Optimal Taxation of Intermediate Goods in A Partially Automated Society *

Hideto Koizumi

October 23, 2020

Abstract

Recent automating technologies have sparked discussions on "robot taxes," aimed to dissuade the displacement of labor and to generate revenue to redistribute to displaced laborers. Implementing such taxes is challenging, however, in part because of the difficulty in clearly separating which technologies substitute for labor from those which complement it. Modeling automating technologies as intermediate goods, I consider the optimal tax policy in this environment. As in standard models, non-linear labor taxes are assigned without the knowledge of a laborer's type. Additionally, due to tax avoidance concerns and arbitrage opportunities, intermediate goods are uniformly and linearly taxed without the knowledge of their complementarity or substitutability with labor. Despite the potential for automating technologies to be complementary to workers, I find that the optimal tax regime includes a strictly positive tax on these intermediate goods. I discuss the implications of these findings for the robustness of robot-tax policy proposals.

Keywords: Automation, Optimal Taxation of Intermediate Goods, Optimal Taxation, Robot Tax

JEL Codes: H21, H25

"I'm sure I can come up with a robot that isn't a robot, according to the [robot] tax code."— Shu-Yi Oei, a Boston College law professor[1]

*The author thanks Benjamin Lockwood, Juuso Toikka, Alexander Rees-Jones, and Kent Smetters for extensive guidance and instructions in this work. The author also thanks Eduardo Azevedo for his comments and guidance, and the seminar and conference discussants and audiences for useful feedback. This project is funded by Mack Institute Research Grants for presentations abroad.

[1]This quote is excerpted from a Wall Street Journal article, "The 'Robot Tax' Debate Heats Up" on January 8, 2020.

1 Introduction

Historically, routine labor has been displaced by automating technologies. This trend has accelerated during the last three decades (Acemoglu and Restrepo (2019)), leading to increasing inequality (Acemoglu and Restrepo (2020)). Furthermore, using the monthly Current Population Survey, Ding et al. (2020) document that the COVID-19 pandemic has further displaced routine labor with automation.[2] The rapidly declining labor share and rising inequality have led some researchers including Costinot and Werning (2018), Thuemmel (2018), and Guerreiro et al. (2020) to consider taxing robots, to redistribute income to these displaced workers.[3]

Implementing taxes such as a "robot tax" is challenging, however, in part because of the difficulty in clearly separating which intermediate goods perfectly substitute for labor from those which complement it. There are numerous examples of these difficult cases such as self-check-out cash registers vs. conventional cash registers, self-driving trucks vs. conventional trucks, and industrial robots that displace assembly line workers vs. robots designed to augment the productivity of assembly line workers. Screening intermediate goods that perfectly substitute for routine labor from those which complement it is administratively costly and may not be even feasible. Moreover, even if the planner decides to pay these administrate costs and imposes a robot tax,[4] tax avoidance will be a concern as encapsulated by the epigraph above and as documented in Slemrod and Kopczuk (2002).[5]

To address these practical issues, this paper studies the welfare consequences of imposing a tax on intermediate goods (such as robots) when their type cannot be determined by the planner. In particular, my model considers a two-by-two scenario: two types of labor and two types of intermediate goods. Workers are categorized into (low-skill) routine and (high-skill) non-routine labor. Intermediate goods are dichotomized into (i) *displacing intermediate*

[2]Some news articles after the inception of the COVID-19 pandemic, such as the ones from Time (`https://time.com/5876604/machines-jobs-coronavirus/`) and the New York Times (`https://www.nytimes.com/2020/04/10/business/coronavirus-workplace-automation.html`) raise a serious concern that the pandemic accelerates the already concerning automation movement.

[3]Bill Gates suggested a robot tax to compensate for tax revenue lost from the displacement, a different reason from Costinot and Werning (2018), Thuemmel (2018), and Guerreiro et al. (2020)

[4]Or "automation tax" as suggested by Acemoglu et al. (2020). They suggest imposing a higher tax on the use of capital in tasks where labor has a comparative advantage. However, this tax policy also has the same issues of task screening and tax avoidance.

[5]Additionally, see Slemrod and Yitzhaki (2002).

goods (e.g., self-check-out cash registers and self-driving trucks), which are complements to non-routine labor but are perfect substitutes for routine labor and (ii) *complementary intermediate goods* (e.g., conventional cash registers and conventional trucks), which are more complementary to routine labor than to non-routine labor. The two types of intermediate goods are perfect substitutes. I believe that this assumption is natural for modeling automation; for instance, if a firm buys a self-driving truck for a certain distribution task, there is no point to also purchase a conventional truck for this task.

As in standard models, labor taxes must be assigned without the knowledge of a laborer's type and inputs, while I go beyond standard models and additionally impose that intermediate goods must be taxed without the knowledge of their complementarity or substitutability with routine labor. The government observes labor income levels and taxes them in a non-linear schedule. An intermediate good tax is imposed with a uniform, proportional rate over different types of intermediate goods. The non-discriminatory tax rate over different intermediate good types addresses the aforementioned screening and tax avoidance concerns. Moreover, following Guesnerie (1998), I focus on a proportional intermediate good tax: non-linear taxes on intermediate goods could generate arbitrage opportunities in the resale market.

Due to asymmetric information on intermediate good types, there are two opposing forces of non-discriminatory intermediate good taxation in welfare. On the one hand, by taxing *only* displacing intermediate goods, the planner can reduce the wage gap between the two labor types. Taxing a complement of non-routine workers will decrease non-routine-worker wage rates while taxing a substitute for routine workers will increase routine-worker wage rates. The reduction of the wage gap will relax the incentive compatibility constraint of non-routine workers to mimic routine workers and reduce their hours of work to that of routine workers—that is, a reduction in the information rent of non-routine types—in the planner's welfare maximization program. On the other hand, taxing *only* complementary intermediate goods will decrease the wage rates of not only non-routine workers but also routine workers since complementary intermediate goods complement both types of labor. A decrease in the routine-workers' wage rates will decrease welfare and possibly dominate the positive redistribution effects from taxing displacing intermediate goods.

Despite these competing forces and complex settings, I find that there is a simple solution in which on top of optimal income taxation, the planner imposes a strictly positive proportional intermediate good tax that is non-discriminatory over types of intermediate goods, for redistributive purposes. Note that this result holds regardless of the degree of complementarity between routine labor and complementary intermediate goods. Therefore, the result is not driven solely by the relative degree of substitutability and complementarity among the two intermediate good types and routine labor.

A novel key force driving the positive result exploits the price difference between the two types of intermediate goods. As a unique feature of an automation model, perfectly substituting intermediate goods such as robots need zero routine labor by definition. This implies that the price of complementary intermediate goods has to be lower than that of displacing intermediate goods in a partially automated economy since complementary intermediate goods require routine labor for potentially automatable tasks.[6] Thus, if we impose a uniform proportional tax on both types of intermediate goods, the tax burden is placed disproportionally on displacing intermediate goods than complementary intermediate goods since displacing intermediate goods are more expensive. This differential tax burden reduces the wage gap between the two worker types. The reduced wage gap relaxes the incentive compatibility constraints of non-routine workers, resulting in first-order informational gain.

The contribution of this paper is twofold. The first contribution is with regard to the modeling. By studying a model of heterogeneous labor *and* intermediate good types with asymmetric information over *both* labor and intermediate good types, this research addresses the significant policy issues surrounding robot tax policy proposals. To the best of my knowledge, this is the first study to analyze a novel setting with asymmetric information

[6]Note that there are two corner solutions: one without any use of routine labor and thus complementary intermediate goods, and the other without any use of displacing intermediate goods and with the composite of routine labor and complementary intermediate goods and non-routine labor. At the former corner solution, only displacing intermediate goods and non-routine labor will be used in production. Then, the setting becomes equivalent to one of the corner solutions from Guerreiro et al. (2020), and the optimal intermediate good tax rate is zero. This is because there is no redistributive gain from intermediate good taxation in the absence of routine labor. In contrast, at the latter corner solution, only the composite of routine labor and complementary intermediate goods together with non-routine labor will be used in production. In this case, the optimal intermediate good tax rate is generally negative since this setting in the absence of displacing intermediate goods tends to be the opposite case of the interior solution of Guerreiro et al. (2020). These are trivial cases and thus will not be explored in the main text.

over both labor and intermediate good types in the optimal taxation literature.

The second contribution is the discovery of a new force to mitigate the aforementioned asymmetric information problems by a non-discriminatory (linear) tax rate. The planner wants to tax the use of automating technologies but wants to subsidize the use of complementing technologies. In automation models, since automating technologies are perfect substitutes for routine labor while complementary technologies require routine labor, the factor price of complementary technologies has to be lower than that of automating counterparts. Then, even if the planner imposes a positive, uniform proportional tax over these different technologies, it results in differential burden on different technology types and thus reduces the asymmetric information problems. Despite the level of complexity my model permits, this novel channel leads to a simple, practical solution for the robustness of robot tax policy proposals.

1.1 Literature Review

As the seminal paper of optimal taxation of intermediate goods, Diamond and Mirrlees (1971a) find no production distortion to be optimal. They demonstrate that if the planner can tax net trades of different goods at different linear rates, then taxing intermediate goods is suboptimal. This condition in Diamond and Mirrlees (1971a) implies that the planner can distinguish between different labor types and can impose different tax rates over different labor types even when these workers earn the same income, violating the premise of my model. In fact, if the government can implement different tax schedules for different labor types, this will achieve perfect redistribution, leaving no room for intermediate good taxation in my model.

And yet, the restriction to the same income tax schedule is insufficient to justify production distortion for redistribution. Atkinson and Stiglitz (1976) find that if workers with different productivities are perfect substitutes, then production efficiency is still optimal. My model assumes complementarity between routine and non-routine labor, and thus their result is not applicable.

Building on the work of Stiglitz (1982), Naito (1999) is the first study to show that sacrificing production efficiency for redistribution may be optimal when heterogeneous labor types are imperfect substitutes. The important channel of redistribution in Naito (1999) is to indirectly increase the wage of low-skill workers and decrease that of high-skill workers by taxing skilled labor-intensive goods that are complements to high-skill workers but substitutes for low-skill workers.

This channel is used by the three recent papers mentioned in the introduction section that find a positive robot tax rate to be optimal for income redistribution. One is Guerreiro et al. (2020), the closest paper to mine.[7] They question if it is welfare-improving to tax robots, using the task-based framework emphasized by Autor et al. (2003) and Acemoglu and Restrepo (2018). In their settings, there are two types of labor, routine and non-routine, between which the policymaker cannot distinguish or cannot impose different tax rates based on labor types, and one type of intermediate goods—robots. Robots are complements to non-routine workers, but substitutes for routine workers. In their static model, occupations are fixed, and routine workers cannot switch to the sector of non-routine labor, while workers are allowed to change labor supply amounts. Guerreiro et al. (2020) find an optimal robot tax to be, in general, strictly positive, using the channel of Naito (1999).

Note that they analyze a dynamic extension, focusing on their rich calibration-based quantitative analysis of a dynamic version of their static model. In their dynamic model, they incorporate an endogenous skill choice process with heterogeneous skill acquisition costs. This introduces an additional negative effect of a positive robot tax since it reduces incentives to acquire non-routine skill sets. Therefore, the optimality of a robot tax becomes a quantitative question in the dynamic model. They calibrate the dynamic model with a geometrically declining robot cost and find a positive robot tax at the beginning when the cost of robots is still high. Given their result, the optimality of an intermediate good tax in the dynamic model with the additional intermediate good type is also a quantitative question. For this reason and partly because of technical and computational difficulties,[8]

[7]Among many related papers such as the direct taxation literature exemplified by Diamond and Mirrlees (1971a,b), Atkinson and Stiglitz (1976), Deaton (1979), Saez (2001, 2004), Rothschild and Scheuer (2013), Scheuer (2014), Stantcheva (2014), and Gomes et al. (2017), (2) the indirect taxation literature exemplified by Naito (1999), Saez (2002), Slavík and Yazici (2014), and (3) the new dynamic public finance literature exemplified by Golosov et al. (2003), Kocherlakota (2005), Golosov et al. (2006), Golosov et al. (2011), Farhi and Werning (2013), Golosov et al. (2016), Stantcheva (2017), I select and discuss on the most closely related ones in this section.

[8]As shown in the 2019 version of Guerreiro et al. (2020), a dynamic model can allow for greater generality exemplified in functional forms of production function. This is true, however, only when

I focus on a static environment and short-term analyses with the same assumption that routine workers of the current generation cannot afford for sufficient education to become non-routine workers, while both labor types can adjust labor supply amounts. In contrast to their static model, I incorporate heterogeneous intermediate good types and asymmetric information over these types to deal with the aforementioned policy issues.[9]

The other two papers, Thuemmel (2018) and Costinot and Werning (2018), quantitatively find a positive robot tax rate to be optimal, using the same channel found by Naito (1999). Thuemmel (2018), a concurrent paper to Guerreiro et al. (2020), is similar to Guerreiro et al. (2020) but has richer elements in that Thuemmel (2018) allows for an additional labor type and heterogeneity within occupations, which are not present in my model. His model further permits endogenous occupational choice that renders the sign of an optimal robot tax rate theoretically ambiguous, and the focus of his paper is to provide a rigorous quantitative analysis. However, he also does not incorporate the additional type of intermediate goods.

Costinot and Werning (2018) study the optimal robot tax rate using a sufficient statistics approach. The goal of their study "is not to sign the tax on robots, nor to explore a particular production structure, but instead to offer tax formulas highlighting key sufficient statistics needed to determine the level of taxes, with fewer structural assumptions" (p. 4). The important assumption in their setting is that the planner knows which firms use "new technology" such as robots and which firms use "old technology" involving conventional non-robotic intermediate goods. One can translate displacing intermediate goods as "new technology firms" and complementary intermediate goods as "old technology firms." In this sense, they study a scenario where there is no asymmetric information on intermediate good types in my model without structural assumptions. In this paper, my main focus is on a

there is only one type of capital/intermediate goods. I have conducted similar exercises of their 2019 version with my model of two intermediate goods and found that it quickly gets analytically intractable and computationally difficult.

[9]Displacing intermediate goods in my model corresponds to robots in their model. Note that Slavík and Yazici (2014) have two different types of intermediate goods in their dynamic model as well: structure and equipment. Yet, they have no asymmetric information on intermediate good types in their settings and thus do not allow for the situation of our interests.

case that involves asymmetric information over these intermediate good types.

2 Model Environment: Key Players in the Economy And Equilibrium Conditions

I focus on a static environment in which intermediate good taxation can be interpreted such that the planner reduces the total amount (undiscounted sum) of allowable deductions by the Internal Revenue Service (IRS) through tax depreciation, a widely practised tax policy in the world.[10] This paper follows the notation of Guerreiro et al. (2020), so that readers can closely compare the two papers.

There are two types of households: routine and non-routine labor households. Their utility is based on the consumption of private goods, while their disutility comes from labor. As assumed in Naito (1999) and the static version of Guerreiro et al. (2020), I focus on a short-term analysis with fixed occupations in which the current generation of routine labor households cannot afford sufficient education to become non-routine labor households, while both households are allowed to change labor supply amounts. This assumption allows me to succinctly delineate the novel key force to mitigate asymmetric information over intermediate good types. Routine labor, non-routine labor, and displacing and complementary intermediate goods all contribute to the production of a single consumption good. Note that in the main body of this paper, to highlight the aforementioned novel key force, I drop the term for the consumption of public goods, while all of the analytical results carry through with the addition of the term, as depicted in the Appendix.

There is a continuum of symmetric tasks in the unit range $[0, 1]$ where routine workers together with complementary intermediate goods compete against displacing intermediate goods.[11] Guerreiro et al. (2020) assume constant efficiency of inputs across tasks in their

[10]For example, the U.S. federal government allows firms to deduct tax for the purchase of depreciable assets by the amount equal to the the asset values over time following the depreciation schedule set by the Internal Revenue Service. In the static version of my model, the federal planner can impose a tax on intermediate goods by reducing the total deduction amounts of such purchase. For example, if a firm purchases \$70,000 depreciable assets that follows 7-year depreciation schedule, currently, the firm is allowed to deduct the entire \$70,000 in year one (written off as an expense) or deduct \$10,000 over seven years. The federal planner can impose 10% intermediate good tax by reducing the total deduction amount to \$63,000.

[11]See Autor et al. (2003) for the importance of tasks performed by routine workers.

static model, which renders their production function equivalent to the production function of Autor et al. (2003). I incorporate tasks in my model not only to closely follow Guerreiro et al. (2020) for comparison, but also to precisely elicit what assumptions warrant the aggregation of production. Readers can skip the task aggregation part and would not lose any important insight. Intermediate good producers for both intermediate good types come from perfectly competitive, external (global) markets. The final good producer faces a production function featuring constant returns to scale and thus zero profits.

2.1 Household

The economy has a continuum of households with a unit measure. These households are decomposed to π_n non-routine worker households and π_r routine worker households, where subscript n and r denote the non-routine and routine labor types, respectively. A household of type $j \in \{n, r\}$ enjoys utility from the consumption of private goods, c_j. Again, note that in the main body of this paper, to highlight the aforementioned novel key force, I drop the term for the household consumption of public goods, while all of the analytical results carry through with the term as depicted in the Appendix. Each household experiences disutility from the hours of labor it supplies, l_j. Every household has one unit of time, leading to $l_j \leq 1$. A household of type j's optimization problem is

$$\begin{aligned} &\underset{c_j, l_j}{\text{maximize}}\ U_j = u(c_j, l_j) \\ &\text{subject to } c_j \leq w_j l_j - T(w_j l_j), \end{aligned}$$

where w_j denotes the wage rate of type j and $T(.)$ indicates the income tax schedule.[12] Note that the price of a single consumption good is normalized to 1.

For convenience, write $u_x = \partial u(c, l)/\partial x$ where $x = c, l$ and $u_{xy} = \partial^2 u(c, l)/\partial x \partial y$. I make

[12] In the appendix for general results, the maximization problem becomes

$$\begin{aligned} &\underset{c_j, l_j}{\text{maximize}}\ U_j = u(c_j, l_j) + v(G) \\ &\text{subject to } c_j \leq w_j l_j - T(w_j l_j), \end{aligned}$$

with assumptions that $v'(G) > 0, v''(G) < 0$.

the standard concavity and convexity assumptions that, $u_c > 0, u_l < 0, u_{cc}, u_{ll} < 0$, and that consumption and leisure are both normal goods. Then, we are given $u_{ll}l/u_l + 1 - u_{cl}l/u_c > 0$. Additionally, I assume that $u(c,l)$ satisfies the standard Inada conditions for interior solutions.

2.2 Intermediate Good Producers

Both types of intermediate goods are produced by perfectly-competitive intermediate good producers in the external (global) market and are used in automatable tasks $i \in [0,1]$. Subscript d corresponds to displacing intermediate goods and subscript c corresponds to complementary intermediate goods. Following Guerreiro et al. (2020), I assume that tasks are symmetric—i.e., there is no difference among tasks in needs for productivity. Then, the price of intermediate good type $k \in \{c,d\}$, $p_k(i)$, is same across tasks—that is, $p_k(i) = p_k \; \forall i$—and is equal to marginal costs of ϕ_k units of output.

Following the literature, a change in the income flows to these producers in response to intermediate good taxation will not be considered. If the welfare function includes such income changes, then the optimal intermediate good tax is expected to be even higher. This is because non-routine workers are expected to own more intermediate goods than routine workers, making greater room for redistribution. Therefore, my main result is a lower bound in this sense.

2.3 Final good producer

The representative producer of the final consumption good employs non-routine workers (N_n) for a single non-automatable task, routine labor ($n(i)$) for symmetric, automatable task i, and buys intermediate goods, displacing intermediate goods ($x_d(i)$) and complementary intermediate goods ($x_c(i)$), for the same automatable task i as well. For routine task i, the final good producer faces the following task production function:

$$y(i) = \kappa_i x_d(i) + \left[\beta o(i) x_c(i)^{\frac{q-1}{q}} + (1-\beta) z(i) n(i)^{\frac{q-1}{q}}\right]^{\frac{q}{q-1}}, \tag{1}$$

where q indicates an elasticity of substitution between routine workers and complementary intermediate goods, while $\kappa(i), o(i), z(i)$ represent the efficiency parameters of displacing intermediate goods, complementary intermediate goods, and routine labor, respectively. Note that as assumed in the static model of Guerreiro et al. (2020), for tractability, I also assume constant efficiency across tasks—that is, $\kappa(i) = o(i) = z(i) = 1$.[13] Furthermore, for routine labor and complementary intermediate goods to be gross-complements, q is assumed to be between 0 and 1. As implicitly assumed in Guerreiro et al. (2020), I assume the task production function is integrable on interval $[0,1]$. With these routine tasks, the producer has the following final production function:

$$Y = A\left(\int_0^1 y(i)^{\frac{\rho-1}{\rho}} di\right)^{\frac{\rho}{\rho-1}(1-\alpha)} N_n^{\alpha} \quad \alpha \in (0,1), \rho \in [0,\infty), \tag{2}$$

where ρ is an elasticity of substitution between tasks. Let m denote the share of task production by displacing intermediate goods in automatable tasks —that is, $m = \frac{\int_0^1 x_d(i)di}{\int_0^1 y(i)di}$. For convenience, since tasks are symmetric and have no difference in needs for productivity, I write that for the range $[0,m]$, the final good producer only uses displacing intermediate goods, while for the range $(m,1]$, the final good producer only uses the composite of routine workers and complementary intermediate goods. Thus, m is a choice variable for the final good producer.

Then, we can divide the integral into the two ranges and rewrite the production function as:

$$Y = A\left[\int_0^m x_d(i)^{\frac{\rho-1}{\rho}} di + \left(\int_m^1 \left[\beta x_c(i)^{\frac{q-1}{q}} + (1-\beta) n(i)^{\frac{q-1}{q}}\right]^{\frac{\rho-1}{\rho}} di\right)^{\frac{q}{q-1}}\right]^{\frac{\rho}{\rho-1}(1-\alpha)} N_n^{\alpha},$$
$$\alpha \in (0,1), \rho \in [0,\infty). \tag{3}$$

[13]Note that this way of defining automated and not-yet automated tasks is different from Acemoglu and Restrepo (2018). This assumption abstracts away from the *quality* of tasks a new technology destroys and creates, and it restriction attention to the *quantity* of tasks being destroyed and created.

With the production function, the final good producer's problem is to maximize profits,

$$Y - w_n N_n - w_r \int_m^1 n(i)di - (1+\tau_x) \int_0^1 \left(\phi_d x_d(i) + \phi_c x_c(i)\right) di, \tag{4}$$

where τ_x is an ad-valorem uniform tax rate on intermediate goods. Notice that my production function features constant returns to scale, leading to the zero profits property.

Proposition 1 below simplifies the production function (3). The intuition behind the derivation is that since all the marginal costs (ϕ_d, ϕ_c, w_r) for automatable tasks are constant across tasks, the parameter for the elasticity of substitution gets dropped. For readability, denote by X_d, X_c, and N_r the total amounts of displacing intermediate goods, complementary intermediate goods, and routine labor, respectively.

Proposition 1. *We can simplify the production function as*

$$Y = A\left(X_d + \left(\beta X_c^{\frac{q-1}{q}} + (1-\beta) N_r^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}}\right)^{1-\alpha} N_n^{\alpha} \tag{5}$$

Note that my production function contains that from Guerreiro et al. (2020) as a special case. To see this, let $q \longrightarrow \infty$ so that it becomes

$$Y = A\left(X_d + \beta X_c + (1-\beta)N_r\right)^{1-\alpha} N_n^{\alpha}.$$

If $\beta = 0$, then the production function becomes equivalent to that in the static model of Guerreiro et al. (2020) and the production function of Autor et al. (2003).

2.4 Government

In the main body for analytical results, while the government sets tax rates, for clarity, it does not provide any public good using the revenue. I bring back the public good provision in the Appendix for general results in which the government faces the budget constraint:

$$G \leq \pi_r T\left(w_r l_r\right) + \pi_n T\left(w_n l_n\right) + \tau_x \int_0^1 \left[\phi_d x_d\left(i\right) + \phi_c x_c(i)\right] di, \tag{6}$$

where G represents the amount of public good provision. Note that I can also include this budget constraint with an exogenously given government spending target G, and all of the theoretical results still hold true.

2.5 Equilibrium

An equilibrium is defined as the collection of a set of allocations $\{c_r, l_r, c_n, l_n, N_r, N_m, X_d, X_c, x_d(i), x_c(i), n(i), m\}$, prices $\{w_r, w_n, p_x\}$, and a tax system $\{T(.), \tau_x\}$ such that: (i) given prices and taxes, allocations solve the households' problem; (ii) given prices and taxes, allocations solve the firms' problem; and (iii) markets clear.[14]

The market-clearing conditions for routine and non-routine labor are given by, for arbitrary task i,

$$(1-m)n(i) = N_r = \pi_r l_r, \tag{7}$$

$$N_n = \pi_n l_n. \tag{8}$$

The market-clearing condition for displacing intermediate goods is, for arbitrary task i,

$$m x_d(i) = X_d, \tag{9}$$

while that for complementing robots is

$$(1-m)x_c(i) = X_c, \tag{10}$$

Similarly, the market-clearing condition for the output market is

$$\pi_r c_r + \pi_n c_n \leq Y - \int_0^1 \left(\phi_d x_d(i) + \phi_c x_c(i)\right) di. \tag{11}$$

Also, note that we get an interior solution only if for a given level of total task-output, the total costs are the same between displacing intermediate goods and the composite of

[14]In the generalized version and numerical analysis, an equilibrium further includes government spending in the collection of a set of allocations and condition (iv) the government budget constraint is satisfied.

routine workers and complementary intermediate goods. Equivalently,

$$(1+\tau_x)\phi_d = \frac{w_r n(i) + (1+\tau_x)\phi_c x_c(i)}{\left(\beta x_c(i)^{\frac{q-1}{q}} + (1-\beta) n(i)^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}}} = \frac{w_r N_r + (1+\tau_x)\phi_c X_c}{\left(\beta X_c^{\frac{q-1}{q}} + (1-\beta) N_r^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}}}. \tag{12}$$

This assumption of the equality of total costs comes from the static model of Guerreiro et al. (2020), and we can interpret this—the producer's indifference between choosing displacing intermediate goods and the composite of routine labor and complementary intermediate goods—in the following way. Suppose there are enough tasks for routine labor and complementary intermediate goods, and suppose initially, the effective cost of displacing intermediate goods is strictly higher than that of the composite of routine labor and complementary intermediate goods. Then, whether by bargaining or by an increase in routine labor demand, the wage of routine labor goes up, and greater routine labor will be supplied. This iterative process continues until the inequality becomes equality. At the equality after the iterative process, I assume that the final good producer does not fire the already-employed routine workers and suddenly substitute displacing intermediate goods for routine labor completely. Rather, the tie-breaking rule is that the producer first hires the full amount of labor supplied at the endogenous wage level w_r and exogenous costs of intermediate goods, and then adopts displacing intermediate goods for the rest of tasks.

The constant marginal cost of each input implies that at an interior solution, $x_d(i) = \left(\beta x_c(i)^{\frac{q-1}{q}} + (1-\beta) n(i)^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}}$ for every i. This implies the following relation at an equilibrium with partial automation ($m \in (0,1)$):

$$x_d(i) = \frac{X_d}{m} = \frac{\left(\beta X_c^{\frac{q-1}{q}} + (1-\beta) N_r^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}}}{(1-m)}, \text{ for } i \in [0,m]. \tag{13}$$

Notice that using the fact that $m x_d(i) = X_d$ and $x_d(i) = \left(\beta x_c(i)^{\frac{q-1}{q}} + (1-\beta) n(i)^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}}$ for every task i, one can also express m as

$$m = \frac{X_d}{X_d + \left(\beta X_c^{\frac{q-1}{q}} + (1-\beta) N_r^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}}}. \tag{14}$$

I provide equilibrium conditions for non-limit cases where $0 < q < 1$, and the limit cases are provided in the Appendix. At an interior solution, by taking the first-order conditions of the profit function with respect to the inputs and plugging in the equilibrium conditions above, we get

$$w_n = \alpha \frac{A^{1/\alpha}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{[(1+\tau_x)\,\phi_d]^{\frac{1-\alpha}{\alpha}}}, \tag{15}$$

$$w_r = (1-\beta)(1+\tau_x)\phi_d \left(\frac{(1-\beta)\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1} - \beta} \right)^{\frac{1}{q-1}}. \tag{16}$$

Note that when $0 < \left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1} < \beta$, $w_r < 0$. In other words, as long as $\frac{\phi_c}{\phi_d} < \beta^{\frac{q}{q-1}}$, $w_r > 0$. Then, we can derive

$$m = 1 - \left(\frac{(1+\tau_x)\phi_d}{(1-\alpha)A}\right)^{\frac{1}{\alpha}} \frac{\pi_r l_r}{\pi_n l_n} \left(\frac{\beta(1-\beta)}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1} - \beta} + (1-\beta) \right)^{\frac{q}{q-1}}. \tag{17}$$

What is important here is that w_r has a positive relation and w_n and m have inverse relations with τ_x.

2.6 Main Result: Asymmetric Information at Both Household And Production Sides

In this section, the planner can observe neither household types nor intermediate good types. What the planner observes are households' reported income levels and purchased amounts of intermediate goods. For the reasons highlighted in Guesnerie (1998), I focus on a linear intermediate good tax. Note that in contrast to Mirrlees (1971), the productivities of different agents are endogenous; they depend upon τ_x. This feature of my model induces trade-offs between redistribution and production efficiency by imposing an intermediate good tax. To avoid degenerate cases, I focus on the case with partial automation, $0 < m < 1$. Furthermore, I assume that $w_n \geq w_r$ in an equilibrium.

In the Mirrlees' settings, the policymaker's problem is to choose allocations $\{c_j, l_j\}_{j=r,n}$ and a uniform intermediate good tax rate τ_x to maximize the following utilitarian social welfare:

$$\pi_r \omega_r U_r + \pi_n \omega_n U_n, \tag{18}$$

where ω_j are social weights for household of type j. These weights are normalized in a way that $\pi_r \omega_r + \pi_n \omega_n = 1$. Following Guerreiro et al. (2020), I also focus on a case $\omega_r \geq 1$, so that the planner puts either equal or more weights on routine workers. After plugging in the equilibrium conditions to the output market clearing condition, we can get the following resource constraint that the planner faces:

$$\pi_r c_r + \pi_n c_n \leq \pi_n w_n l_n \left(\frac{\tau_x + \alpha}{\alpha(1+\tau_x)} \right) + \frac{\pi_r w_r l_r}{(1+\tau_x)}. \tag{19}$$

The planner also faces households' incentive compatibility problems. For households of type j to choose bundles $\{c_j, l_j\}$, the bundle must yield at least as high as the utility level of any other arbitrary bundle choice $\{c, l\}$ that satisfies the budget constraint $c \leq w_j l - T(w_j l)$. This implies that $u(c_j, l_j) \geq u(c, l)$. In particular, routine workers must prefer their own-type bundle, $\{c_r, l_r\}$, to the bundle that they would obtain from masquerading the non-routine type by adjusting their hours of work, $\{c_n, w_n l_n / w_r\}$. Also, non-routine workers must prefer their own-type bundle, $\{c_n, l_n\}$, to the bundle that they would obtain from masquerading the routine type by adjusting their hours of work, $\{c_r, w_r l_r / w_n\}$. Note that an important assumption here is that mimicking the other type will not alter that worker's productivity.[15] We can write these points as the following two incentive compatibility constraints

$$u\left(c_n, l_n\right) \geq u\left(c_r, \frac{w_r}{w_n} l_r\right), \tag{20}$$

$$u\left(c_r, l_r\right) \geq u\left(c_n, \frac{w_n}{w_r} l_n\right). \tag{21}$$

Recall that the wages of the two types of households are given by (15) and (16). These are necessary conditions for the household optimality and thus for an equilibrium.

On the other hand, given that the exogenously given costs of intermediate goods, there

[15]See Scheuer and Werning (2016) for discussion on this point.

is no need for an explicit incentive compatibility constraint of firms to be written. This is because the restriction to uniform linear tax rates allows τ_x to determine firms' behavior and eliminates the incentive issues of firms' tax avoidance attempts by misreporting types of intermediate goods.

In the following lemma, I show that the resource constraint (19) and the incentive compatibility constraints (20) and (21) are also sufficient conditions for an equilibrium. To establish that, the planner can set an income tax schedule such that for example, the government will appropriate the entire income if a household reports income levels other than $w_r l_r$ and $w_n l_n$. The proof is provided in the appendix.

Lemma 1. *The resource constraint (19) and the incentive compatibility constraints (20) and (21) are necessary and sufficient conditions for an equilibrium.*

With this result, the Mirrleedian planning problem is to choose the allocations $\{c_j, l_j\}_{j=n,r'}$ and intermediate good tax rate τ_x. Following Guerreiro et al. (2020), I focus on cases where the IC constraint for non-routine workers binds, while that for routine workers slacks. This is a scenario that Stiglitz (1982) calls a *normal case*. This assumption holds in my numerical exercises that are available upon request, as long as $w_n \geq w_r$.

Note that the expression for the net output in the right-hand side of (19) can be rewritten as:

$$\frac{\tau_x + \alpha}{\alpha\left(1+\tau_x\right)^{1/\alpha}} \frac{\alpha A^{1/\alpha}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\phi_d^{\frac{1-\alpha}{\alpha}}} \pi_n l_n + \pi_r l_r (1-\beta)\phi_d \left(\frac{(1-\beta)\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1} - \beta} \right)^{\frac{1}{q-1}} \tag{22}$$

Notice that the term $\dfrac{\tau_x + \alpha}{\alpha\left(1+\tau_x\right)^{1/\alpha}}$ is equal to one when $\tau_x = 0$ and strictly less than one when $\tau_x \neq 0$. This term measures the production inefficiency caused by the tax on intermediate goods.

As discussed in Guerreiro et al. (2020), in the absence of complementary intermediate goods, the key force to make the intermediate good tax positive is the reduction in the wage gap. This, in turn, relaxes the planner's information constraint from the non-routine workers' incentive compatibility, and therefore the planner can improve welfare by this informational gain, a similar intuition to Naito (1999). The problem with the uniform intermediate good tax is that it may lower the wage of routine workers. This negative effect can dominate the informational gain, and thus it appears infeasible to determine the sign of the uniform tax with generality, at a glance.

Recall the intuition behind the main result mentioned in the introduction. Due to the price differences between the two types of intermediate goods, a uniform intermediate good tax indirectly imposes differential tax burden on the two intermediate good types. This additional key force renders the sign of the uniform tax theoretically unambiguous at an interior solution.

Proposition 2. *Assume that the optimal allocation is such that the incentive compatibility constraint for non-routine workers binds, but that for routine workers does not bind. Then, at an interior solution ($0 < m < 1$ and $l_r > 0$), the optimal uniform linear intermediate good tax is strictly positive, regardless of the value of $q \in [0,1]$. The optimal intermediate good tax rate generally satisfies*

$$\frac{\tau_x}{1+\tau_x} = \frac{\alpha}{1-\alpha}\frac{\pi_r l_r}{\pi_n}\left[\widetilde{\omega}_r\left(-u_l\left(c_r, l_r\right)\right) - (1-\beta)\phi_d\left(\frac{(1-\beta)\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1} - \beta}\right)^{\frac{1}{q-1}}\right] \tag{23}$$

where $\widetilde{\omega}_r = \omega_r/\mu$, for μ being the Lagrange multiplier on the resource constraint.

3 Extension

3.1 Dynamic Model

Guerreiro et al. (2020) analyze a dynamic extension, focusing on their calibration-based quantitative analysis. In their dynamic model, they incorporate an endogenous skill choice process with heterogeneous skill acquisition costs. This addition introduces an additional negative effect of a positive robot tax since it reduces incentives to acquire non-routine skill sets. Therefore, the optimality of a robot tax becomes a quantitative question in the dynamic model. Given their result, the optimality of an intermediate good tax in the dynamic model with the additional intermediate good type is also a quantitative question. For this reason and because of technical and computational difficulties, I do not explore the dynamic extension in this paper. While my exploration on the dynamic extension is available upon request, the analysis does not provide any further theoretical insight and is thus omitted from this paper.

3.2 Second-best: No Asymmetric Information over Intermediate Good Types

I also omitted from this paper my analysis on a situation where the government cannot perfectly distinguish/discriminate between two types of labor but can perfectly distinguish between two types of intermediate goods. This perfect differentiation assumption renders my model similar to Guerreiro et al. (2020). Consequently, the corresponding theoretical results are similar: a positive tax rate on displacing intermediate goods and a negative tax rate on complementary intermediate goods. Optimality can be achieved by either of (i) just a positive tax on displacing intermediate goods, (ii) just a negative tax on complementary intermediate goods, or (iii) the combination of these, depending on the relative cost of intermediate goods and the degree of complementarities between routine labor and complementary intermediate goods. These theoretical results are available upon request.

3.3 Numerical Analyses

Since my main focus is to succinctly delineate the novel key force to mitigate asymmetric information over intermediate good types, I also omitted from this paper my simulation analyses. My numerical analyses follow the settings used by the simulation analyses of the static model in the 2019 version of Guerreiro et al. (2020) that utilize parameter estimates from other papers. Depending on the relative cost of intermediate goods and the degree of complementarities between routine labor and complementary intermediate goods, I demonstrate my third-best outcome can be comparable to the aforementioned second-best outcome in terms of the welfare improvement measured by an increase in the routine laborer's consumption amount, through a positive intermediate good tax. These numerical results are available upon request as well.

4 Conclusion & Limitations

This paper finds that despite the asymmetric information problems, the optimal uniform intermediate good tax rate over different types of intermediate goods is strictly positive, as long as the solution is interior. My main focus is to succinctly delineate the novel key force to mitigate asymmetric information over intermediate good types. Here, I acknowledge the limitations of this study. First, my model focuses on short-term analyses and assumes away occupational choices between routine and non-routine sectors for analytical tractability. As mentioned in the previous sections, the optimality of a robot tax becomes a quantitative question with endogenous occupational choice in both static and dynamic cases. Given the focus of this study on a new channel to mitigate asymmetric information, and given technical and computational difficulties, I do not relax this assumption in this paper.

Another limitation is that I assume perfectly competitive external global markets for intermediate goods producers. One possible extension is to relax this assumption and to incorporate the R&D process of intermediate good producers along the lines of Aghion et al. (2013). In this case, a uniform intermediate good tax that results in differential tax burden on different intermediate good types would create a resource wedge in R&D between displacing and complementary intermediate goods, leading to differential technological progress over time. This could further drive up the optimal uniform intermediate good tax rate in the long run.

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Appendices

A Proof of Proposition 1

By the integrability assumption mentioned above, we can differentiate the profit function (4) with respect to $x_d(i), x_c(i)$, and $n(i)$. Then, the optimal choices of N_n, $x_d(i)$ for $i \in [0, m]$, $n_i, x_c(i)$ for $i \in (m, 1]$ require the following first-order conditions:

$$w_n = \frac{\alpha Y}{N_n}, \tag{24}$$

$$\begin{aligned}(1+\tau_x)\,\phi_d =& (1-\alpha)Y\left(\int_0^m x_d\,(s)^{\frac{\rho-1}{\rho}}\,ds + \left(\int_m^1 \left[x_c(s)^{\frac{q-1}{q}} + n(s)^{\frac{q-1}{q}}\right]^{\frac{\rho-1}{\rho}} ds\right)^{\frac{q}{q-1}}\right)^{-1} \\ & x_d(i)^{-\frac{1}{\rho}},\ \text{for } i \in [0, m],\end{aligned} \tag{25}$$

$$\begin{aligned}(1+\tau_x)\phi_c =& (1-\alpha)Y\left(\int_0^m x_d\,(s)^{\frac{\rho-1}{\rho}}\,ds + \left(\int_m^1 \left[\beta x_c(s)^{\frac{q-1}{q}} + (1-\beta)n(s)^{\frac{q-1}{q}}\right]^{\frac{\rho-1}{\rho}} ds\right)^{\frac{q}{q-1}}\right)^{-1} \\ & \left(\int_m^1 \left[\beta x_c(z)^{\frac{q-1}{q}} + (1-\beta)n(z)^{\frac{q-1}{q}}\right]^{\frac{\rho-1}{\rho}} dz\right)^{\frac{1}{q-1}} \beta x_c(i)^{-\frac{1}{q}},\ \text{for } i \in (m, 1].\end{aligned} \tag{26}$$

$$\begin{aligned}w_r =& (1-\alpha)Y\left(\int_0^m x_d\,(s)^{\frac{\rho-1}{\rho}}\,ds + \left(\int_m^1 \left[\beta x_c(s)^{\frac{q-1}{q}} + (1-\beta)n(s)^{\frac{q-1}{q}}\right]^{\frac{\rho-1}{\rho}} ds\right)^{\frac{q}{q-1}}\right)^{-1} \\ & \left(\int_m^1 \left[\beta x_c(z)^{\frac{q-1}{q}} + (1-\beta)n(z)^{\frac{q-1}{q}}\right]^{\frac{\rho-1}{\rho}} dz\right)^{\frac{1}{q-1}} (1-\beta)n(i)^{-\frac{1}{q}},\ \text{for } i \in (m, 1].\end{aligned} \tag{27}$$

From (25), it follows that in the m automated tasks, the optimal level of displacing robots is equal across tasks. From (26) and (27), it also follows that in the $1-m$ unautomated tasks, the optimal level of routine labor and cobots are also the same across tasks. Hitherto, whenever possible, we drop the task index i for simplicity. Note that the final producer will choose $m = 0$ whenever the effective price of displacing intermediate goods is higher than that of the combination of complementary intermediate goods and routine labor:

$$(1+\tau_x)\phi_d > \frac{w_r n(i) + (1+\tau_x)\phi_c x_c(i)}{\left(\beta x_c(i)^{\frac{q-1}{q}} + (1-\beta) n(i)^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}}}. \tag{28}$$

They will choose $m = 1$ instead if the inequality above is the opposite.[16] We will observe $0 < m < 1$ if the inequality is equality. The interpretation of this assumption that is also made in the static model of Guerreiro et al. (2020) is explained in the main body of this paper above.

Then, as partial automation, we get $x_d(i) = \left[\beta x_c(i)^{\frac{q-1}{q}} + (1-\beta) n(i)^{\frac{q-1}{q}}\right]$. Thus, at partial automation, we can drop the substitution parameters for tasks, ρ. Then, we get

$$Y = A\left[\int_0^m x_d(i)di + \left(\int_m^1 \left[\beta x_c(i)^{\frac{q-1}{q}} + (1-\beta) n(i)^{\frac{q-1}{q}}\right] di\right)^{\frac{q}{q-1}}\right]^{(1-\alpha)} N_n^{\alpha},$$
$$\alpha \in (0,1), \rho \in [0,\infty). \tag{29}$$

Also, since the amount of $x_c(i)$ is the same across tasks, and since that of $n(i)$ is the same across tasks, we can write[17]

$$Y = A\left[\int_0^m x_d(i)di + \int_m^1 \left[\beta x_c(i)^{\frac{q-1}{q}} + (1-\beta) n(i)^{\frac{q-1}{q}}\right]^{\frac{q}{q-1}} di\right]^{(1-\alpha)} N_n^{\alpha},$$
$$\alpha \in (0,1), \rho \in [0,\infty). \tag{30}$$

What's more, at partial automation, we get

$$m x_d(i) = X_d, \text{ for } i \in [0,m], (1-m)x_c(i) = X_c, \text{ and } (1-m)n(i) = N_r,$$
$$\text{for } i \in (m,1], \tag{31}$$

[16] Note that the denominator in (28) is a certain output level $y(i)$

[17] It might be easier to see this with finite summation. If the sum is finite, then the integral is $\left(\sum_{i=m}^1 x_c(i)^{\frac{q-1}{q}} + \sum_{i=m}^1 n(i)^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}}$. Since $x_c(i) = x_c(i')$ and $n(i) = n(i')$ for $i \neq i'$, the sum equals to $\left([(1-m)x_c(i)]^{\frac{q-1}{q}} + [(1-m)n(i)]^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}} = (1-m)[x_c(i)^{\frac{q-1}{q}} + n(i)^{\frac{q-1}{q}}]^{\frac{q}{q-1}}$

where X_d is the total amount of displacing robots, X_c is that of complementing robots, and N_r is the total employment of routine labor, as defined in the main text[18]. Note that by definition, we have $m = \frac{X_d}{X_d + \left(\beta X_c^{\frac{q-1}{q}} + (1-\beta) N_r^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}}}$ if $m > 0$. Then, we can rewrite the production function as

$$Y = A \left(X_d + \left(\beta X_c^{\frac{q-1}{q}} + (1-\beta) N_r^{\frac{q-1}{q}} \right)^{\frac{q}{q-1}} \right)^{1-\alpha} N_n^{\alpha}$$

.

[18]Note that $(1-m)x_c(i) = X_c$, and $(1-m)n(i) = N_r$ implies $(1-m)[x_c^{\frac{q-1}{q}} + n^{\frac{q-1}{q}}]^{\frac{q}{q-1}} = \left([(1-m)x_c]^{\frac{q-1}{q}} + [(1-m)n]^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}} = \left(X_c^{\frac{q-1}{q}} + N_r^{\frac{q-1}{q}}\right)^{\frac{q}{q-1}}$.

B Proof of Lemma 1

A $q \longrightarrow 1$

Proof. The final good producer optimizes inputs and get

$$x_c(i) = \begin{cases} \frac{\pi_r l_r}{(1-m)} \left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{1}{\beta-1}}, & i \in (m,1] \\ 0, & \text{otherwise} \end{cases} \tag{32}$$

$$n(i) = \begin{cases} \frac{\pi_r l_r}{(1-m)}, & i \in (m,1] \\ 0, & \text{otherwise} \end{cases} \tag{33}$$

$$x_d(i) = \begin{cases} \frac{\pi_r l_r}{(1-m)} \left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{\beta}{\beta-1}}, & i \in [0,m] \\ 0, & \text{otherwise} \end{cases} \tag{34}$$

$$m = \max\left\{1 - \left(\frac{(1+\tau_x)\phi_d}{(1-\alpha)A}\right)^{\frac{1}{\alpha}} \left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{\beta}{\beta-1}} \frac{\pi_r l_r}{\pi_n l_n}, 0\right\} \tag{35}$$

$$Y = A\left[\int_0^m x_d(i)^{\frac{\rho-1}{\rho}} di + \int_m^1 \left[x_c(i)^\beta n(i)^{1-\beta}\right]^{\frac{\rho-1}{\rho}} di\right]^{\frac{\rho}{\rho-1}(1-\alpha)} N_n^\alpha, \quad \alpha \in (0,1)\,, \rho \in [0,\infty)\,. \tag{36}$$

At an interior solution, while w_n is the same in the main text,

$$w_r = (1-\beta)(1+\tau_x)\phi_d \left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{\beta}{\beta-1}}. \tag{37}$$

Combining these results, after plugging in the equilibrium conditions to the output market clearing condition and after some cumbersome algebra, we can rewrite the resource constraint (15) as:

$$\pi_r c_r + \pi_n c_n \leq \pi_n w_n l_n \left(\frac{\tau_x + \alpha}{\alpha(1+\tau_x)}\right) + \frac{\pi_r w_r l_r}{1+\tau_x} \tag{38}$$

This captures all the equilibrium conditions of the production side.

Given that households cannot change their types, the optimality of hosuehold choice gives us

$$u\left(c_j, l_j\right) \geq u(c, l), \quad \forall (c, l) : c \leq w_j l - T\left(w_j l\right). \tag{39}$$

Then, the following IC constraints are necessary conditions for the optimality:

$$u\left(c_n, l_n\right) \geq u\left(c_r, \frac{w_r}{w_n} l_r\right)$$
$$u\left(c_r, l_r\right) \geq u\left(c_n, \frac{w_n}{w_r} l_n\right)$$

Notice that these two IC constraints are also sufficient conditions for the household side at an equilibrium since the government can freely adjust the tax schedule $T(.)$ so that for all $Y \notin \{Y_n, Y_r\}$, both household types receive worse allocations than their respective allocation. The government can achieve this by setting the tax schedule to be:

$$T(y) = y - \max\left\{c | u\left(c_i, l_i\right) \geq u\left(c, \frac{y}{w_i}\right), \text{ for } i = r, n\right\} \tag{40}$$

■

B $q \longrightarrow 0$

Proof. The final good producer optimizes inputs and get

$$x_c(i) = \begin{cases} \frac{\pi_r l_r}{(1-m)}, & i \in (m, 1] \\ 0, & \text{otherwise} \end{cases} \tag{41}$$

$$n(i) = \begin{cases} \frac{\pi_r l_r}{(1-m)}, & i \in (m, 1] \\ 0, & \text{otherwise} \end{cases} \tag{42}$$

$$x_d(i) = \begin{cases} \frac{\pi_r l_r}{(1-m)}, & i \in [0, m] \\ 0, & \text{otherwise} \end{cases} \tag{43}$$

$$m = \max\left\{1 - \left(\frac{(1+\tau_x)\phi_d}{(1-\alpha)A}\right)^{\frac{1}{\alpha}} \frac{\pi_r l_r}{\pi_n l_n}, 0\right\} \tag{44}$$

$$Y = A\left[\int_0^m x_d(i)^{\frac{\rho-1}{\rho}}di + \int_m^1 \min[x_c(i), n(i)]^{\frac{\rho-1}{\rho}}di\right]^{\frac{\rho}{\rho-1}(1-\alpha)} N_n^{\alpha}, \quad \alpha \in (0,1), \rho \in [0,\infty). \tag{45}$$

$$w_r = (1+\tau_x)(\phi_d - \phi_c) \tag{46}$$

Combining these results, again, we can rewrite the resource constraint (15) to

$$\pi_r c_r + \pi_n c_n \leq \pi_n w_n l_n \frac{\tau_x + \alpha}{\alpha(1+\tau_x)} + \frac{\pi_r w_r l_r}{1+\tau_x} \tag{47}$$

This captures all the equilibrium conditions of the production side.

The rest of the proof is the same with $q \longrightarrow 1$. ■

C $0 < q < 1$

Proof. The final good producer optimizes inputs and get

$$x_c(i) = \begin{cases} \frac{\pi_r l_r}{(1-m)}\left(\frac{1-\beta}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}-\beta}\right)^{\frac{q}{q-1}}, & i \in (m,1] \\ 0, & \text{otherwise} \end{cases} \tag{48}$$

$$n(i) = \begin{cases} \frac{\pi_r l_r}{(1-m)}, & i \in (m,1] \\ 0, & \text{otherwise} \end{cases} \tag{49}$$

$$x_d(i) = \begin{cases} \frac{\pi_r l_r}{(1-m)}\left(\beta\left(\frac{1-\beta}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}-\beta}\right) + (1-\beta)\right), & i \in [0,m] \\ 0, & \text{otherwise} \end{cases} \tag{50}$$

$$m = \max\left\{1 - \left(\frac{(1+\tau_x)\phi_d}{(1-\alpha)A}\right)^{\frac{1}{\alpha}} \frac{\pi_r l_r}{\pi_n l_n}, 0\right\} \tag{51}$$

$$Y = A\left[\int_0^m x_d(i)^{\frac{\rho-1}{\rho}}di + \int_m^1 \min[x_c(i), n(i)]^{\frac{\rho-1}{\rho}}di\right]^{\frac{\rho}{\rho-1}(1-\alpha)} N_n^{\alpha}, \quad \alpha \in (0,1), \rho \in [0,\infty). \tag{52}$$

$$w_r = (1-\alpha)(1-m)\frac{Y}{\pi_r l_r}, \tag{53}$$

Combining these results, after plugging in the equilibrium conditions to the output market clearing condition and after some cumbersome algebra, we can rewrite the resource constraint (15) to

$$\pi_r c_r + \pi_n c_n \leq \pi_n w_n l_n \frac{\tau_x + \alpha}{\alpha(1+\tau_x)} + \frac{\pi_r w_r l_r}{1+\tau_x} \tag{54}$$

This captures all the equilibrium conditions of the production side.

The rest of the proof is the same with $q \longrightarrow 1$. This concludes the whole proof. ■

C Proof of Proposition 2

Proof. Define $W(\tau_x) = \max \pi_r \omega_r u(c_r, l_r) + \pi_n \omega_n u(c_n, l_n)$. Then, the social planner's optimization problem is

$$\underset{\tau_x}{\text{maximize}}\; W(\tau_x)$$

subject to

$$[\eta_r \pi_r] \quad u(c_r, l_r) \geq u\left(c_n, \frac{w_n}{w_r} l_n\right),$$

$$[\eta_n \pi_n] \quad u(c_n, l_n) \geq u\left(c_r, \frac{w_r}{w_n} l_r\right),$$

$$[\mu] \quad \pi_r c_r + \pi_n c_n \leq \pi_n w_n l_n \left(\frac{\tau_x + \alpha}{\alpha(1+\tau_x)}\right) + \frac{\pi_r w_r l_r}{(1+\tau_x)}.$$

Letting $\eta_r = 0$, we get

$$\begin{aligned} W'(\tau_x) = & -\eta_n \pi_n u_l \left(c_r, \frac{w_r}{w_r} l_r\right) \frac{d\log(w_r/w_n)}{d\log(1+\tau_x)} \frac{1}{1+\tau_x} \frac{w_r l_r}{w_n} \\ & + \mu \Bigg[\pi_n w_n l_n \frac{\tau_x + \alpha}{\alpha(1+\tau_x)^2} \left(\frac{d\log w_n}{d\log(1+\tau_x)} + \frac{1-\alpha}{(\tau_x + \alpha)} \right) + \\ & \frac{\pi_r w_r l_r}{(1+\tau_x)^2} \left(\frac{d\log w_r}{d\log(1+\tau_x)} - 1 \right) \Bigg]. \end{aligned} \tag{55}$$

I separately prove the limit cases ($q \longrightarrow 0$ and $q \longleftrightarrow 1$) and interior case ($0 < q < 1$).

A $\quad q \longrightarrow 0$

With the equilibrium wages we have gotten above, we get:

$$w_r = (1+\tau_x)(\phi_d - \phi_c) \Rightarrow \frac{d\log w_r}{d\log(1+\tau_x)} = 1$$

$$w_n = \alpha \frac{A^{1/\alpha}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{[(1+\tau_x)\phi_d]^{\frac{1-\alpha}{\alpha}}} \Rightarrow \frac{d\log w_n}{d\log(1+\tau_x)} = -\frac{1-\alpha}{\alpha}$$

$$\frac{w_r}{w_n} = \frac{(1+\tau_x)^{\frac{1}{\alpha}} \left[\phi_d^{\frac{1}{\alpha}} - \phi_c \phi_d^{\frac{1-\alpha}{\alpha}}\right]}{\alpha A^{1/\alpha}(1-\alpha)^{\frac{1-\alpha}{\alpha}}} \Rightarrow \frac{d\log w_r/w_n}{d\log(1+\tau_x)} = \frac{1}{\alpha}$$

Plugging these back into the envelope condition (55), we get:

$$\begin{aligned} W'(\tau_x) &= -\eta_n \pi_n u_l \left(c_r, \frac{w_r}{w_r} l_r\right) \frac{1}{\alpha(1+\tau_x)} \frac{w_r l_r}{w_n} + \mu \pi_n w_n l_n \frac{\tau_x + \alpha}{\alpha(1+\tau_x)^2} \left[-\frac{1-\alpha}{\alpha} + \frac{1-\alpha}{\tau_x + \alpha} \right] \\ &= \frac{1}{\alpha(1+\tau_x)} \left[-\eta_n \pi_n u_l \left(c_r, \frac{w_r}{w_n} l_r\right) \frac{w_r l_r}{w_n} - \mu \pi_n w_n l_n \frac{\tau_x}{1+\tau_x} \frac{1-\alpha}{\alpha} \right] \end{aligned}$$

Since $\mu > 0$, if $\tau_x \leq 0$ and if $w_r > 0$—that is, $\phi_c < \phi_d$—, then we are guaranteed to have:

$$W'(\tau_x) > 0.$$

Thus, the social planner can always improve welfare by marginally increasing τ_x. Furthermore, the optimal level of τ_x implies $W'(\tau_x) = 0$, so we get:

$$\frac{\tau_x}{1+\tau_x} = \frac{\alpha}{1-\alpha} \frac{\eta_n \left(-u_l \left(c_r, \frac{w_r}{w_n} l_r\right) \frac{w_r l_r}{w_n}\right)}{\mu w_n l_n} \tag{56}$$

The first order condition with respect to l_r yields:

$$\begin{aligned} -\frac{\eta_n}{\mu} u_l \left(c_r, \frac{w_r}{w_r} l_r\right) \frac{w_r l_r}{w_n} &= -\left(\frac{\widetilde{\omega}_r \pi_r u_l (c_r, l_r) l_r + \frac{\pi_r w_r l_r}{1+\tau_x}}{\pi_n}\right) \\ &= \frac{\pi_r l_r}{\pi_n} (\phi_d - \phi_c) \left[\frac{\widetilde{\omega}_r (-u_l (c_r, l_r))}{\phi_d - \phi_c} - 1\right] \end{aligned} \tag{57}$$

Thus, we get

$$\frac{\tau_x}{1+\tau_x} = \frac{\alpha}{1-\alpha} \frac{\pi_r l_r}{\pi_n w_n l_n} (\phi_d - \phi_c) \left[\frac{\widetilde{\omega}_r (-u_l (c_r, l_r))}{\phi_d - \phi_c} - 1\right] \tag{58}$$

B $q \longrightarrow 1$ Case

Here, we analyze the Cobb-Douglas case. We have

$$X_c = \left(\frac{\phi_c}{\beta \phi_d}\right)^{\frac{1}{\beta-1}} \pi_r l_r. \tag{59}$$

Also,

$$\begin{aligned} m &= 1 - \left(\frac{(1+\tau_x)\phi_d}{(1-\alpha)A}\right)^{\frac{1}{\alpha}} \frac{(\pi_r l_r)^{(1-\beta)}}{\pi_n l_n} X_c^{\beta} \\ &= 1 - \left(\frac{(1+\tau_x)\phi_d}{(1-\alpha)A}\right)^{\frac{1}{\alpha}} \frac{(\pi_r l_r)^{(1-\beta)}}{\pi_n l_n} \left(\left(\frac{\phi_c}{\beta \phi_d}\right)^{\frac{1}{\beta-1}} \pi_r l_r\right)^{\beta} \\ &= 1 - \left(\frac{(1+\tau_x)\phi_d}{(1-\alpha)A}\right)^{\frac{1}{\alpha}} \frac{\pi_r l_r}{\pi_n l_n} \left(\frac{\phi_c}{\beta \phi_d}\right)^{\frac{\beta}{\beta-1}}. \end{aligned} \tag{60}$$

Furthermore,

$$
\begin{aligned}
w_r &= (1-\beta)(1+\tau_x)\phi_d X_c^{\beta}(\pi_r l_r)^{-\beta} \\
&= (1-\beta)(1+\tau_x)\phi_d \left(\left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{1}{\beta-1}} \pi_r l_r\right)^{\beta} (\pi_r l_r)^{-\beta} \\
&= (1-\beta)(1+\tau_x)\phi_d \left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{\beta}{\beta-1}}. \qquad (61)
\end{aligned}
$$

With the equilibrium wages we have gotten above, we get:

$$
w_r = (1+\tau_x)(1-\beta)\phi_d \left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{\beta}{\beta-1}} \Rightarrow \frac{d\log w_r}{d\log(1+\tau_x)} = 1
$$

$$
w_n = \alpha \frac{A^{1/\alpha}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\left[(1+\tau_x)\phi_d\right]^{\frac{1-\alpha}{\alpha}}} \Rightarrow \frac{d\log w_n}{d\log(1+\tau_x)} = -\frac{1-\alpha}{\alpha}
$$

$$
\frac{w_r}{w_n} = \frac{\left[(1+\tau_x)\phi_d\right]^{\frac{1}{\alpha}}}{\alpha A^{1/\alpha}(1-\alpha)^{\frac{1-\alpha}{\alpha}}} \left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{\beta}{\beta-1}} \Rightarrow \frac{d\log w_r/w_n}{d\log(1+\tau_x)} = \frac{1}{\alpha}
$$

Plugging these back into the envelope condition (55), we get:

$$
\begin{aligned}
W'(\tau_x) &= -\eta_n\pi_n u_l\left(c_r, \frac{w_r}{w_r} l_r\right)\frac{1}{\alpha(1+\tau_x)}\frac{w_r l_r}{w_n} + \mu\pi_n w_n l_n \frac{\tau_x+\alpha}{\alpha(1+\tau_x)^2}\left[-\frac{1-\alpha}{\alpha} + \frac{1-\alpha}{\tau_x+\alpha}\right] \\
&= \frac{1}{\alpha(1+\tau_x)}\left[-\eta_n\pi_n u_l\left(c_r, \frac{w_r}{w_n} l_r\right)\frac{w_r l_r}{w_n} - \mu\pi_n w_n l_n \frac{\tau_x}{1+\tau_x}\frac{1-\alpha}{\alpha}\right]
\end{aligned}
$$

Since $\mu > 0$, if $\tau_x \le 0$, we have:

$$
W'(\tau_x) > 0.
$$

Thus, the social planner can always improve welfare by marginally increasing τ_x. Furthermore, the optimal level of τ_x implies $W'(\tau_x) = 0$, so we get:

$$
\frac{\tau_x}{1+\tau_x} = \frac{\alpha}{1-\alpha}\frac{\eta_n\left(-u_l\left(c_r, \frac{w_r}{w_n} l_r\right)\frac{w_r l_r}{w_n}\right)}{\mu w_n l_n} \qquad (62)
$$

The first order condition with respect to l_r yields:

$$-\frac{\eta_n}{\mu}u_l\left(c_r,\frac{w_r}{w_r}l_r\right)\frac{w_r l_r}{w_n}=-\left(\frac{\widetilde{\omega}_r\pi_r u_l\left(c_r,l_r\right)l_r+\frac{\pi_r w_r l_r}{1+\tau_x}}{\pi_n}\right)$$
$$=\frac{\pi_r\phi_d l_r}{\pi_n}\left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{\beta}{\beta-1}}\phi_d(1-\beta)\left[\frac{\widetilde{\omega}_r\left(-u_l\left(c_r,l_r\right)\right)}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{\beta}{\beta-1}}\phi_d(1-\beta)}-1\right] \tag{63}$$

Thus, we get

$$\frac{\tau_x}{1+\tau_x}=\frac{\alpha}{1-\alpha}\frac{\pi_r l_r}{\pi_n}\left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{\beta}{\beta-1}}\phi_d(1-\beta)\left[\frac{\widetilde{\omega}_r\left(-u_l\left(c_r,l_r\right)\right)}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{\frac{\beta}{\beta-1}}\phi_d(1-\beta)}-1\right] \tag{64}$$

C $0<q<1$ case

We showed that the statement holds in the limit cases of $q\longrightarrow 0$ and $q\longrightarrow 1$. Thus, we are left with non-limit cases. The equilibrium conditions are stated in the main text for $0<q<1$ case. Note

$$X_c=\left(\frac{1-\beta}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}-\beta}\right)^{\frac{q}{q-1}}\pi_r l_r. \tag{65}$$

Recall that w_n is not a function of q, and that

$$w_r=(1-\beta)(1+\tau_x)\phi_d\left(\frac{(1-\beta)\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}-\beta}\right)^{\frac{1}{q-1}} \tag{66}$$

Letting $\eta_r=0$, we get

$$W'\left(\tau_x\right)=-\eta_n\pi_n u_l\left(c_r,\frac{w_r}{w_r}l_r\right)\frac{d\log\left(w_r/w_n\right)}{d\log\left(1+\tau_x\right)}\frac{1}{1+\tau_x}\frac{w_r l_r}{w_n}$$
$$+\mu\left[\pi_n w_n l_n\frac{\tau_x+\alpha}{\alpha(1+\tau_x)^2}\left(\frac{d\log w_n}{d\log(1+\tau_x)}+\frac{1-\alpha}{(\tau_x+\alpha)}\right)+\right.$$
$$\left.\frac{\pi_r w_r l_r}{(1+\tau_x)^2}\left(\frac{d\log w_r}{d\log(1+\tau_x)}-1\right)\right]. \tag{67}$$

With the equilibrium wages we have gotten above, we get:

$$w_r = (1-\beta)(1+\tau_x)\phi_d \left(\frac{(1-\beta)\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1} - \beta} \right)^{\frac{1}{q-1}} \Rightarrow \frac{d\log w_r}{d\log(1+\tau_x)} = 1$$

$$w_n = \alpha \frac{A^{1/\alpha}(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\left[(1+\tau_x)\phi_d\right]^{\frac{1-\alpha}{\alpha}}} \Rightarrow \frac{d\log w_n}{d\log(1+\tau_x)} = -\frac{1-\alpha}{\alpha}$$

$$\frac{d\log w_r/w_n}{d\log(1+\tau_x)} = \frac{1}{\alpha}$$

Plugging these back into the envelope condition (55), we get:

$$\begin{aligned} W'(\tau_x) &= -\eta_n\pi_n u_l\left(c_r, \frac{w_r}{w_r}l_r\right)\frac{1}{\alpha(1+\tau_x)}\frac{w_r l_r}{w_n} + \mu\pi_n w_n l_n \frac{\tau_x+\alpha}{\alpha(1+\tau_x)^2}\left[-\frac{1-\alpha}{\alpha} + \frac{1-\alpha}{\tau_x+\alpha}\right] \\ &= \frac{1}{\alpha(1+\tau_x)}\left[-\eta_n\pi_n u_l\left(c_r, \frac{w_r}{w_n}l_r\right)\frac{w_r l_r}{w_n} - \mu\pi_n w_n l_n \frac{\tau_x}{1+\tau_x}\frac{1-\alpha}{\alpha}\right] \end{aligned}$$

Since $\mu > 0$, if $\tau_x \leq 0$, then we are guaranteed to have:

$$W'(\tau_x) > 0.$$

Thus, the social planner can always improve welfare by marginally increasing τ_x.

Furthermore, the optimal level of τ_x implies $W'(\tau_x) = 0$, so we get:

$$\frac{\tau_x}{1+\tau_x} = \frac{\alpha}{1-\alpha}\frac{\eta_n\left(-u_l\left(c_r, \frac{w_r}{w_n}l_r\right)\frac{w_r l_r}{w_n}\right)}{\mu w_n l_n} \tag{68}$$

The first order condition with respect to l_r yields:

$$
\begin{aligned}
-\frac{\eta_n}{\mu} u_l\left(c_r, \frac{w_r}{w_r} l_r\right) \frac{w_r l_r}{w_n} &= -\left(\frac{\widetilde{\omega}_r \pi_r u_l\left(c_r, l_r\right) l_r + \frac{\pi_r w_r l_r}{1+\tau_x}}{\pi_n}\right) \\
&= \frac{\pi_r l_r}{\pi_n}(1-\beta)\phi_d \left(\frac{(1-\beta)\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1} - \beta}\right)^{\frac{1}{q-1}} \\
&\quad \left[\frac{\widetilde{\omega}_r\left(-u_l\left(c_r, l_r\right)\right)}{(1-\beta)\phi_d \left(\frac{(1-\beta)\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1} - \beta}\right)^{\frac{1}{q-1}}} - 1\right]
\end{aligned}
$$

Thus, we get

$$
\begin{aligned}
\frac{\tau_x}{1+\tau_x} =& \frac{\alpha}{1-\alpha} \frac{\pi_r l_r}{\pi_n}(1-\beta)\phi_d \left(\frac{(1-\beta)\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1} - \beta}\right)^{\frac{1}{q-1}} \\
& \left[\frac{\widetilde{\omega}_r\left(-u_l\left(c_r, l_r\right)\right)}{(1-\beta)\phi_d \left(\frac{(1-\beta)\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1}}{\left(\frac{\phi_c}{\beta\phi_d}\right)^{q-1} - \beta}\right)^{\frac{1}{q-1}}} - 1\right]
\end{aligned}
$$

$\blacksquare$

D General Model with Government Spending

The results of this paper carry through with the incorporation of government spending in the model. First, the household problem becomes

$$\underset{c_j, l_j}{\text{maximize}}\ U_j = u(c_j, l_j) + v(G)$$
$$\text{subject to } c_j \leq w_j l_j - T(w_j l_j),$$

where $v'(G) > 0, v''(G) < 0$. And now, we have the government budget constraint represented by equation (6). Furthermore, the definition of equilibrium needs the following additional condition, (iv) the government budget constraint is satisfied.

Accordingly, the output market equilibrium condition becomes

$$\pi_r c_r + \pi_n c_n + G \leq Y - \int_0^1 (\phi_d x_d(i) + \phi_c x_c(i))\, di. \tag{69}$$

These additions will not affect any of the derivation and proofs for the propositions and lemma above.