

Week 1: Descriptive Analytics

- ◆ An Operational Decision Problem
- ◆ Forecasting with Past Historical Data
- ◆ Moving Averages
- ◆ Exponential Smoothing
- ◆ Thinking about Trends and Seasonality
- ◆ Forecasting for new Products
- ◆ Fitting distributions

Week 1: Descriptive Analytics

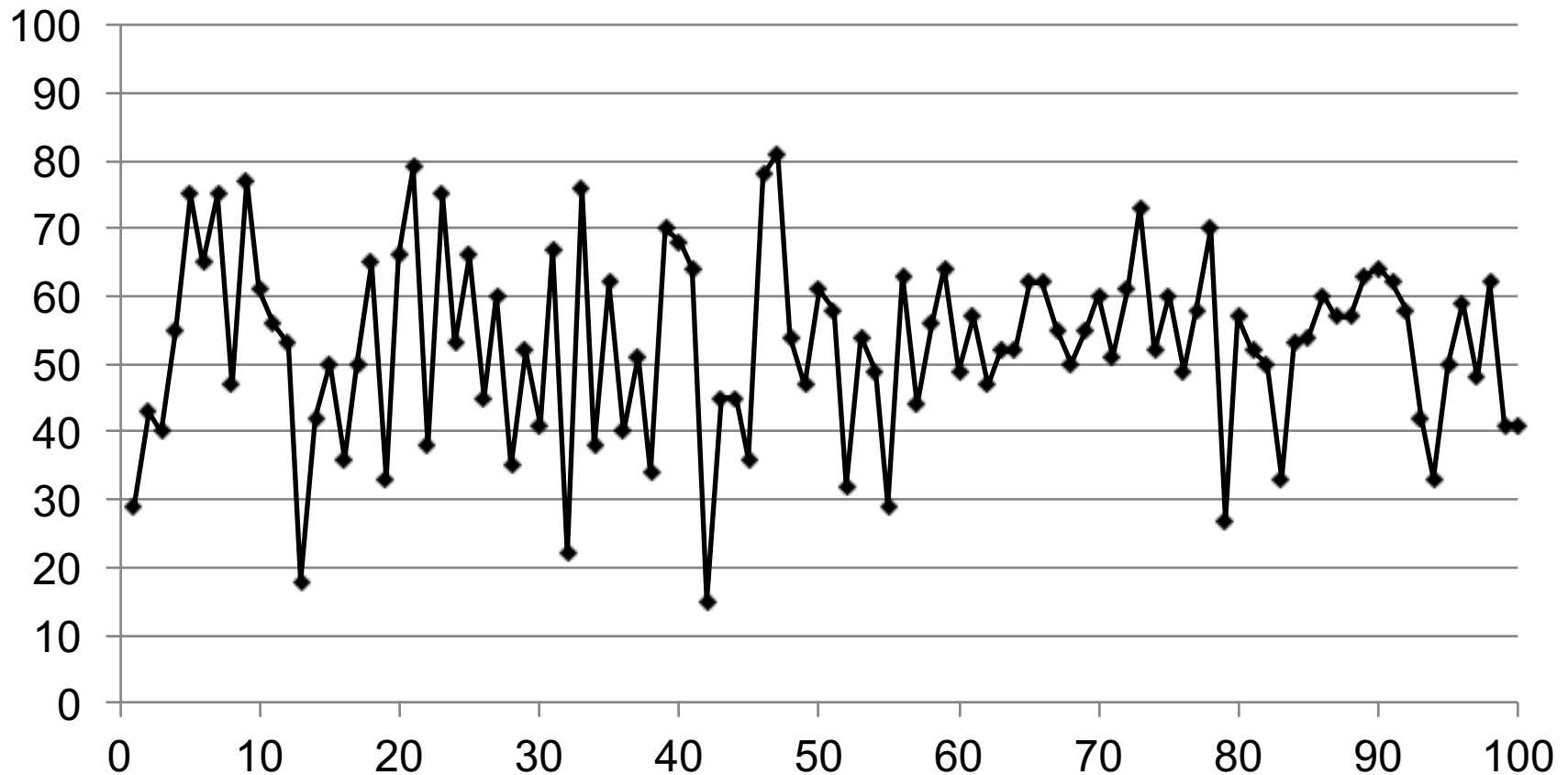
- ◆ An Operational Decision Problem

- ◆ Forecasting with Past Historical Data
- ◆ Moving Averages
- ◆ Exponential Smoothing

Session 2

- ◆ Thinking about Trends and Seasonality
- ◆ Forecasting for new Products
- ◆ Fitting distributions

Recall our past demand data



- ◆ Let D_t denote demand observed in period t
- ◆ From the past data, we have D_1, D_2, \dots, D_{100}

Descriptive Statistics: Mean and Standard Deviation

◆ Sample Mean or Sample Average:

- Arithmetic average of all the data points $\mu = (D_1 + D_2 + \dots + D_n)/n$
- Tells us (roughly) what you expect the next observation to be
- Can be calculated using excel **average()** function.
- Whatever your average is, the demand in future will deviate from the average.

◆ Sample Standard Deviation

- A measure of how much noise or variation (from the average) there is in your data.
- Standard deviation $s = \sqrt{\sum (D_t - \mu)^2 / (n-1)}$, $t = 1, \dots, n$
- Can be calculated using excel **stdev()** function.

Descriptive Statistics for our Data

- ◆ Let past data be $D_1, D_2, \dots, D_{99}, D_{100}$
- ◆ Sample Average: 52.81
- ◆ Sample Standard deviation: 13.73

- ◆ In the excel file *DemandData.xlsx*, I show how to calculate these two for our data on an excel sheet.

- ◆ These are two descriptive statistics of our data.

- ◆ Note 1: If our data were normally distributed, these two statistics would be sufficient to describe the demand.

- ◆ Predictive Statistics: We need to adjust our sample standard deviation for forecasting purposes. *More on this later.*

Notation: Look Ahead Forecasts

- ◆ Recall that D_1, D_2, \dots, D_t are the past values (demands observed).
 - When we are making a forecast in period t , we have demands up to period t .
- ◆ We call $F_{t, t+\tau}$ = forecast made in period t for demand in future period $t + \tau$ where $\tau = 1, 2, 3, \dots$
 - E.g. $F_{t, t+3}$ is the forecast made in period t for 3 periods ahead.
 - E.g. $F_{100, 100+3}$ is the forecast made in period 100, for the period 103. This is called a 3-step forecast. Why?
 - Because, we are looking and forecasting for three periods ahead.

One-Step Forecast

- ◆ Typically, we are interested in the next outcome, or simply one-step ahead forecasts.
 - $F_{t, t+1}$ is the forecast made at t for period $t+1$
 - $F_{t-1, t}$ is the forecast made at $t-1$ for the next period t
- ◆ We will use the shorthand notation F_{t+1} for one-step forecast made at t for period $t+1$.
 - Simply, F_{t+1} stands for $F_{t, t+1}$

Forecasting for Stationary Series

- ◆ Stationary data shows no trend behavior.
 - Roughly speaking the future resembles the past.
 - Example: Our Newsvendor Demand data (DemandData.xls) which shows no perceptible trend.
- ◆ A stationary time series has the form:
 $D_t = \mu + \varepsilon_t$ where μ is a constant and ε_t is a random variable with mean 0 and some standard deviation σ .
- ◆ We use past data for forecasting.
- ◆ Two common methods for forecasting stationary series are **moving averages** and **exponential smoothing** (advanced material slides).

Moving Averages

- ◆ A moving averages forecast is the arithmetic average of the n most recent observations.
- ◆ We will denote the Moving Averages method that uses n data points as
 - MA (n)
- ◆ For a one-step-ahead forecast for period t :
 - $F_t = (D_{t-1} + D_{t-2} + \dots + D_{t-n})/n$
- ◆ For moving averages, a multi-step forecast is the same as the one-step forecast.

Moving Averages

- ◆ One-step-ahead forecast for period t :

- $F_t = (D_{t-1} + D_{t-2} + \dots + D_{t-(n-1)} + D_{t-n}) / n$

- ◆ It is called “moving” average because the chosen data points “move” and are always the most recent n data points.

- ◆ Forecasting for period $t+1$,

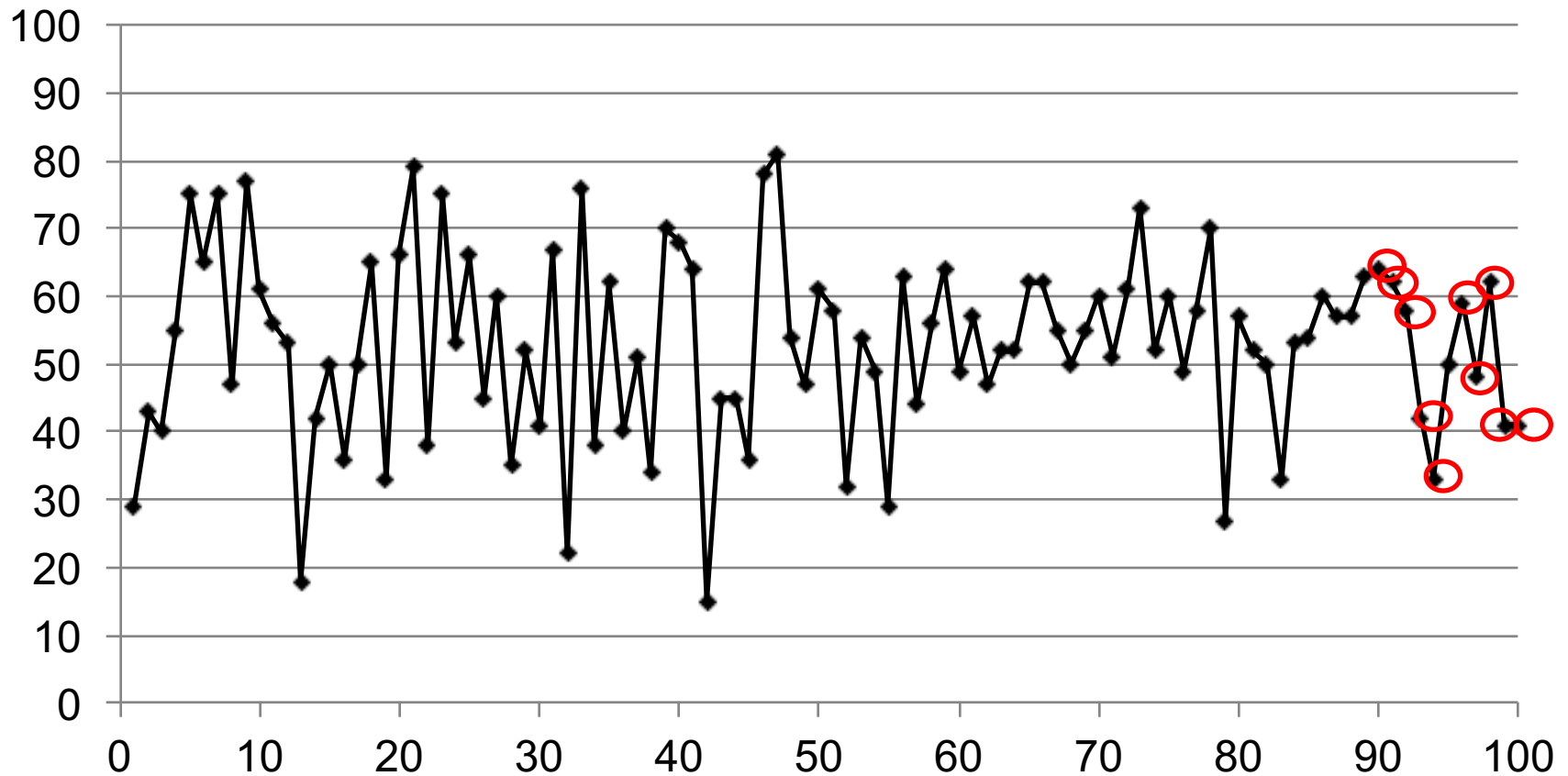
- $F_{t+1} = (D_t + D_{t-1} + \dots + D_{t-(n-1)}) / n$

- We have moved the data used by 1 period (D_t is added and D_{t-n} is discarded).

Moving Averages Example 1:

- ◆ Let's calculate the Moving Average for past 10 periods
- ◆ I simply call this MA(10).
- ◆ First we look at Descriptive Statistics.
- ◆ Then, I'll show you Predictive Statistics which you can use for Forecasting. (You will see more of the predictive analysis in Week 4).

Last 10 Data points.



◆ Last 10 data points are marked in red.

Example 1: Descriptive Statistics

- ◆ Sample Mean: Average of data from last 10 points.

$$\mu = 49.60$$

- ◆ Sample Standard Deviation:

$$s = 10.28$$

- ◆ Again, I'll show you how to calculate the descriptive statistics using the template [Week1MATemplate.xlsx](#)
- ◆ If our data were normally distributed (like a bell curve) the above statistics - mean and standard deviation – would be sufficient to **describe** the demand distribution.

Predictive Statistics

- ◆ We can use the statistics calculated for prediction or forecasting.
- ◆ Mean for Prediction = Descriptive Sample mean
 - In other words, descriptive sample mean is an unbiased estimator for mean of the true demand distribution, and hence can be used for prediction.
- ◆ However, the standard deviation for prediction needs to be adjusted because of insufficient data.
- ◆ When the data is normally distributed,
Standard Deviation for Prediction = $s + s/\sqrt{n}$

Recall that s is the descriptive standard deviation that we calculated
and n is the total number of data points used for calculation.

Predictive Statistics (Example 1 continued..)

- ◆ When we use $n=10$ data points:
- ◆ When the data is Normally distributed,
- ◆ Mean for Prediction $= \mu = 49.60$
- ◆ Standard Deviation for Prediction,

$$\sigma = s + \frac{s}{\sqrt{n}} = 10.28 + \frac{10.28}{\sqrt{10}} = 13.53$$

Example 1 (continued): Descriptive Statistics

- ◆ Suppose we use $n = 20$ data points. MA (20)
- ◆ Sample Mean: Average of data from last 20 points.

$$\mu = 51.95$$

- ◆ Sample Standard Deviation:

$$s = 9.62$$

- ◆ We have sufficient descriptive statistics for normally distributed data.

Predictive Statistics (Example 1 continued..)

- ◆ We have $n = 20$ data points.
- ◆ When the data is Normally distributed,
- ◆ Mean for Prediction $= \mu = 51.95$
 - We expect that the demand in the period would be 51.95 on average.
- ◆ Standard Deviation for Prediction,
$$\sigma = s + \frac{s}{\sqrt{n}} = 9.62 + \frac{9.62}{\sqrt{20}} = 11.77$$
 - For forecasting, we can assume that the actual demand will deviate from the average with the above standard deviation.

Using More Data

- ◆ As we have more data for forecasting
 - Descriptive statistics approach predictive statistics.
 - As we have more data, we gain more confidence for prediction.
 - Note as the number of data points increases, the descriptive standard deviation approaches standard deviation for prediction.
 - $\sigma = s + \frac{s}{\sqrt{n}}$

Statistics for our entire data

- ◆ Using all data available, i.e. $n = 100$

- ◆ Descriptive statistics:

$$\mu = 52.81$$

$$s = 13.73$$

- ◆ Predictive Statistics (for a normal distribution)

$$\mu = 52.81$$

$$\sigma = s + \frac{s}{\sqrt{n}} = 13.73 + \frac{13.73}{\sqrt{100}} = 15.10$$

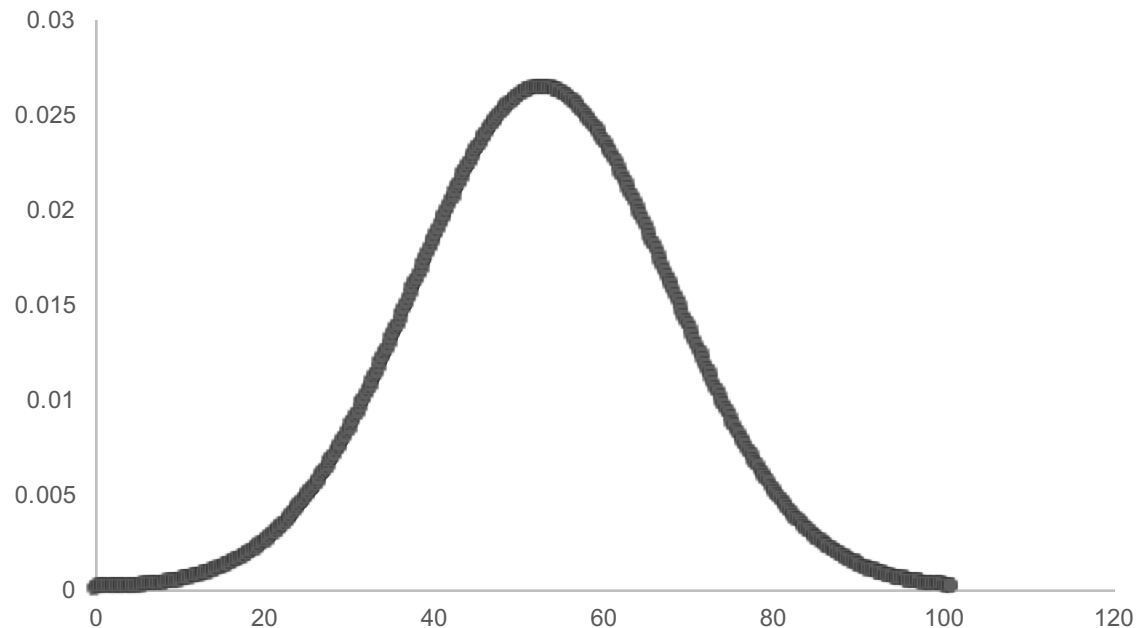
- ◆ We will use the above mean as 52.81 and standard deviation of 15.10 for prediction in the coming weeks.

- ◆ All Solutions documented in the excel file [Week1MASolution.xlsx](#)

Data Visualization

- ◆ We can generate a Normal Distribution graph to visualize how our demand data is distributed...
- ◆ Using mean as 52.81 and standard deviation of 15.10 for prediction

Normal Distribution for Prediction for our Data



Moving Averages: A Discussion

- ◆ Advantages of Moving Average Method
 - Easy to understand.
 - Easy to compute.
 - Provides stable forecasts.
- ◆ Disadvantages of Moving Average Method
 - Lags behind a trend (as we will see in Session 3).
 - It is not a causal model, i.e., it won't explain *why* realizations in the future behave in a certain way.
- ◆ Note that Moving Average method “drops” all data older than the n data points you use.
 - How do you think about how to choose n ?

Moving Averages: What data to use?

- ◆ If you choose to use moving average method of last 10 data points,
 - all the older data is ignored
 - » (e.g. data from 12 periods back is not used at all).
 - all the recent 10 data points are weighed the same.
 - » (e.g. yesterday's data has the same weight as the data from a week before).
- ◆ You may want to give more weight to more recent data and less weight to older data.
- ◆ Exponential smoothing is based on this precise idea.
 - Advanced slides.

Evaluation of Forecasts

- ◆ The forecast error in period t is denoted by e_t ,
 - The difference between the forecast for demand in period t and the actual value of demand realized in t .
 - For one step ahead forecast: $e_t = F_t - D_t$.

- ◆ Three ways to measure errors:
 - Mean Absolute Deviation $MAD = (1/n) \sum |e_t|$
 - Mean Squared Error $MSE = (1/n) \sum e_t^2$
 - Mean Absolute Percentage Error $MAPE = (1/n) \sum |e_t/D_t| \times 100$.

- ◆ Lower the errors, better the forecasting process is.
- ◆ Biases in Forecasts:
 - A bias occurs when the *average* value of a forecast error tends to be positive or negative.

Measuring Errors: An Example

- ◆ Using our dataset again, we will go through different methods of calculating errors in our forecasts.
- ◆ Assume we have data up to 80 periods.
- ◆ We use Moving Averages of 10 periods to calculate our forecasts.
- ◆ Once we have forecast for period 81, demand for 81 occurs.
- ◆ We then forecast for 82, and demand for period 82 occurs and so on.

Errors Example

- ◆ This allows us to calculate the errors from using MA(10)
 - By comparing demands and forecasts over periods 81 through 100.
- ◆ Using, **Week1ErrorsTemplate.xlsx**, I'll walk through an example exercise.
- ◆ For MA(10) moving averages of $n=10$, we get:
 - Mean Absolute Deviation $MAD = (1/n) \sum |e_i|$ = 8.9
 - Mean Squared Error $MSE = (1/n) \sum e_i^2$ =113.15
 - Mean Absolute Percentage Error $MAPE = (1/n) \sum |e_i/D_i| \times 100.$ =19.72%
- ◆ For MA(20) moving averages of $n=20$, we get:
 - Mean Absolute Deviation $MAD = (1/n) \sum |e_i|$ = 7.66
 - Mean Squared Error $MSE = (1/n) \sum e_i^2$ =92.61
 - Mean Absolute Percentage Error $MAPE = (1/n) \sum |e_i/D_i| \times 100.$ =17.29%

Errors Example: Continued...

- ◆ The solution is available in [Week1ErrorsSolution.xlsx](#)
- ◆ It may be preferable to use MA(20) over MA(10) in this case
 - By comparing corresponding error terms in the data
- ◆ Measuring errors allows us to understand better the choice of which method to use.
- ◆ Finally, in the data there is no evidence of any bias.

Wrapping up

- ◆ We saw how to forecast using the Moving Averages method
- ◆ We saw how to measure errors and biases in Forecasting.
- ◆ We learned about Descriptive Statistics...
- ◆ ... and how to adapt the Descriptive Statistics for Prediction.

See you in the next Session.

Next...

- ◆ An Operational Decision Problem
- ◆ Forecasting with Past Historical Data
- ◆ Moving Averages
- ◆ Exponential Smoothing

Session 3

- ◆ Thinking about Trends and Seasonality
- ◆ Forecasting for new products
- ◆ Fitting distributions