Modeling of COVID-19 death cases in Spain by Differential Equation

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Data

This paper uses data from `Our World in Data COVID-19 dataset`[1].
The period chosen is April 1 to April 30, 2020. Table 1 gives some samples from that period.

Date	Total number death cases
April 1, 2020	9387
April 2, 2020	10348
April 3, 2020	11198
April 4, 2020	11947
April 5, 2020	12641

Table 1: Total number of COVID-19 death cases in Spain from April 1 to April 5, 2020.

Model

Let $P(t)$ be the total number of death cases t days after April 1, 2020. The growth rate $\frac{dP}{dt}$, which represents daily new death cases, is calculated using the formula:

$$\frac{dP}{dt}(t_i) = \begin{cases} \frac{1}{2}\left[P(t_{i+1}) - P(t_{i-1})\right] & \text{if } 0 < i < n \\ P(t_1) - P(t_0) & \text{if } i = 0 \\ P(t_n) - P(t_{n-1}) & \text{if } i = n \end{cases} \tag{1}$$

t (days)	0	2	4	6	8	10
P (observed)	9387	11198	12641	14045	15447	16606
$\frac{dP}{dt}$	961.0	799.5	697.0	725.5	644.5	564.0
$\frac{1}{P}\frac{dP}{dt}$	0.102376	0.071397	0.055138	0.051655	0.041723	0.033964

Table 2: Sample values of P, $\frac{dP}{dt}$, and the relative growth rate $\frac{1}{P}\frac{dP}{dt}$.

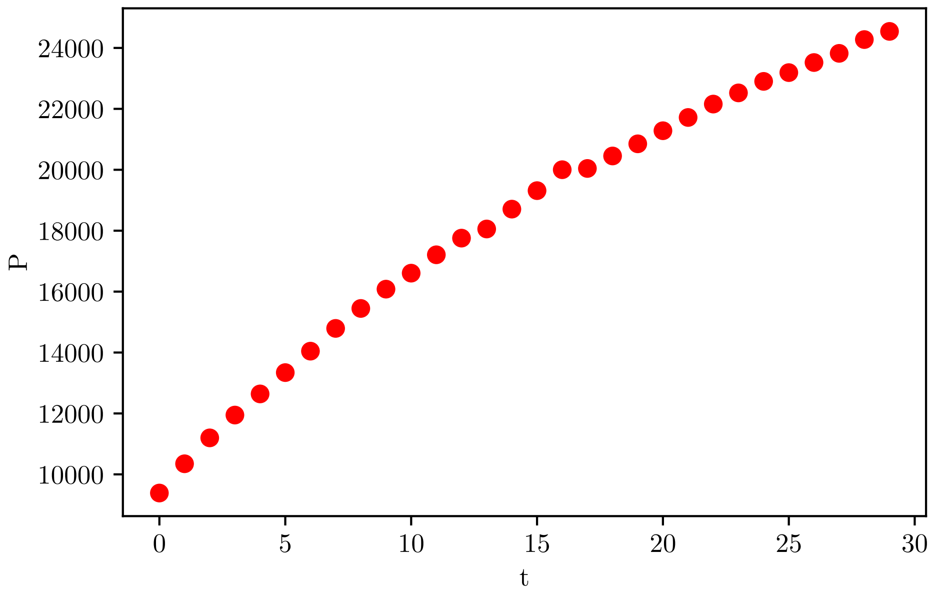

Figure 1: Total number of COVID-19 death cases in Spain from April 1 to April 30, 2020.

For this paper, we use a variant of the Logistic Growth Model:

$$\frac{1}{P} \cdot \frac{dP}{dt} = aP + b \tag{2}$$

The coefficients a and b are determined using least squares polynomial fit:

$$\begin{aligned} a &= -4.827879002776788 \times 10^{-6} \\ b &= 0.12215521367938273 \end{aligned} \tag{3}$$

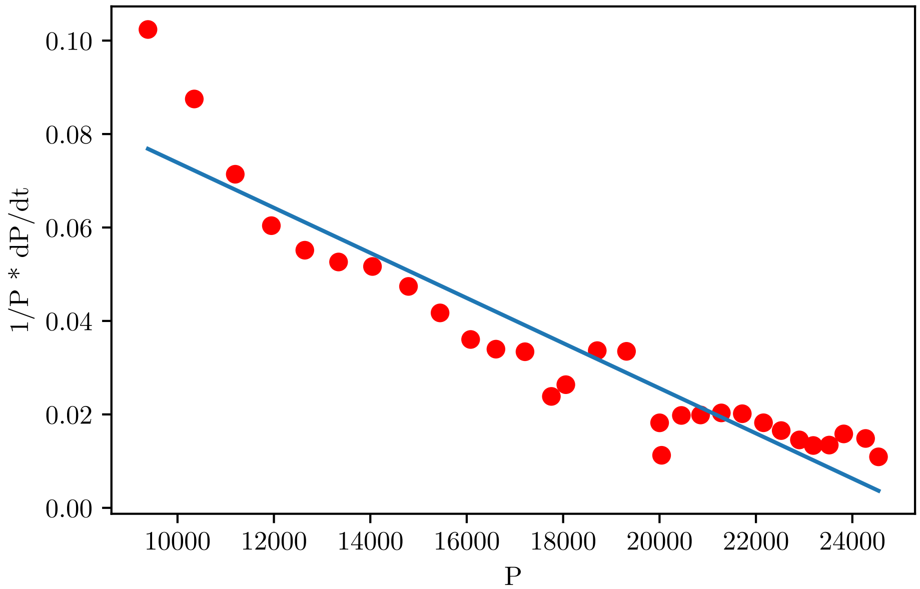

Figure 2: Relative growth rate, $\frac{1}{P} \cdot \frac{dP}{dt}$ (red points), as a function of P and the best fitted line (blue).

Solving Differential Equation

Equation 2 is separable. We have:

$$\frac{dP}{P(aP+b)} = dt \tag{4}$$

Integrate both sides;

$$\int \frac{dP}{P(aP+b)} = \int dt \tag{5}$$

To evaluate the integral on the left side, we use partial fraction decomposition:

$$\frac{1}{P(aP+b)} = \frac{X}{P} + \frac{Y}{aP+b}$$

Multiplying both sides of this equation by $P(aP+b)$ and rearranging the terms, we have:

$$1 = (Xa+Y)P + Xb$$

The polynomials are identical, so their coefficients must be equal:

$$\begin{aligned} Xa + Y &= 0 \\ Xb &= 1 \end{aligned}$$

We get $X = \frac{1}{b}$ and $Y = \frac{-a}{b}$. Therefore:

$$\frac{1}{P(aP+b)} = \frac{1}{b} \cdot \frac{1}{P} - \frac{a}{b} \cdot \frac{1}{aP+b}$$

We can now rewrite equation 5 as:

$$\begin{aligned}
\int \left(\frac{1}{b} \cdot \frac{1}{P} - \frac{a}{b} \cdot \frac{1}{aP+b} \right) dP &= \int dt \\
\frac{1}{b} \ln |P| - \frac{1}{b} \ln |aP+b| &= t + C \\
\ln \left| \frac{P}{aP+b} \right| &= bt + bC \\
\left| \frac{P}{aP+b} \right| &= e^{bt+bC} = e^{bt} e^{bC} \\
\frac{P}{aP+b} &= Ae^{bt} \qquad (6)
\end{aligned}$$

where $A = \pm e^{bC}$. Solving for P, we get:

$$P = \frac{bAe^{bt}}{1 - aAe^{bt}} \qquad (7)$$

The growth rate is:

$$\frac{dP}{dt} = \frac{b^2 Ae^{bt}}{(1 - aAe^{bt})^2} \qquad (8)$$

We find the value of A by putting $t = 0$ in equation 6:

$$A = \frac{P_0}{aP_0 + b}$$

Substituting coefficients from 3 and $P_0 = 9387$:

$$A = 122169.433209181$$

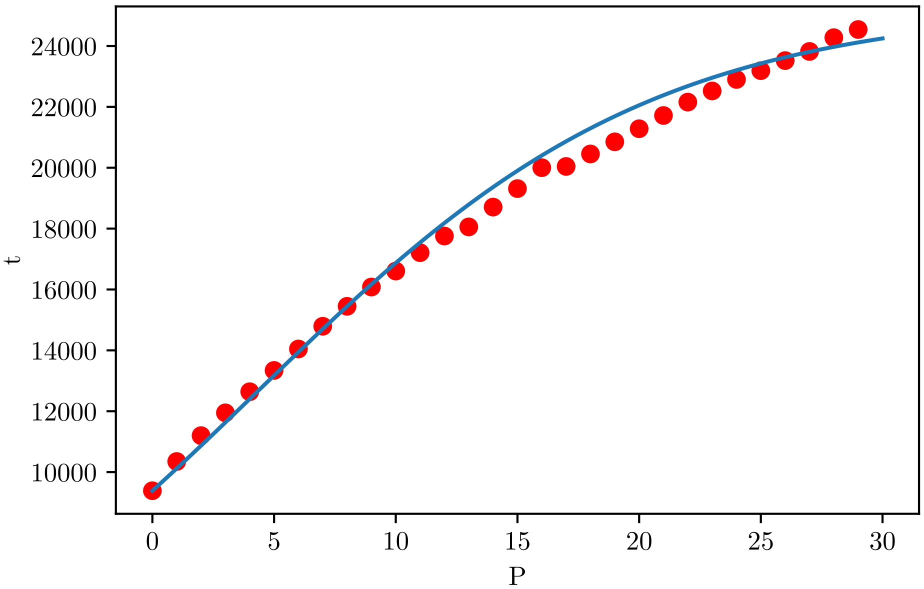

Figure 3: Total number of COVID-19 death cases in Spain from April 1 to April 30, 2020 from official data (red points). The blue line is the theoretical prediction using equation 7 with $a = -4.827879002776788 \times 10^{-6}$, $b = 0.12215521367938273$, and $A = 122169.433209181$.

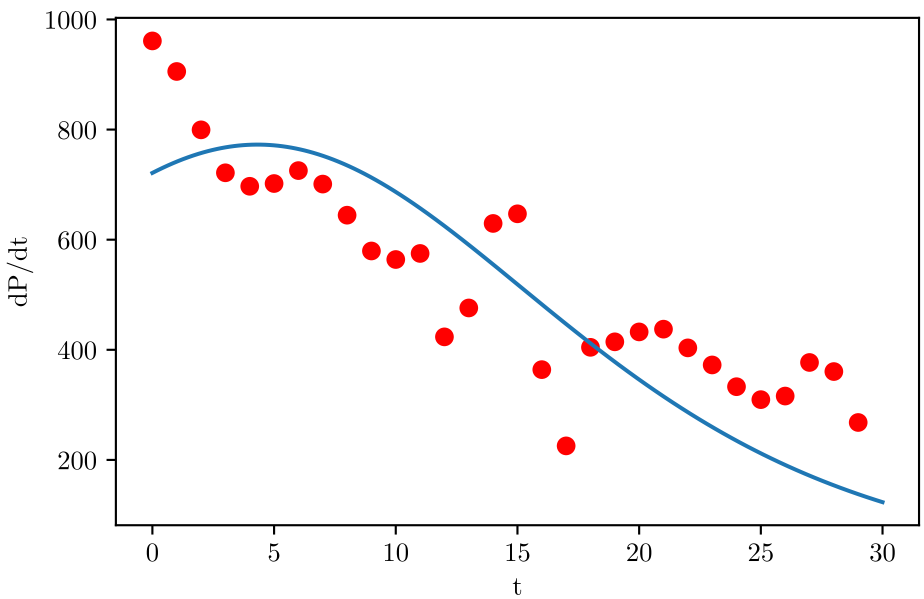

Figure 4: The growth rate of $P(t)$ calculated using formula 1 (red points) and theoretical prediction using equation 8.

Discussion

The source code of this paper is published at https://github.com/hieplpvip/mth252-assignment.

References

1. *Our World in Data COVID-19 dataset* https://ourworldindata.org/coronavirus-source-data.