Minimum Edit Distance

Computing Minimum Edit Distance

Dynamic Programming for Minimum Edit Distance

- **Dynamic programming**: A tabular computation of D(*n*,*m*)
- Solving problems by combining solutions to subproblems.
- Bottom-up
  - We compute D(i,j) for small *i*,*j*
  - And compute larger D(i,j) based on previously computed smaller values
  - i.e., compute D(*i*,*j*) for all *i* ($0 < i < n$) and *j* ($0 < j < m$)

Defining Min Edit Distance (Levenshtein)

- Initialization

  $D(i,0) = i$

  $D(0,j) = j$

- Recurrence Relation:

  For each $i = 1 \ldots M$

  For each $j = 1 \ldots N$

  $$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } X(i) \neq Y(j) \\ 0; & \text{if } X(i) = Y(j) \end{cases} \end{cases}$$

- Termination:

  $D(N,M)$ is distance

The Edit Distance Table

N	9									
O	8									
I	7									
T	6									
N	5									
E	4									
T	3									
N	2									
I	1									
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

The Edit Distance Table

$$D(i,j) = \min \begin{cases} D(i-1,j) + 1 \\ D(i,j-1) + 1 \\ D(i-1,j-1) + \begin{cases} 2; & \text{if } S_1(i) \neq S_2(j) \\ 0; & \text{if } S_1(i) = S_2(j) \end{cases} \end{cases}$$

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Edit Distance

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	#	E	X	E	C	U	T	I	O	N

The Edit Distance Table

N	9	8	9	10	11	12	11	10	9	**8**
O	8	7	8	9	10	11	10	9	8	9
I	7	6	7	8	9	10	9	8	9	10
T	6	5	6	7	8	9	8	9	10	11
N	5	4	5	6	7	8	9	10	11	10
E	4	3	4	5	6	7	8	9	10	9
T	3	4	5	6	7	8	7	8	9	8
N	2	3	4	5	6	7	8	7	8	7
I	1	2	3	4	5	6	7	6	7	8
#	**0**	1	2	3	4	5	6	7	8	9
	#	E	X	E	C	U	T	I	O	N

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