

# THE CALCULUS OF COMPUTATION

HIDENORI SHINOHARA

## 1. CHAPTER 1

### Exercise (1.1).

(a) Assume that there is a falsifying interpretation  $I$ .

1.  $I \models P \wedge Q \rightarrow P \rightarrow Q$  (assumption)
2.  $I \models P \wedge Q$  (by 1 and semantics of  $\rightarrow$ )
3.  $I \not\models P \rightarrow Q$  (by 1 and semantics of  $\rightarrow$ )
4.  $I \models Q$  (by 2 and semantics of  $\wedge$ )
5.  $I \not\models Q$  (by 3 and semantics of  $\rightarrow$ )
6.  $I \models \perp$  (4 and 5 are contradictory)

There is only one branch and it is closed. Thus  $F$  is valid.