

| Protocol | Stage | Spacetime volume | Latency |
|-----------|--------------------|--|------------------------|
| This work | 3 prep | $17 \cdot 18 + 13 \cdot 4$ | Hook injection to 3, a |
| | Measure H_{XY} | $2 \times [2(3 \cdot 4) + 16 \cdot 3]$ | 3 encoding, decoding, |
| | 3 SE | $25 \cdot 8$ | One round of SE at 3 |
| | $3 \rightarrow 5$ | $13 \cdot 4$ | Unitary growth from |
| | Hybrid escape | $3 \times (97 \cdot 8) + 169 \cdot 4$ | Three rounds of SE at |
| [?] | Encode $ T\rangle$ | $13 \cdot 5$ | Stages defined |
| | Stabilize | $14 \cdot 10$ | |
| | Check T | $13 \cdot 7$ | |
| | Stabilizer | $3 \times (337 \cdot 12)$ | |
| | Escape & SE (gap) | $5 \times (337 \cdot 12)$ | |
| [?] | Inject $ T\rangle$ | $9 \cdot 3 + 19 \cdot 9$ | Stages defined |
| | SE | $19 \cdot 9$ | |
| | Morph T | $15 \cdot 4$ | |
| | Check T | $2 \times (24 \cdot 9)$ | |
| | Morph & Expand | $343 \cdot 10$ | |
| | SE | $5 \times (337 \cdot 8)$ | |

Table 1: **Spacetime volume breakdown of different protocols.** Each protocol is divided into multiple stages with their corresponding overhead. Dot-product notation indicates qubit \cdot steps, while expressions with an explicit ‘ \times ’ indicate repeated execution of the circuit. For example, $A \times (Q \cdot T)$ denotes a circuit with Q qubits and T time steps repeated A times. ‘SE’ stands for syndrome extraction. Note that in all circuits, the preparation of a $|+\rangle$ state is implemented as preparation of $|0\rangle$ followed by a Hadamard gate, taking two steps. Similarly, each X -basis measurement is counted as two steps.