection if and only if for any pair of quorums  $U_1, U_2$ ,  $U_1 \cap U_2 \neq \emptyset$ .

**Definition 1.7.** Let  $\langle V, Q \rangle$  be an FBAS and  $B \subset V$ . Then the FBAS  $\langle V, Q \rangle^B$  is defined to be  $\langle V \setminus B, Q^B \rangle$  where  $\forall v \in V, Q^B(v) = \{q \setminus B \mid q \in Q(v)\}$ .

**Theorem 1.8.** Definition 1.7 is well-defined. In other words, if  $\langle V, Q \rangle$  is an FBAS and  $B \subset V$ , then  $\langle V, Q \rangle^B$  is an FBAS.

*Proof.* Let  $v \in V \setminus B, q' \in Q^B(v)$  be given. Then  $q' = q \setminus B$  for some  $q \in Q(v)$ . By the definition of an FBAS,  $v \in q$ . Since  $v \notin B$ ,  $v \in q \setminus B = q'$ . Therefore,  $\langle V, Q \rangle^B$  is an FBAS.  $\square$

**Theorem 1.9.** *Let  $U$  be a quorum in FBAS  $\langle V, Q \rangle$ , let  $B \subset V$  be a set of nodes, and let  $U' = U \setminus B$ . If  $U' \neq \emptyset$ , then  $U'$  is a quorum in  $\langle V, Q \rangle^B$ .*

*Proof.* Since  $U' \neq \emptyset$ , it suffices to show that  $\forall v \in U', \exists q \in Q^B(v), q \subset U'$ . Let  $v \in U'$ . Then  $v \in U$ . Since  $U$  is a quorum in  $\langle V, Q \rangle$ , we can find  $q \in Q(v)$  such that  $q \subset U$ . Then  $q' = q \setminus B \in Q^B(v)$ , and  $q' = q \setminus B \subset U \setminus B = U'$ . Therefore,  $U'$  is a quorum in  $\langle V, Q \rangle^B$ .  $\square$

**Definition 1.10.** Let  $\langle V, Q \rangle$  be an FBAS and  $B \subset V$  be a set of nodes.  $B$  is called a dispensable set, or DSet, if and only if the following conditions are satisfied:

- $\langle V, Q \rangle^B$  enjoys quorum intersection.
- $B = V$  or  $V \setminus B$  is a quorum in  $\langle V, Q \rangle$ .

## 2. VOTING

**Theorem 2.1.** *If an FBAS enjoys quorum intersection and contains no ill-behaved node, then two contradictory statements cannot be both ratified.*

*Proof.* Suppose the statement is false and let  $a, \bar{a}$  denote two contradictory statements ratified in such an FBAS. Let  $U_a, U_{\bar{a}}$  denote quorums ratifying such statements, respectively. By the definition of quorum intersection,  $U_a \cap U_{\bar{a}} \neq \emptyset$ . Let  $v \in U_a \cap U_{\bar{a}}$ . This implies that  $v$  voted for both  $a$  and  $\bar{a}$ . However, this goes against the definition of voting. In other words,  $v$  must be ill-behaved, which is a contradiction to our assumption.  $\square$

**Definition 2.2.** Let  $\langle V, Q \rangle$  be an FBAS and  $v \in V$ .  $v$  is said to be intact if and only if there exists a DSet  $B$  containing all ill-behaved nodes and  $v \notin B$ .  $v$  is said to be befouled if and only if  $v$  is not intact.

**Theorem 2.3.** *Let  $\langle V, Q \rangle$  be an FBAS. Let  $B \subsetneq V$  be a subset containing all the ill-behaved nodes and suppose that  $\langle V, Q \rangle^B$  enjoys quorum intersection. Let  $v_1 \neq v_2 \in V \setminus B$ . If  $v_1$  ratifies a statement  $a$ , then  $v_2$  cannot ratify any statement that contradicts  $a$ .*

*Proof.* Suppose that the theorem is false and let  $U_1, U_2$  be quorums of  $v_1, v_2$  that ratify  $a, \bar{a}$ , respectively where  $a$  and  $\bar{a}$  are contradictory. Since  $v_1 \in U_1 \setminus B$ ,  $U_1 \setminus B \neq \emptyset$ . By Theorem 1.9,  $U'_1 = U_1 \setminus B$  is a quorum in  $\langle V, Q \rangle^B$ . Similarly,  $U'_2 = U_2 \setminus B$  is a quorum in  $\langle V, Q \rangle^B$ . Since  $\langle V, Q \rangle^B$  enjoys quorum intersection,  $U'_1 \cap U'_2 \neq \emptyset$ . Let  $v \in U'_1 \cap U'_2$ . Then  $v \in U_1 \cap U_2$ . In order for  $U_1, U_2$  to ratify  $a, \bar{a}$ , respectively,  $v$  must vote for both  $a$  and  $\bar{a}$ . However, this is against the definition of voting.  $v$  must be an ill-behaved node, so  $v \in B$ , which is a contradiction because  $v \in U_1 \setminus B$ .  $\square$