

Protocol	Stage	Spacetime volume	
This work	3 prep	$17 \cdot 18 + 13 \cdot 4$	Hook injection to 3, a 3 encoding, decoding, One round of SE at 3 Unitary growth from 3 Three rounds of SE at 3
	Measure H_{XY}	$2 \times [2(3 \cdot 4) + 16 \cdot 3]$	
	3 SE	$25 \cdot 8$	
	$3 \rightarrow 5$	$13 \cdot 4$	
	Hybrid escape	$3 \times (97 \cdot 8) + 169 \cdot 4$	
[?]	Encode $ T\rangle$	$13 \cdot 5$	Stages defined
	Stabilize	$14 \cdot 10$	
	Check T	$13 \cdot 7$	
	Stabilizer	$3 \times (337 \cdot 12)$	
	Escape & SE (gap)	$5 \times (337 \cdot 12)$	
[?]	Inject $ T\rangle$	$9 \cdot 3 + 19 \cdot 9$	Stages defined
	SE	$19 \cdot 9$	
	Morph T	$15 \cdot 4$	
	Check T	$2 \times (24 \cdot 9)$	
	Morph & Expand	$343 \cdot 10$	
	SE	$5 \times (337 \cdot 8)$	

Table 1: **Spacetime volume breakdown of different protocols.** Each protocol is divided into multiple stages with their corresponding overhead. Dot-product notation indicates qubit · steps, while expressions with an explicit ‘ \times ’ indicate repeated execution of the circuit. For example, $A \times (Q \cdot T)$ denotes a circuit with Q qubits and T time steps repeated A times. ‘SE’ stands for syndrome extraction. Note that in all circuits, the preparation of a $|+\rangle$ state is implemented as preparation of $|0\rangle$ followed by a Hadamard gate, taking two steps. Similarly, each X -basis measurement is counted as two steps.