Q.1 Given an array of size N &
`Q` queries of the format $\underline{s}$ & $\underline{e}$

$\downarrow$ $\downarrow$
start   end

Return sum of elements from index `s` to `e`.

A : [-3, 6, 2, 4, 5, 2, 8, -9, 3, 1]   N=10

(indices: 0 1 2 3 4 5 6 7 8 9)

Q : 4

s	e	
1	3	⟹ 12
2	7	⟹ 12
4	8	⟹ 9
0	2	⟹ 5

Brute approach :

```
for (i=0 ; i < Q ; i++)
{
  scan{s,e}
  sum = 0
  for (j=s ; j <= e ; j++)      } O(N)
  {
    sum = sum + a[j]
  }
  print (sum)

}
```

T.C ⟹ O(NQ)
S.C ⟹ O(1)
(extra)

Given the scores of last 10 overs of a match.

41	42	43	44	45	46	47	48	49	50

$$[288, 312, 330, 349, 360, 383, 394, 406, 436, 439]$$

Runs scored in last 5 overs $= 439 - 360$
$[46 \rightarrow 50]$ $R[50] - R[45]$

Runs scored in 50th over $= 439 - 436 = 3$
$[50 \rightarrow 50]$ $R[50] - R[49]$

Runs scored in 49th over $= 436 - 406$
$[49 \rightarrow 49]$ $R[49] - R[48]$

Runs scored from 42nd to 45th $= 360 - 288$
$[42 \rightarrow 45]$ $R[45] - R[41]$

ABD $\Rightarrow$ 100 (31)

$\underline{Prefix \ Sum} \atop \underline{Array}$ $\rightarrow$ Every index should store the sum of all elements from start till that index.

$PS[i] \Rightarrow$ Sum of elements from $[0 \rightarrow i]$

$$A[]: \begin{array}{ccccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ [& -3, & 6, & 2, & 4, & 5, & 2, & 8, & -9, & 3, & 1 &] \end{array}$$

$$PS[]: \quad [-3, 3, 5, 9, 14, 16, 24, 15, 18, 19] \leftarrow$$

$$PS[0] = A[0]$$

$$PS[1] = A[0] + A[1]$$

$$PS[2] = A[0] + A[1] + A[2] \Leftarrow$$

$$PS[3] = \underline{A[0] + A[1] + A[2]} + A[3]$$
$$= PS[2] + A[3]$$

$$\vdots$$

$$PS[i] = \underline{A[0] + A[1] + A[2] \cdots + A[i-1]} + A[i]$$
$$\underline{PS[i-1]}$$

$$\boxed{PS[i] = PS[i-1] + A[i]}$$

$$PS[0] = A[0] \leftarrow$$
for (i=1 ; i<N ; i++)
{
$$\quad PS[i] = PS[i-1] + A[i];$$
}

iterations = N-1
TC : O(N)
S.C = O(N)

$$A[]: \begin{array}{ccccccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ [& -3, & 6, & 2, & 4, & 5, & 2, & 8, & -9, & 3, & 1 &] \end{array}$$

$$PS[]: \quad [-3, 3, 5, 9, 14, 16, 24, 15, \boxed{18}, 19]$$

Q:

S	e		
		$\Rightarrow$	$PS[e] - PS[s-1]$
1	3	$\Rightarrow$	$9 - (-3) \Rightarrow 12$
2	7	$\Rightarrow$	$15 - 3 \Rightarrow 12$
4	8	$\Rightarrow$	$18 - 9 \Rightarrow 9$
0	2	$\Rightarrow$	$5 - 0 \Rightarrow 5$

A:
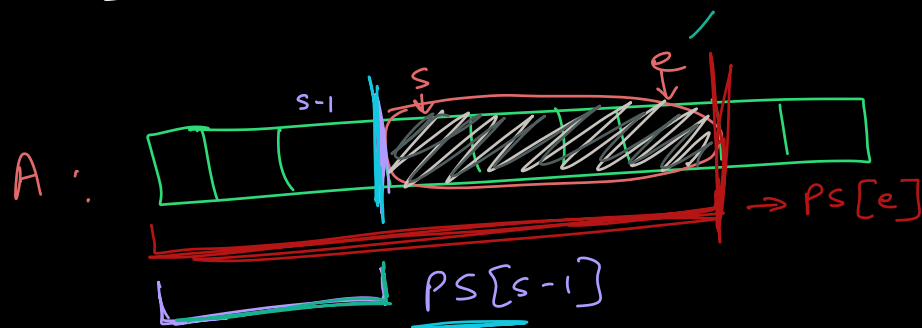
$$PS[e] - PS[s-1]$$

$$sum(0-e) = sum(0, s-1) + sum(s, e)$$

$$\boxed{sum(s,e) = PS[e] - PS[s-1]}$$

$$PS[e] \Leftarrow if(s == 0)$$

#TC : Build P.S. $\Rightarrow O(N)$
        Ans for Q queries $\Rightarrow O(Q)$

T.C $\Rightarrow O(N+Q)$

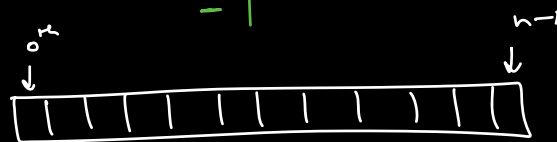S.C $\Rightarrow O(N)$.

Range Sum $\Rightarrow$ Prefix Sum

Q2 Given an array, return `true` if there exists an equilibrium index in array.

E.I $\Rightarrow$ index for which, (excluding itself)

sum of all no. on left side $=$ sum of all no. on right side.

A : [1, 2, 3, 4, 8, 10] $\Rightarrow$ True.

      10   10

Quiz : [-7, 1, 5, 2, -4, 3, 0]

     0 1 2 3 4 5 6

     -1   -1

0th $n-1$

sum on left side for 0th $\Rightarrow 0$ $\Rightarrow S_L$

sum on right side $\Rightarrow 0$ $\Rightarrow S_R$

Brute force For every index `i`

check if $\sum [0 \to i-1] = \sum [i+1 \to n-1]$

return true ;

PS [ ]

for ($i = 0$; $i < N$; $i{+}{+}$)

for ($0 \to i-1$)
sum $+= A[j]$
$\Leftarrow$ $O(N)$

// S_L = sum of 0 to $(i-1)$ $\Rightarrow$

index Out of Bound $\Rightarrow$ $S_L = \boxed{PS[i-1]}$ // $O(1)$

// S_R = sum of $(i+1)$ to $(n-1)$ $\Rightarrow$ for ($i+1 \to N-1$)
sum $+= A[j]$
$S_R = PS[n-1] - PS[i]$ // $O(1)$ $\Leftarrow O(N)$

if ($S_L == S_R$)
return True;

}

T.C $\Rightarrow$ Build P.S. + Find E.I.
$\Downarrow$ $\Downarrow$
$O(N)$

$\underset{\quad\quad\quad 0\quad\quad 1\quad\quad 2\quad\quad 3\quad\quad 4\quad\quad 5\quad\quad 6}{\text{Eg} \cdot A \cdot [-7 , 1 , 5, 2, -4, 3, 0]}$

P.S : $[-7, -6, -1, 1, -3, 0, 0]$

$i = 0$ $S_L = \cancel{PS[-1]^0}$ 0
$\underline{S_R} = PS[n-1] - PS[0] = 0 - (-7) = 7$

$i = 1$ $S_L = PS[0] = -7$
$S_R = PS[n-1] - PS[1] = 0 - (-6) = 6$

$i = 3$ $S_L = PS[2] = -1$
$S_R = PS[n-1] - PS[3] = 0 - 1 = -1$

$i \ldots$

H.W : Do this without using P.S.

S.C ⟹ O(1)

T.C ⟹ O(N)

20 min

Q. Given array, given Q queries.

⟹ s, e, O ⟹ Sum of all the odd indexed elements

⟹ s, e, E ⟹ Sum of all the even indexed elements

from 's' to 'e'

$$A : \begin{bmatrix} \overset{0}{2}, & \overset{1}{3}, & \overset{2}{1}, & \overset{3}{-1}, & \overset{4}{0}, & \overset{5}{8}, & \overset{6}{5}, & \overset{7}{4} \end{bmatrix}$$

s	e	O/E	
⟹ 3	6	O	⟹ $A[3] + A[5]$ ⟹ $\underline{7}$
⟹ 1	5	E	⟹ $A[2] + A[4]$ ⟹ **1**

Range Sum ⟹ Prefix Sum.

$$A : \quad \overset{0}{2}, \quad \overset{1}{3}, \quad \overset{2}{1}, \quad \overset{3}{6}, \quad \overset{4}{4}, \quad \overset{5}{5}$$

s	e
1	7

$PS_E[7] - PS_E[0]$

PS_E : 2, 2, 3, 3, 7, 7

⟹ $PS_E(i)$ ⟹ Sum of all even indexed elements from 0 to i

PS_O : 0, 3, 3, 9, 9, 14

⟹ $PS_O(i)$ ⟹ sum of all odd indexed (0 → i)

Build PS_E & PS_O $\Rightarrow O(N)$

Each Query $\rightarrow O(1)$ time

Q4 Given an array. Count the no. of special index in the array.

Directi
CodeNation
Google
JP Morgan

Special index : An index after removing which,

Sum of all odd indexed elements $=$ Sum of all even indexed elements.

remaing array.

$$
\begin{array}{ccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 \\
A: & 4 & 3 & 2 & 7 & 6 & -2
\end{array}
$$

Remove.

$i=0 \Rightarrow \quad 3 \quad 2 \quad 7 \quad 6 \quad -2 \Rightarrow S_e=8 \mid S_o=8$ ✓

$O(N)$ $i=1 \Rightarrow \quad 4 \quad 2 \quad 7 \quad 6 \quad -2 \Rightarrow S_e=9 \mid S_o=8$ ✗

$i=2 \Rightarrow \quad 4 \quad 3 \quad 7 \quad 6 \quad -2 \Rightarrow S_e=9 \mid S_o=9$ ✓

$\vdots$

Quiz $\begin{bmatrix} \overset{0}{4}, & \overset{1}{1}, & \boxed{\overset{2}{5}}, & \overset{3}{3}, & \overset{4}{7}, & \overset{5}{10} \end{bmatrix}$

$$
\begin{array}{ccccc}
4 & 1 & 3 & 7 & 10 \\
0 & 1 & 2 & 3 & 4
\end{array}
$$

$x \rightarrow x-1$
odd $\rightarrow$ even
even $\rightarrow$ odd.

Quiz: $[\overset{0}{2}, \overset{1}{3}, \overset{2}{1}, \overset{3}{\boxed{4}}, \overset{4}{0}, \overset{5}{-1}, \overset{6}{2}, \overset{7}{-2}, \overset{8}{10}, \overset{9}{8}]$

$$\underline{S_0} = S_0[0 \to 2] + \underline{S_E[4 \to 9]}$$

$\hookrightarrow$ Because after removed of index 3
all index in this range will toggle.

$\underline{S_0} \Rightarrow [\overset{0}{2}, \overset{1}{3}, \overset{2}{1}, \overset{3}{0}, \overset{4}{-1}, \overset{5}{2}, \overset{6}{-2}, \overset{7}{10}, \overset{8}{8}]$

Quiz: $[\overset{0}{2}, \overset{1}{3}, \overset{2}{1}, \overset{3}{\boxed{4}}, \overset{4}{0}, \overset{5}{-1}, \overset{6}{2}, \overset{7}{-2}, \overset{8}{10}, \overset{9}{8}]$

$$\underline{S_E} = S_E[0 \to 2] + S_0[4 \to 9]$$

After removing index i,
$$S_E = S_E[0, i-1] + S_0[i+1, N-1]$$
$$S_0 = S_0[0, i-1] + S_E[i+1, N-1]$$

$S(s,e)$
$= P(e) - P(s-1)$

$\underline{PS_E} \Rightarrow$ Sum of element at Even index $(0 \to i)$

$PS_0 \Rightarrow$ Sum of elments at odd index $(0 \to i)$

$O(1)$

$$S_E[0, i-1] = PS_E[i-1]$$
$$S_0[0, i-1] = PS_0[i-1]$$
$$S_0[i+1, N-1] = PS_0[N-1] - PS_0[i]$$

$$S_E \, [i+1, N-1] = PS_E \, [N-1] - PS_E \, [i] \, \Big)$$

Psendo Code.

```
// Build PSₑ        ⎫
                    ⎬ O(N)
// Build PS₀        ⎭

⟹ ans = 0;
    for ( i = [0]; i < N ; i++ )          ⟹ O(N)
    {
        // Sₑ ⟹ Sum of even indexed elem after removal of i
index Out of Bounds
    ⟹  Sₑ = PSₑ [i-1] + ( PS₀ [N-1] - PS₀ [i] )   ⟹ O(1)

        // S₀ ⟹ Sum of odd indexed elem after removal of i
        S₀ = PS₀ [i-1] + ( PSₑ [N-1] - PSₑ [i] )   ⟹ O(1)

        if ( Sₑ == S₀ )
            ans ++;
    }
}
```

$$T.C \implies \begin{array}{l} O(N) \Rightarrow PS_E \\ + O(N) \Rightarrow PS_0 \\ + O(N) \end{array} \bigg\} \implies \underline{O(N)}$$

check special
for all i

$$S.C \implies O(N)$$

(extra)

Doubts.

$$A : [7, 7, 4, 3, 2, 1] \implies Ans = 4.$$

$O(1)$.

$$i \longrightarrow n \implies swap\ (A[i], A[j])$$

Java

$temp = A[i]$

H.W.

$A[i] = 0$

XOR way

$A[i]$