



Text Classification and Naïve Bayes

Naïve Bayes: Learning



Learning the Multinomial Naïve Bayes Model

- First attempt: maximum likelihood estimates
 - simply use the frequencies in the data

$$\hat{P}(c_j) = \frac{\text{doccount}(C = c_j)}{N_{doc}}$$

$$\hat{P}(w_i | c_j) = \frac{\text{count}(w_i, c_j)}{\sum_{w \in V} \text{count}(w, c_j)}$$



Parameter estimation

$$\hat{P}(w_i | c_j) = \frac{\text{count}(w_i, c_j)}{\sum_{w \in V} \text{count}(w, c_j)}$$

fraction of times word w_i appears
among all words in documents of topic c_j

- Create mega-document for topic j by concatenating all docs in this topic
 - Use frequency of w in mega-document



Problem with Maximum Likelihood

- What if we have seen no training documents with the word ***fantastic*** and classified in the topic **positive** (***thumbs-up***)?

$$\hat{P}(\text{"fantastic"} \mid \text{positive}) = \frac{\text{count}(\text{"fantastic"}, \text{positive})}{\sum_{w \in V} \text{count}(w, \text{positive})} = 0$$

- Zero probabilities cannot be conditioned away, no matter the other evidence!

$$c_{MAP} = \operatorname{argmax}_c \hat{P}(c) \prod_i \hat{P}(x_i \mid c)$$



Laplace (add-1) smoothing for Naïve Bayes

$$\begin{aligned}\hat{P}(w_i | c) &= \frac{\text{count}(w_i, c) + 1}{\sum_{w \in V} (\text{count}(w, c) + 1)} \\ &= \frac{\text{count}(w_i, c) + 1}{\left(\sum_{w \in V} \text{count}(w, c) \right) + |V|}\end{aligned}$$



Multinomial Naïve Bayes: Learning

- From training corpus, extract *Vocabulary*
- Calculate $P(c_j)$ terms
 - For each c_j in C do
 - $docs_j \leftarrow$ all docs with class $= c_j$
$$P(c_j) \leftarrow \frac{|docs_j|}{|\text{total \# documents}|}$$
- Calculate $P(w_k | c_j)$ terms
 - $Text_j \leftarrow$ single doc containing all $docs_j$
 - For each word w_k in *Vocabulary*
 - $n_k \leftarrow$ # of occurrences of w_k in $Text_j$
$$P(w_k | c_j) \leftarrow \frac{n_k + \alpha}{n + \alpha |Vocabulary|}$$



Laplace (add-1) smoothing: unknown words

Add one extra word to the vocabulary, the “unknown word” w_u

$$\begin{aligned}\hat{P}(w_u | c) &= \frac{\text{count}(w_u, c) + 1}{\left(\sum_{w \in V} \text{count}(w, c) \right) + |V + 1|} \\ &= \frac{1}{\left(\sum_{w \in V} \text{count}(w, c) \right) + |V + 1|}\end{aligned}$$



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