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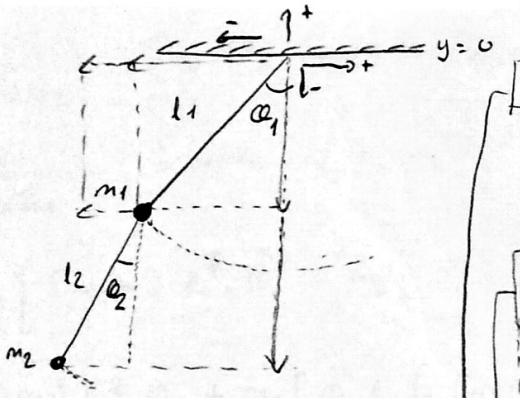
Sy - 1



m_1 kütlesi için

$V = -m_1 \cdot g \cdot l_1 \cdot \cos\theta_1$

$\vec{r}_1 = (-l_1 \cdot \cos\theta_1 , -l_1 \cdot \sin\theta)$

$\dot{\vec{r}}_1 = \dot{\theta}_1 l_1 \cdot \sin\theta_1 , \dot{\theta}_1 (-l_1 \cdot \cos\theta_1)$

$T = \frac{1}{2} m_1 \left[l_1^2 \cdot \dot{\theta}_1^2 \cdot \sin^2\theta + l_2^2 \cdot \dot{\theta}_1^2 \cdot \cos^2\theta_1 \right]$

$= \frac{1}{2} m_1 \cdot \dot{\theta}_1^2 \cdot l_1^2$

$L = \frac{1}{2} m_1 \dot{\theta}_1^2 \cdot l_1^2 + m_1 g l_1 \cdot \cos\theta_1$

$\frac{dL}{d\theta_1} = -m_1 \cdot g \cdot l_1 \cdot \sin\theta_1$

$\frac{dL}{d\dot{\theta}_1} = m_1 \dot{\theta}_1 \cdot l_1^2$

$\frac{d}{dt}\left[m_1 \dot{\theta}_1 l_1^2 \right] = -m_1 g \, l_1 \cdot \sin\theta_1$

$m_1 \ddot{\theta}_1 l_1^2 = -m_1 \cdot g \cdot l_1 \cdot \sin\theta_1$

$\frac{d^2\theta_1}{dt^2} + \frac{g}{l_1} \cdot \sin\theta_1 = 0$ → Tek sarkacın sonuçu işleme yansıtılmıştır

m_2 kütlesi için

$V = m_2 \cdot g \left(-l_1 \cdot \cos\theta_1 - l_2 \cdot \cos\theta_2 \right)$

$\vec{r}_2 = \left(-l_1 \cdot \sin\theta_1 - l_2 \cdot \sin\theta_2 , -l_1 \cdot \cos\theta_1 - l_2 \cdot \cos\theta_2 \right)$

$\dot{\vec{r}}_2 = \left(-\dot{\theta}_1 \cdot l_1 \cdot \cos\theta_1 - \dot{\theta}_2 l_2 \cdot \cos\theta_2 , + \dot{\theta}_1 l_1 \cdot \sin\theta_1 + \dot{\theta}_2 l_2 \cdot \sin\theta_2 \right)$

$$L = \frac{1}{2} m_1 \cdot \dot{\theta}_1^2 l_1^2 + \frac{1}{2} m_2 \left[\dot{\theta}_1^2 l_1^2 \cos^2\theta_1 + \dot{\theta}_2^2 l_2^2 \cos^2\theta_2 + 2 \dot{\theta}_1 l_1 \dot{\theta}_2 l_2 \cdot \cos\theta_1 \cdot \cos\theta_2 + \dot{\theta}_1^2 l_1^2 \sin^2\theta_1 + \dot{\theta}_2^2 l_2^2 \sin^2\theta_2 \right.$$

$$\left. + 2 \dot{\theta}_1 l_1 \cdot \dot{\theta}_2 \cdot l_2 \cdot \sin\theta_1 \cdot \sin\theta_2 \right] + m_2 g \left(l_1 \cdot \cos\theta_1 + l_2 \cdot \cos\theta_2 \right) + m_1 g l_1 \cdot \cos\theta_1$$

$$L = \frac{1}{2} m_1 \cdot \dot{\theta}_1^2 \cdot l_1^2 + \frac{1}{2} m_2 \left[\dot{\theta}_1^2 l_1^2 + \dot{\theta}_2^2 l_2^2 + 2 \cdot \dot{\theta}_1 l_1 \cdot \dot{\theta}_2 l_2 \left[\cos(\theta_1 - \theta_2) \right] \right] + m_2 g \cdot l_2 \cdot \cos\theta_2 + \left[m_1 + m_2 \right] l_1 \cdot g \cdot \cos\theta_1$$

$$L = \frac{1}{2} l_1^2 \dot{\theta}_1^2 \left[m_1 + m_2 \right] + \frac{1}{2} m_2 \left[\dot{\theta}_2^2 l_2^2 + 2 \dot{\theta}_1 \dot{\theta}_2 l_1 l_2 \left(\cos(\theta_1 - \theta_2) \right) \right] + m_2 g \cdot l_2 \cdot \cos\theta_2 + \left[m_1 + m_2 \right] l_1 \cdot g \cdot \cos\theta_1$$

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$$\frac{dL}{d\theta_1} = -m_2 \dot{\theta}_1 \dot{\theta}_2 l_1 l_2 \sin(\theta_1-\theta_2) - [m_1+m_2] g l_1 \sin\theta_1$$

$$\frac{dL}{d\dot{\theta}_1} = [m_1+m_2] l_1^2 \dot{\theta}_1 + [\dot{\theta}_2 l_1 l_2 \cos(\theta_1-\theta_2)] m_2$$

$$\frac{d}{dt}\left[[m_1+m_2] l_1^2 \dot{\theta}_1 + m_2 [\dot{\theta}_2 l_1 l_2 \cos(\theta_1-\theta_2)] \right] = -m_2 \dot{\theta}_1 \dot{\theta}_2 l_1 l_2 \sin(\theta_1-\theta_2) - [m_1+m_2] g l_1 \sin\theta_1$$

$$[m_1+m_2] l_1^2 \ddot{\theta}_1 + m_2 [\ddot{\theta}_2 l_1 l_2 \cos(\theta_1-\theta_2) - \dot{\theta}_2 l_1 l_2 \sin(\theta_1-\theta_2)\cdot(\dot{\theta}_1-\dot{\theta}_2)] = -m_2 \dot{\theta}_1 \dot{\theta}_2 l_1 l_2 \sin(\theta_1-\theta_2) - [m_1+m_2] g l_1 \sin\theta_1$$

$$[m_1+m_2] l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \ddot{\theta}_2 \cos(\theta_1-\theta_2) + \dot{\theta}_2^2 m_2 l_1 l_2 \sin(\theta_1-\theta_2) + [m_1+m_2] g l_1 \sin\theta_1 = 0$$

Denklem 1

$$\frac{dL}{d\theta_2} = +m_2 \dot{\theta}_1 \dot{\theta}_2 l_1 l_2 \sin(\theta_1-\theta_2) - m_2 g l_2 \sin\theta_2$$

$$\frac{dL}{d\dot{\theta}_2} = m_2 [\dot{\theta}_2 l_2^2 + \dot{\theta}_1 l_1 l_2 \cos(\theta_1-\theta_2)] \Rightarrow \frac{d}{dt}\left[\frac{dL}{d\dot{\theta}_2}\right] = m_2 [\ddot{\theta}_2 l_2^2 + \ddot{\theta}_1 l_1 l_2 \cos(\theta_1-\theta_2) - \dot{\theta}_1 l_1 l_2 \sin(\theta_1-\theta_2)\cdot(\dot{\theta}_1-\dot{\theta}_2)]$$

$$m_2 \ddot{\theta}_2 l_2^2 + m_2 \ddot{\theta}_1 l_1 l_2 \cos(\theta_1-\theta_2) - m_2 \dot{\theta}_1^2 l_1 l_2 \sin(\theta_1-\theta_2) + m_2 \dot{\theta}_1 \dot{\theta}_2 l_1 l_2 \sin(\theta_1-\theta_2) = m_2 \dot{\theta}_1 \dot{\theta}_2 l_1 l_2 \sin(\theta_1-\theta_2) - m_2 g l_2 \sin\theta_2$$

$$m_2 \ddot{\theta}_2 l_2^2 + m_2 \ddot{\theta}_1 l_1 l_2 \cos(\theta_1-\theta_2) - m_2 \dot{\theta}_1^2 l_1 l_2 \sin(\theta_1-\theta_2) + m_2 g l_2 \sin\theta_2 = 0$$

denklem 2

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$\frac{d\theta}{dt} = \omega \qquad \theta = \theta_0 + \omega t$

$[m_1+m_2]\, l_1\, \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cdot \cos(\theta_1-\theta_2) + \dot{\theta}_2^2 \cdot m_2 \cdot l_2 \sin(\theta_1-\theta_2) + [m_1+m_2]\, g \sin\theta_1 = 0$

$\ddot{\theta}_2 l_2 + \ddot{\theta}_1 l_1 \cdot \cos(\theta_1-\theta_2) - \dot{\theta}_1^2 l_1 \cdot \sin(\theta_1-\theta_2) + g \sin\theta_2 = 0$

$\dot{\theta}_1 = \omega_1, \quad \dot{\theta}_2 = \omega_2, \quad \ddot{\theta}_1 = \dot{\omega}_1, \quad \ddot{\theta}_2 = \dot{\omega}_2$

$[m_1+m_2]\, l_1 \cdot \dot{\omega}_1 + m_2 \cdot l_2 \cdot \dot{\omega}_2 \cdot \cos(\theta_1-\theta_2) + \omega_2^2 m_2 \cdot l_2 \cdot \sin(\theta_1-\theta_2) + [m_1+m_2]\, g \cdot \sin\theta_1 = 0$

$\dot{\omega}_2 l_2 + \dot{\omega}_1 l_1 \cdot \cos(\theta_1-\theta_2) + \omega_1^2 l_1 \cdot \sin(\theta_1-\theta_2) + g \cdot \sin\theta_2 = 0$

$\frac{d\omega_1}{dt} = -\, m_2 \cdot l_2 \cdot \dot{\omega}_2 \cdot \cos(\theta_1-\theta_2) - \omega_2^2 \cdot m_2 \cdot l_2 \cdot \sin(\theta_1-\theta_2) + [m_1+m_2] \cdot g \cdot \sin\theta_1$

$\omega_1 = (\omega_1)_0 - \frac{\left[m_2 l_2 \dot{\omega}_2 \cdot \cos(\theta_1-\theta_2) + \omega_1^2 \cdot m_2 \cdot l_2 \sin(\theta_1-\theta_2) + [m_1+m_2] g \cdot \sin\theta_1 \right]}{(m_1+m_2) \cdot l_1} \cdot t$ → Δt alınacak. (t_2-t_1)

$\frac{d\theta_1}{dt} = \omega_1 \Rightarrow \theta_1 = (\theta_1)_0 + \omega_1 \cdot t$ → $\Delta t (t-t_1)$ alınacak

$\frac{d\omega_2}{dt} = -\, \frac{\left[\dot{\omega}_1 \cdot l_1 \cdot \cos(\theta_1-\theta_2) + \omega_1^2 \cdot l_1 \cdot \sin(\theta_1-\theta_2) - g \sin\theta_2 \right] \cdot 1}{l_2}$

$\omega_2 = (\omega_2)_0 - \frac{\left[\dot{\omega}_1 \cdot l_1 \cdot \cos(\theta_1-\theta_2) + \omega_1^2 \cdot l_1 \cdot \sin(\theta_1-\theta_2) - g \cdot \sin\theta_2 \right]}{l_2} \cdot t$ → $\Delta t (t_2-t_1)$ alınacak.

$\frac{d\theta_2}{dt} = \omega_2 \Rightarrow \theta_2 = (\theta_2)_0 + \omega_2 \cdot t$ → $\Delta t (t_2-t_1)$ alınacak

$\omega_1, \dot{\omega}_1, \omega_2, \dot{\omega}_2$ denklemde yerine konulacak