Assignment 3: Camera Calibration and Fundamental Matrix Estimation with RANSAC

Overview

The goal of this project is to introduce you to camera and scene geometry. Specifically we will estimate the camera projection matrix, which maps 3D world coordinates to image coordinates, as well as the fundamental matrix, which relates points in one scene to epipolar lines in another. The camera projection matrix and the fundamental matrix can be estimated using point correspondences:

- To estimate the projection matrix—intrinsic and extrinsic camera calibration—the input is corresponding 3d and 2d points.
- To estimate the fundamental matrix the input is corresponding 2d points across two images.

You will start out by estimating the projection matrix and the fundamental matrix for a scene with ground truth correspondences. Then you'll move on to estimating the fundamental matrix using point correspondences from SIFT and RANSAC.

Remember these challenging images of Gaudi's Episcopal Palace from Project 2? By using RANSAC to find the fundamental matrix with the most inliers, we can filter away spurious matches and achieve near perfect point to point matching as shown below:

Details and Starter Code

This project consists of three parts:

1. Estimating the projection matrix,
2. Estimating the fundamental matrix, and
3. Estimating the fundamental matrix reliably with RANSAC from unreliable SIFT matches.

These three tasks can be implemented in `student.py`.

Part I: Camera Projection Matrix

Given known 3D to 2D point correspondences, how can we recover a projection matrix that transforms from world 3D coordinates to 2D image coordinates? Recall that using homogeneous coordinates the equation for moving from 3D world to 2D camera coordinates is:

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \cong \begin{pmatrix} su \\ sv \\ s \end{pmatrix} = M \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Our task is to find M. We are going to set up a system of linear equations to find the least squares regression solution for these camera matrix parameters, given correspondences between 3D world points and 2D image points. In class, we saw two solutions to this problem, which both require rearranging the coefficients of the 3D and 2D point evidence:

$$u = \frac{m_{11}X + m_{12}Y + m_{13}Z + m_{14}}{m_{31}X + m_{32}Y + m_{33}Z + m_{34}}$$

$$\rightarrow (m_{31}X + m_{32}Y + m_{33}Z + m_{34})u = m_{11}X + m_{12}Y + m_{13}Z + m_{14}$$

$$\rightarrow 0 = m_{11}X + m_{12}Y + m_{13}Z + m_{14} - m_{31}uX - m_{32}uY - m_{33}uZ - m_{34}u$$

$$v = \frac{m_{21}X + m_{22}Y + m_{23}Z + m_{24}}{m_{31}X + m_{32}Y + m_{33}Z + m_{34}}$$

$$\rightarrow (m_{31}X + m_{32}Y + m_{33}Z + m_{34})v = m_{21}X + m_{22}Y + m_{23}Z + m_{24}$$

$$\rightarrow 0 = m_{21}X + m_{22}Y + m_{23}Z + m_{24} - m_{31}vX - m_{32}vY - m_{33}vZ - m_{34}v$$

With these rearrangements, we're able to set up our linear regression to find the elements of the matrix M. However, in projective geometry, the matrix M is only defined up to an unknown scale. In effect, these equations have many different possible solutions. In the degenerate solution case, M = all zeros, which is not very helpful in our context.

We will consider two ways around this:

1. We can fix the scale by setting the last element m_{34} to **1 (one)** and then find the remaining coefficients,
2. We can use the singular value decomposition to directly solve the constrained optimization problem in which the scale is set:

$$min\|Ax\|$$
$$s.t.\|x\| = 1$$

Having recovered a vector of values with one of these methods, we must reshape it into the estimated camera projection matrix M.

To help you validate M, we provide evaluation code which computes the total "residual error" between the projected 2d location of each 3d point and the actual location of that point in the 2d image. The residual is the Euclidean distance in the image plane (square root of the sum of squared differences in u and v). This should be very small—in the order of a pixel. We also provide a set of "normalized points" in the files `./pts2d-norm-pic_a.txt` and `./pts3d-norm.txt`. The starter code will load these 20 corresponding normalized 2D and 3D points. If you solve for M using all the points, you should receive a matrix that is a scaled equivalent of the following matrix:

$$M_{normA} = \begin{pmatrix} -0.4583 & 0.2947 & 0.0139 & -0.0040 \\ 0.0509 & 0.0546 & 0.5410 & 0.0524 \\ -0.1090 & -0.1784 & 0.0443 & -0.5968 \end{pmatrix}$$

Given this matrix, we can project 3D points in the world onto our camera plane. For example, this matrix will take the normalized 3D point < 1.2323, 1.4421, 0.4506, 1.0> and project it to 2D image < u, v> of < 0.1419, –0.4518> (after converting the homogeneous 2D point < us, vs, s> to its nonhomogeneous version by dividing by s).

Once we have an accurate projection matrix M, it is possible to tease it apart into the more familiar and more useful matrix K of intrinsic parameters and matrix [R | T] of extrinsic parameters. For this project we will only ask you to estimate one particular extrinsic parameter: the camera center in world coordinates. Let us define M as being made up of a 3x3 matrix called Q, with 4th column m_4 :

$$M = (Q|m_4)$$

The center of the camera C could be found by:

$$C = -Q^{-1}m_4$$

If we use the normalized 3D points and M given above, we would receive camera center:

$$C_{normA} =< -1.5125, -2.3515, 0.2826 >$$

We've also provided a visualization which will show the estimated 3d location of the camera with respect to the normalized 3d point coordinates.

Part II: Fundamental Matrix Estimation

The next part of this project is to estimate the mapping of points in one image to lines in another by means of the fundamental matrix. This will require you to use similar

methods to those in part 1. We will make use of the corresponding point locations listed in `pts2d-pic_a.txt` and `pts2d-pic_b.txt`. Recall that the definition of the fundamental matrix is:

$$\begin{pmatrix} u' & v' & 1 \end{pmatrix} \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix} \begin{pmatrix} u \\ u \\ 1 \end{pmatrix} = 0$$

The fundamental matrix is sometimes defined as the transpose of the above matrix with the left and right image points swapped. Both are valid fundamental matrices, but the visualization functions in the starter code assume you use the above form.

Another way of writing this matrix equations is:

$$\begin{pmatrix} u' & v' & 1 \end{pmatrix} \begin{pmatrix} f_{11}u & f_{12}v & f_{13} \\ f_{21}u & f_{22}v & f_{23} \\ f_{31}u & f_{32}v & f_{33} \end{pmatrix} = 0$$

Which is the same as:

$$(f_{11}uu' + f_{12}vu' + f_{13}u' + f_{21}uv' + f_{22}vv' + f_{23}v' + f_{31}u + f_{32}v + f_{33}) = 0$$

With this, we can build a linear system to solve with least squares regression. For each unknown variable, we need one equation to constrain our solution. A pair of corresponding points will produce one equation for our system. However, as in part I, this matrix is only defined up to scale and the degenerate zero solution solves these equations. We need to fix the scale and solve using the same methods as in part I. This leaves us able to solve the system with 8 point correspondences (or more, as we recover the best least squares fit to all points).

The least squares estimate of F is full rank; however, the fundamental matrix is a rank 2 matrix. As such we must reduce its rank. To do this, we can decompose F using singular value decomposition into the matrices U ΣV' = F. We can then estimate a rank 2 matrix by setting the smallest singular value in Σ to zero thus generating Σ2. The fundamental matrix is then easily calculated as F = U Σ2 V'. We can check our fundamental matrix estimation by plotting the epipolar lines using the plotting function provided in the starter code.

Part III: Fundamental Matrix with RANSAC

Given two photographs of a scene, it is unlikely that our point corresponence with which to perform regression for the fundamental matrix would all be correct. Our next task is to use RANSAC to reliably estimate a fundamental matrix from unreliable point correspondences computed with a feature point detector. We simulate noise (because we use ground truth points) by two kinds of noise functions.

We first ask you to write a positional noise function. This will accept an interval and will add some value (sampled uniformly at random) from the interval, basically perturbing the coordinates of the matches by some amount. This must be done to only some percentage of the points, specified in the function arguments.

Our other functions introduces noise in the matches by randomly swapping some subset of the matches. So if you have 100 points and are asked to add noise to 30%, then 30 of the points should be shuffled in the list. Note we want to switch up which points in image A are matched with other points in image B, not swap the indices of the points itself.

The starter code uses ground truth matches (along with a flag to perform feature point matching with the ORB descriptor) for an image pair. Optionally, we will perturb these matches with noise using your functions you just wrote. We'll use these possible point correspondences and RANSAC to try and find a good fundamental matrix. We will iteratively choose a random set of point correspondences (e.g., 8, 9, or some small number of points), solve for the fundamental matrix using the function you wrote for part II, and then count the number of inliers. Inliers in this context will be point correspondences that "agree" with the estimated fundamental matrix. To count how many inliers a fundamental matrix has, you will need a distance metric based on the fundamental matrix. (Hint: For a point correspondence (x',x) what properties does the fundamental matrix have?). You will also need to pick a threshold between inliers and outliers. The results will be very sensitive to this threshold, so explore a range of values which strikes a balance between permission and exclusion. We must also decide on a stopping criteria for RANSAC, e.g., considering thousands of iterations of RANSAC. Upon stopping, return the fundamental matrix with the most inliers.

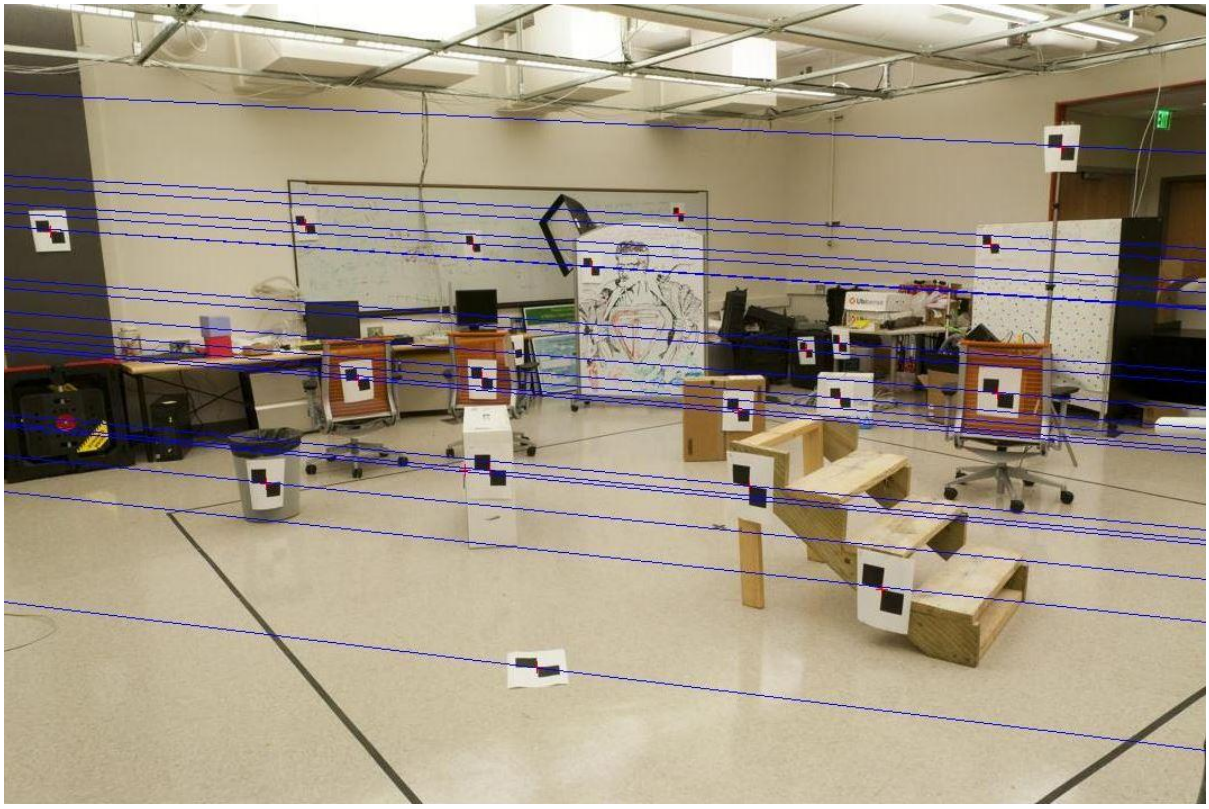
Recall from lecture the expected number of iterations of RANSAC to find the "right" solution in the presence of outliers. For example, if half of your input correspondences are wrong, then you have a 0.5^8 = 0.39% chance to randomly pick 8 incorrect correspondences when estimating the fundamental matrix. The correct fundamental matrix must have more inliers than outliers created from spurious matches, else the problem is not solvable.

For some real images, the fundamental matrix that is estimated may imply an impossible relationship between the cameras, e.g., an epipole in the center of one image when the cameras actually have a large translation parallel to the image plane. The estimated fundamental matrix may also be incorrect because the world points are coplanar, or because the cameras are not actually pinhole cameras and have lens distortions. Still, these "incorrect" fundamental matrices tend to remove incorrect feature point matches (and, unfortunately, many correct ones too).

Evaluation and Visualization

For part I, we have provided expected output (matrix M and camera center C). These are numerical estimates so we won't be checking for exact numbers, just approximately correct locations.

For part II, you will be evaluated based on your estimate of the fundamental matrix. You can test how good your estimate of the fundamental matrix is by drawing the epipolar lines on one image which correspond to a point in the other image. You should see all of the epipolar lines crossing through the corresponding point in the other image, like this:



We provide you with a function in the starter code that draws an epipolar line in an image given the fundamental matrix, and a point from the other image.

Part III is the most open ended part of this assignment. Your goal should be to demonstrate that you can estimate a fundamental matrix for a real image pair and use it to reject spurious keypoint matches. We may check the fundamental matrix you estimate, and you should visualize the rectified image in your writeup. In addition, we require you to implement noise functions and investigate the effects of noise on how well RANSAC performs.

We do not include the keypoint accuracy evaluation used in project 2. You should be able to get near-perfect accuracy (very few outliers) among the keypoints you designate as inliers.

Data

We provide 2D and 3D ground truth point correspondences for the base image pair (`pic_a.jpg` and `pic_b.jpg`) and for the three images used.

Banned Functions

You must construct the matrices for camera calibration and F matrix estimation yourself, but you can use off-the-shelf linear solvers. You also must implement RANSAC yourself. You also have to write the noise functions yourself. Feel free to use numpy functions.

Coordinate Normalization Notes

Your estimate of the fundamental matrix can be improved by normalizing the coordinates before computing the fundamental matrix. It is suggested you perform the normalization through linear transformations as described below to make the mean of the points zero and the average magnitude about 1.0 or some other small number (square root of 2 is also recommended).

$$\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = \begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -c_u \\ 0 & 1 & -c_v \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

The transform matrix T is the product of the scale and offset matrices. c_u and c_v are the mean coordinates. To compute a scale s you could estimate the standard deviation after subtracting the means. Then the scale factor s would be the reciprocal of whatever estimate of the scale you are using. Or you could find the maximum absolute value. Or you could scale the coordinates such that the average squared distance from the origin (after centering) is 2. You could use one scale matrix based on the statistics of the coordinates from both images or you could do it per image.

However you scale your coordinates, you will need to use the scaling matrices to adjust your fundamental matrix so that it can operate on the original pixel coordinates as follows:

$$F_{orig} = T_b^T @ F_{norm} @ T_a$$

This can be implemented in your function `estimate_fundamental_matrix()` written for part II, but you won't actually notice a difference in part II because the input correspondences are pretty much perfect. In part III (which calls `estimate_fundamental_matrix()`) you are more likely to notice an improvement. Alternatively, you could implement the scaling based on the distribution of all feature coordinates and not just the handful passed into `estimate_fundamental_matrix()`. In your writeup you should highlight one before and after case where normalization improved your results.

Writeup

Describe your process and algorithm, show your results, describe any extra credit, and tell us any other information you feel is relevant. We provide you with a LaTeX

template `writeup/writeup.tex`. Please compile it into a PDF and submit it along with your code.

Make sure to include the following details in writing up your method:

1. Your estimate of the projection matrix and the camera center.
2. Your estimate of the fundamental matrix for the base image pair (`pic_a.jpg` and `pic_b.jpg`).
3. Show us the correspondences (and rectified image) with increasing levels of noise of both kinds. Compare how RANSAC vs vanilla estimation does on each level. Write about your thoughts on how well RANSAC does.
4. Show us what happens if ORB features are used instead of cheat interest point matches. Detail your conclusions.

Rubric

- +10 pts: Correctly setting up the system of equations for the least squares regression for the projection matrix.
- +10 pts: Correctly solving for the projection matrix and correctly solving for the camera center.
- +10 pts: Correctly setting up the fundamental matrix regression.
- +10 pts: Correctly solving for the fundamental matrix and generating good epipoles on the sample images.
- +8 pts: Correctly implementing the noise functions.
- +27 pts: Correctly combining RANSAC with fundamental matrix estimation and generating correctly rectified images.
- +25 pts: Write up.

Extra Credit

Please document in your write what extra credit you've done and detail it's outcome.

- up to 10 pts: Use a different RANSAC routine (and compare its performance to basic RANSAC), such as:
    - MAGSAC (C++ implementation) (Python implementation)
    - MLESAC (C++ implementation via PCL) (Python bindings for PCL)

Credits

This project is based on a project from Aaron Bobick's offering of CS 4495 at GATech, and has been expanded and edited by Henry Hu, Grady Williams, and James Hays. Eleanor Tursman adapted the code for Python, with help from Anna Sabel.