Oh

We have used the big-Oh notation somewhat casually in this chapter to describe the growth behavior of a function. Here is the formal definition of the big-Oh notation: Suppose we have a function $T(n)$. Usually, it represents the processing time of an algorithm for a given input of size n. But it could be any function. Also, suppose that we have another function $f(n)$. It is usually chosen to be a simple function, such as $f(n) = n^k$ or $f(n) = \log(n)$, but it too can be any function. We write

$$T(n) = O(f(n))$$

if $T(n)$ grows at a rate that is bounded by $f(n)$. More formally, we require that for all n larger than some threshold, the ratio $T(n)/f(n) \le C$ for some constant value C.

If $T(n)$ is a polynomial of degree k in n, then one can show that $T(n) = O(n^k)$. Later in this chapter, we will encounter functions that are $O(\log(n))$ or $O(n \log(n))$. Some algorithms take much more time. For example, one way of sorting a sequence is to compute all of its permutations, until you find one that is in increasing order. Such an algorithm takes $O(n!)$ time, which is very bad indeed.

Table 1 shows common big-Oh expressions, sorted by increasing growth.

Strictly speaking, $T(n) = O(f(n))$ means that T grows no faster than f. But it is permissible for T to grow much more slowly. Thus, it is technically correct to state that $T(n) = n^2 + 5n - 3$ is $O(n^3)$ or even $O(n^{10})$.

Table 1 Common Big-Oh Growth Rates

Big-Oh Expression	Name
$O(1)$	Constant
$O(\log(n))$	Logarithmic
$O(n)$	Linear
$O(n \log(n))$	Log-linear
$O(n^2)$	Quadratic
$O(n^3)$	Cubic
$O(2^n)$	Exponential
$O(n!)$	Factorial