# Week 2: Making Best Decisions in Settings with Low Uncertainty

- A resource allocation example: Zooter Industries
- Converting a verbal problem description into an algebraic model: decisions, objective, constraints
- From an algebraic model to a spreadsheet implementation: optimizing with Excel Solver
- Matching demand and supply across space: Keystone Dry Goods Logistics

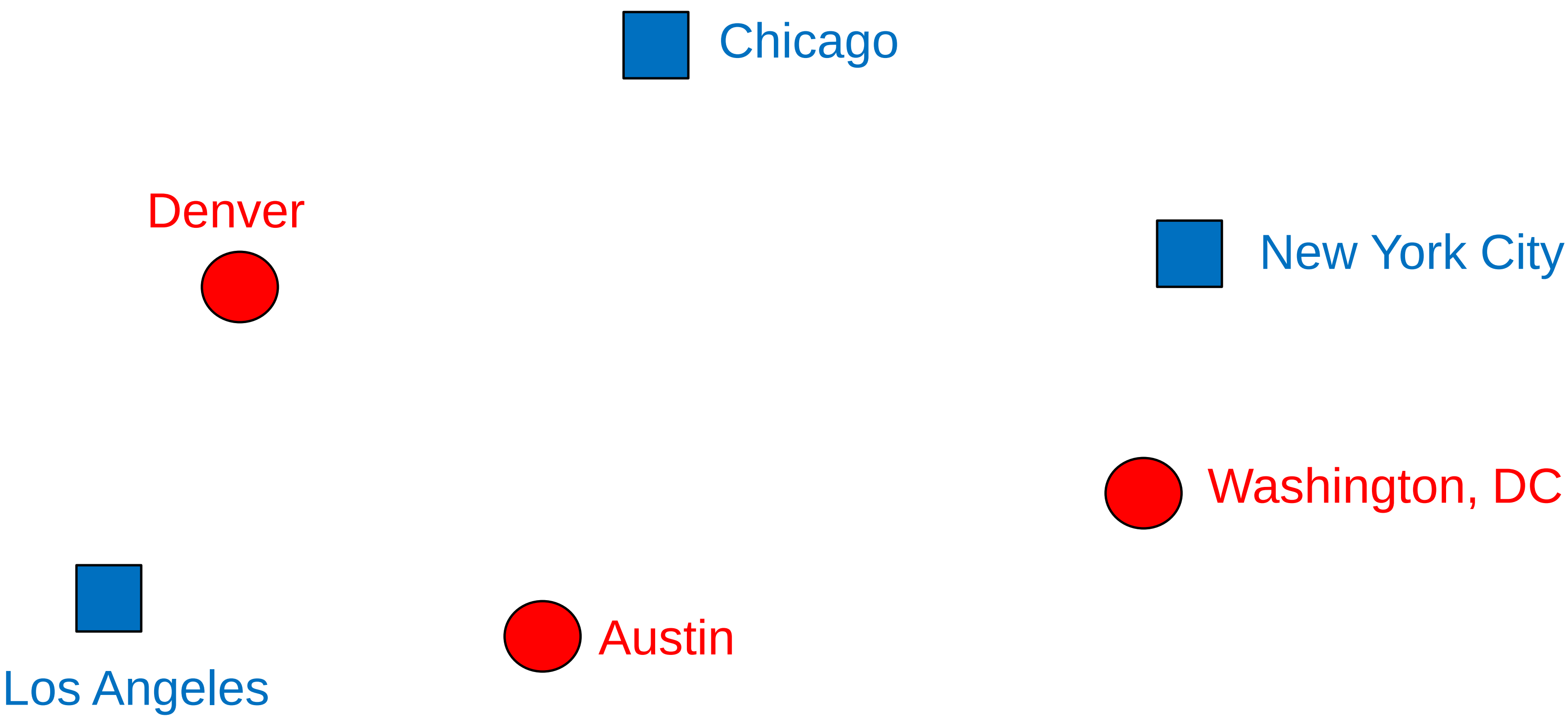
# Matching Demand and Supply Across a Network: Keystone Dry Goods Logistics

- The Keystone Dry Goods Logistics Company (KDGL) provides transportation and logistics services to a range of retail companies

- For the next month, KDGL needs to ensure on-time transportation of powdered drink shipments for one of its clients

- KDGL's client owns three warehouses, located in Los Angeles (L), Chicago (C), and New York City (N), and it uses three regional distribution centers located in Denver (D), Austin (A), and Washington, DC (W).

# KDGL: Warehouses and Distribution Centers



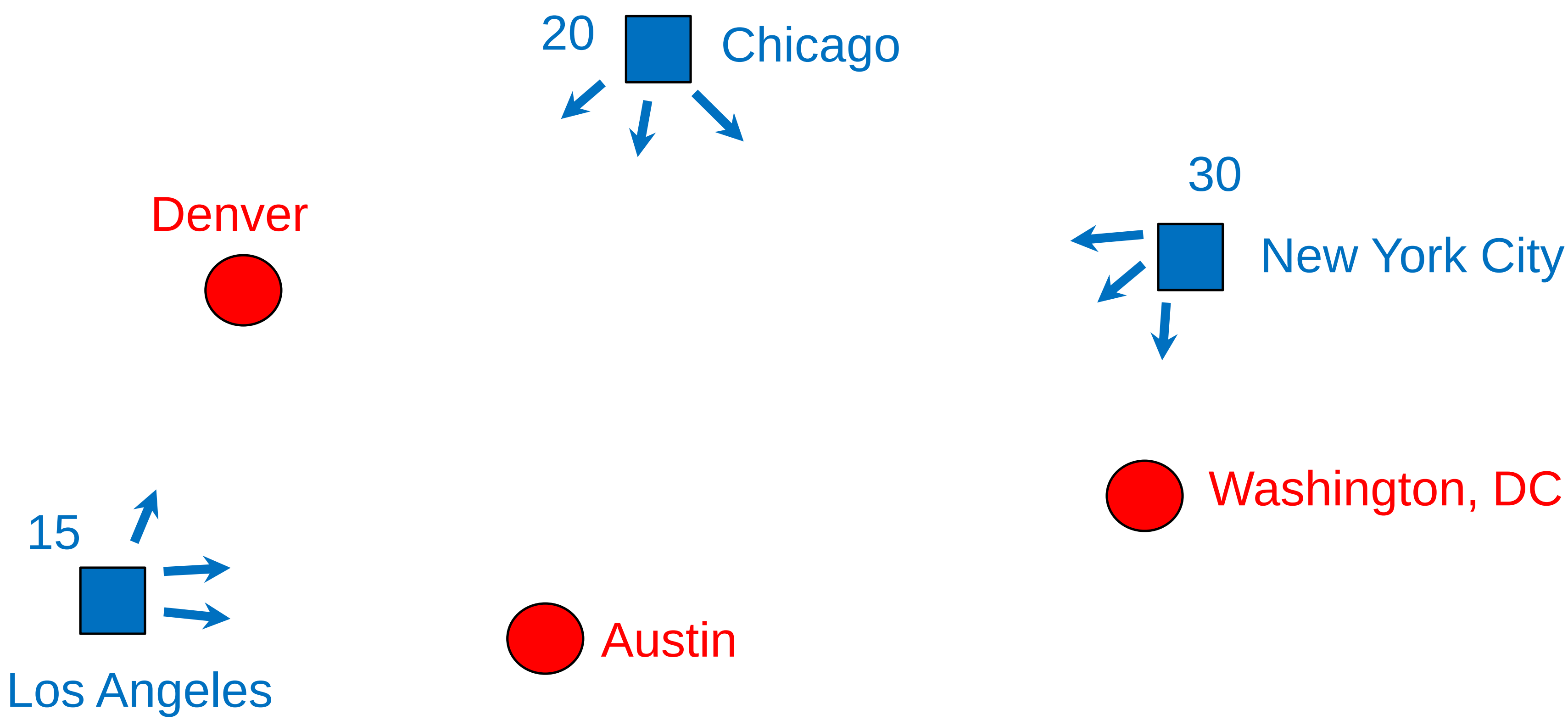
# Amounts to Be Shipped Out of Warehouses

- The warehouses have a total supply of 65 tons of powdered drink that must be transported to the distribution centers next month to make room for new warehouse inventory.

| **Warehouse** | **To Be Shipped Out (tons)** |
|---|---|
| **Los Angeles (L)** | 15 |
| **Chicago (C)** | 20 |
| **New York City (N)** | 30 |

- For example, exactly 20 tons of powdered drink must be transported out of the Chicago warehouse

# KDGL: Warehouses and Distribution Centers



# Minimum Amounts to Be Shipped to Distribution Centers

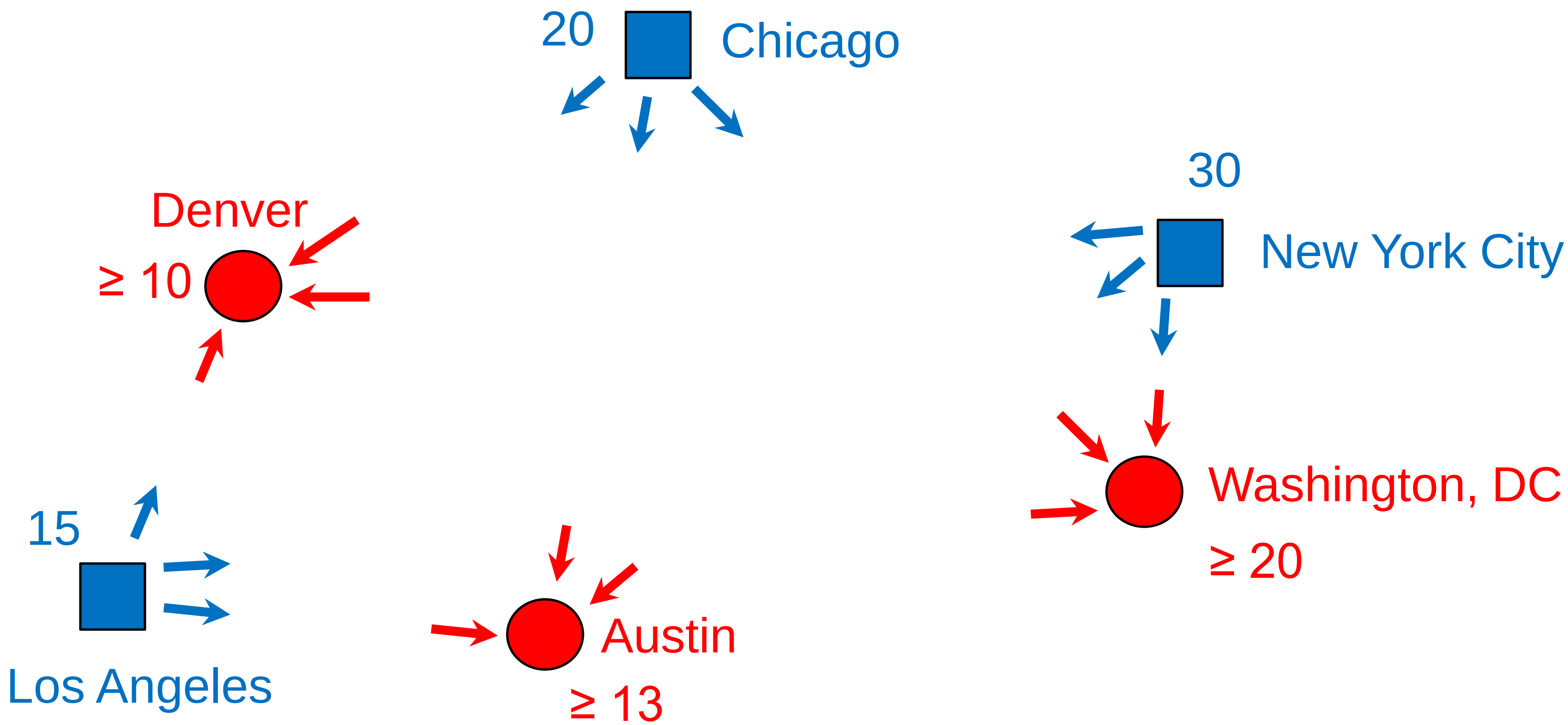
- There are minimum quantities that must be shipped to each of the distribution centers.

- KDGL may ship more than these quantities to ensure that all 65 tons of powdered drink are removed from the warehouses, but it cannot ship less

| **Distribution Center** | **Minimum To Be Shipped In (tons)** |
|---|---|
| **Denver** | 10 |
| **Austin** | 13 |
| **Washington, DC** | 20 |

- For example, at least 10 tons of powdered drink must be shipped to the Denver distribution center

# KDGL: Warehouses and Distribution Centers



# Shipping Costs

- The transportation costs (in $ per ton) to ship powdered drink between each warehouse and each distribution center:

| **From / To** | **Denver (D)** | **Austin (A)** | **Washington, DC (W)** |
|---|---|---|---|
| **Los Angeles (L)** | $105.00 | $135.00 | $153.00 |
| **Chicago (C)** | $110.00 | $140.00 | $137.00 |
| **New York City (N)** | $130.00 | $132.00 | $115.00 |

- For example, if KDGL decides to ship 10 tons from the New York City warehouse to the Austin distribution center, it will incur a shipping cost of $132*10 = $1320

# Task Formulation

- KDGL’s management needs to determine the amounts of cargo to transport from each warehouse to each distribution center to minimize the total shipping cost while satisfying the warehouse supply and distribution center demand constraints

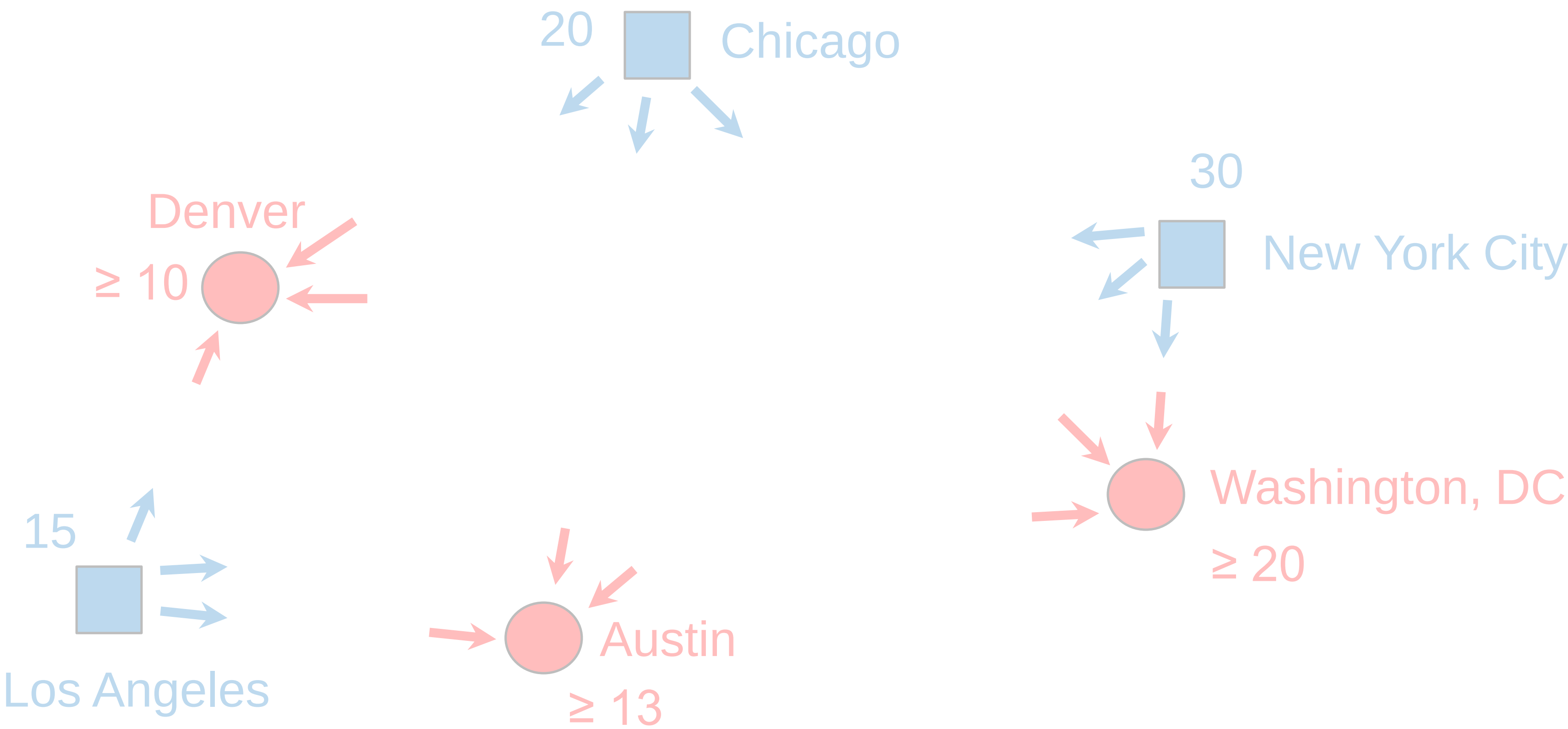
# Verbal Formulation of KDGL Problem

- What does KDGL's management needs to determine?
  - Amounts of cargo to transport from each warehouse to each distribution center

- What does it want to achieve?
  - Lowest possible total shipping cost

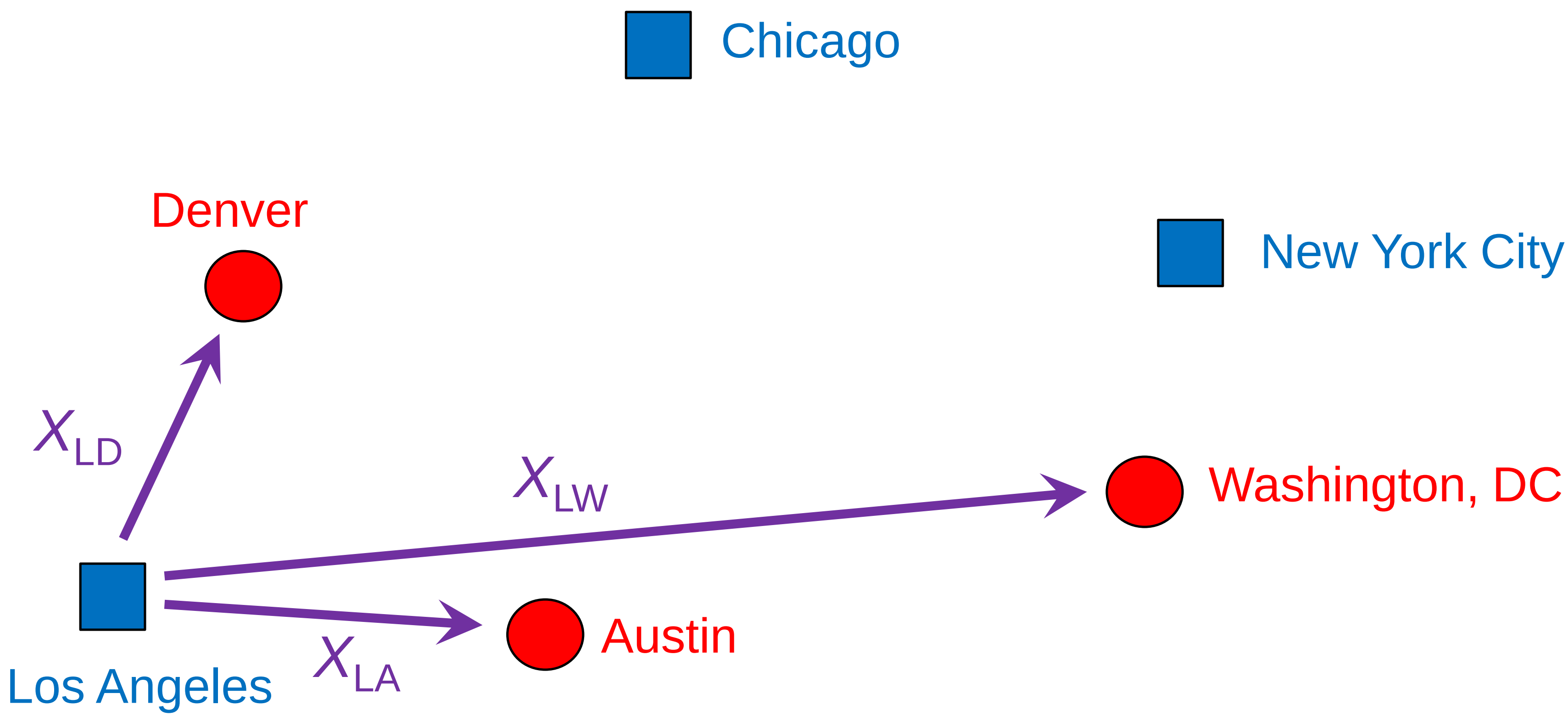
- What business requirements must it satisfy?
  - It has to satisfy warehouse supply constraints and distribution center minimum demand constraints

# KDGL: Decision Variables




- KDGL's management needs to determine the amounts of cargo to transport

# KDGL: Decision Variables

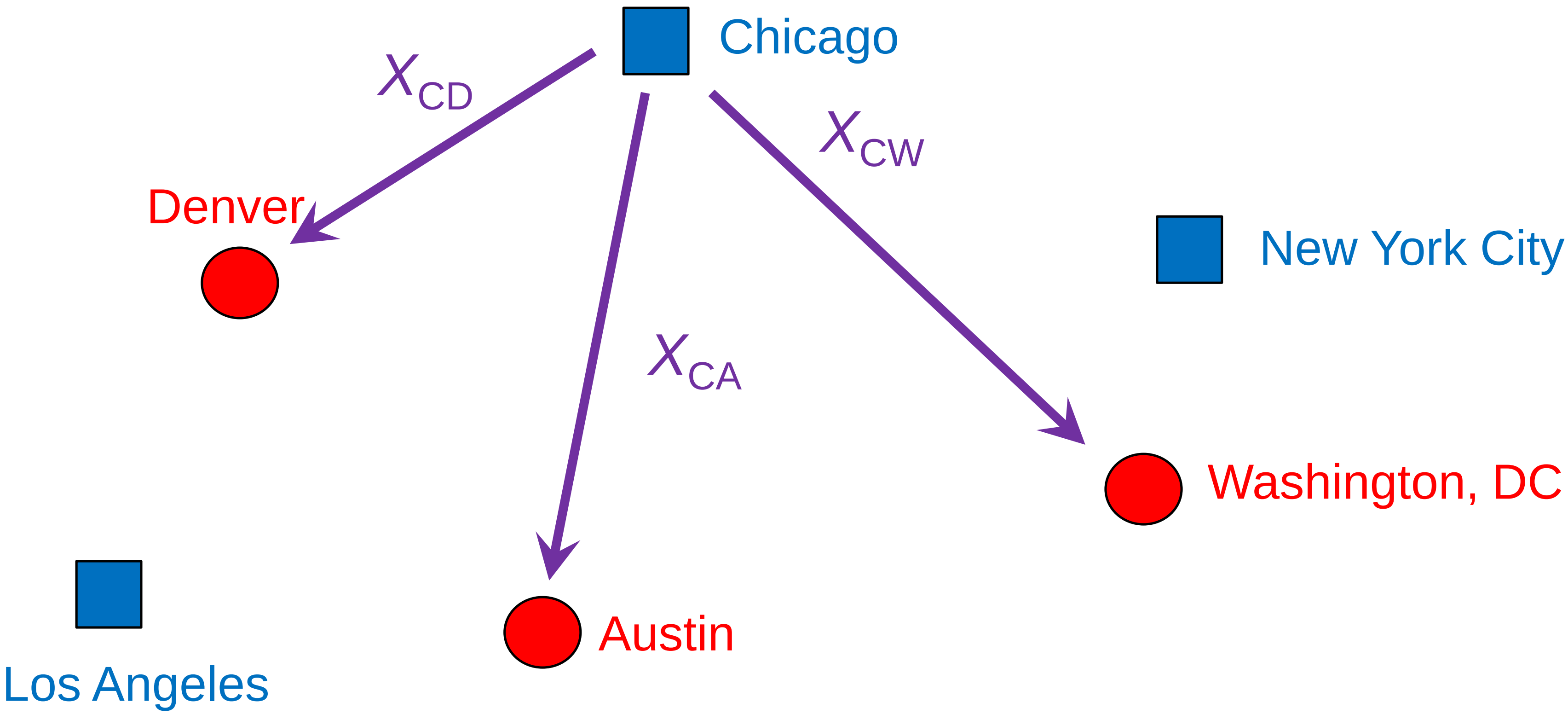



$X_{LD}$ = number of tons shipped from Los Angeles to Denver

$X_{LA}$ = number of tons shipped from Los Angeles to Austin

$X_{LW}$ = number of tons shipped from Los Angeles to Washington, DC

# KDGL: Decision Variables

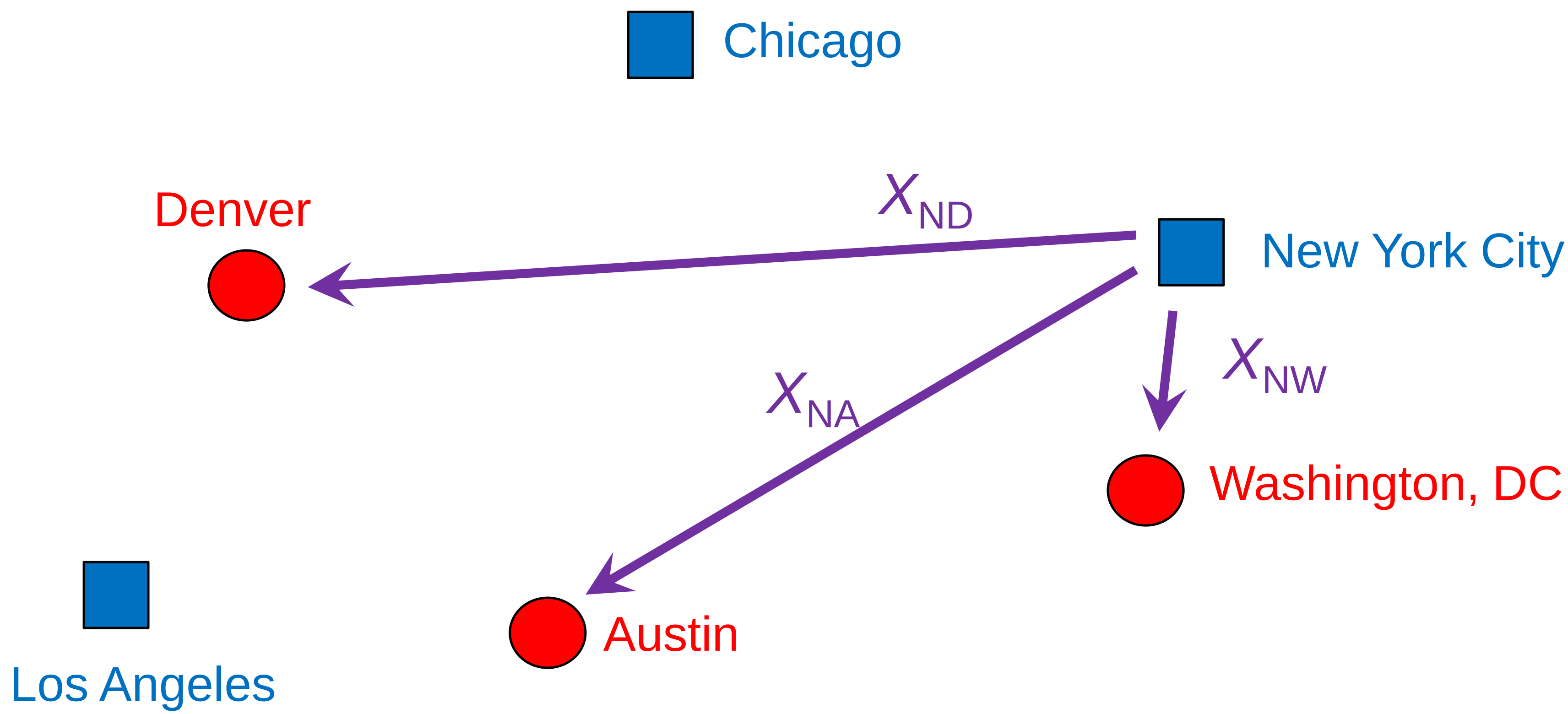



$X_{CD}$ = number of tons shipped from Chicago to Denver

$X_{CA}$ = number of tons shipped from Chicago to Austin

$X_{CW}$ = number of tons shipped from Chicago to Washington, DC

# KDGL: Decision Variables




$X_{ND}$ = number of tons shipped from New York City to Denver

$X_{NA}$ = number of tons shipped from New York City to Austin

$X_{NW}$ = number of tons shipped from New York City to Washington, DC

# KDGL: 9 Decision Variables

$X_{LD}$ = number of tons shipped from Los Angeles to Denver

$X_{LA}$ = number of tons shipped from Los Angeles to Austin

$X_{LW}$ = number of tons shipped from Los Angeles to Washington, DC

$X_{CD}$ = number of tons shipped from Chicago to Denver

$X_{CA}$ = number of tons shipped from Chicago to Austin

$X_{CW}$ = number of tons shipped from Chicago to Washington, DC

$X_{ND}$ = number of tons shipped from New York City to Denver

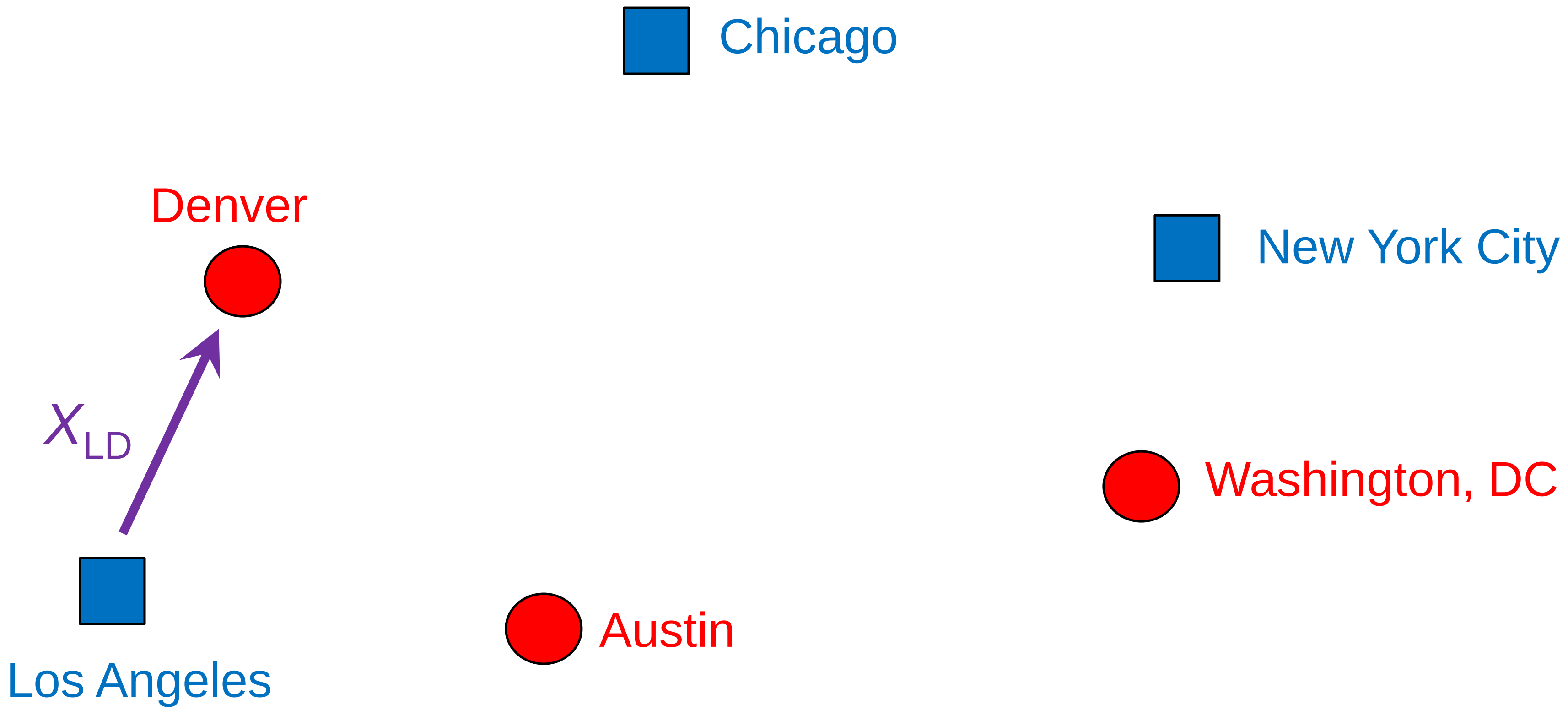
$X_{NA}$ = number of tons shipped from New York City to Austin

$X_{NW}$ = number of tons shipped from New York City to Washington, DC

# KDGL: Objective Function

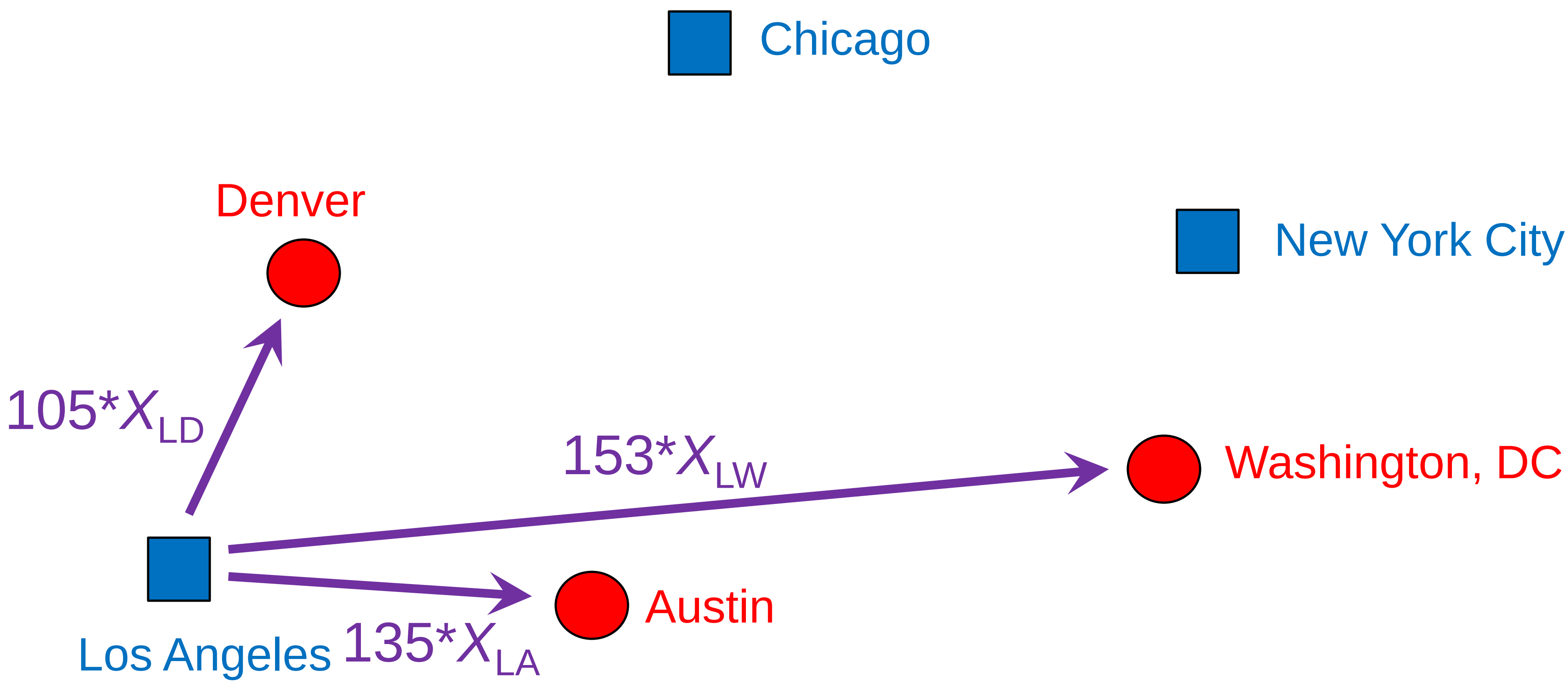
- KDGL’s management needs to determine the **amounts of cargo to transport from each warehouse to each distribution center** to

# KDGL: Objective Function



| From / To | Denver (D) | Austin (A) | Washington, DC (W) |
|---|---|---|---|
| Los Angeles (L) | $105.00 | $135.00 | $153.00 |
| Chicago (C) | $110.00 | $140.00 | $137.00 |
| New York City (N) | $130.00 | $132.00 | $115.00 |

# KDGL: Objective Function




$105*X_{LD}$ = the cost to ship $X_{LD}$ tons from Los Angeles to Denver

$135*X_{LA}$ = the cost to ship $X_{LA}$ tons from Los Angeles to Austin

$153*X_{LW}$ = the cost to ship $X_{LW}$ tons from Los Angeles to Washington, DC

# KDGL: Objective Function

Total Shipping Cost to Be Minimized:

$105 * X_{LD} + 135 * X_{LA} + 153 * X_{LW}$ (Cost for shipping from Los Angeles)

$+ 110 * X_{CD} + 140 * X_{CA} + 137 * X_{CW}$ (Cost for shipping from Chicago)

$+ 130 * X_{ND} + 132 * X_{NA} + 115 * X_{NW}$ (Cost for shipping from New York City)

# KDGL: Constraints

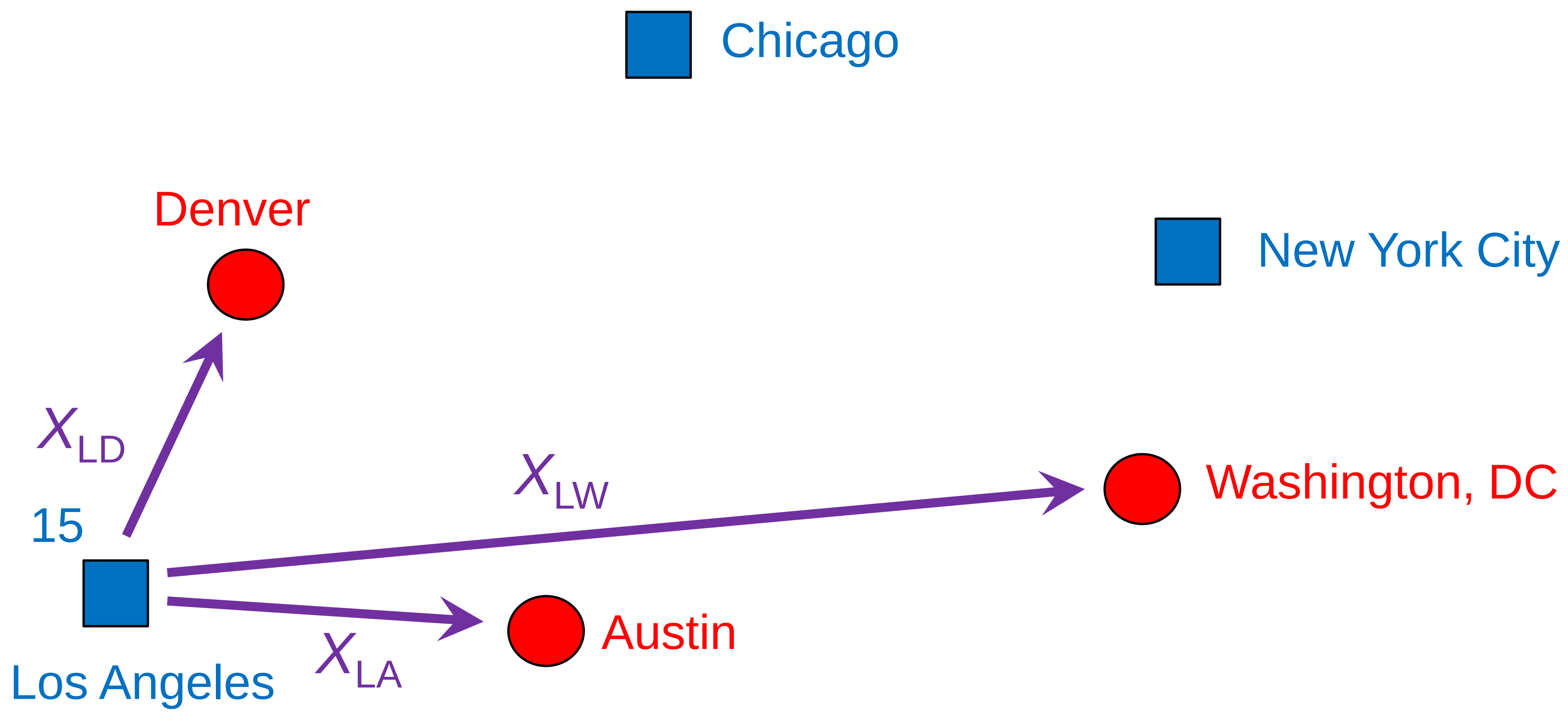
- KDGL's management needs to determine the **amounts of cargo to transport from each warehouse to each distribution center** to **minimize the total shipping cost**

# KDGL: Supply Constraints

- The warehouses have a total supply of 65 tons of powdered drink that must be transported to the distribution centers next month to make room for new warehouse inventory.

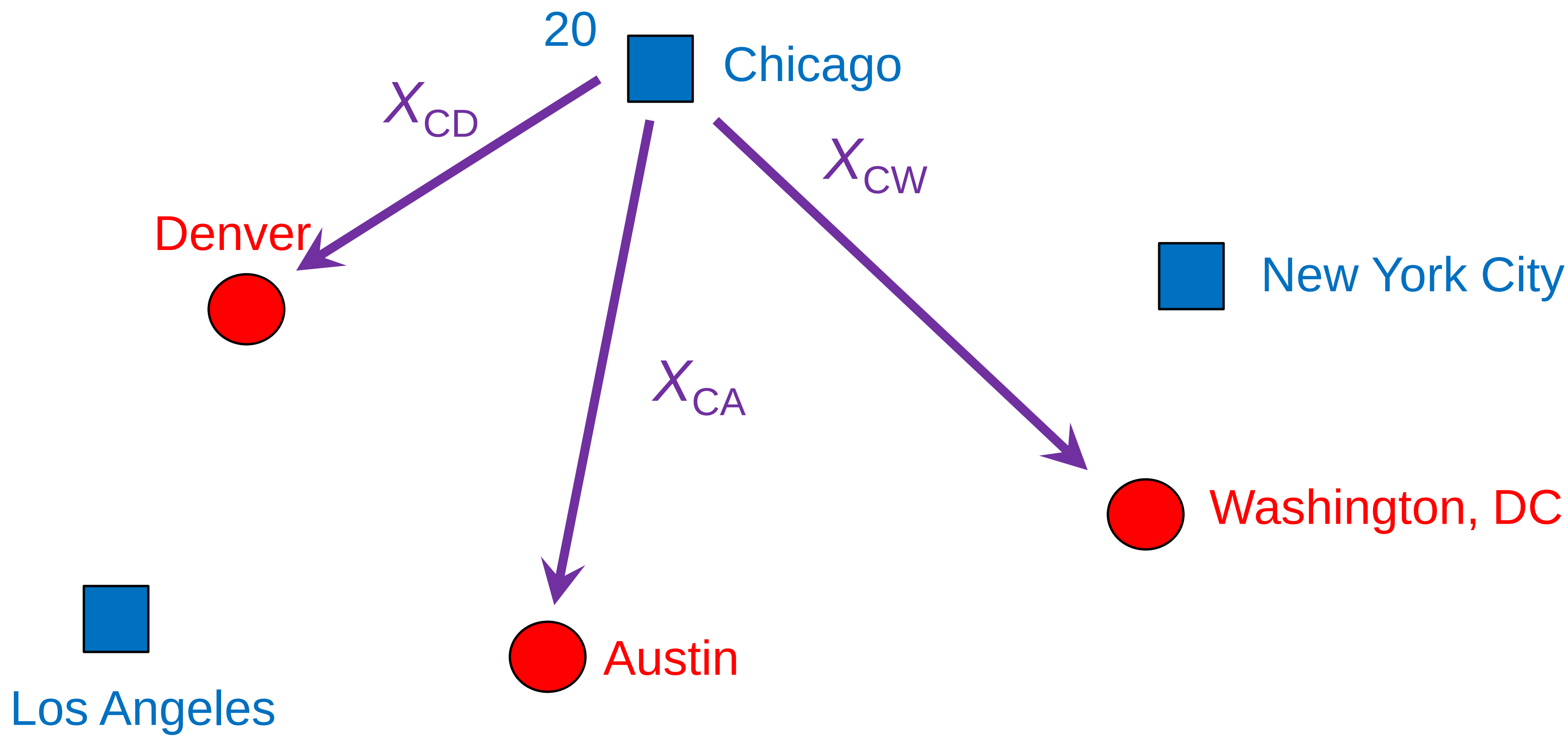
| Warehouse | To Be Shipped Out (tons) |
|---|---|
| **Los Angeles (L)** | 15 |
| **Chicago (C)** | 20 |
| **New York City (N)** | 30 |

# KDGL: Supply Constraints



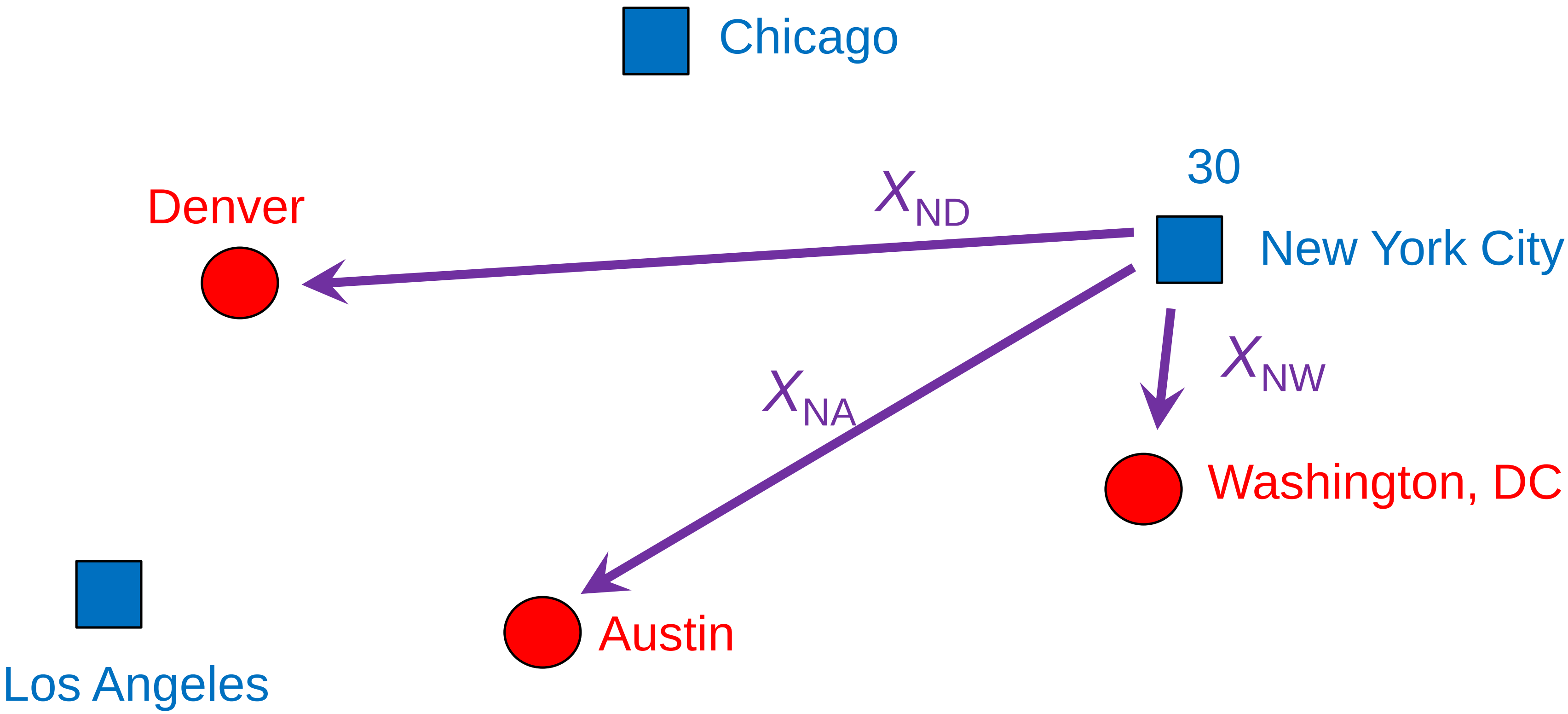

Number of tons shipped from Los Angeles = $\boxed{X_{LD} + X_{LA} + X_{LW} = 15}$

# KDGL: Supply Constraints




Number of tons shipped from Chicago = $\boxed{X_{CD} + X_{CA} + X_{CW} = 20}$

# KDGL: Supply Constraints




Number of tons shipped from New York City = $\boxed{X_{ND} + X_{NA} + X_{NW} = 30}$

# KDGL: Supply Constraints

$$X_{LD} + X_{LA} + X_{LW} = 15 \qquad \text{(Los Angeles supply)}$$

$$X_{CD} + X_{CA} + X_{CW} = 20 \qquad \text{(Chicago supply)}$$

$$X_{ND} + X_{NA} + X_{NW} = 30 \qquad \text{(New York supply)}$$

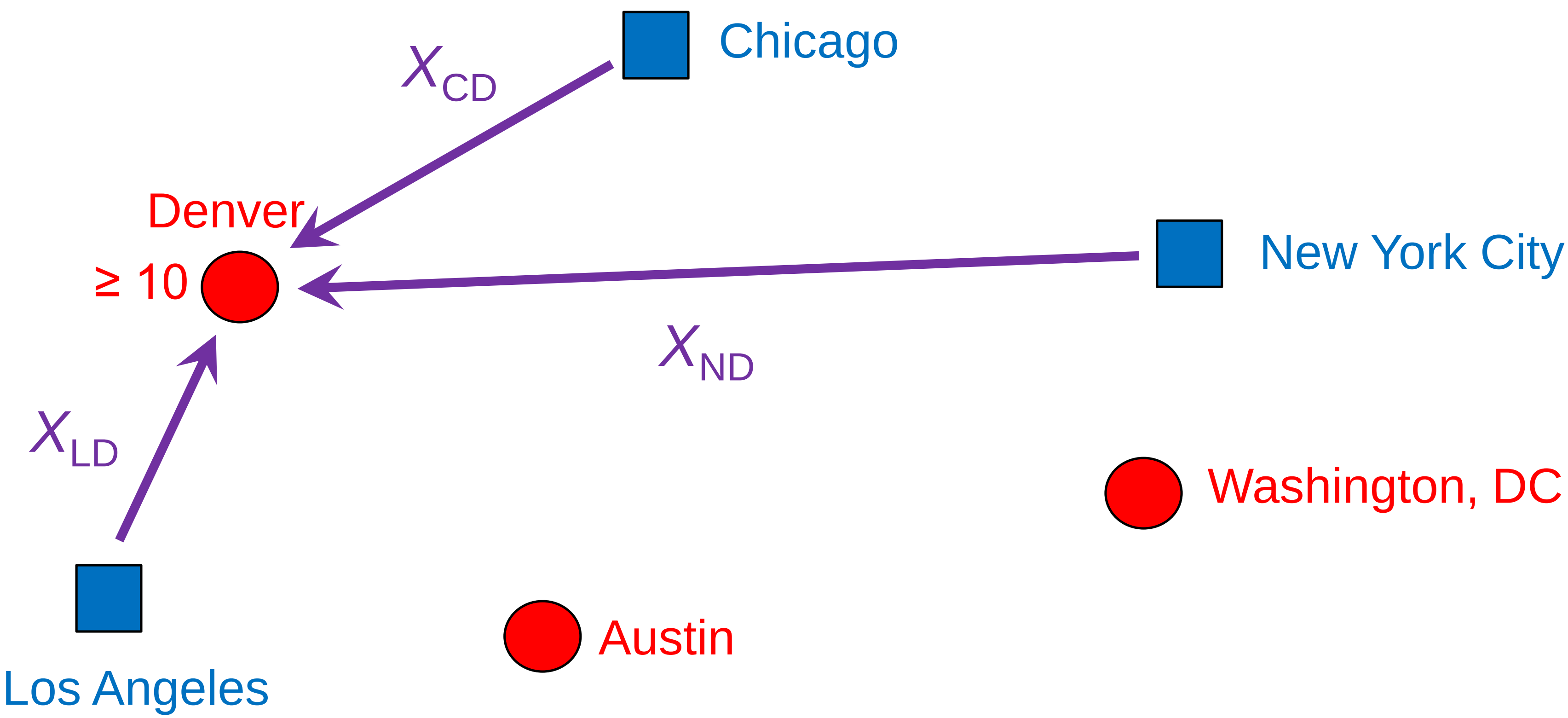
# KDGL: Demand Constraints

- There are minimum quantities that must be shipped to each of the distribution centers.

- KDGL may ship more than these quantities to ensure that all 65 tons of powdered drink are removed from the warehouses, but it cannot ship less

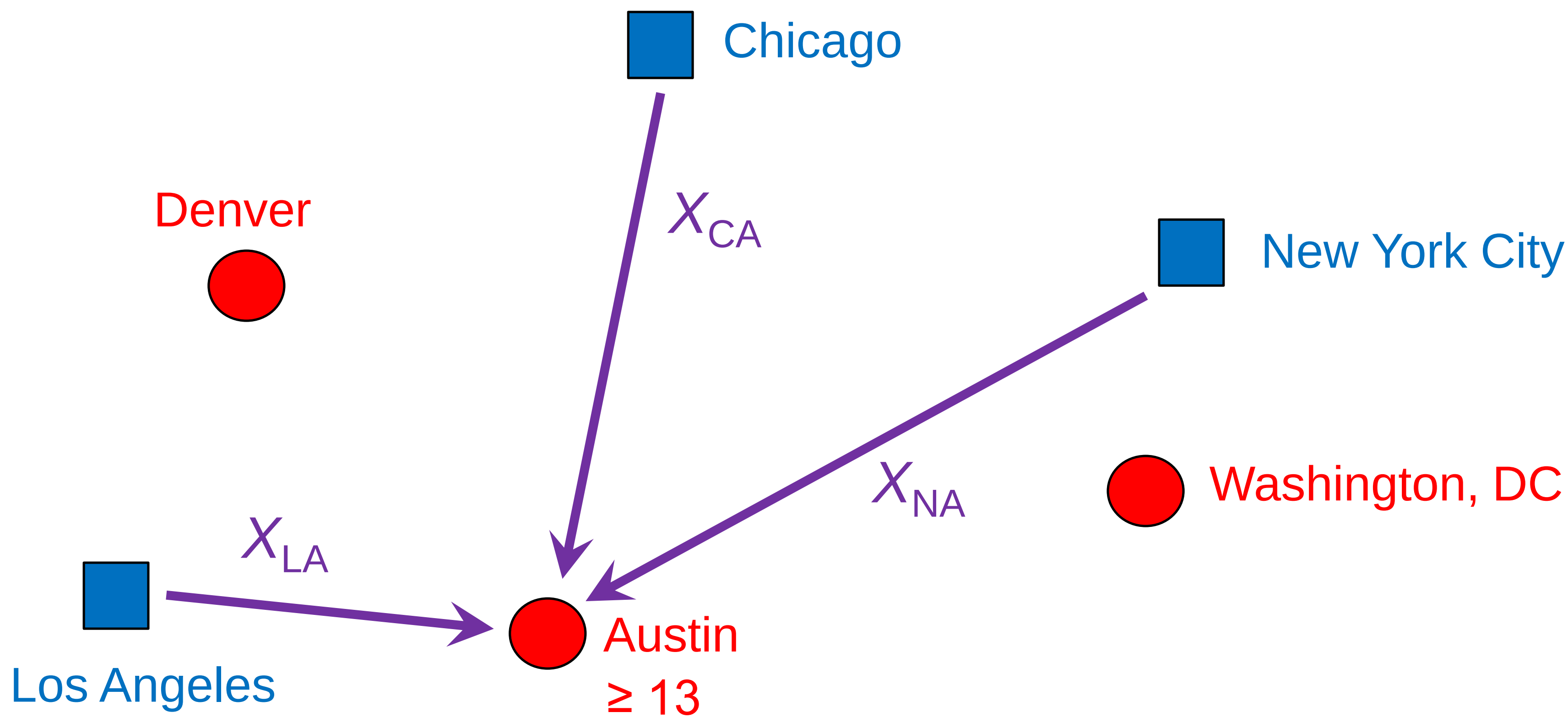
| **Distribution Center** | **Minimum To Be Shipped In (tons)** |
|---|---|
| **Denver** | 10 |
| **Austin** | 13 |
| **Washington, DC** | 20 |

# KDGL: Demand Constraints



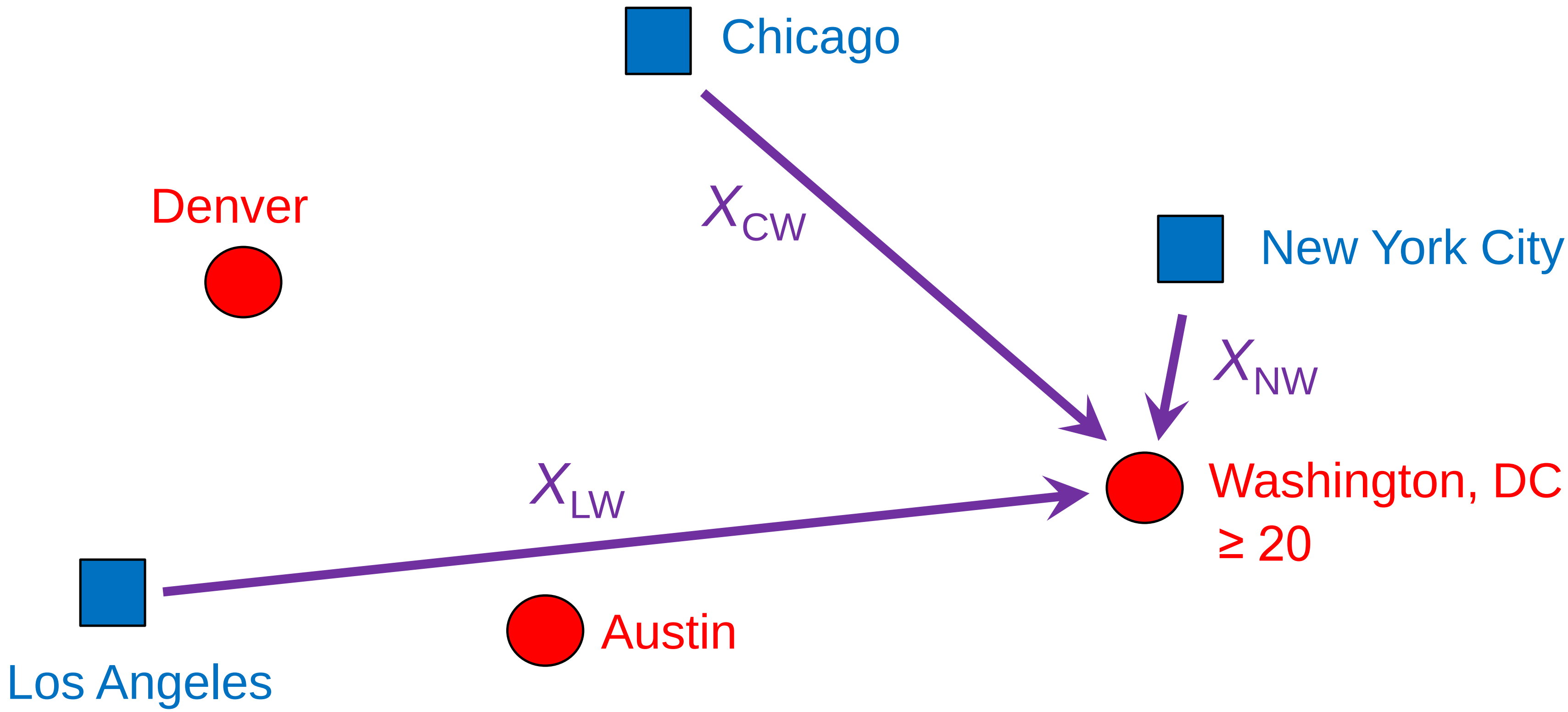

Number of tons shipped to Denver = $\boxed{X_{LD} + X_{CD} + X_{ND} \geq 10}$

# KDGL: Demand Constraints




Number of tons shipped to Austin = $\boxed{X_{LA} + X_{CA} + X_{NA} \geq 13}$

# KDGL: Demand Constraints




Number of tons shipped to Washington, DC = $\boxed{X_{LW} + X_{CW} + X_{NW} \geq 20}$

# KDGL: Demand Constraints

$X_{LD} + X_{CD} + X_{ND} \geq 10$ (Minimum Denver demand)

$X_{LA} + X_{CA} + X_{NA} \geq 13$ (Minimum Austin demand)

$X_{LW} + X_{CW} + X_{NW} \geq 20$ (Minimum Washington, DC demand)

# KDGL: Model Formulation

Minimize the Total Shipping Cost

$$105*X_{LD} + 135*X_{LA} + 153*X_{LW} + 110*X_{CD} + 140*X_{CA} + 137*X_{CW}$$
$$+ 130*X_{ND} + 132*X_{NA} + 115*X_{NW}$$

$X_{LD} + X_{LA} + X_{LW} = 15$ (Los Angeles supply)

$X_{CD} + X_{CA} + X_{CW} = 20$ (Chicago supply)

$X_{ND} + X_{NA} + X_{NW} = 30$ (New York supply)

$X_{LD} + X_{CD} + X_{ND} \geq 10$ (Minimum Denver demand)

$X_{LA} + X_{CA} + X_{NA} \geq 13$ (Minimum Austin demand)

$X_{LW} + X_{CW} + X_{NW} \geq 20$ (Minimum Washington, DC demand)

$X_{LD}, \ldots, X_{NW} \geq 0$ (Non-negativity)

# Spreadsheet Solution

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **Keystone.xlsx** | | ***Keystone Dry Goods Logistics Problem*** | | | | |
| 2 | **Operations Analytics MOOC** | | | | | | |
| 3 | | | | | **Total Cost (in $)** | 7,485 | |
| 4 | ***Shipping Cost ($ per ton)*** | | | | | =SUMPRODUCT(B6:D8,B12:D14) | |
| 5 | From/To | Denver | Austin | Washington | | | |
| 6 | Los Angeles | 105.00 | 135.00 | 153.00 | | | |
| 7 | Chicago | 110.00 | 140.00 | 137.00 | | | |
| 8 | New York City | 130.00 | 132.00 | 115.00 | | | |
| 9 | | | | | =SUM(B12:D12) | | |
| 10 | ***Shipping quantities (tons)*** | | | | | | |
| 11 | From/To | Denver | Austin | Washington | Total Shipped | | ***Supply*** |
| 12 | Los Angeles | **15** | **0** | **0** | 15 | = | 15 |
| 13 | Chicago | **17** | **3** | **0** | 20 | = | 20 |
| 14 | New York City | **0** | **10** | **20** | 30 | = | 30 |
| 15 | Total | 32 | 13 | 20 | | | |
| 16 | =SUM(B12:B14) | => | => | => | | | |
| 17 | ***Minimum Demand*** | 10 | 13 | 20 | | | |

- Solver states that the best decision is to send the entire Los Angeles supply to Denver, split the supply in Chicago between Denver and Austin, and split New York City's supply between Austin and Washington
- The shipping cost associated with this decision is $7485

# Making Optimal Decisions in Practice: Commercial Optimization Packages

- Solver is a useful tool for learning optimization, but for problems of practical size, with thousands of variables and constraints, commercial alternatives may offer an advantage

- Whether using Solver, or commercial packages, optimization requires specifying decision variables, an objective function and constraints

- A recent comparison of optimization software by the *OR/MS Today:* http://www.orms-today.org/surveys/LP/lp12.html (while the table is called "linear programming survey", the software packages mentioned can handle non-linear models)

# Optimization in Practice

- Examples of the use of optimization methods: articles published in *Interfaces*

Chevron

Optimizing Chevron's Refineries
Ted Kutz, Mark Davis, Robert Creek, Nick Kenaston, Craig Stenstrom, and Margery Connor
Interfaces 2014, 44:1, 39-54.

ZARA

Zara Uses Operations Research to Reengineer Its Global Distribution Process
Felipe Caro, Jérémie Gallien, Miguel Díaz, Javier García, José Manuel Corredoira, Marcos Montes, José Antonio Ramos, and Juan Correa
Interfaces 2010, 40:1, 71-84.

- In practice, optimization often needs to be done in uncertain environments