THE CALCULUS OF COMPUTATION

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1. Chapter 1

Exercise (1.1).

(a) Assume that there is a falsifying interpretation I.
1. $I \models P \wedge Q \to P \to Q$ (assumption)
2. $I \models P \wedge Q$ (by 1 and semantics of $\to$)
3. $I \not\models P \to Q$ (by 1 and semantics of $\to$)
4. $I \models Q$ (by 2 and semantics of $\wedge$)
5. $I \not\models Q$ (by 3 and semantics of $\to$)
6. $I \models \perp$ (4 and 5 are contradictory)

There is only one branch and it is closed. Thus F is valid.

Exercise (1.2).

(a) To prove that $\top \Leftrightarrow \neg\perp$, we prove that $\top \leftrightarrow \neg\perp$ is valid. Assume that there is a falsifying interpretation I such that $I \not\models \top \leftrightarrow \neg\perp$. We apply the semantics of $\leftrightarrow$.

The first branch is:

1a. $I \models \top \wedge \neg(\neg\perp)$
2a. $I \models \neg(\neg\perp)$ (by 1a and semantics of $\wedge$)
3a. $I \not\models \neg\perp$ (by 2a and semantics of $\neg$)
4a. $I \models \perp$ (by 3a and semantics of $\neg$)

The second branch is:

1b. $I \models \neg\top \wedge \neg\perp$
2b. $I \models \neg\top$ (by 1b and semantics of $\wedge$)
3b. $I \not\models \top$ (by 2b and semantics of $\neg$)
4b. $I \models \top$ (Under any interpretation, $\top$ has value true)
5b. $I \models \perp$ (3b and 4b are contradictory)

Thus both branches are closed, and thus $\top \leftrightarrow \neg\perp$ is valid.

Exercise (1.5).

(a) By using the list of template equivalences on P.19, we can obtain the negation normal form of the original formula as following:
- $F : \neg(P \to Q)$.
- $F' : \neg(\neg P \vee Q)$.
- $F'' : \neg(\neg P) \wedge \neg Q$.
- $F''' : P \wedge \neg Q$.

The only connectives in F''' are $\neg, \wedge$, and $\vee$ and the negations appear only in literals. Thus F''' is in NNF. Furthermore, F''' is actually in CNF and DNF since it is the disjunction of one conjunction, and it is the conjunction of two clauses.

Exercise (1.6).

(a) $\bigwedge_{v \in V} \bigvee_{c \in C} P_v^c$.

(b) $\bigwedge_{(v,w) \in E} \bigvee_{c_1 \neq c_2 \in C} (\neg P_v^{c_1} \vee \neg P_w^{c_2})$.

(c) $\bigwedge_{(v,w) \in E} \bigvee_{c \in C} (\neg P_v^c \vee \neg P_w^c)$.

(d) No clue. The problem statement seems too ambiguous.

(e) They are already in CNF. We have $N \cdot M$ variables and $N \cdot M + K \cdot M \cdot (M-1) + K \cdot M$ clauses in the encoding above.