

微分積分学・同演習 A

演習問題 10

- 1.† (1) $\begin{cases} \pi & (n = m) \\ 0 & (n \neq m) \end{cases}$ (2) 0 (3) $\begin{cases} \pi & (n = m) \\ 0 & (n \neq 0) \end{cases}$
2. (1) $x \operatorname{Arcsin} x + \sqrt{1-x^2}$ (2) $x \log x - x$ (3) $\frac{1}{2}(x^2+1) \operatorname{Arctan} x - \frac{x}{2}$
3. (1) $\operatorname{Arcsin} \frac{x}{a}$ (2) $\operatorname{Arctan}(x-a)$ (3) $\frac{1}{2} \left(x\sqrt{a^2-x^2} + a^2 \operatorname{Arcsin} \frac{x}{a} \right)$
4. (1) $\operatorname{Arctan} x$ (2) $\frac{1}{2} \left(\frac{x}{x^2+1} + \operatorname{Arctan} x \right)$ (3) $\frac{1}{8} \left(\frac{x(3x^2+5)}{(x^2+1)^2} + 3 \operatorname{Arctan} x \right)$
- 5.† (1) $\frac{1}{2} \left(\frac{1}{x-1} - \frac{1}{x+1} \right)$ (2) $\frac{1}{3} \left(\frac{x+2}{x^2+x+1} - \frac{1}{x-1} \right)$ (3) $\frac{1}{x} - \frac{x-1}{x^2+1}$
 (4) $\frac{1}{3} \left(\frac{1}{x-1} - \frac{x-1}{x^2+x+1} \right)$ (5) $x + \frac{1}{2} \left(\frac{3}{x-1} - \frac{1}{x+1} \right)$
- 6.† (1) $\frac{1}{2} \log \left| \frac{x-1}{x+1} \right|$ (2) $\frac{1}{6} \log(x^2+x+1) - \frac{1}{3} \log|x-1| + \frac{1}{\sqrt{3}} \operatorname{Arctan} \frac{2x+1}{\sqrt{3}}$
 (3) $\operatorname{Arctan} x + \log|x| - \frac{1}{2} \log(x^2+1)$
 (4) $\frac{1}{6} \log(x^2-x+1) - \frac{1}{3} \log|x+1| + \frac{1}{\sqrt{3}} \operatorname{Arctan} \frac{2x-1}{\sqrt{3}}$
 (5) $\frac{x^2}{2} + \frac{3}{2} \log|x-1| - \frac{1}{2} \log|x+1|$
7. (1) $\frac{1}{2\sqrt{2}} \left(\frac{x+\sqrt{2}}{x^2+\sqrt{2}x+1} - \frac{x-\sqrt{2}}{x^2-x+1} \right)$ (2) $\frac{1}{2\sqrt{2}} \left(\frac{x}{x^2-\sqrt{2}x+1} - \frac{x}{x^2+\sqrt{2}x+1} \right)$
 (3) $\frac{1}{2} \left(\frac{x}{x^2+\sqrt{2}x+1} + \frac{x}{x^2-\sqrt{2}x+1} \right)$
- 8.† (1) $\frac{e^{ax}}{a} \left(x^3 - \frac{3}{a}x^2 + \frac{6}{a^2}x - \frac{6}{a^3} \right)$ (2) $\frac{1}{2}(x^2+1)e^{-x^2}$ (3) $\frac{1}{3} \log \frac{e^x+1}{e^x+4}$
 (4) $\frac{1}{2} \operatorname{Arctan} \frac{x^2\sqrt{a^4-x^4}}{x^4-a^4}$ (5) $\log(\sqrt{x^2-a^2}+x)$ (6) $\frac{1}{4} \left(\log \left| \frac{x+1}{x-1} \right| - \frac{2x}{x^2-1} \right)$
 (7) $\frac{x^2}{2} \log|x| - \frac{x^2}{4}$ (8) 出題ミスです．正しくは $(\operatorname{Arcsin} x)^2$ でした．この場合は部分積分を用いて $x(\operatorname{Arcsin} x)^2 + 2\sqrt{1-x^2} \operatorname{Arcsin} x - 2x$ となります．元の場合は初等関数で表すことができないようです．
- (解法略説) (1), (2) 部分積分を三回行う．(3) $t = e^x$ (4) $t = x^2$ (5) $s = \sqrt{x^2-a^2} + x$
 (6) 部分分数分解 (7) 部分積分