Introduction to Machine Learning

Loss Functions, Bias and Variance

Loss functions

- 0/1 loss
  - Perceptron

Any hope?

- Consider modifying function to add a regularizer
- R(w,b)

- Optimize trade-off between low training error and simplicity

Relationship between margin and loss

- Small changes to w,b have large impact on objective function

Relationship between margin and loss

- Sigmoid function

Surrogate loss functions

- 0/1
- Hinge
- Logistic
- Exponential
- Squared

$$\ell^{(0/1)}(y, \hat{y}) = \mathbf{1}[y\hat{y} \leq 0]$$

$$\ell^{(\text{hin})}(y, \hat{y}) = \max\{0, 1 - y\hat{y}\}$$

$$\ell^{(\log)}(y, \hat{y}) = \frac{1}{\log 2} \log\left(1 + \exp[-y\hat{y}]\right)$$

$$\ell^{(\exp)}(y, \hat{y}) = \exp[-y\hat{y}]$$

$$\ell^{(\text{sqr})}(y, \hat{y}) = (y - \hat{y})^2$$

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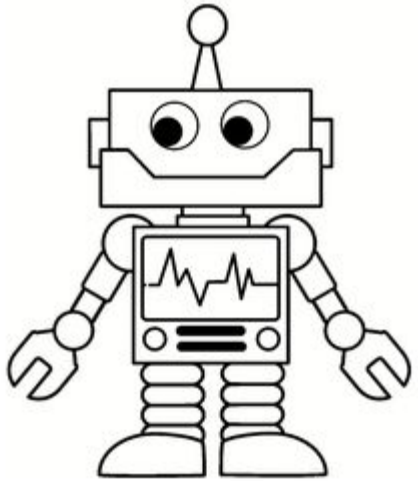
Regularizer

- Helps to prevent overfit
- Perceptron $\min_{w,b} \sum_n \mathbf{1}[y_n(\boldsymbol{w} \cdot \boldsymbol{x}_n + b) \leq 0] + \lambda R(\boldsymbol{w}, b)$
- What are good choices of R(w,b) for hyperplanes?



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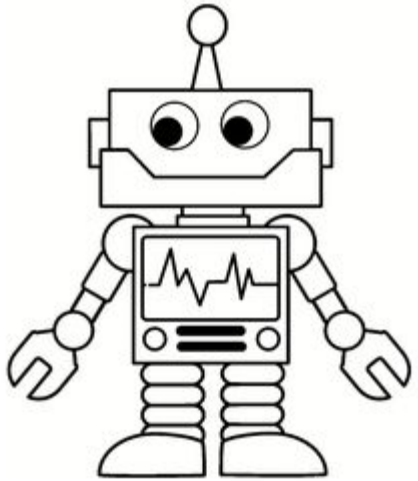


Norm

- Popular choice
- Keeps function (weights) from changing too quickly
- $R^{(\text{norm})}(\boldsymbol{w}, b) = ||\boldsymbol{w}|| = \sqrt{\sum_d w_d^2}$
  - Convex
  - Smooth
- l-norm (p-norm)

$$||\boldsymbol{w}||_p = \left(\sum_d |w_d|^p \right)^{\frac{1}{p}}$$

P-norm

$$||\boldsymbol{w}||_p = \left(\sum_d |w_d|^p \right)^{\frac{1}{p}}$$

Bias and fairness

Bias and Fairness

Business

Apple Card algorithm sparks gender bias allegations against Goldman Sachs

COMPUTING

Racial Bias Found in a Major Health Care Risk Algorithm

When a Computer Program Keeps You in Jail

What types of bias can exist?

- Adaptation

Fairness and data bias

- Labeling
- Sample selection
- Task
- Feedback loops

Error

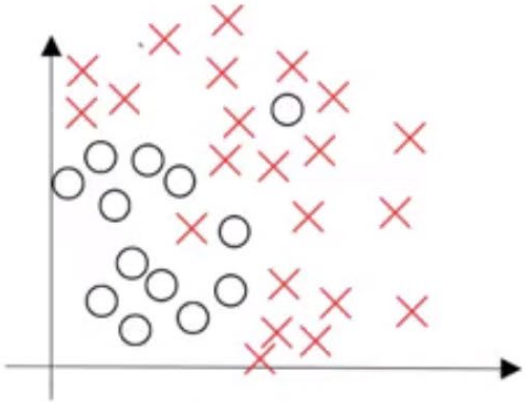
$$\text{error}(f) = \underbrace{\left[\text{error}(f) - \min_{f^* \in \mathcal{F}} \text{error}(f^*)\right]}_{\text{estimation error}} + \underbrace{\left[\min_{f^* \in \mathcal{F}} \text{error}(f)\right]}_{\text{approximation error}}$$

Error

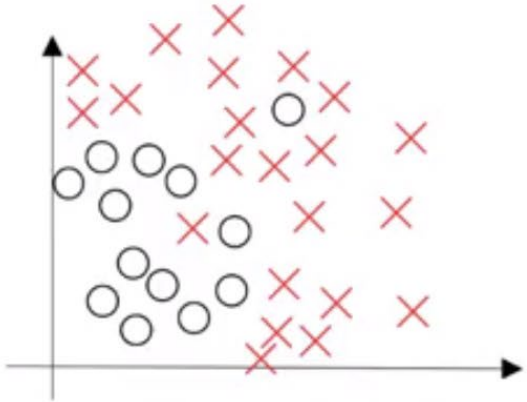
- Bias error
  - Simplifying assumptions that make target function easier to learn
- Variance error
  - Amount that estimated target function will change if change training data

Bias-Variance tradeoff

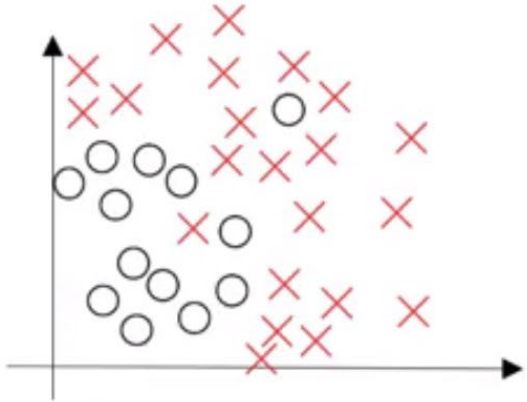
- Goal: low bias and low variance
- As we increase complexity
  - Bias decreases (better fit to data)
  - Variance increases (fit varies more with data)



high bias



"just right"



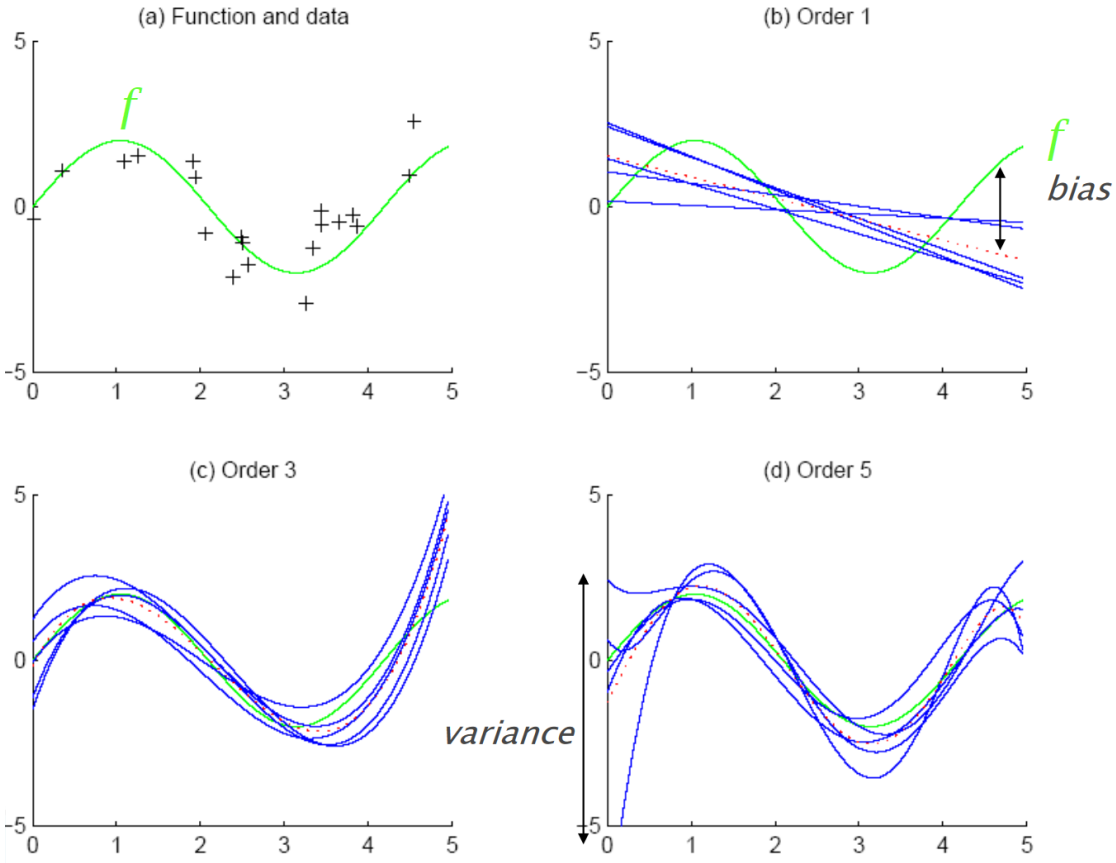
high variance

Bias and variance

Cat classification

Example

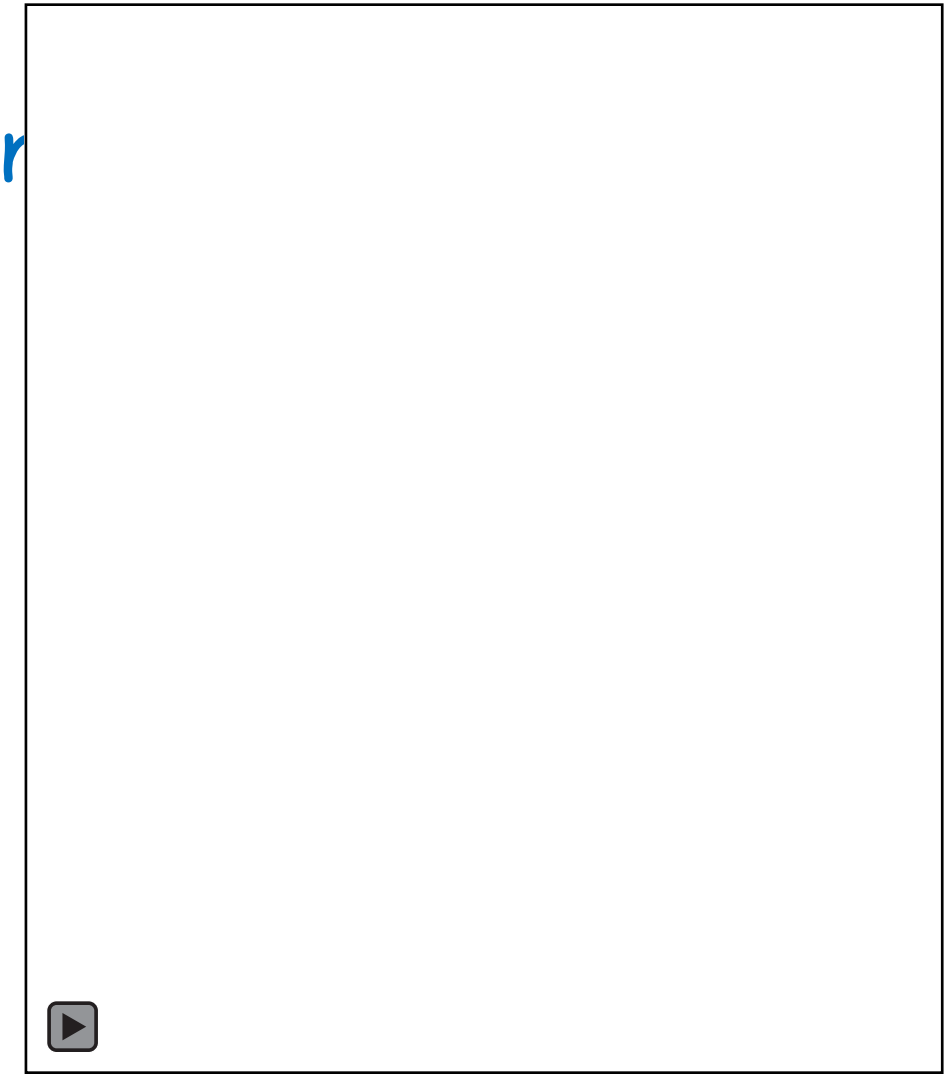
- f(x) = 2 sin(1.5x)



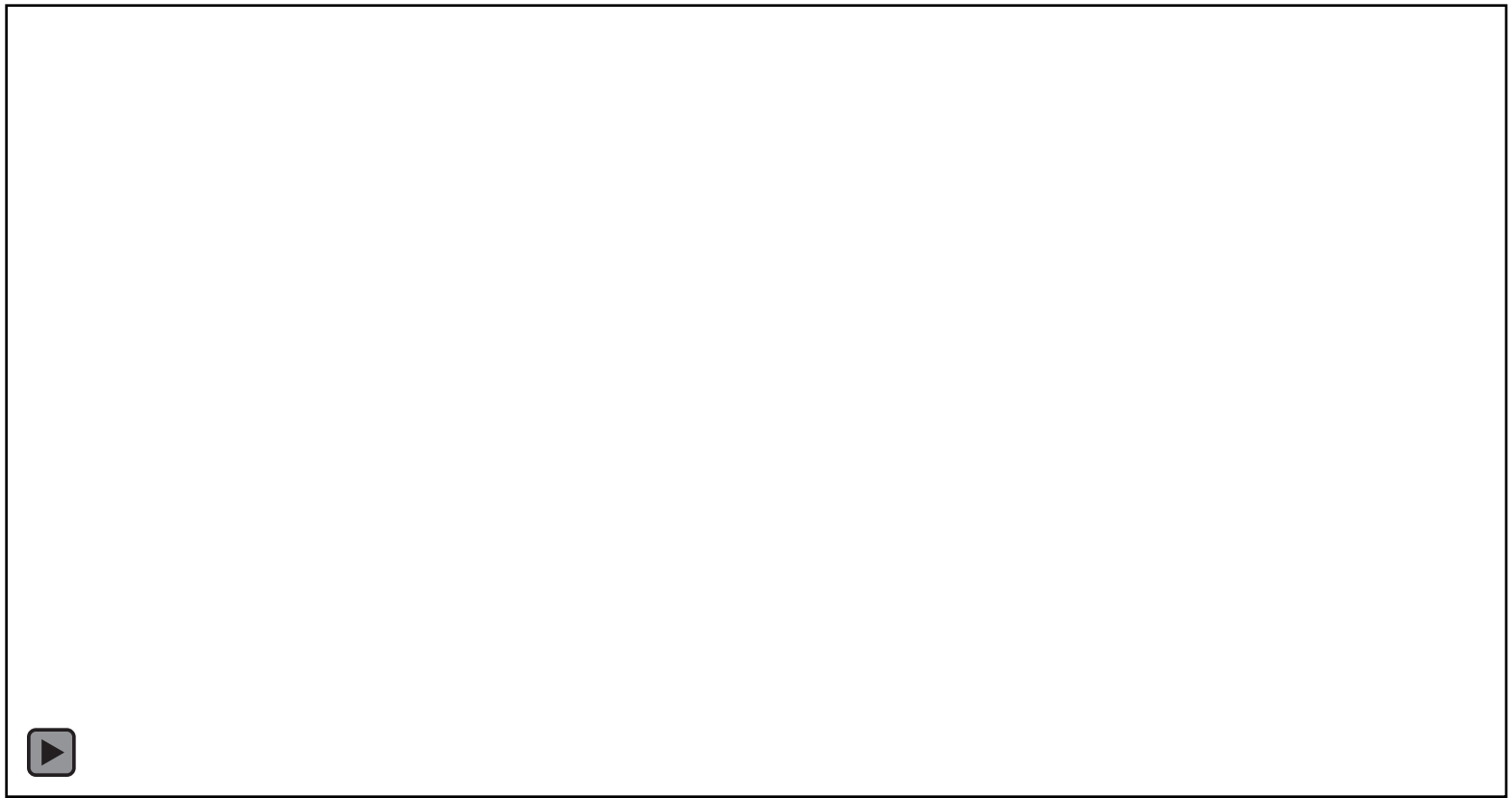
Regularization

- Path to bias-variance trade-off

DL Bias and Variar



Decision stump



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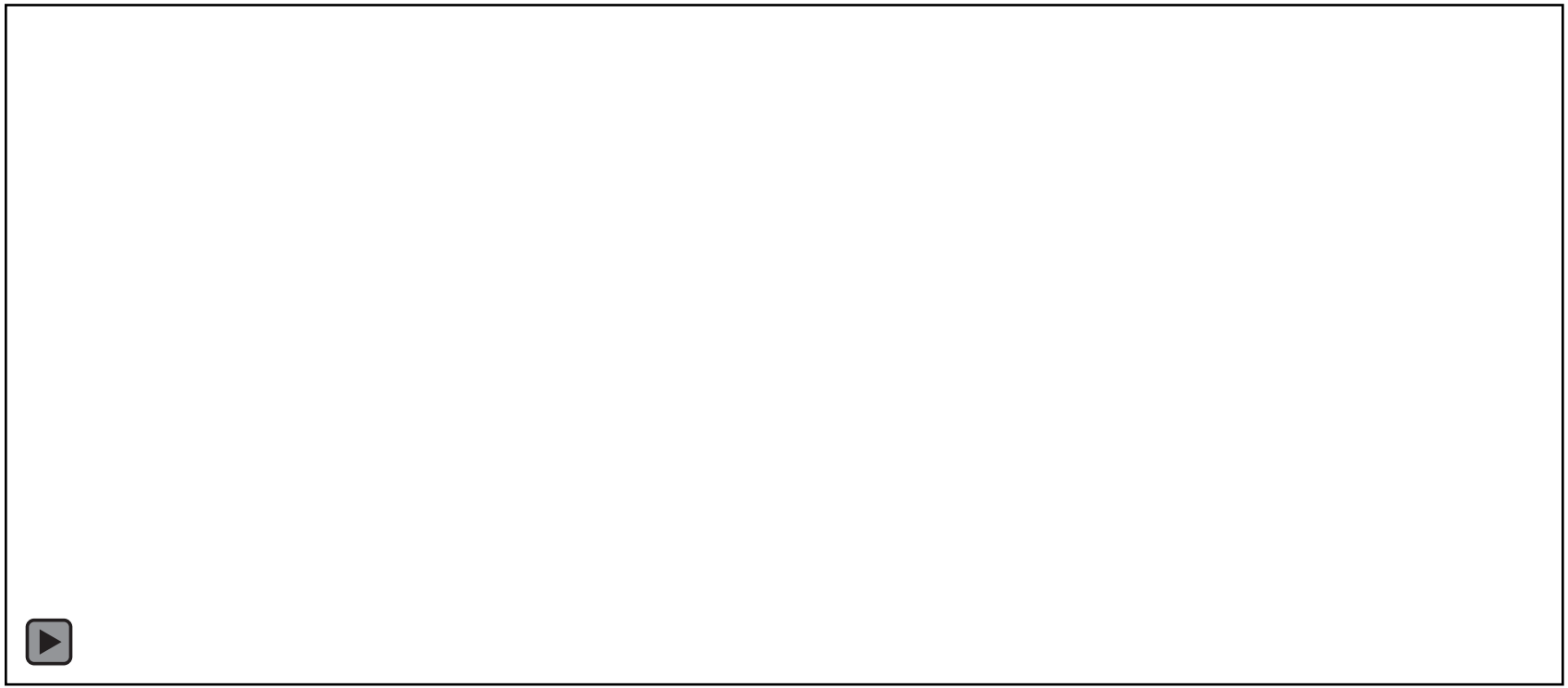
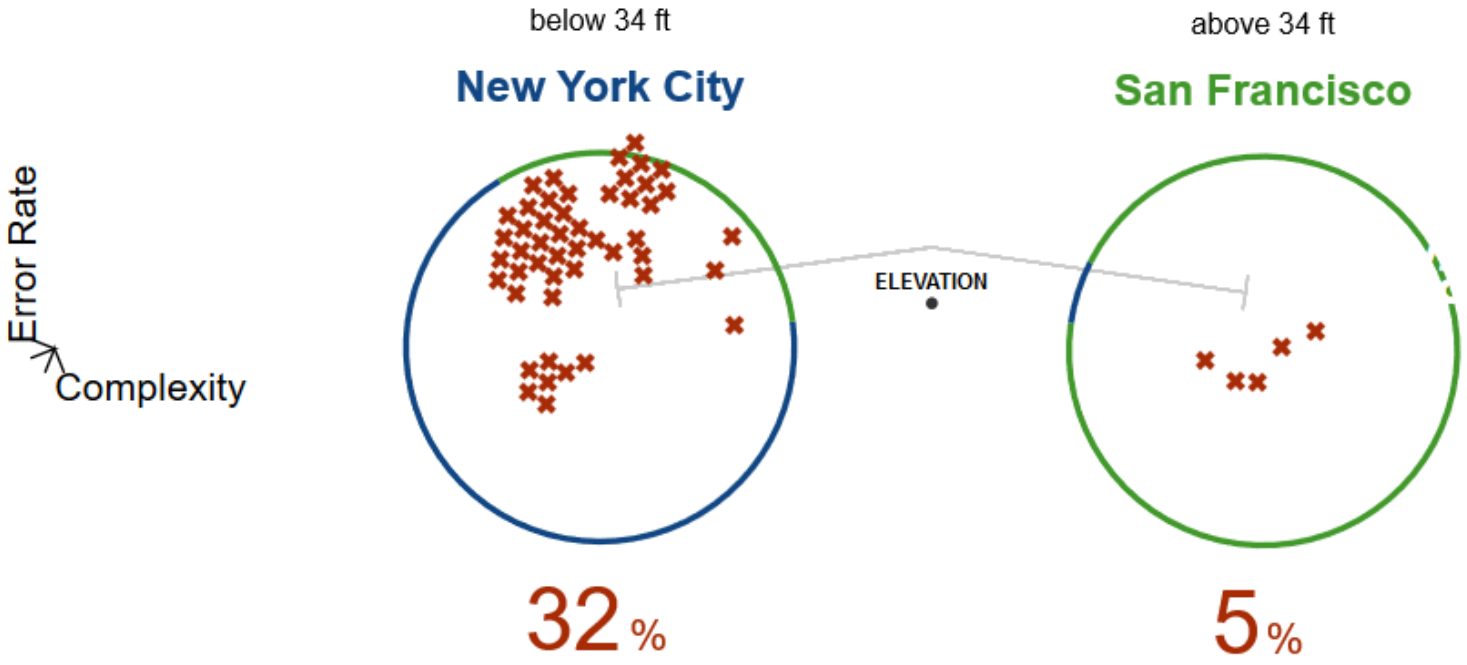
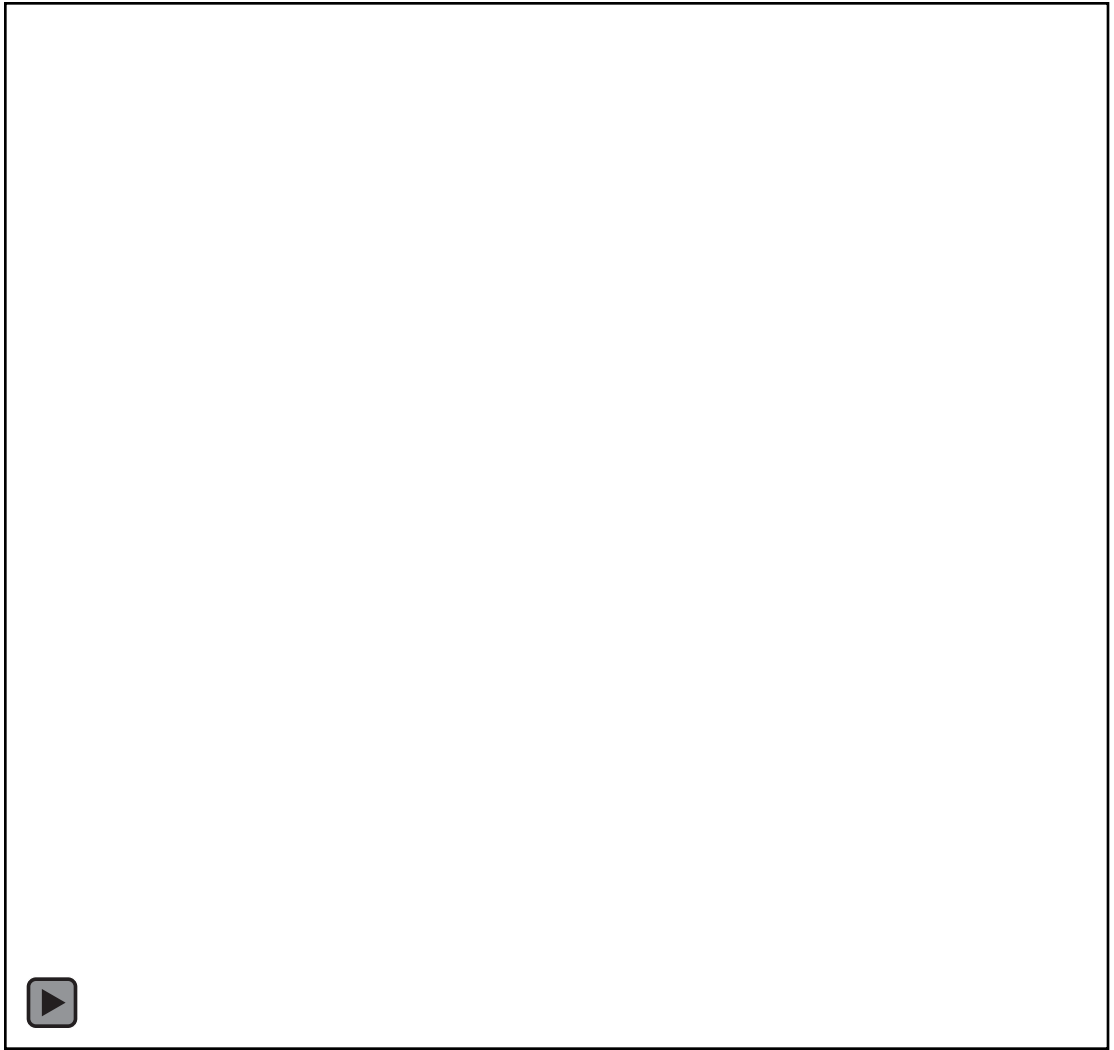


Errors due to bias



Decrease error due to bias



Example of var

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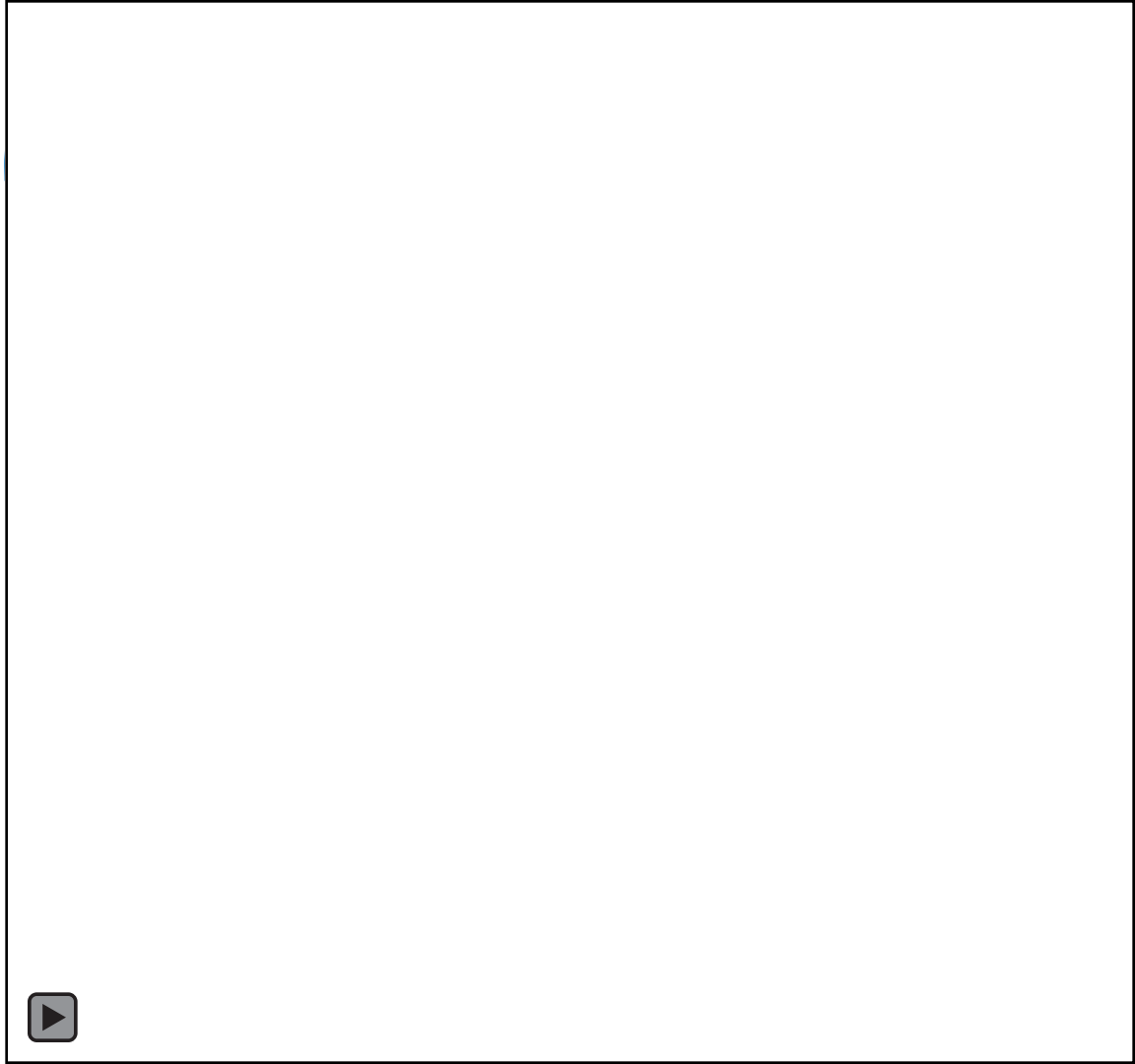

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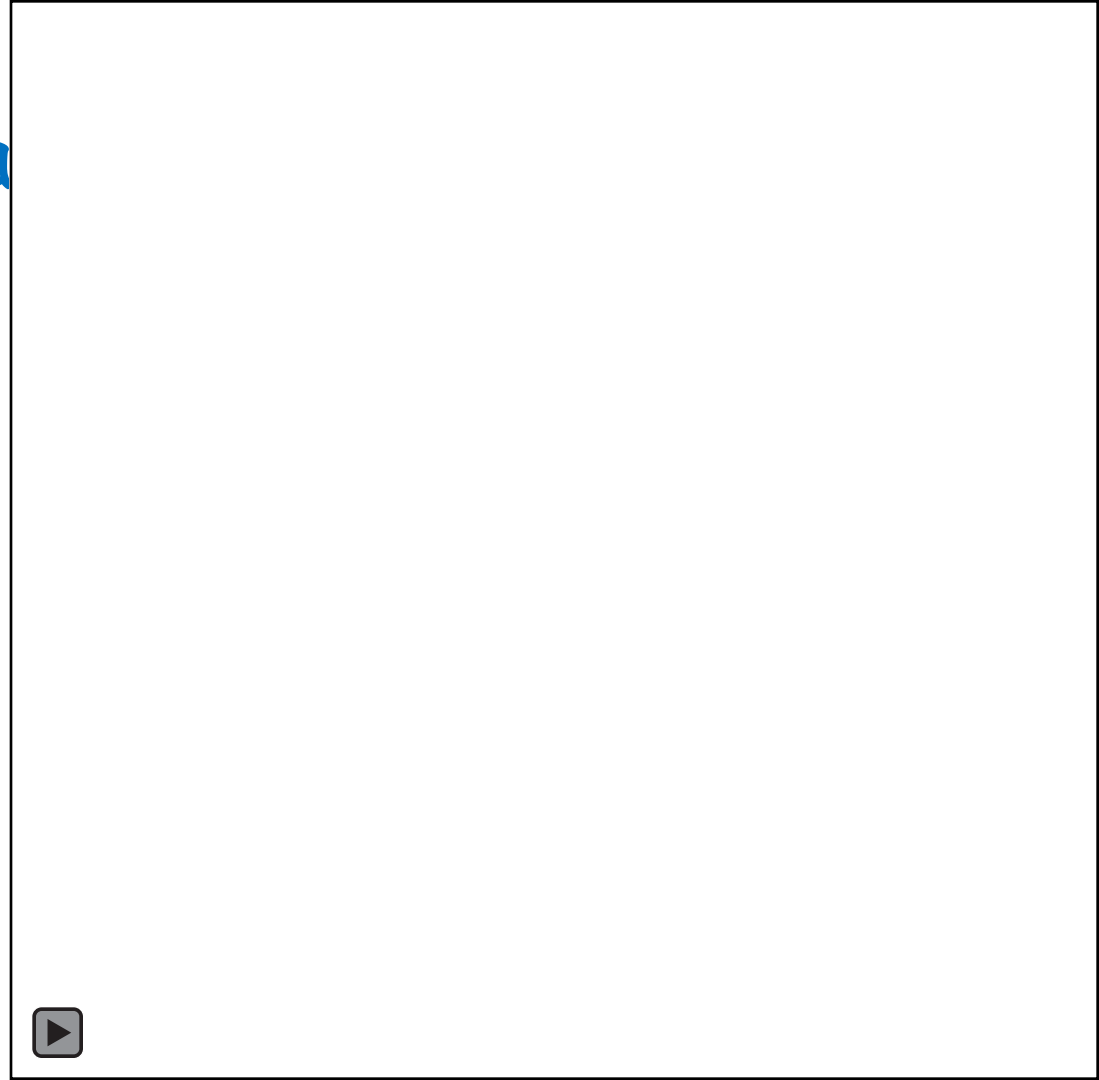
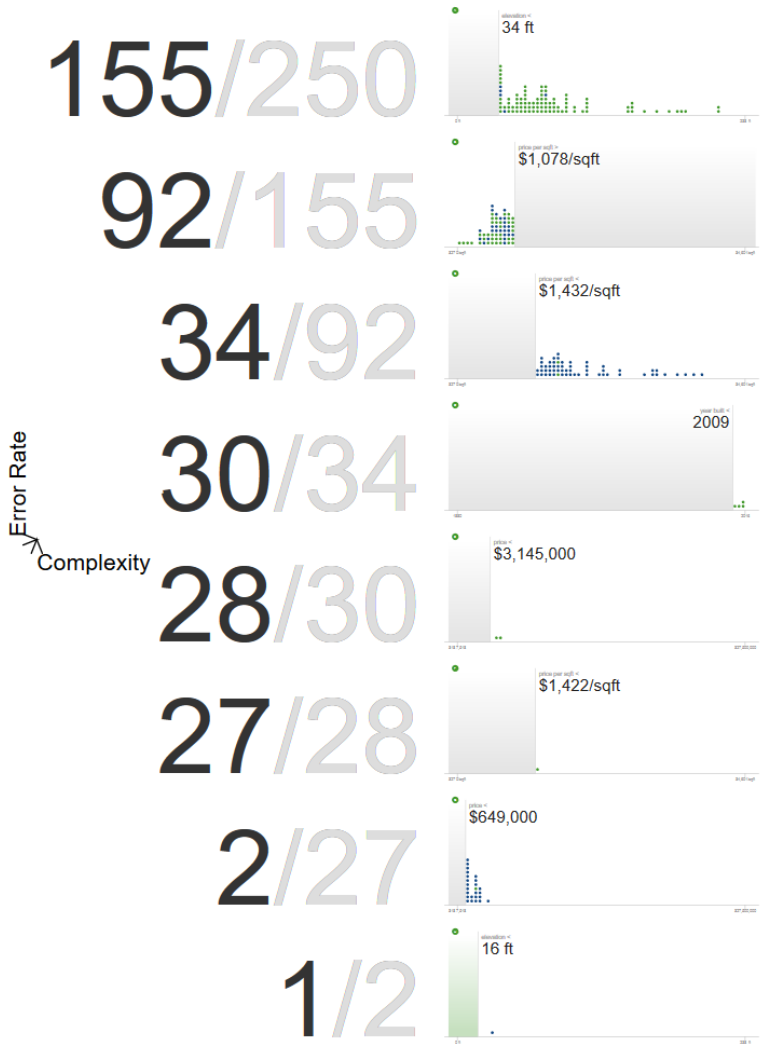
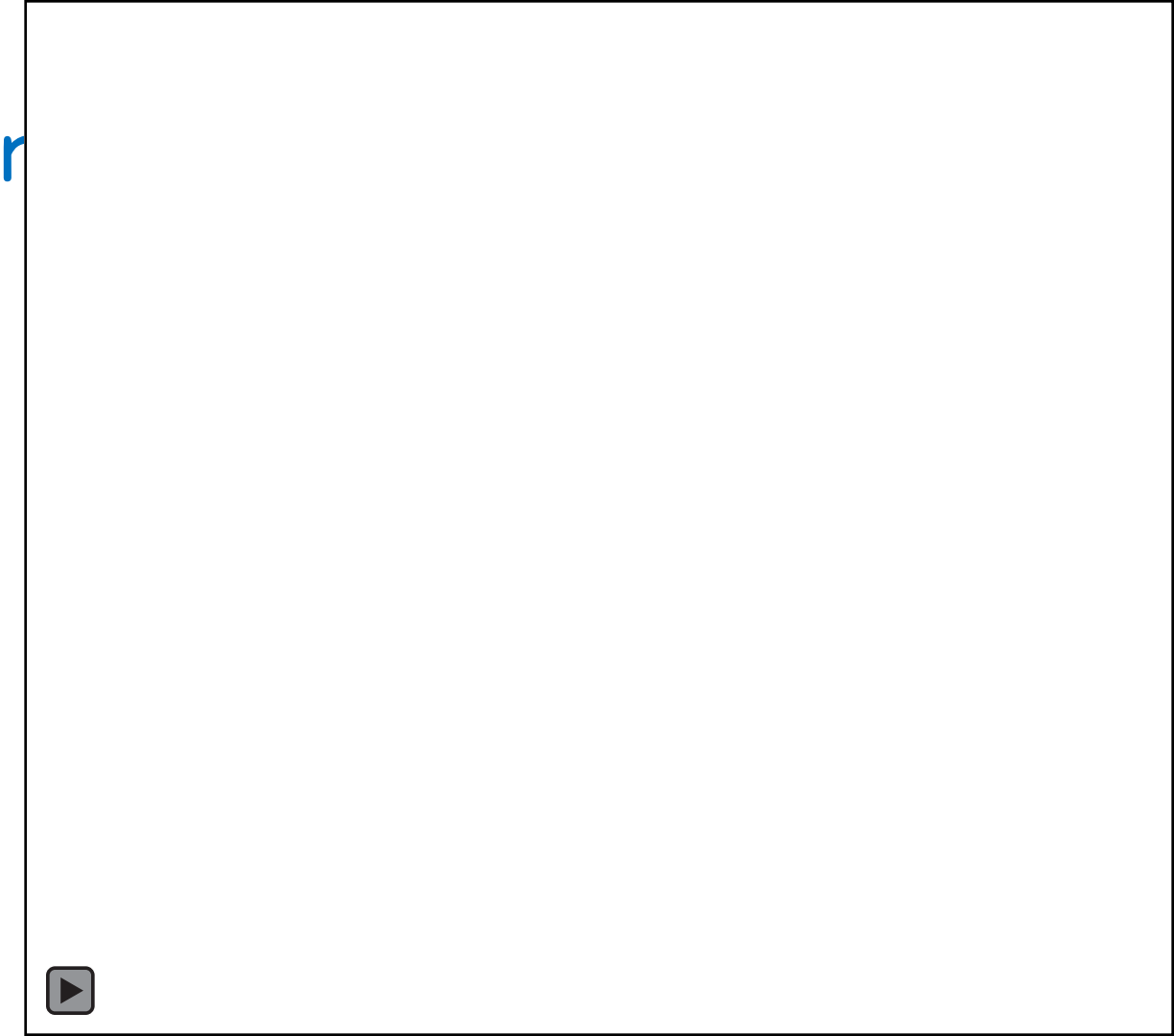


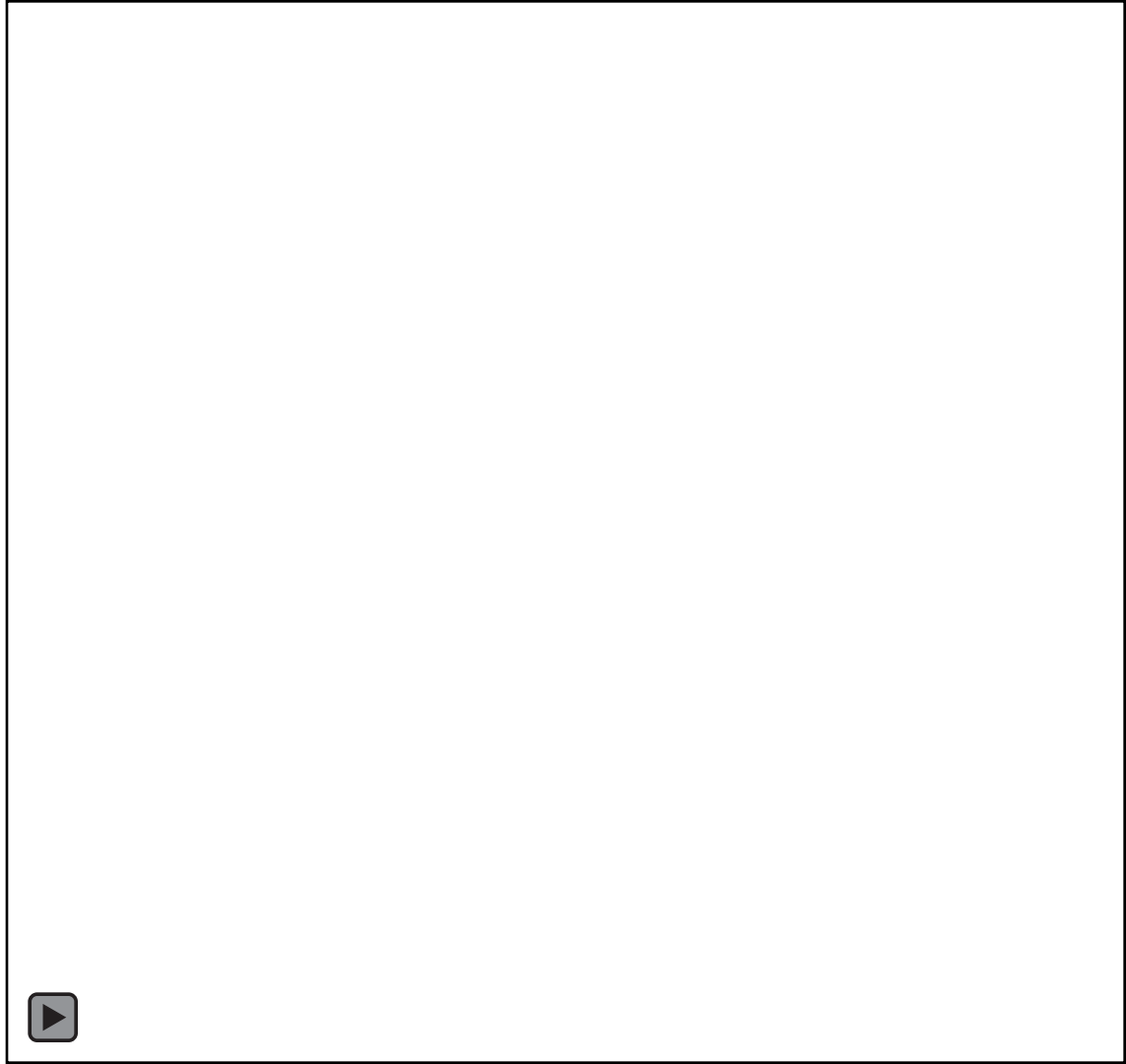
Example of variance



Flukes are nor



Bias-variance tradeoff



Bias-va
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Bias-var
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