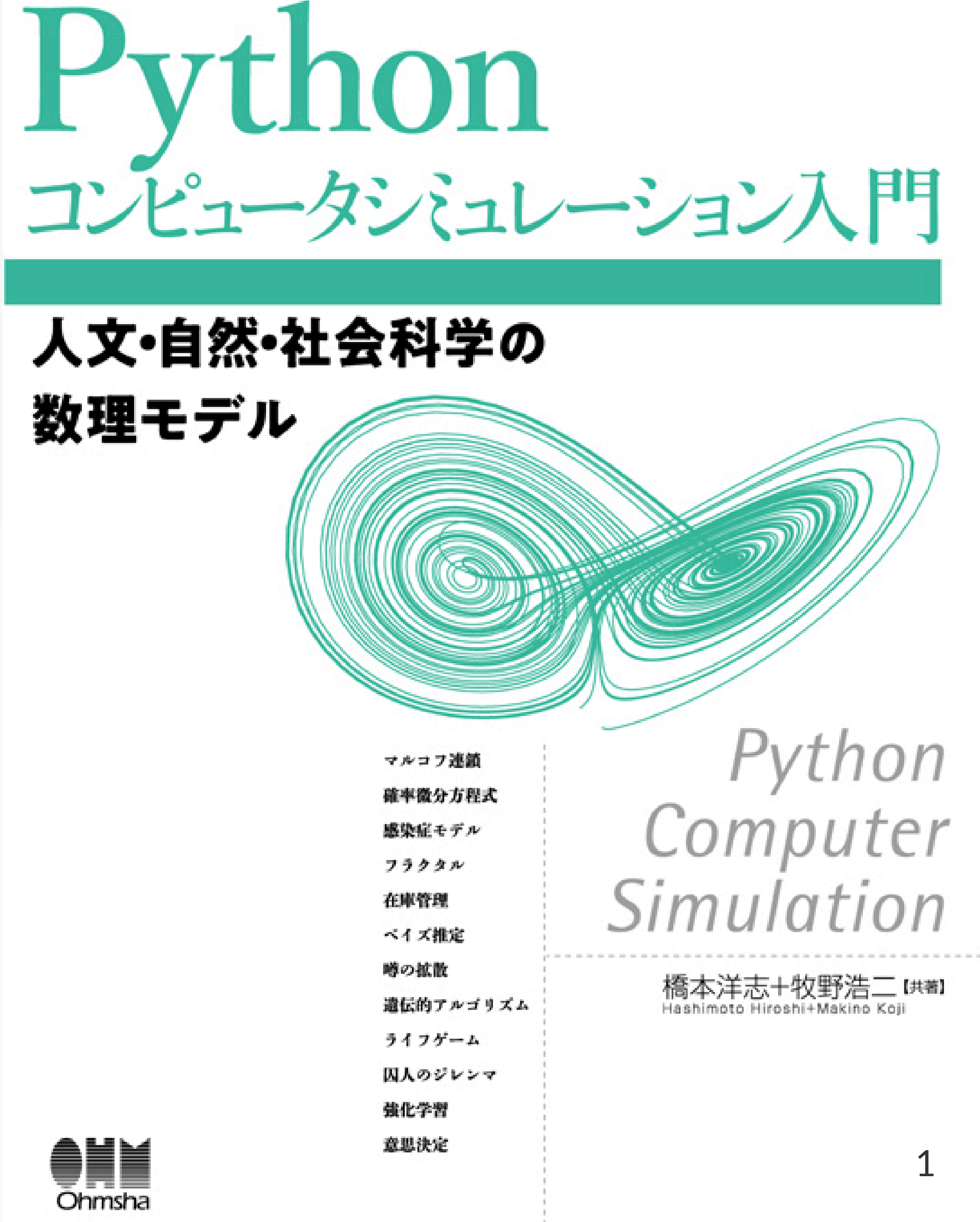
Python Computer Simulation



Chapter 5 Natural science Model

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5.1 Population preditction

5.2 Epidemic

5.3 Predator-prey relationship

5.4 Fractal

5.5 Chaos

5.6 Sound and frequency

5.1 Population preditction

- ***MOTIVATION***: Predict future population with a mathmatical model
- ***APPROACH***: Ideas of **Population Model**
  - $\frac{dP(t)}{dt} = \gamma P(t)(\frac{P\infty - P(t)}{P\infty})$ #1 | $P(t)$: population
  - The population growth rate in a given year is proportional to the population in that year.
  - Population growth has a upper limit
  - The population growth rate decreases as the population gets close to the upper limitation
- Analitical solution of Equation #1
  - $P(t) = \frac{P\infty}{1+(\frac{P\infty}{P0}-1)e^-\gamma t}$ #2
  - The solution is a logistic function
- ***EXERCISE***: Parameter estimation
  - Determine $P0$, $P\infty$, and γ with real data

5.2 Epidemic

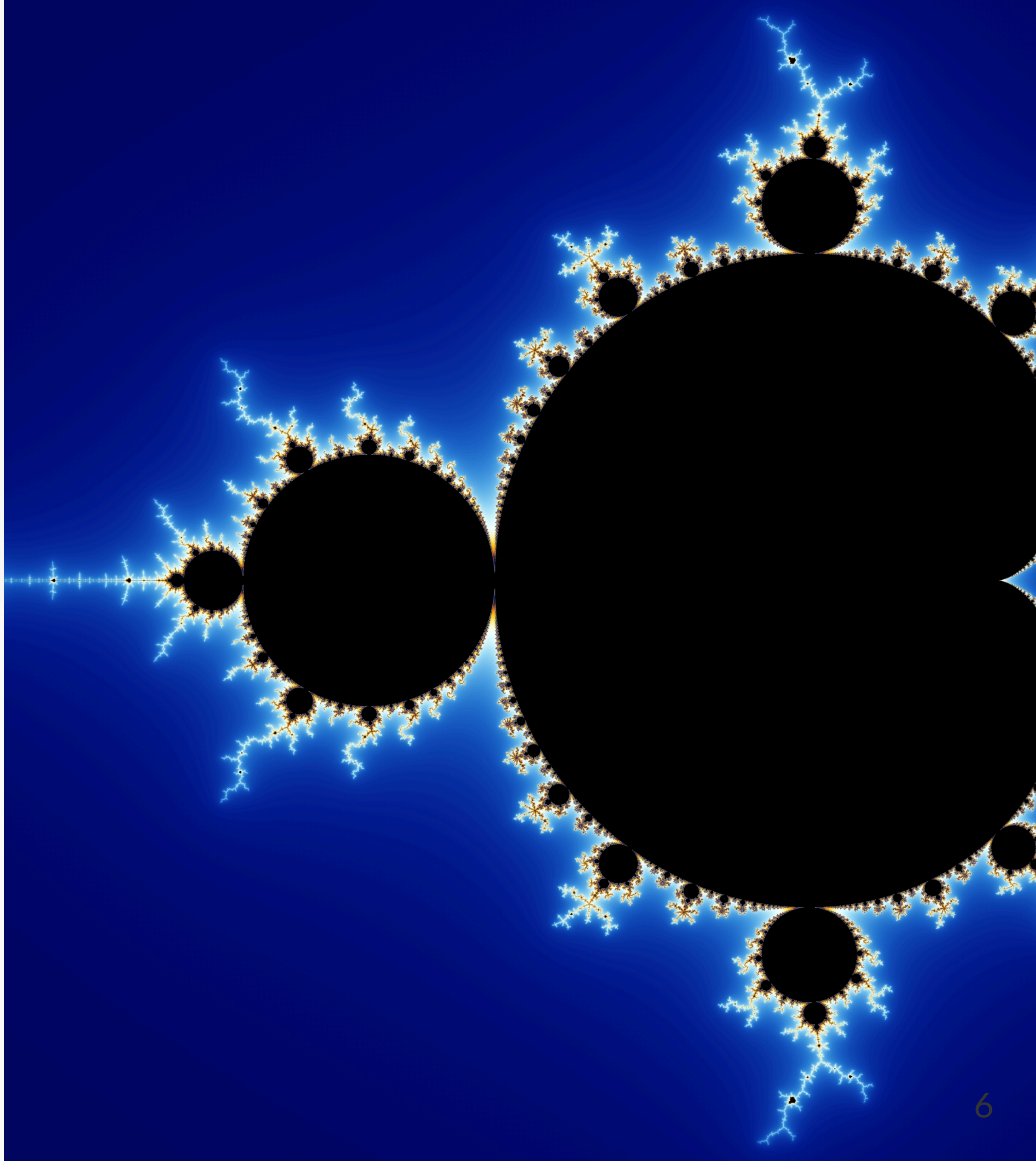
- ***MOTIVATION***: Predict and analyze processes of epidemics with a mathmatical model
- ***APPROACH***: Ideas of **SIR Model**
  - $\frac{dS(t)}{dt} = -\beta S(t)I(t)$ #3 | $S(t)$: Susceptibles
  - $\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t)$ #4 | $I(t)$: Infectives
  - $\frac{dR(t)}{dt} = \gamma I(t)$ #5 | $R(t)$: Recovered
  - β stands for infection rate and γ stands for recovery rate
  - $S(t) * I(t)$ represents contacts between the susceptibles and the infectives
  - New infetives are proportional to Infection rate
  - Variation of the infectives is the difference between new infectives and new people who recovered
  - New people who recovered are propotional to $I(t)$
- ***EXERCISE***: Simulation and analysis focusing on $S(0)$
  - SIR Model behaves in a different way depending on the initial value of $S(t)$
  - $S(0) <= \frac{\gamma}{\beta}$ or $S(0) > \frac{\gamma}{\beta}$?

5.3 Predator-prey relationship

- ***MOTIVATION***: Expalain population variation of organisms with a mathmatical model
- ***APPROACH***: Ideas of **Predator-prey Model**
  - $\frac{dx(t)}{dt} = ax(t) - bx(t)y(t)$ #6 | $x(t)$: Preys
  - $\frac{dy(t)}{dt} = -cy(t) + dx(t)y(t)$ #7 | $y(t)$: Predators
  - Regarding Equation #6, the Preys increase exponentially without the predators
  - Regarding Equation #6, the preys decrease depending on the numbers of chances to encounter the predators
  - Regarding Equation #7, the predators exponentially decrease without the preys
  - Regarding Equation #7, the predators increase depending on the numbers of chances to encounter the preys
- ***EXERCISE***: Conduct simulations
  - Check the trend on the numbers of the predators and preys
  - What if both of the predators and preys are overfished simultaneously?
  - What if both of the predators and preys are not fished so much simultaneously?

5.4 Fractal #1

- **Fractals** are geometric shapes which have similar structure at a given scale (**self similarity**)
- ***EXERCISE #1***: Generate Mandelbrot Set
    - **Mandelbrot Set** is a fractal resulted from calculation for the divergence condition for a complex sequence
    - Check the details of the figure at a given scale
    - Find self similarity



5.4 Fractal #2

- ***MOTIVATION***: Depict complex fern leaves (シダの葉) with a few simple rules
- ***APPROACH***: Repetition and randomization
    - prepare four specific affine tranformations to get the values of x and y for the next step
    1. $f1$ is for stems
    2. $f2$ is for continuous smaller leaves
    3. $f3$ is for larger leaves on left
    4. $f4$ is for larger leaves on right
    - Set probabilities for each transformation
- ***EXERCISE #2***: Depict fern leaves on computer
    - Generate **Barnsley Fern**
    - Focus on self similarity
    - Can you depict different types of fern leaves with different parameters?

5.5 Chaos

- **Chaos** is complex system which is deterministic but hard to its predict future behavior due to **sensitive dependence on the initial conditions (初期値鋭敏性)**
- ***EXERCISE #1***: Depict **Lorenz attractor**
    - $\frac{dx(t)}{dt} = s(y - x)$ #8
    - $\frac{dy(t)}{dt} = rx - y - xz$ #9
    - $\frac{dz(t)}{dt} = xy - bz$ #10
    - A famous set of values (s, r, b) is $(10, 28, 8/3)$
    - Change the initial values by just a bit (e.g. x(0)=0 -> x(0)=0.0001)
- ***EXERCISE #2***: Depict chaos of **logistic equation**
    - $x(n+1) = ax(n)(1 - x(n)), (n = 0, 1, 2, \dots)$ #11
    - Check different types of behavior for the value of $x(n), with different a
    - Change the initial value of $x(0)$ to check sensitive dependence on the initial conditions (e.g. x(0)=0.01 -> x(0)=0.010001)
    - Generate figures for the values of a and $x(n)$ (**logistic maps**)

5.6 Sound and frequency

- **Twelve equal temperament** is a musical note system deviding sounds into twelve notes
- When a note gets a one round, the frequency of the note doubles
- When the frequency of a note is f, that of the next note is $\sqrt[12]{2}f$
- ***EXERCISE***: Calculate frequency of notes
  - What is the frequency for the next Do?
  - What is the frequency for the Ti on the table?
    - Hint: You can find **La#** between La and Ti

Note	Do	Re	Mi	Fa	So	La	Ti
Frequency (Hz)	262	294	330	349	392	440	???