THE CALCULUS OF COMPUTATION

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1. Chapter 1

Exercise (1.1).

(a) Assume that there is a falsifying interpretation I.
1. $I \models P \wedge Q \to P \to Q$ (assumption)
2. $I \models P \wedge Q$ (by 1 and semantics of $\to$)
3. $I \not\models P \to Q$ (by 1 and semantics of $\to$)
4. $I \models Q$ (by 2 and semantics of $\wedge$)
5. $I \not\models Q$ (by 3 and semantics of $\to$)
6. $I \models \perp$ (4 and 5 are contradictory)

There is only one branch and it is closed. Thus F is valid.

Exercise (1.2).

(a) To prove that $\top \Leftrightarrow \neg\perp$, we prove that $\top \leftrightarrow \neg\perp$ is valid. Assume that there is a falsifying interpretation I such that $I \not\models \top \leftrightarrow \neg\perp$. We apply the semantics of $\leftrightarrow$.

The first branch is:

1a. $I \models \top \wedge \neg(\neg\perp)$
2a. $I \models \neg(\neg\perp)$ (by 1a and semantics of $\wedge$)
3a. $I \not\models \neg\perp$ (by 2a and semantics of $\neg$)
4a. $I \models \perp$ (by 3a and semantics of $\neg$)

The second branch is:

1b. $I \models \neg\top \wedge \neg\perp$
2b. $I \models \neg\top$ (by 1b and semantics of $\wedge$)
3b. $I \not\models \top$ (by 2b and semantics of $\neg$)
4b. $I \models \top$ (Under any interpretation, $\top$ has value true)
5b. $I \models \perp$ (3b and 4b are contradictory)

Thus both branches are closed, and thus $\top \leftrightarrow \neg\perp$ is valid.