Language Modeling

Advanced:
Kneser-Ney Smoothing

Resulting Good-Turing numbers

- Numbers from Church and Gale (1991)
- 22 million words of AP Newswire

$$c* = \frac{(c+1)N_{c+1}}{N_c}$$

- It sure looks like c* = (c - .75)

Count c	Good Turing c*
0	.0000270
1	0.446
2	1.26
3	2.24
4	3.24
5	4.22
6	5.19
7	6.21
8	7.24
9	8.25

Absolute Discounting Interpolation

- Save ourselves some time and just subtract 0.75 (or some d)!

discounted bigram

Interpolation weight

$$P_{\text{AbsoluteDiscounting}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) - d}{c(w_{i-1})} + \lambda(w_{i-1})P(w)$$

unigram

- (Maybe keeping a couple extra values of d for counts 1 and 2)
- But should we really just use the regular unigram P(w)?

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Kneser-Ney Smoothing I

- Better estimate for probabilities of lower-order unigrams!
  - Shannon game: *I can't see without my reading_____* ? Francisco glasses
  - "Francisco" is more common than "glasses"
  - … but "Francisco" always follows "San"
- The unigram is useful exactly when we haven't seen this bigram!
- Instead of P(w): "How likely is w"
- P_{continuation}(w): "How likely is w to appear as a novel continuation?
  - For each word, count the number of bigram types it completes
  - Every bigram type was a novel continuation the first time it was seen

$$P_{CONTINUATION}(w) \propto \left| \{ w_{i-1} : c(w_{i-1}, w) > 0 \} \right|$$

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Kneser-Ney Smoothing II

- How many times does w appear as a novel continuation:

$$P_{CONTINUATION}(w) \propto \left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|$$

- Normalized by the total number of word bigram types

$$\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|$$

$$P_{CONTINUATION}(w) = \frac{\left|\{w_{i-1} : c(w_{i-1}, w) > 0\}\right|}{\left|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}\right|}$$

Kneser-Ney Smoothing III

- Alternative metaphor: The number of # of word types seen to precede w

$$| \{ w_{i-1} : c(w_{i-1}, w) > 0 \} |$$

- normalized by the # of words preceding all words:

$$P_{CONTINUATION}(w) = \frac{\left| \{ w_{i-1} : c(w_{i-1}, w) > 0 \} \right|}{\sum_{w'} \left| \{ w'_{i-1} : c(w'_{i-1}, w') > 0 \} \right|}$$

- A frequent word (Francisco) occurring in only one context (San) will have a low continuation probability

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Kneser-Ney Smoothing IV

$$P_{KN}(w_i \mid w_{i-1}) = \frac{\max(c(w_{i-1}, w_i) - d, 0)}{c(w_{i-1})} + \lambda(w_{i-1})P_{CONTINUATION}(w_i)$$

λ is a normalizing constant; the probability mass we've discounted

$$\lambda(w_{i-1}) = \frac{d}{c(w_{i-1})} \left| \{w : c(w_{i-1}, w) > 0\} \right|$$

the normalized discount

The number of word types that can follow w_{i-1}
= # of word types we discounted
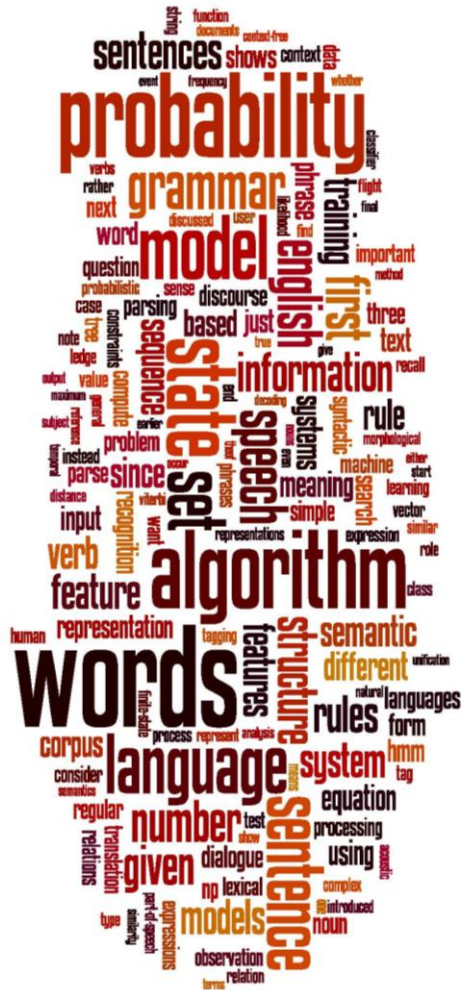= # of times we applied normalized discount

Kneser-Ney Smoothing: Recursive formulation

$$P_{KN}(w_i \mid w_{i-n+1}^{i-1}) = \frac{\max(c_{KN}(w_{i-n+1}^i) - d, 0)}{c_{KN}(w_{i-n+1}^{i-1})} + /(w_{i-n+1}^{i-1})P_{KN}(w_i \mid w_{i-n+2}^{i-1})$$

$$c_{KN}(\cdot) = \begin{cases} count(\cdot) & \text{for the highest order} \\ continuationcount(\cdot) & \text{for lower order} \end{cases}$$

Continuation count = Number of unique single word contexts for •

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