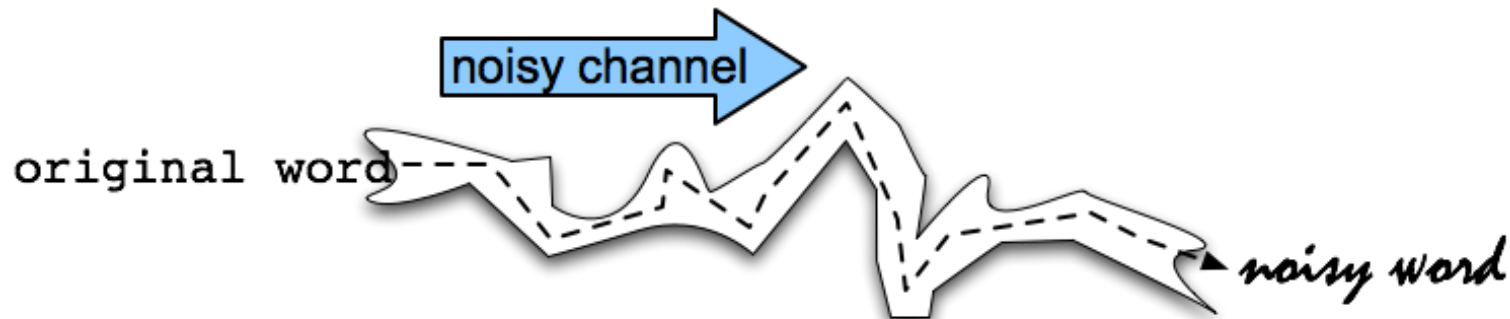


Spelling Correction and the Noisy Channel

The Noisy Channel Model of Spelling



Noisy Channel Intuition





Noisy Channel

- We see an observation x of a misspelled word
- Find the correct word w

$$\begin{aligned}\hat{w} &= \operatorname{argmax}_{w \in V} P(w | x) \\ &= \operatorname{argmax}_{w \in V} \frac{P(x | w)P(w)}{P(x)} \\ &= \operatorname{argmax}_{w \in V} P(x | w)P(w)\end{aligned}$$



History: Noisy channel for spelling proposed around 1990

- **IBM**

- Mays, Eric, Fred J. Damerau and Robert L. Mercer. 1991. Context based spelling correction. *Information Processing and Management*, 23(5), 517–522

- **AT&T Bell Labs**

- Kernighan, Mark D., Kenneth W. Church, and William A. Gale. 1990. A spelling correction program based on a noisy channel model. Proceedings of COLING 1990, 205-210



Non-word spelling error example

acress



Candidate generation

- Words with similar spelling
 - Small edit distance to error
- Words with similar pronunciation
 - Small edit distance of pronunciation to error



Damerau-Levenshtein edit distance

- Minimal edit distance between two strings, where edits are:
 - Insertion
 - Deletion
 - Substitution
 - Transposition of two adjacent letters



Words within 1 of across

Error	Candidate Correction	Correct Letter	Error Letter	Type
acress	actress	t	-	deletion
acress	cress	-	a	insertion
acress	caress	ca	ac	transposition
acress	access	c	r	substitution
acress	across	o	e	substitution
acress	acres	-	s	insertion
acress	acres	-	s	insertion



Candidate generation

- 80% of errors are within edit distance 1
- Almost all errors within edit distance 2
- Also allow insertion of **space** or **hyphen**
 - thisidea → this idea
 - inlaw → in-law



Language Model

- Use any of the language modeling algorithms we've learned
- Unigram, bigram, trigram
- Web-scale spelling correction
 - Stupid backoff



Unigram Prior probability

Counts from 404,253,213 words in Corpus of Contemporary English (COCA)

word	Frequency of word	$P(\text{word})$
actress	9,321	.0000230573
cress	220	.0000005442
caress	686	.0000016969
access	37,038	.0000916207
across	120,844	.0002989314
acres	12,874	.0000318463



Channel model probability

- **Error model probability, Edit probability**
- *Kernighan, Church, Gale 1990*
- *Misspelled word $x = x_1, x_2, x_3 \dots x_m$*
- *Correct word $w = w_1, w_2, w_3, \dots, w_n$*
- $P(x|w)$ = probability of the edit
 - (deletion/insertion/substitution/transposition)



Computing error probability: confusion matrix

$\text{del}[x,y]$: count(xy typed as x)

$\text{ins}[x,y]$: count(x typed as xy)

$\text{sub}[x,y]$: count(x typed as y)

$\text{trans}[x,y]$: count(xy typed as yx)

Insertion and deletion conditioned on previous character



Confusion matrix for spelling errors

sub[X, Y] = Substitution of X (incorrect) for Y (correct)

X	Y (correct)																									
	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
a	0	0	7	1	342	0	0	2	118	0	1	0	0	3	76	0	0	1	35	9	9	0	1	0	5	0
b	0	0	9	9	2	2	3	1	0	0	0	5	11	5	0	10	0	0	2	1	0	0	8	0	0	0
c	6	5	0	16	0	9	5	0	0	0	1	0	7	9	1	10	2	5	39	40	1	3	7	1	1	0
d	1	10	13	0	12	0	5	5	0	0	2	3	7	3	0	1	0	43	30	22	0	0	4	0	2	0
e	388	0	3	11	0	2	2	0	89	0	0	3	0	5	93	0	0	14	12	6	15	0	1	0	18	0
f	0	15	0	3	1	0	5	2	0	0	0	3	4	1	0	0	0	6	4	12	0	0	2	0	0	0
g	4	1	11	11	9	2	0	0	0	1	1	3	0	0	2	1	3	5	13	21	0	0	1	0	3	0
h	1	8	0	3	0	0	0	0	0	0	2	0	12	14	2	3	0	3	1	11	0	0	2	0	0	0
i	103	0	0	0	146	0	1	0	0	0	0	6	0	0	49	0	0	0	2	1	47	0	2	1	15	0
j	0	1	1	9	0	0	1	0	0	0	0	2	1	0	0	0	0	0	5	0	0	0	0	0	0	0
k	1	2	8	4	1	1	2	5	0	0	0	0	5	0	2	0	0	0	6	0	0	0	4	0	0	3
l	2	10	1	4	0	4	5	6	13	0	1	0	0	14	2	5	0	11	10	2	0	0	0	0	0	0
m	1	3	7	8	0	2	0	6	0	0	4	4	0	180	0	6	0	0	9	15	13	3	2	2	3	0
n	2	7	6	5	3	0	1	19	1	0	4	35	78	0	0	7	0	28	5	7	0	0	1	2	0	2
o	91	1	1	3	116	0	0	0	25	0	2	0	0	0	0	14	0	2	4	14	39	0	0	0	18	0
p	0	11	1	2	0	6	5	0	2	9	0	2	7	6	15	0	0	1	3	6	0	4	1	0	0	0
q	0	0	1	0	0	0	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
r	0	14	0	30	12	2	2	8	2	0	5	8	4	20	1	14	0	0	12	22	4	0	0	1	0	0
s	11	8	27	33	35	4	0	1	0	1	0	27	0	6	1	7	0	14	0	15	0	0	5	3	20	1
t	3	4	9	42	7	5	19	5	0	1	0	14	9	5	5	6	0	11	37	0	0	2	19	0	7	6
u	20	0	0	0	44	0	0	0	64	0	0	0	0	0	2	43	0	0	4	0	0	0	2	0	8	0
v	0	0	7	0	0	3	0	0	0	0	0	1	0	0	1	0	0	0	8	3	0	0	0	0	0	0
w	2	2	1	0	1	0	0	2	0	0	1	0	0	0	0	7	0	6	3	3	1	0	0	0	0	0
x	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0	0
y	0	0	2	0	15	0	1	7	15	0	0	0	2	0	6	1	0	7	36	8	5	0	0	1	0	0
z	0	0	0	7	0	0	0	0	0	0	0	7	5	0	0	0	0	2	21	3	0	0	0	0	3	0



Generating the confusion matrix

- Peter Norvig's list of errors
- Peter Norvig's list of counts of single-edit errors



Channel model

Kernighan, Church, Gale 1990

$$P(x|w) = \begin{cases} \frac{\text{del}[w_{i-1}, w_i]}{\text{count}[w_{i-1} w_i]} , & \text{if deletion} \\ \frac{\text{ins}[w_{i-1}, x_i]}{\text{count}[w_{i-1}]} , & \text{if insertion} \\ \frac{\text{sub}[x_i, w_i]}{\text{count}[w_i]} , & \text{if substitution} \\ \frac{\text{trans}[w_i, w_{i+1}]}{\text{count}[w_i w_{i+1}]} , & \text{if transposition} \end{cases}$$



Channel model for across

Candidate Correction	Correct Letter	Error Letter	$x w$	$P(x word)$
actress	t	-	c ct	.000117
cress	-	a	a #	.00000144
caress	ca	ac	ac ca	.00000164
access	c	r	r c	.000000209
across	o	e	e o	.0000093
acres	-	s	es e	.0000321
acres	-	s	ss s	.0000342



Noisy channel probability for across

Candidate Correction	Correct Letter	Error Letter	$x w$	$P(x word)$	$P(word)$	$10^9 * P(x w)P(w)$
actress	t	-	c ct	.000117	.0000231	2.7
cress	-	a	a #	.00000144	.000000544	.00078
caress	ca	ac	ac ca	.00000164	.00000170	.0028
access	c	r	r c	.000000209	.0000916	.019
across	o	e	e o	.0000093	.000299	2.8
acres	-	s	es e	.0000321	.0000318	1.0
acres	-	s	ss s	.0000342	.0000318	1.0



Noisy channel probability for across

Candidate Correction	Correct Letter	Error Letter	$x w$	$P(x \text{word})$	$P(\text{word})$	$10^9 * P(x w)P(w)$
actress	t	-	c ct	.000117	.0000231	2.7
cress	-	a	a #	.00000144	.000000544	.00078
caress	ca	ac	ac ca	.00000164	.00000170	.0028
access	c	r	r c	.000000209	.0000916	.019
across	o	e	e o	.0000093	.000299	2.8
acres	-	s	es e	.0000321	.0000318	1.0
acres	-	s	ss s	.0000342	.0000318	1.0



Using a bigram language model

- “a stellar and versatile **acress** whose combination of sass and glamour...”
- Counts from the Corpus of Contemporary American English with add-1 smoothing
- $P(\text{actress}|\text{versatile}) = .000021$ $P(\text{whose}|\text{actress}) = .0010$
- $P(\text{across}|\text{versatile}) = .000021$ $P(\text{whose}|\text{across}) = .000006$
- $P(\text{“versatile actress whose”}) = .000021 * .0010 = 210 \times 10^{-10}$
- $P(\text{“versatile across whose”}) = .000021 * .000006 = 1 \times 10^{-10}$



Using a bigram language model

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Evaluation

- Some spelling error test sets
 - [Wikipedia's list of common English misspelling](#)
 - [Aspell filtered version of that list](#)
 - [Birkbeck spelling error corpus](#)
 - [Peter Norvig's list of errors \(includes Wikipedia and Birkbeck, for training or testing\)](#)

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The Noisy Channel
Model of Spelling

