# Minimum Edit Distance

Minimum Edit Distance in Computational Biology



# **Sequence Alignment**

AGGCTATCACCTGACCTCCAGGCCGATGCCC
TAGCTATCACGACCGCGGTCGATTTGCCCGAC

-AGGCTATCACCTGACCTCCAGGCCGA--TGCCC--TAG-CTATCAC--GACCGC--GGTCGATTTGCCCGAC



## Why sequence alignment?

- Comparing genes or regions from different species
  - to find important regions
  - determine function
  - uncover evolutionary forces
- Assembling fragments to sequence DNA
- Compare individuals to looking for mutations





# Alignments in two fields

- In Natural Language Processing
  - We generally talk about distance (minimized)
    - And weights
- In Computational Biology
  - We generally talk about similarity (maximized)
    - And scores



## The Needleman-Wunsch Algorithm

Initialization:

$$D(i,0) = -i * d$$
  
 $D(0,j) = -j * d$ 

Recurrence Relation:

$$D(i,j) = \max \begin{cases} D(i-1,j) & - d \\ D(i,j-1) & - d \\ D(i-1,j-1) & + s[x(i),y(j)] \end{cases}$$

Termination:



## The Needleman-Wunsch Matrix



(Note that the origin is at the upper left.)

Slide adapted from Serafim Batzoglou



## A variant of the basic algorithm:

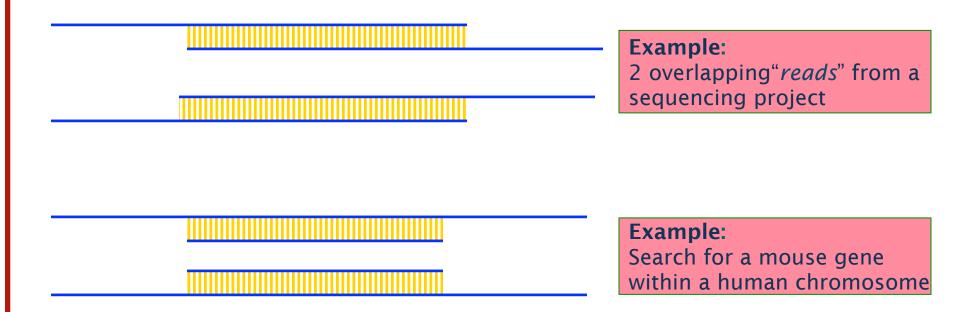
 Maybe it is OK to have an unlimited # of gaps in the beginning and end:

-----CTATCACCTGACCTCCAGGCCGATGCCCCTTCCGGC
GCGAGTTCATCTATCAC--GACCGC--GGTCG------

If so, we don't want to penalize gaps at the ends

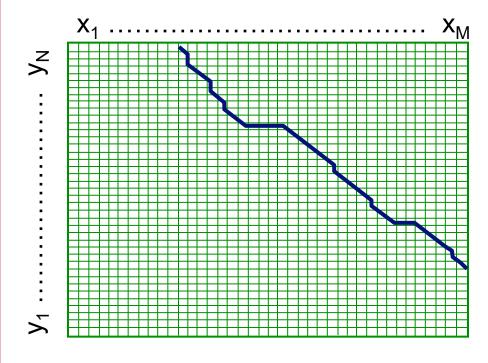


# **Different types of overlaps**





# **The Overlap Detection variant**



Slide from Serafim Batzoglou

## Changes:

1. Initialization

For all i, j,  

$$F(i, 0) = 0$$
  
 $F(0, j) = 0$ 

2. Termination

$$F_{OPT} = \max \begin{cases} \max_{i} F(i, N) \\ \max_{j} F(M, j) \end{cases}$$



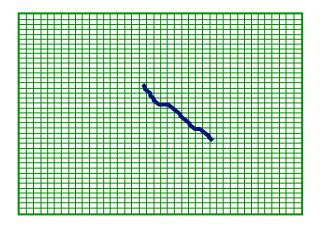
# **The Local Alignment Problem**

Given two strings

$$x = x_1 \dots x_M$$

$$y = y_1 \dots y_N$$

Find substrings x', y' whose similarity (optimal global alignment value) is maximum





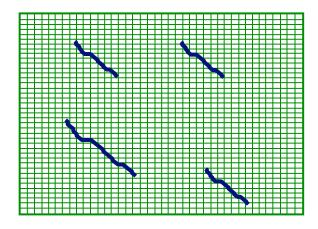
# The Smith-Waterman algorithm

Idea: Ignore badly aligning regions

Modifications to Needleman-Wunsch:

Initialization: F(0, j) = 0

$$F(i, 0) = 0$$



Iteration: 
$$F(i, j) = \max \begin{cases} 0 \\ F(i - 1, j) - d \\ F(i, j - 1) - d \\ F(i - 1, j - 1) + s(x_i, y_j) \end{cases}$$



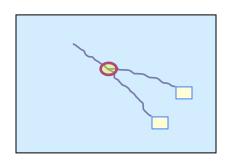
# The Smith-Waterman algorithm

## **Termination:**

1. If we want the best local alignment...

$$F_{OPT} = max_{i,j} F(i, j)$$

Find F<sub>OPT</sub> and trace back



2. If we want all local alignments scoring > t

?? For all i, j find F(i, j) > t, and trace back?

Complicated by overlapping local alignments



# Local alignment example

X = ATCAT

Y = ATTATC

Let:

m = 1 (1 point for match)

d = 1 (-1 point for del/ins/sub)

		A	Т	Т	A	Т	C
	0	0	0	0	0	0	0
A	0						
T	0						
C	0						
A	0						
Т	0						





# Local alignment example

X = ATCAT

Y = ATTATC

		A	T	T	A	T	C
	0	0	0	0	0	0	0
A	0	1	0	0		0	0
Т	0	0	2	1	0	2	0
C	0	0	1	1	0	1	3
A	0	1	0	0	2	1	2
$\mathbf{T}$	0	0	2	0	1	3	2





# Local alignment example

X = ATCAT

Y = ATTATC

		A	T	T	A	Т	C
	0	0	0	0	0	0	0
A	0	1	0	0	1	0	0
T	0	0	2	1	0	2	0
C	0	0	1	1	0	1	3
A	0	1	0	0	2	1	2
$\mathbf{T}$	0	0	2	0	1	3	2





# Local alignment example

X = ATCAT

Y = ATTATC

		A	T	Т	A	T	C
	0	0	0	0	0	0	0
A	0	1	0	0	1	0	0
Т	0	0	2	1	0	2	0
C	0	0	1	1	0	1	3
A	0	1	0	0	2	1	2
T	0	0	2	0	1	3	2

# Minimum Edit Distance

Minimum Edit Distance in Computational Biology