Week 2: Making Best Decisions in Settings with Low Uncertainty

- A resource allocation example: Zooter Industries
- Converting a verbal problem description into an algebraic model: decisions, objective, constraints

Session 1

- From an algebraic model to a spreadsheet implementation: optimizing with Excel Solver

Session 2

- Matching demand and supply across space: Keystone Dry Goods Logistics

Session 3

Zooter Industries: Products, Profits, Demand

- *Zooter Industries* (ZI) manufactures high-end kick-scooters for the North American market
- ZI's main product models are Razor and Navajo, with profit contributions of \$150 and \$160 per unit
- At present, ZI's scooters are so popular that the company can sell all the units it makes

Zooter Industries: Manufacturing Process

- The production process for each model includes three main steps:
  - frame manufacturing
  - wheels and deck assembly
  - quality assurance and packaging

- Each unit of the two scooter models requires the following processing times in these production steps:

Model	**Frame Manufacturing (hours)**	**Wheels and Deck Assembly (hours)**	**Quality Assurance and Packaging (hours)**
Razor	4.0	1.5	1.0
Navajo	5.0	2.0	0.8

Zooter Industries: Supply Side

- ZI's capacity available at each production step is shown below for the coming week

Production Step	**Available Time in the Coming Week (hours)**
Frame Manufacturing	5610
Wheels and Deck Assembly	2200
Quality Assurance and Packaging	1200

- How many units of each model should ZI produce in the coming week in order to maximize its weekly profit?

Assuming Away Uncertainty: Pros and Cons

- The Zooter example treats profit contributions, manufacturing requirements, supply availabilities as non-random quantities
- If ZI decides to make a certain number of units of each scooter model in the coming week, it will know for sure
  - How much profit it will make
  - Whether it will have sufficient supply of each resource
- The “no uncertainty” assumption simplifies the search for the best production plan
- In practice, it allows us to tackle analytics models with large numbers of products and resources

Assuming Away Uncertainty: Pros and Cons

- May be a reasonable assumption when a decision maker has substantial control over his/her business environment
  - Short-term planning
  - Longer-term planning when existing contracts ensure stability of prices, costs, and demand and supply parameters
- May result in problematic recommendations in settings with significant data uncertainty
- When uncertainty is significant and must be included in the analysis, the task of finding the best decision may become far more complex
- In Weeks 3 and 4 we will look at how to evaluate choices and make best decisions in such settings

Evaluating a Production Plan: Decision Variables

- Before approaching a task of finding the best production plan, or **optimizing** production, we must know how to evaluate any given production plan

- In optimization lingo, the term **"decision variables"** describes the quantities that a decision maker can change to achieve a desired performance.

- In the ZI example, there are two decision variables:
  - *R*, the number of Razor scooters to produce in the coming week
  - *N*, the number of Navajo scooters to produce in the coming week

- A particular choice of values for decision variables is called a **"solution"**. For example, *R*=500 and *N*=500 is a solution

Evaluating a Production Plan: Objective Function

- If ZI decides to produce *R*=500 Razor and *N*=500 Navajo scooters in the coming week, how much profit will ZI make in this case?
  - Profit (in $) = $150*500*+$160*500 = $75000 + $80000 = $155000

- The **"objective"** is a performance metric we want to maximize or minimize. In this example, profit is an objective to be maximized

- For *R*=500 and *N*=500, the profit value is $155000. How much profit will ZI make for an arbitrary pair of values *R* and *N*?
  - Profit (in $) = 150**R* + 160**N*

- 150**R* + 160**N* is an **"objective function"**, i.e., an objective expressed as a function of decision variables

- $155000 is an "**objective function value"** (OFV) for solution *R*=500, *N*=500

Evaluating a Production Plan: Constraints

- If ZI decides to produce *R*=500 Razor and *N*=500 Navajo scooters in the coming week, how much of each resource will it require?

- Required number of **frame manufacturing hours**:

  4*500+5*500 = 4500 – does not exceed 5610 hours available

- In general, for any potential production plan, the required number of frame manufacturing hours may not exceed the number of hours available

- In the optimization lingo, we use the term "**constraint**" to describe this requirement

Evaluating a Production Plan: Constraints

- Does the *R*=500 and *N*=500 production plan have enough of other resources to be implemented?

- Required number of **wheels and deck assembly hours**:
  1.5*500+2.0*500 = 1750 – does not exceed 2200 hours available

- Required number of **quality assurance and packaging hours**:
  1.0*500+0.8*500 = 900 – does not exceed 1200 hours available

- A production plan that, like *R*=500 and *N*=500, satisfies all constraints is called **feasible**

Evaluating a Production Plan: Constraints

- What if ZI decides to produce R=500 Razor and N=750 Navajo scooters?

- Required number of **frame manufacturing hours**:
  4*500+5*750 = 5750 – exceeds 5610 hours available

- Required number of **wheels and deck assembly hours**:
  1.5*500+2.0*750 = 2250 – exceeds 2200 hours available

- Required number of **quality assurance and packaging hours**:
  1.0*500+0.8*750 = 1100 – does not exceed 1200 hours available

- A production plan that, like R=500 and N=750, violates at least one constraint is called **infeasible**

Evaluating a Production Plan: Constraints

- For a production plan that makes R Razor and N Navajo scooters, how does one express a constraint on the number of available frame manufacturing hours?

- In words, we have "number of required frame manufacturing hours may not exceed the number of available hours"

- Using variables R and N, we can write this statement as

$$4*R + 5*N \le 5610$$

- In the same way, the constraints on the number of available wheels and deck assembly hours and the number of available quality assurance and packaging hours can be written as

$$1.5*R + 2.0*N \le 2200$$

$$1.0*R + 0.8*N \le 1200$$

Other Constraints?

- Numbers R and N must be integer

$$R, N = \text{integer}$$

- Numbers R and N cannot be negative

$$R, N \geq 0$$

Searching for the Best Production Plan: A Complete Model

- Putting the decision variables, objective function and constraints together, we can express our model as

Maximize $150*R + 160*N$

subject to

$4*R + 5*N \leq 5610$ (frame manufacturing hours)

$1.5*R + 2.0*N \leq 2200$ (wheel and deck manufacturing hours)

$1.0*R + 0.8*N \leq 1200$ (QA and packaging hours)

$R, N =$ integer

$R, N \geq 0$

- We will use Solver to “optimize” this model

A Comment on Objective and Constraints

- An optimization model can have any number of decision variables and constraints but it must have one objective to be maximized or minimized
- In practice, there could be a number of quantities, **key performance indicators**, that a manager may want to keep track of: profit, cost, customer service levels, utilization of resources, etc.
- If one of the key performance indicators, such as profit, is selected as the objective, the rest of the key performance indicators can be used in constraints
- For example, the model can “maximize profit while making sure that the resource utilization does not exceed 95%”

Model Types: “Easier” …

- Zooter model involves only
  - constant parameters, like 5610
  - products of decision variables and constant parameters, like 150*R and 0.8*N
  - adding and/or subtracting the resulting expressions, like 1.5*R + 2.0*N

- Such models are called **“linear”** and easier to optimize in practice

… And "Harder"

- Sometimes it is necessary to use models that involve **"nonlinear"** expressions of decisions variables, for example, $R*N$, R^2, $N/(R+N)$ or $\sqrt{N}$
- Nonlinear models are much harder to optimize, especially as the number of variables and constraints grows
- In the Zooter model, the numbers of scooters produced must be round, or **integer**
- In general, imposing such a requirement can significantly complicate optimization even in linear models, especially in models with large numbers of variables and constraints

Additional References

- More on optimization, linear, non-linear models as well as models with integer variables:
  - "Business Analytics" by James R. Evans
  - "Spreadsheet Modeling and Decision Analysis: A Practical Introduction to Business Analytics" by C. Ragsdale