Directed and Undirected Graphs

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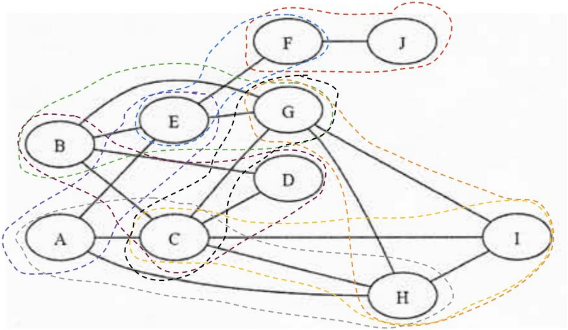

C_1 : F, J	C_4 : G, H, I	C_7 : B, C, D
C_2 : F, E	C_5 : C, H, I	C_8 : A, E
C_3 : B, E, G	C_6 : A, C, H	C_9 : C, G

a) construction of the moral graph

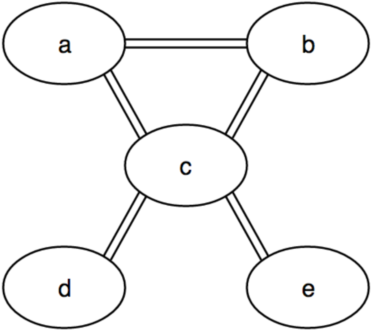

Figure: moral graph of given DAG

b) pdf from cliques and seperators

- we show that the DAG representation equals the given formula for the joint distribution:

$$
\begin{aligned}
P(a,b,c,d,e) &= P(a)P(b)P(c|a,b)P(d|c)P(e|c) && (1)\\
&= \frac{P(a,b)P(a,b,c)P(d,c)P(e,c)}{P(c)^2 P(a,b)} && (2)\\
&= \frac{P(a,b,c)P(d,c)P(e,c)}{P(c)P(c)} && (3)\\
&= \frac{P(C_1)P(C_2)P(C_3)}{P(S_1)P(S_2)} = \frac{\prod_i P(C_i)}{\prod_j P(S_j)} && (4)\\
& && (5)
\end{aligned}
$$

a) potential functions and the clique-marginal representation

- used to represent joint probabilities of cliques
- potential functions are strictly positive, real-valued and only depend on random variables in their assigned clique
- don't need to be normalised so that $\psi_n(C_n) = \alpha\, P(C_n),\ \alpha > 0$
- link between joint probability distribution and clique-marginal representation:

$$P(V \in \mathcal{V}) \;=\; \prod_{V \in \mathcal{V}} P(V | V_{pa(V)}) \;=\; \frac{\prod_i P(C_i)}{\prod_j P(S_j)} \;=\; \alpha \frac{\prod_i \psi_i(C_i)}{\prod_j \psi_j(S_j)} \qquad (6)$$

b) exploitation of potential functions to make inferences

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c) benefit in comparison to DAG factorisation

- simpler and clearer models due to substitution of subgraphs by cliques
- much better representation for inference

Literature

'Machine Intelligence I', Lecture Notes by Prof. Dr. Klaus Obermayer, 2015

'Probabilistic Networks and Expert Systems', Chapter 3, Cowell et al., 1999