Modeling of COVID-19 death cases in Spain by Differential Equation

MTH252 Assignment

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Data

This paper uses data from `Our World in Data COVID-19 dataset`[1].
The period chosen is April 1 to April 30, 2020. Table 1 gives some samples from that period.

Date	Total number death cases
April 1, 2020	9387
April 2, 2020	10348
April 3, 2020	11198
April 4, 2020	11947
April 5, 2020	12641

Table 1: Total number of COVID-19 death cases in Spain from April 1 to April 5, 2020.

Model

Let $P(t)$ be the total number of death cases t days after April 1, 2020. The growth rate $\frac{dP}{dt}$, which represents daily new death cases, is calculated using the formula:

$$\frac{dP}{dt}(t_i) = \begin{cases} \frac{1}{2}\left[P(t_{i+1}) - P(t_{i-1})\right] & \text{if } 0 < i < n \\ P(t_1) - P(t_0) & \text{if } i = 0 \\ P(t_n) - P(t_{n-1}) & \text{if } i = n \end{cases} \tag{1}$$

t (days)	0	2	4	6	8	10
P (observed)	9387	11198	12641	14045	15447	16606
$\frac{dP}{dt}$	961.0	799.5	697.0	725.5	644.5	564.0
$\frac{1}{P}\frac{dP}{dt}$	0.102376	0.071397	0.055138	0.051655	0.041723	0.033964

Table 2: Sample values of P, $\frac{dP}{dt}$, and the relative growth rate $\frac{1}{P}\frac{dP}{dt}$.

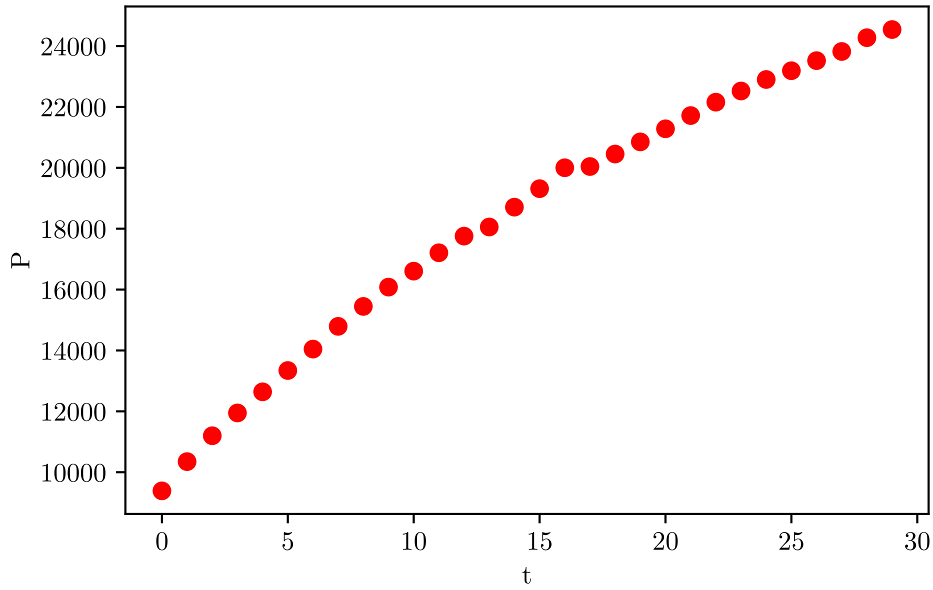

Figure 1: Total number of COVID-19 death cases in Spain from April 1 to April 30, 2020.

For this paper, we use a variant of the Logistic Growth Model:

$$\frac{1}{P}\frac{dP}{dt} = aP + b \tag{2}$$

The coefficients a and b are determined using least squares polynomial fit:

$$\begin{aligned} a &= -4.827879002776788 \times 10^{-6} \\ b &= 0.12215521367938273 \end{aligned} \tag{3}$$

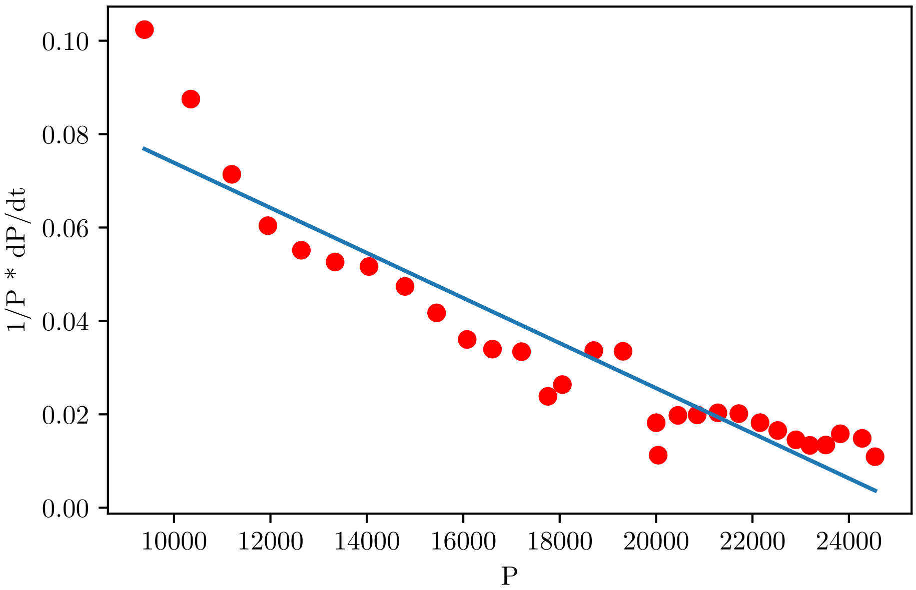

Figure 2: Relative growth rate, $\dfrac{1}{P}\dfrac{dP}{dt}$ (red points), as a function of P and the best fitted line (blue).

Solving Differential Equation

Equation 2 is separable. We have:

$$\frac{dP}{P(aP+b)} = dt \tag{4}$$

Integrate both sides;

$$\int \frac{dP}{P(aP+b)} = \int dt \tag{5}$$

To evaluate the integral on the left side, we use partial fraction decomposition:

$$\frac{1}{P(aP+b)} = \frac{X}{P} + \frac{Y}{aP+b}$$

Multiplying both sides of this equation by $P(aP+b)$ and rearranging the terms, we have:

$$1 = (Xa+Y)P + Xb$$

The polynomials are identical, so their coefficients must be equal:

$$\begin{aligned} Xa + Y &= 0 \\ Xb &= 1 \end{aligned}$$

We get $X = \frac{1}{b}$ and $Y = \frac{-a}{b}$. Therefore:

$$\frac{1}{P(aP+b)} = \frac{1}{b} \cdot \frac{1}{P} - \frac{a}{b} \cdot \frac{1}{aP+b}$$

We can now rewrite equation 5 as:

$$\begin{aligned}
\int \left(\frac{1}{b} \cdot \frac{1}{P} - \frac{a}{b} \cdot \frac{1}{aP+b} \right) dP &= \int dt \\
\frac{1}{b} \ln|P| - \frac{1}{b} \ln|aP+b| &= t + C \\
\ln\left|\frac{P}{aP+b}\right| &= bt + bC \\
\left|\frac{P}{aP+b}\right| &= e^{bt+bC} = e^{bt}e^{bC} \\
\frac{P}{aP+b} &= Ae^{bt} \qquad (6)
\end{aligned}$$

where $A = \pm e^{bC}$. Solving for P, we get:

$$P = \frac{bAe^{bt}}{1 - aAe^{bt}} \qquad (7)$$

The growth rate is:

$$\frac{dP}{dt} = \frac{b^2Ae^{bt}}{(1 - aAe^{bt})^2} \qquad (8)$$

We find the value of A by putting $t = 0$ in equation 6:

$$A = \frac{P_0}{aP_0 + b}$$

Substituting coefficients from 3 and $P_0 = 9387$:

$$A = 122169.433209181$$

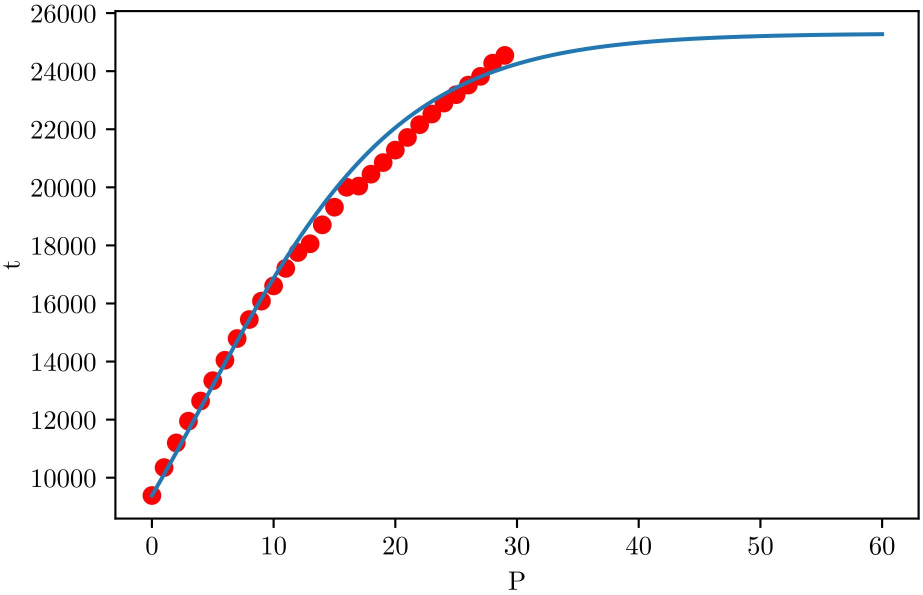

Figure 3: Total number of COVID-19 death cases in Spain from April 1 to April 30, 2020 from official data (red points) and theoretical prediction using equation 7 (blue line).

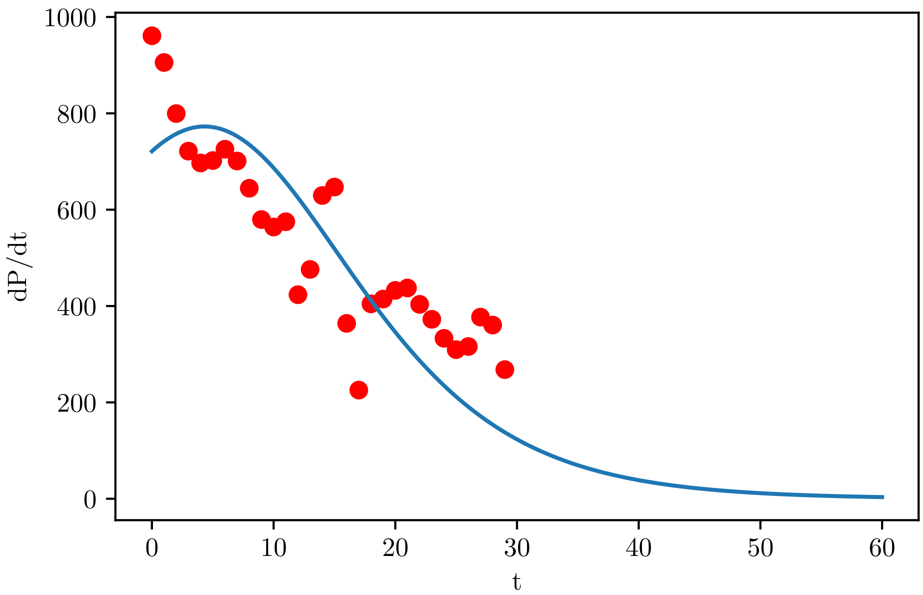

Figure 4: The growth rate of $P(t)$ calculated using formula 1 (red points) and theoretical prediction using equation 8 (blue line).

Discussion

This paper presents a simple logistic model for predicting COVID-19 fatality in Spain. As showed in figure 3, the total number of death cases increased exponentially in the early stages but began to level off at the end of April. The model predicts that daily death cases would decrease significantly in May, which indeed happened in reality. Therefore, the logistic model is a suitable choice for predicting COVID-19 fatality.

The source code of this paper is available at `https://github.com/hieplpvip/mth252-assignment`.

References

1. *Our World in Data COVID-19 dataset* `https://ourworldindata.org/coronavirus-source-data`.

Appendix: Data used in this paper

Date	t	P	$\frac{dP}{dt}$	$\frac{1}{P}\frac{dP}{dt}$
4/1/2020	0	9387.0	961.0	0.10237562586555875
4/2/2020	1	10348.0	905.5	0.08750483185156552
4/3/2020	2	11198.0	799.5	0.0713966779782104
4/4/2020	3	11947.0	721.5	0.06039173014145811
4/5/2020	4	12641.0	697.0	0.055138042876354716
4/6/2020	5	13341.0	702.0	0.052619743647402746
4/7/2020	6	14045.0	725.5	0.051655393378426485
4/8/2020	7	14792.0	701.0	0.04739048134126555
4/9/2020	8	15447.0	644.5	0.041723311969961804
4/10/2020	9	16081.0	579.5	0.03603631614949319
4/11/2020	10	16606.0	564.0	0.03396362760448031
4/12/2020	11	17209.0	575.0	0.03341274914289035
4/13/2020	12	17756.0	423.5	0.023851092588420816
4/14/2020	13	18056.0	476.0	0.026362428001772263
4/15/2020	14	18708.0	629.5	0.03364870643574941
4/16/2020	15	19315.0	647.0	0.033497281905254986
4/17/2020	16	20002.0	364.0	0.018198180181981802
4/18/2020	17	20043.0	225.5	0.011250810756872724
4/19/2020	18	20453.0	404.5	0.019777049821542072
4/20/2020	19	20852.0	414.5	0.019878189142528296
4/21/2020	20	21282.0	432.5	0.020322338126115967
4/22/2020	21	21717.0	437.5	0.020145508127273566
4/23/2020	22	22157.0	403.5	0.01821094913571332
4/24/2020	23	22524.0	372.5	0.016537915112768604
4/25/2020	24	22902.0	333.0	0.014540214828399267
4/26/2020	25	23190.0	309.5	0.013346269943941355
4/27/2020	26	23521.0	316.0	0.013434802942051783
4/28/2020	27	23822.0	377.0	0.015825707329359416
4/29/2020	28	24275.0	360.5	0.014850669412976313
4/30/2020	29	24543.0	268.0	0.010919610479566475