Environmental Policy and Economic Takeoff: What Underdeveloped Countries Can Learn from a Schumpeterian Model*

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Abstract

Economic development and environmental pollution have been inseparable companions throughout human history. Countries in their early development stage face a challenging trade-off between a better environment and economic growth. This study incorporates pollution and environmental policy into an endogenous growth framework to evaluate the impact of environmental policy in the form of emissions trading on the economic takeoff of an economy. We find that a stricter environmental policy with a lower cap for pollution permits delays industrialization and decreases the economic growth rate along the transition path, but it has no effect on long-run economic growth. Finally, we calibrate the model to current underdeveloped countries to examine whether the trade-off between environmental preservation and early industrialization is substantial for these countries. We find that due to their sufficiently high population growth, underdeveloped countries can safeguard the environment, notwithstanding a mere delay of a few years in the industrialization process.

Keywords: Schumpeterian growth; Emissions trading; Industrialization; Underdeveloped countries

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1 Introduction

Today, there exists an extensive body of literature on the relationship between environmental degradation, environmental policies, and economic growth. One reason for the proliferation of this literature over the past few decades is the multitude of alarming environmental issues we have been facing. Our world currently finds itself amidst the hottest period in our entire recorded history. Global warming is gradually killing the human species without proactive measures to stem its advance. However, climate change is solely one of many environmental challenges confronting us. Notably, the sight of urban skylines veiled in smog has become symbolic in developing countries these days. Such atmospheric degradation mirrors the environmental struggles experienced by developed countries during their own stages of industrialization.

Numerous developed and emerging economies are actively directing efforts and resources towards the goal of decarbonization. A prevalent strategy involves prioritizing clean and renewable energy sources, offering sustained economic growth without environmental degradation. This pursuit promises to break the trade-off between economic development and environmental preservation, offering a potential win-win scenario for these countries. However, for underdeveloped countries, it is a whole other story. Faced with limited options, they confront a daunting trade-off between economic growth and environmental preservation. Although a better environment is needed for the survival of the human species, it is hard for them to see why they should worry about the environment when they are just trying to survive day-to-day. In an effort to illuminate the impact of stringent environmental policies on industrialization delay in underdeveloped countries, which is less discussed in the literature, we integrate pollution and emissions trading system into a Schumpeterian growth model akin to the framework proposed by Peretto (2015).

In the comprehensive framework detailed by Peretto (2015), the model economy consists of three integral sectors: the production of a unique final good by firms operating under perfect competition, an intermediate good sector responsible for both the manufacture of intermediate goods and the execution of R&D activities, and a household sector, which provides the labor force essential for production. Extending this framework, our study introduces the environmental dimension to the model by adopting a cap-and-trade system. Pollution is a consequential by-product of final good production and imposes a cost on the environment. Under this emissions trading system, firms must internalize environmental externalities from their production process. This necessitates a choice between implementing pollution abatement measures and incurring the costs associated with additional pollution permits beyond a regulatory cap.

In Peretto (2015), the industrialization of the economy is characterized by two distinct innovation phases within the intermediate good sector once the economy takes off. The first industrialization era is characterized by variety-expanding innovations, where new intermediate good firms introduce novel products to the market. This variety expansion intensifies competition among monopolistic firms, laying the foundational dynamics for industrial growth. Progressing to the second era of industrialization, the economy features not only the continuation of variety expansion but also the advent of quality-improving innovations. Here, established firms invest in R&D to maintain or grow their market share by offering higher quality goods. The way industrial stages unfold in the model—beginning with a burst of new products, followed by improvements in quality and even more diverse goods—mirrors the historical progression observed since the Industrial Revolution. Initially, the Revolution introduced many new tools and techniques in production, significantly expanding the variety of goods. Over time, not only were these tools and techniques refined and improved, but we also witnessed the creation of entirely new tools, validating the model's assumption against the backdrop of economic history.

Considering the phases of industrialization discussed in Peretto (2015), it is clear that the underlying dynamics driving the economy through these industrial periods depend crucially on the market size for each intermediate good firm. The market size, fundamentally influenced by the interaction between population growth and the number of new firms, acts as the major determinant of the incentives for intermediate good firms to engage in R&D activities within the economy. The transition from pre-industrial stagnation to a vibrant industrial economy, characterized initially by variety expansion and subsequently by both product expansion and quality enhancement, underscores the critical role of market size. This variable decides the threshold moments leading firms to innovate, marking the transitions between the economic development phases. Over time, the economy converges to a balanced growth path (BGP) where innovation and economic growth rates remain constant. See figure 1.

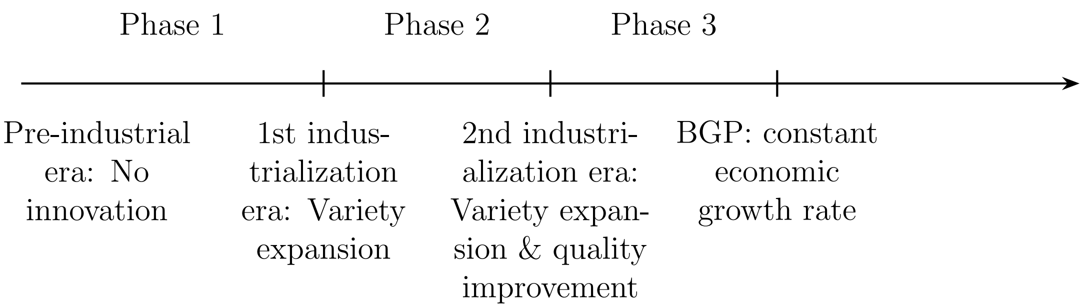

Figure 1: The dynamics of the model economy.

Our analysis reveals that implementing a more stringent environmental policy by lowering the cap for pollution permits delays the economy's transition from pre-industrial stagnation to innovation-driven growth. The policy also reduces the economic growth rate during the transition path; however, it does not affect the growth rate in the long run. The intuition behind our theoretical results can be explained as follows. A stricter environmental policy lowers the final output level, which in turn decreases the demand for intermediate goods and reduces the monopolistic profit for a given market size. As a result, fewer firms choose to enter the market. Therefore, the market size must be bigger to provide entrants with sufficient incentives to introduce new products. In other words, the economy enters the industrial era at a later time point. On the one hand, the reduced entry of firms slows economic growth for a given market size. On the other hand, with fewer new entrants into the market, leading to less intense market competition, incumbent firms experience an expansion in their market size. This expansion incentivizes incumbents to engage in quality-improving innovation, ultimately fostering economic growth. The two opposing effects cancel out each other in the long run, rendering the steady-state economic growth rate constant, irrespective of the environmental policy.

We calibrate our theoretical model to reflect the circumstances of the current least developed countries (LDCs). Our numerical exercise indicates that industrialization delays in these countries induced by simulated stringent environmental policies are surprisingly brief, spanning just a few years. This result is significant, especially considering the conventional belief that environmental regulation could substantially hinder economic progress in regions facing development challenges. In LDCs, which are typically characterized by high population growth rates, this rapid demographic expansion can, to some extent, mitigate industrialization delays attributed to stringent environmental policies and the delays are minimal. The basis for this phenomenon lies in the straightforward link between population growth and the expansion of market size. Simply put, a growing population speeds up market growth, leading to an earlier start of industrialization. The result of our numerical exercise offers an optimistic perspective for these countries, suggesting that environmental sustainability does not have to come at a significant cost to economic growth.

Our study relates to several strands of research. The first pertains to the relatively large literature on growth and innovation. A seminal study of this field is Romer (1990) which developed the first endogenous growth model with variety expansion. Other seminal papers are Aghion and Howitt (1992) and Grossman and Helpman (1991), which built the solid foundation for quality-ladder Schumpeterian growth models thereafter. In the literature, R&D on variety development is often referred to as horizontal innovation, while quality-improving R&D is vertical innovation. Both horizontal and vertical innovation growth models remain highly relevant and are frequently discussed in leading textbooks on economic growth (e.g., Acemoglu, 2009; Barro and Sala-i Martin, 2004).

Despite their popularity, neither model type is without limitations. They are prone to scale effects, whereby the economic growth rate increases proportionally with the number of working people or the number of R&D researchers. The skepticism towards scale effects is well-documented, notably in the work of Jones (1995), and further corroborated by more contemporary research, including that of Bloom et al. (2020) in which they provide solid time series evidence against scale effects. Specifically, despite a roughly exponential increase in the number of researchers in the U.S. since 1930, total factor productivity (TFP) growth has consistently declined. Furthermore, the quality-ladder models have faced another criticism. This type of model is based on the idea of creative destruction from Schumpeter (1942) in the sense that entrants with higher quality goods replace incumbents. However, Garcia-Macia et al. (2019) provides concrete evidence supporting creative accumulation, indicating that most innovation is conducted by incumbents, not entrants.

Given the limitations of earlier endogenous growth models, several authors including Peretto (1998), Young (1998), and Segerstrom (2000) developed the second-generation innovation-driven growth models that feature both types of R&D: variety expansion and quality improvement. Laincz and Peretto (2006), Ha and Howitt (2007), and Madsen (2008) provide empirical support for this model generation. We contribute to the literature by incorporating pollution flow and emissions trading system into a variant of this second-generation Schumpeterian model class to examine the impact of environmental policy on the transition from pre-industrial stagnation to innovation-based growth.

The second strand is the large theoretical literature on the link between environmental policies and economic growth. Ricci (2007) provides one of the comprehensive surveys of this literature. We focus on the branch of endogenous growth models here. There is no clear consensus in the literature since we have both pessimistic and optimistic schools. The former school emphasizes that a more ambitious environmental policy hurts economic growth (e.g., Gradus and Smulders, 1993; Grimaud, 1999). The latter school seems dominant, especially recently; see Afonso and Afonso (2015), Bianco and Salie (2017), Bovenberg and Smulders (1995), Grimaud and Rougé (2008), Hamaguchi (2021), Hart (2004), Hettich (1998), and Nakada (2004). They build various endogenous growth models and show that a stricter environmental policy, at least under appropriate conditions, can create a win-win situation with a better environment and enhanced economic growth. It is worth noting that papers in the literature analyze the impact of environmental policy on steady-state economic growth rate or on the transitional path between an economy's initial steady state and final steady state under the effects of environmental policy. Our paper studies the policy impact on the transitional path from pre-industrial economic stagnation to a modern R&D-based economy, which is rather less discussed in the literature.

Finally, our study relates to the literature on endogenous economic takeoff, that is, the transition to rapid economic growth originates from within the economy's dynamics, rather than being triggered by external factors. The first branch that develops unified growth theory includes seminal studies from Galor and Weil (2000) and Galor and Moav (2002). See, for example, Galor (2011) for a comprehensive review of unified growth theory. Unified growth theory illustrates how societies transition from focusing on having more children to investing in the education and well-being of fewer children. This shift from quantity to quality in child-rearing, alongside the accumulation of knowledge and skills, allows a society to break free from the limitations of early economic stagnation—where any growth is canceled out by a growing population—and start on a journey towards sustained economic development. There is also a recent branch of the Schumpeterian growth model in which economic takeoff is driven by innovations instead of human capital. This branch started from Peretto (2015) where the scale-free model features both horizontal and vertical innovations. Subsequent papers examine different mechanisms that affect endogenous economic takeoff in that Schumpeterian economy; see, for example, Iacopetta and Peretto (2021) on corporate governance, Chu et al. (2020) on intellectual property rights, Chu et al. (2022) on rent-seeking government, and Chu et al. (2023) on export demand. The contribution of our study is to incorporate pollution and environmental policy to examine the potential impact of policy stringency on economic takeoff in underdeveloped countries.

The rest of the paper is organized as follows. Section 2 presents the model and characterizes the equilibrium of the decentralized economy. Section 3 analyzes the dynamics of the model through the transition path and the balanced growth path of the economy. We then conduct a quantitative analysis in section 4. Section 5 discusses and concludes.

2 The model economy

We employ the Schumpeterian framework with endogenous economic takeoff in Peretto (2015). Our model economy begins in a pre-industrial era without innovations and gradually transitions into an industrial era with variety-expanding and quality-improving innovations.

To enrich the model, we integrate pollution and environmental policy, specifically through the mechanism of tradable pollution permits. The economic structure comprises three sectors: a final good sector, an intermediate good sector engaged in both production of intermediate goods and R&D activities, and a household sector. In this setup, firms producing the final good operate in a perfectly competitive market, combining household labor services and intermediate goods from monopolistic firms. This production process, however, generates pollution and imposes a cost on the environment. Consequently, the introduction of pollution permits compels firms to account for the environmental impact of their operations, facing a choice between purchasing permits for excess emissions or participating in costly abatement activities to reduce emissions.

Our exposition continues by presenting all components of the model's structure without addressing the era(s) to which they belong. We will discuss the model's dynamics in section 3, examining how innovation-driven growth occurs within this enriched Schumpeterian setting.

2.1 Final good sector

There is a unit mass of perfectly competitive firms $i \in [0, 1]$ that produce the unique final good by the following production function:

$$Y_t(i) = \int_0^{N_t} X_t^\theta(i,j)[Z_t^\alpha(j)Z_t^{1-\alpha}L_t(i)/N_t^{1-\sigma}]^{1-\theta}dj, \tag{1}$$

where $Y_t(i)$ is the output of the final good by firm i at time t. $X_t(i,j)$ is the quantity of intermediate good j used in production by firm i at time t and $Z_t(j)$ is the corresponding quality of good j at time t. N_t is the mass of intermediate goods at time t and we let $Z_t \equiv \frac{1}{N_t}\int_0^{N_t} Z_t(j)dj$ denote the aggregate quality or technology of all intermediate goods.[1] The presence of Z_t in the production function (1) captures the

[1] Like most endogenous growth models, the study has no physical capital. However, we can treat intermediate goods as capital that fully depreciates after use.

idea of technology spillover and the parameter $\alpha \in (0,1)$ decides the degree $1-\alpha$ of that spillover. Next, $L_t(i)$ is labor services hired by firm i at time t and parameter $1-\theta \in (0,1)$ captures the labor share of income. The parameter $\sigma \in (0,1]$ determines the degree of congestion (or rivalry) of the labor services across intermediate goods.[2] In the extreme case when $\sigma = 1$, there is no congestion and the labor services $L_t(i)$ can be shared by all intermediate goods without productivity loss. In this model, we treat the final good as the numeraire and all quantities are expressed in units of the final good.

Pollution is a by-product of the final good production process. We follow Acemoglu et al. (2012) to assume that pollution flow is linearly increasing in output level. Furthermore, pollution can be reduced by costly abatement expenditure (e.g., Gradus and Smulders, 1993). Combining these assumptions, pollution flow $E_t(i)$ at time t is produced by the following linear mechanism:

$$E_t(i) = \xi[Y_t(i) - D_t(i)], \tag{2}$$

with parameter $\xi > 0$ and $D_t(i) < Y_t(i)$ is the abatement expenditure by firm i at time t. Acemoglu et al. (2012) call ξ as the rate of environmental degradation from the production process.

The authorities must implement environmental policies to make final good firms internalize the negative externalities from the production process. This study considers cap-and-trade, that is, an emissions trading system. Each final good firm i is distributed permit quotas $\bar{E}Y_t(i)$ at time t. There is a competitive market for pollution permits where final good firms trade their distributed quotas. The unit price of the permits is denoted by $p_{E,t}$. Any firm i that emits excessively (i.e., $E_t(i) > \bar{E}Y_t(i)$) must purchase additional permits of $E_t(i) - \bar{E}Y_t(i)$ at the market price $p_{E,t}$. On the other hand, firms that emit less (i.e., $E_t(i) < \bar{E}Y_t(i)$) can sell unused permits of $\bar{E}Y_t(i) - E_t(i)$ to earn revenue. We call $\bar{E}$ as the policy parameter in the model. From the pollution function in eq. (2), it makes sense to set $\bar{E} < \xi$. If we had $\bar{E} \geq \xi$, the cap-and-trade would not impact final good firms' behavior since the emissions cap for each firm is no less than the pollution flow the firm emits.

Final good firm i chooses labor $L_t(i)$, quantity of intermediate goods $X_t(i,j)$, and abatement expenditure $D_t(i)$ to maximize the following profit $\pi_t(i)$ at each time t:

$$\pi_t(i) = Y_t(i) - w_t L_t(i) - \int_0^{N_t} p_t(j) X_t(i,j)dj - D_t(i) - p_{E,t}[E_t(i) - \bar{E}Y_t(i)], \tag{3}$$

where w_t denotes labor wage at time t and $p_t(j)$ refers to the price of intermediate good variety j at time t. The Appendix shows the detailed derivation of the firm's

[2]For a more detailed discussion on the congestion parameter, see Peretto (2015).

profit maximization problem. We obtain the following first-order conditions:

$$w_t = \left(\frac{\bar{E}}{\xi}\right)(1-\theta)\frac{Y_t(i)}{L_t(i)}, \tag{4}$$

$$X_t(i,j) = \left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)}\left(\frac{\theta}{p_t(j)}\right)^{1/(1-\theta)} Z_t^{\alpha}(j)Z_t^{1-\alpha}L_t(i)/N_t^{1-\sigma}, \tag{5}$$

$$p_{E,t} = \frac{1}{\xi}, \tag{6}$$

where eq. (4) shows the labor demand function and eq. (5) is the demand function for intermediate good $j \in [0, N_t]$ by firm i. The equilibrium permit price is shown in eq. (6).

Summing up eqs. (4)-(5) for each firm within the final good sector to derive aggregated equations:

$$w_t \int_0^1 L_t(i)di = \left(\frac{\bar{E}}{\xi}\right)(1-\theta)\int_0^1 Y_t(i)di,$$

$$\int_0^1 X_t(i,j)di = \left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)}\left(\frac{\theta}{p_t(j)}\right)^{1/(1-\theta)} Z_t^{\alpha}(j)Z_t^{1-\alpha}\int_0^1 L_t(i)di/N_t^{1-\sigma}.$$

Hence,

$$w_t = \left(\frac{\bar{E}}{\xi}\right)(1-\theta)\frac{Y_t}{L_t}, \tag{7}$$

$$X_t(j) = \left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)}\left(\frac{\theta}{p_t(j)}\right)^{1/(1-\theta)} Z_t^{\alpha}(j)Z_t^{1-\alpha}L_t/N_t^{1-\sigma}, \tag{8}$$

where $Y_t = \int_0^1 Y_t(i)di$, $L_t = \int_0^1 L_t(i)di$, and $X_t(j) = \int_0^1 X_t(i,j)di$ denote the total final output, the aggregate labor and the final good sector's demand for intermediate good j, respectively.

2.2 Intermediate good sector

2.2.1 Incumbents

A monopolistic firm $j \in [0, N_t]$ uses one unit of final good to produce one unit of intermediate good j and incurs a fixed operating cost $\phi Z_t^{\alpha}(j)Z_t^{1-\alpha}$ units of the final good, where $\phi > 0$ is the operating cost parameter.[3] The firm increases the quality of its product by investing in in-house R&D:

$$\dot{Z}_t(j) = I_t(j), \tag{9}$$

[3] We follow Peretto (2015) and use this cost function for simplicity where parameter α is defined in eq. (1). The more general form $\phi Z_t^{\varphi}(j)Z_t^{1-\varphi}$ would slightly change eq. (14) without changing the qualitative results.

where $I_t(j)$ is research outlays spent by firm j at time t. Note that this occurs only in the second era of industrialization where incumbents have sufficient incentives to improve their products' quality to raise demand.

The firm's gross profit flow (before R&D) at time t is the total revenue $p_t(j)X_t(j)$ less the sum of the variable production cost $X_t(j)$ and the fixed operating cost:

$$\begin{aligned}\Pi_t(j) &= p_t(j)X_t(j) - X_t(j) - \phi Z_t^{\alpha}(j)Z_t^{1-\alpha} \\ &= \left[\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)}(p_t(j)-1)\left(\frac{\theta}{p_t(j)}\right)^{1/(1-\theta)} L_t/N_t^{1-\sigma} - \phi\right] Z_t^{\alpha}(j)Z_t^{1-\alpha}.\end{aligned} \tag{10}$$

The value of this monopolistic firm is simply the sum of discounted dividend flow (i.e., net profit) over its lifetime:

$$V_t(j) = \int_t^{\infty} \exp\left(-\int_t^s r_u du\right)[\Pi_s(j) - I_s(j)]ds, \tag{11}$$

where r_t is the interest rate (i.e., return to saving) at time t.

We adopt the standard approach in the literature, focusing on a symmetric equilibrium where quality (i.e., technology level) is the same across the whole intermediate good sector $Z_t(j) = Z_t$ and the demand for each intermediate good firm is identical $X_t(j) = X_t$. Firm j maximizes its firm value in eq. (11) subject to eqs. (8)-(10). The Appendix shows the details of this maximization problem. The optimal monopoly price is

$$p_t(j) = p_t = 1/\theta. \tag{12}$$

Substituting this into eq. (8) yields the demand function for each intermediate good under symmetric equilibrium as follows:

$$X_t(j) = X_t = \left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)} \theta^{2/(1-\theta)} Z_t L_t/N_t^{1-\sigma}. \tag{13}$$

The maximization problem also leads to the following lemma:

Lemma 1. *The rate of return on in-house R&D, that is, on quality-improving innovation is*

$$r_t^Z = \alpha\frac{\Pi_t}{Z_t} = \alpha\left[\frac{1-\theta}{\theta}\frac{X_t}{Z_t} - \phi\right]. \tag{14}$$

Proof. See the Appendix.

2.2.2 Entrants

Once the economy enters the initial industrialization phase, entrepreneurs enter the market and develop new varieties of intermediate goods during both industrial eras. We follow Peretto (2007) and assume that a new firm is subject to a setup cost of

βX_t, where $\beta > 0$ is an entry-cost parameter, to start its operation.[4] New firms pay this initial cost by issuing equity. To ensure symmetry of the model, we assume that entrants at time t have access to the aggregate technology Z_t and enter the market at the technology level. When entry is positive (i.e., $\dot{N}_t > 0$), the entry condition is

$$V_t = \beta X_t. \tag{15}$$

The idea behind this entry condition is that once a new firm enters the market, it is expected to earn a total discounted value of V_t, which is the firm value in our symmetric equilibrium. Every entrepreneur can find his/her place in the market as long as he/she pays the entry cost of βX_t. Entrepreneurs keep making their entries until the expected net profit is zero.

We have the no-arbitrage condition (i.e., the Hamilton-Jacobi-Bellman equation)

$$r_t^N = \frac{\Pi_t - I_t}{V_t} + \frac{\dot{V}_t}{V_t}, \tag{16}$$

where we use r_t^N to denote the rate of return on entry, that is, on variety-expanding innovation. The right-hand side of eq. (16) is the return rate on investment in a firm which consists of net monopolistic profit $\Pi_t - I_t$ and capital gain $\dot{V}_t$. This return rate, which is the same across all intermediate good firms, must equal the saving rate r_t of the household in equilibrium since the household invests in all those firms. Moreover, when entry is positive, the rate on variety innovation r_t^N is also equal to the saving rate. Eq. (16) conveys this argument. We obtain the following lemma:

Lemma 2. *The rate of return on variety-expanding innovation is*

$$r_t^N = \frac{\frac{1-\theta}{\theta}\frac{X_t}{Z_t} - \phi - \frac{\dot{Z}_t}{Z_t}}{\beta \frac{X_t}{Z_t}} + \frac{\dot{X}_t}{X_t}. \tag{17}$$

Proof. See the Appendix.

2.3 Household

There is a representative household in the model. It has $L_t = L_0 e^{\lambda t} (L_0 \equiv 1)$ members in total at time t where $\lambda > 0$ is the population growth rate. Each member consumes the final good and suffers from pollution. The lifetime utility of the household is

$$U(0) = \int_0^\infty e^{-(\rho-\lambda)t} [\ln c_t - f(E_t)]\, dt, \tag{18}$$

[4] We see that the entry cost is proportional to the new firm's initial volume of output. Peretto (2007) argues that this assumption captures the idea that the entry cost depends on the amount of productive assets required to start production.

where $\rho > \lambda$ is the subjective discount rate. Denote $c_t \equiv C_t/L_t$ as the individual consumption of the final good where C_t is the total consumption. $E_t = \int_0^1 E_t(i)di$ is the total pollution flow at time t where $f(E_t) > 0, f'(E_t) > 0$. The second derivative $f''(E_t) > 0$ captures the idea of increasing marginal damage of pollution.[5] In this model, we assume that each member supplies a unit of labor inelastically and earns the wage of w_t. The household owns the total amount of assets A_t that earn the market interest of r_t.

The instantaneous change in assets is

$$\dot{A}_t = r_t A_t + w_t L_t - C_t$$

which says that the households' savings $\dot{A}_t$ are equal to the sum of investment interest $r_t A_t$ and labor income $w_t L_t$ less consumption C_t. We transform the equation to obtain the expression in per capita terms as follows:

$$\begin{aligned}
\dot{(a_t L_t)} &= r_t a_t L_t + w_t L_t - c_t L_t \\
\dot{a}_t L_t + a_t \dot{L}_t &= r_t a_t L_t + w_t L_t - c_t L_t \\
\frac{\dot{a}_t}{a_t} + \underbrace{\frac{\dot{L}_t}{L_t}}_{=\lambda} &= r_t + \frac{w_t - c_t}{a_t} \\
\dot{a}_t &= (r_t - \lambda)a_t + w_t - c_t, \qquad (19)
\end{aligned}$$

where $a_t \equiv A_t/L_t$ is the asset value of one individual member. Given the above budget constraint, the household chooses consumption to maximize utility. The Appendix shows the detailed derivation of the household's optimization problem.

We obtain the familiar intertemporal Euler equation which characterizes the optimal path of consumption growth for a given interest rate

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \qquad (20)$$

2.4 Aggregation

Under symmetric equilibrium, the demand for intermediate good j from final good firm i in eq. (5) can be rewritten as

$$X_t(i,j) = \left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)} \theta^{2/(1-\theta)} Z_t L_t(i)/N_t^{1-\sigma}, \qquad (21)$$

where we use the optimal monopoly price in eq. (12). Substituting this expression into the production function in eq. (1) and again imposing the symmetric equilibrium give

$$Y_t(i) = \left(\frac{\bar{E}}{\xi}\right)^{\theta/(1-\theta)} \theta^{2\theta/(1-\theta)} N_t^{\sigma} Z_t L_t(i), \qquad (22)$$

[5]We follow the conventional idea of convex damage function. See, for example, Carleton et al. (2022) and Deng and Mendelsohn (2021).

which we aggregate over the final good sector to yield the reduced-form expression for the aggregate output as follows:

$$\begin{aligned}\int_0^1 Y_t(i)di &= \left(\frac{\bar{E}}{\xi}\right)^{\theta/(1-\theta)} \theta^{2\theta/(1-\theta)} N_t^{\sigma} Z_t \int_0^1 L_t(i)di \\ Y_t &= \left(\frac{\bar{E}}{\xi}\right)^{\theta/(1-\theta)} \theta^{2\theta/(1-\theta)} N_t^{\sigma} Z_t L_t. \end{aligned} \tag{23}$$

Furthermore, we can employ eqs. (21) and (22) to show that final good firms earn zero profit. It is straightforward from eq. (4) that the wage payment of firm i equals to

$$w_t L_t(i) = (1-\theta)\frac{\bar{E}}{\xi} Y_t(i). \tag{24}$$

Next, we utilize eqs. (12) and (21)-(22) to show the total payment to intermediate good firms as a fraction of the final output:

$$\begin{aligned}\int_0^{N_t} p_t(j) X_t(i,j)dj &= N_t \frac{1}{\theta}\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)} \theta^{2/(1-\theta)} Z_t L_t(i)/N_t^{1-\sigma} \\ &= \theta\frac{\bar{E}}{\xi}\left(\frac{\bar{E}}{\xi}\right)^{\theta/(1-\theta)} \theta^{2\theta/(1-\theta)} N_t^{\sigma} Z_t L_t(i) \\ &= \theta\frac{\bar{E}}{\xi} Y_t(i). \end{aligned} \tag{25}$$

Substituting eqs. (2), (6) and (24)-(25) into (3) yields

$$\Pi_t(i) = \left(1 - \frac{\bar{E}}{\xi}\right) Y_t(i) - D_t(i) - \frac{1}{\xi}\left\{\xi[Y_t(i) - D_t(i)] - \bar{E} Y_t(i)\right\} = 0.$$

Hence, final good firms earn zero profit for any level of abatement expenditure $D_t(i)$. Moreover, if $D_t(i) < \left(1 - \frac{\bar{E}}{\xi}\right) Y_t(i)$, final good firm i must purchase additional permits. Firm i can sell unused permits if $D_t(i) > \left(1 - \frac{\bar{E}}{\xi}\right) Y_t(i)$.

2.5 Equilibrium

We define the decentralized equilibrium for the economy. The equilibrium consists of a time path of aggregate quantities $\{Y_t, A_t, C_t, X_t, I_t, D_t\}$ where aggregate abatement cost $D_t = \int_0^1 D_t(i)di$ and prices $\{r_t, w_t, p_t, V_t\}$ such that in each period

1. The representative household chooses C_t to maximize utility taking $\{r_t, w_t\}$ as given. The optimal consumption and labor follow the Euler eq. (20).

2. Final good firm $i \in [0,1]$ chooses $\{L_t(i), X_t(i,j), D_t(i)\}$ to maximize profit in eq. (3) taking $\{w_t, p_t(j), p_{E,t}\}$ as given. It earns zero profit under perfect competition.

3. The incumbent in the industry j chooses $\{X_t(j), p_t(j), I_t(j)\}$ to maximize the firm value in eq. (11) given the demand from the final good firms in eq. (8), taking r_t as given.

4. Entrants make entry decisions, taking V_t as given. They earn expected zero profit due to the entry condition in eq. (15).

5. The final good market clears such that the final output Y_t is used to finance consumption C_t, abatement expenditure D_t, variable production cost and fixed operating cost $N_t(X_t + \phi Z_t)$, and investment on innovations $N_t I_t + \dot{N}_t \beta X_t$ where the first term is investment in in-house R&D (i.e., quality improvement) and the second term shows investment in variety development:

$$Y_t = C_t + D_t + N_t(X_t + \phi Z_t + I_t) + \dot{N}_t \beta X_t. \tag{26}$$

6. The labor market clears such that labor demanded by final good firms is equal to the household's endowed labor:

$$L_t = \int_0^1 L_t(i) di. \tag{27}$$

7. The asset market clears such that the value of the household's assets is equal to the total value of all monopolistic firms:

$$A_t = N_t V_t. \tag{28}$$

8. The competitive market for pollution permits clears such that the aggregate pollution flow is equal to the total pollution permits in each period:

$$\int_0^1 E_t(i) di = E_t = \int_0^1 \bar{E} Y_t(i) di.$$

   Therefore, from eq. (2) we obtain the equilibrium aggregate abatement expenditure as a fraction of the final output:

$$\xi(Y_t - D_t) = \bar{E} Y_t \Leftrightarrow D_t = \left(1 - \frac{\bar{E}}{\xi}\right) Y_t. \tag{29}$$

Denote $g_t \equiv \dot{y}_t / y_t$ as the growth rate of per capita output $y_t \equiv Y_t / L_t$. Eq. (23) gives the reduced-form expression for output per capita as follows:

$$y_t \equiv \frac{Y_t}{L_t} = \left(\frac{\bar{E}}{\xi}\right)^{\theta/(1-\theta)} \theta^{2\theta/(1-\theta)} N_t^{\sigma} Z_t$$

which we take logs and time derivatives to obtain

$$g_t = \sigma n_t + z_t, \tag{30}$$

where $n_t \equiv \dot{N}_t / N_t$ is the variety growth rate and $z_t \equiv \dot{Z}_t / Z_t$ is the quality growth rate. Eq. (30) is the reason why Peretto (2015) calls σ as the social return of variety expansion.

We define a state variable χ_t as follows:

$$\chi_t \equiv \theta^{2/(1-\theta)} \frac{L_t}{N_t^{1-\sigma}}. \tag{31}$$

On one side, when the population size L_t increases, so does χ_t. Although monopolistic firms primarily supply to producers of final goods rather than directly to consumers, a larger population size means a higher demand for the final good, which in turn boosts the demand for intermediate goods. On the other side, an increase in the number of firms within the economy (reflected by a higher N_t) results in a reduction of χ_t, illustrating the diffusion of market power. Essentially, χ_t helps us understand the market size available to each firm that produces intermediate goods. This variable plays a crucial role in our model because it influences how and when the economy progresses through different stages, affecting how firms respond to changing market conditions. For clarity, we refer to variable χ_t simply as the market size hereafter.

Taking logs and time derivatives of eq. (31) yields

$$\frac{\dot{\chi}_t}{\chi_t} = \lambda - (1-\sigma)n_t \tag{32}$$

which demonstrates the relationship between the growth rates of the market size and the variety expansion.

Furthermore, we can express the quality-adjusted demand X_t/Z_t for each intermediate firm in terms of χ_t using eqs. (13) and (31):

$$\frac{X_t}{Z_t} = \left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)} \theta^{2/(1-\theta)} L_t/N_t^{1-\sigma} = \left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)} \chi_t. \tag{33}$$

In the absence of pollution permits cap, meaning $\frac{\bar{E}}{\xi} = 1$, there is a direct equivalence between the market size and the quality-adjusted demand. However, the situation changes significantly once the government introduces a cap on pollution permits. For a given market size χ_t, a more stringent policy (i.e., a reduced $\bar{E}$) decreases the (quality-adjusted) demand for each intermediate good. This happens because final good firms now face pollution-related costs, which in turn negatively affects demand and profits for each intermediate good firm.

Finally, from eqs. (14) and (17) the innovation returns can also be expressed using the variable χ_t as follows:

$$r_t^Z = \alpha \left[\frac{1-\theta}{\theta} \left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)} \chi_t - \phi\right], \tag{34}$$

$$r_t^N = \frac{1}{\beta} \left[\frac{1-\theta}{\theta} - \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} \frac{\phi + z_t}{\chi_t}\right] + z_t + \frac{\dot{\chi}_t}{\chi_t}. \tag{35}$$

3 Dynamics of the economy

We assume the economy starts in a pre-industrial period characterized by a specific number of product varieties N_0 (with the initial population normalized to $L_0 \equiv 1$). Similar to the original model in Peretto (2015), the dynamics of the economy depend on the magnitude of the market size χ_t. Once χ_t grows sufficiently large and surpasses a certain threshold, market entry and the introduction of new product varieties appeal to new entrants. This phase marks the activation of variety-expanding innovation. As the market size χ_t further expands, it eventually reaches a point where existing monopolistic firms have incentives to improve their products' quality to increase the demand for their products. That is when the quality-improving innovation kicks off. The economy converges gradually to a balanced growth path (BGP) afterward.

3.1 Pre-industrial era

Strictly speaking, the market-clearing condition in eq. (26) is for the industrial period. In the pre-industrial stage, the corresponding condition is

$$Y_t = C_t + D_t + N_0(X_t + \phi Z_0).$$

Note that the number of products N_t and quality Z_t before industrialization remain constant at N_0 and Z_0, respectively, since there is no innovation in this period. A constant number of existing monopolistic firms produce intermediate goods and earn profits.

Since the innovation growth rates are zero in this era (i.e., $n_t = z_t = 0$), we must have from eq. (30) that the growth rate of per capita output is also equal to zero, that is, output per capita remains constant in this period

$$g_t = \sigma n_t + z_t = 0. \tag{36}$$

Given the initial level $\chi_0 = \theta^{2/(1-\theta)} L_0 / N_0^{1-\sigma}$, the market size

$$\chi_t = \theta^{2/(1-\theta)} L_t / N_0^{1-\sigma}$$

which increases according to the dynamics

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{\dot{L}_t}{L_t} = \lambda. \tag{37}$$

We notice that the market size χ_t increases over time during the pre-industrial period solely due to population growth and is eventually large enough to activate innovation.

3.2 First phase of industrialization: Variety expansion

When the economy enters this industrial phase, variety-expanding innovation is activated as new firms enter the market and introduce new products. The entry condition in eq. (15) holds. We first prove that the consumption-output ratio is constant once innovation kicks off.

Lemma 3. *When entry is positive, the consumption-output ratio jumps to*

$$\frac{c_t}{y_t} = \frac{\bar{E}}{\xi}[1 - \theta + \beta\theta^2(\rho - \lambda)] \tag{38}$$

and remains constant thereafter.

Proof. See the Appendix.

In other words, consumption and output grow at the same rate from the outset of industrialization in this model. Therefore, from the Euler equation (20) we have

$$g_t = r_t - \rho. \tag{39}$$

Another worth-mentioning point from eq. (38) is that a stricter environmental policy in the form of a lower cap for pollution permits (i.e., a lower $\bar{E}$) decreases the household's consumption as a fraction of the final output. This is understandable due to the presence of higher abatement costs to final good firms.

Next, we derive the relationship between the variety growth rate n_t and the market size χ_t. In the first phase of the industrial era, the market rate of return on assets (i.e., the interest rate) is equal to the innovation returns on variety development, that is, $r_t = r_t^N$. Combining eqs. (30), (32), (35), and (39) yields the variety growth rate as a function of the market size as follows:

$$n_t = \frac{1}{\beta}\left[\frac{1-\theta}{\theta} - \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} \frac{\phi}{\chi_t}\right] - \rho + \lambda. \tag{40}$$

Note that to derive this expression, we take into account that the initial phase of industrialization does not feature quality-improving innovation, that is, $z_t = 0$. Eq. (40) shows that n_t is monotonically increasing in χ_t. Therefore, once the market size χ_t rises above a threshold, the variety innovation starts, that is, n_t goes above zero. We can solve for that threshold analytically, namely χ_N, from eq. (40) by setting $n_t = 0$

$$\chi_N = \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} \frac{\phi}{\frac{1-\theta}{\theta} - \beta(\rho - \lambda)}, \tag{41}$$

where we assume that parameter values satisfy the condition that $\chi_N > \chi_0$. We obtain the following proposition:

Proposition 1. *The first stage of the industrial era starts at time* $t_N = \ln(\chi_N/\chi_0)/\lambda$ *where*

$$\chi_N = \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} \frac{\phi}{\frac{1-\theta}{\theta} - \beta(\rho - \lambda)}.$$

During this stage, the market size evolves according to the dynamics

$$\dot{\chi}_t = \frac{1-\sigma}{\beta}\left\{\left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} \phi - \left[\frac{1-\theta}{\theta} - \beta\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right]\chi_t\right\} > 0. \tag{42}$$

Proof. See the Appendix.

A clear conclusion from this proposition is that implementing a stricter environmental policy, characterized by a reduced cap on pollution permits (i.e., a lower $\bar{E}$), leads to an increase in both the critical market size threshold χ_N and the corresponding time t_N at which this threshold is reached, thereby delaying the onset of the industrialization phase. This is because a more stringent environmental policy curtails the demand and profit achievable by monopolistic firms for a given market size χ_t, necessitating a larger market size to generate sufficient incentives for new firms to enter the market and introduce new products.

The per capita output growth rate can be pinned down using eq. (30):

$$g_t = \sigma n_t = \frac{\sigma}{\beta}\left[\frac{1-\theta}{\theta} - \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} \frac{\phi}{\chi_t}\right] - \sigma(\rho - \lambda) > 0,$$

where we again utilize the fact that quality-improving innovation is absent during this first phase of industrialization, that is, $z_t = 0$. As the market size χ_t expands, there is a corresponding gradual increase in the economic growth rate g_t. This relationship underscores the importance of market size in fueling the economy's expansion, as a larger market size leads to a higher rate of output growth through increased demand for intermediate goods.

However, the introduction of a lower cap on pollution permits significantly alters the dynamic, particularly affecting the intermediate good sector. By restricting the number of permissible emissions, the policy directly imposes additional environmental costs on final good firms, which in turn reduces demand and profit for intermediate good firms. The consequence of lowered profitability in the intermediate good sector is a reduction in the incentive for new firms to enter the market. This reduced firm entry has direct implications for the economic growth rate. Given a market size χ_t, a more stringent environmental policy adversely affects the growth rate of output g_t. The reason behind this outcome is that economic growth hinges exclusively upon the development of new products during the first phase of industrialization. The reduction in the flow of new entrants leads to a slowdown in economic growth during the first era of industrialization.

3.3 Second phase of industrialization: Variety expansion and quality improvement

As the market size χ_t continues growing, it eventually reaches another critical threshold. From this juncture, not only do entrants have incentives to expand the range of differentiated products, but incumbents also find the need to improve the quality of their goods to boost their demand. In this phase, all rates of returns are equal $r_t = r_t^N = r_t^Z$. Substituting eq. (34) into eq. (39) yields the economic growth rate g_t

as a function of the market size χ_t:

$$g_t = \alpha \left[\frac{1-\theta}{\theta} \left(\frac{\bar{E}}{\xi} \right)^{1/(1-\theta)} \chi_t - \phi \right] - \rho. \tag{43}$$

Like the first phase of industrialization, a larger market size χ_t speeds up the economic growth rate g_t. This happens because there is growing demand and profit for the intermediate good sector due to market expansion. Entrants find it more attractive to enter the market now, while established firms have higher incentives to make their products better and sell even more. As a result, the economy grows faster with more firms innovating and improving. Additionally, for a given market size χ_t, a more restrictive environmental policy lowers demand and profitability for intermediate good firms. This stricter policy renders R&D activities less rewarding, thereby reducing the output growth rate.

In this industrialization phase, quality-improving innovation is activated, that is, $z_t > 0$. We employ eq. (30) to obtain

$$z_t = g_t - \sigma n_t = \alpha \left[\frac{1-\theta}{\theta} \left(\frac{\bar{E}}{\xi} \right)^{1/(1-\theta)} \chi_t - \phi \right] - \rho - \sigma n_t. \tag{44}$$

Combining eqs. (30), (32), (35), and (39) yields

$$n_t = \frac{1}{\beta} \left[\frac{1-\theta}{\theta} - \left(\frac{\bar{E}}{\xi} \right)^{-1/(1-\theta)} \frac{\phi + z_t}{\chi_t} \right] - \rho + \lambda. \tag{45}$$

We obtain the following proposition:

Proposition 2. *The second phase of the industrial era is activated at time $t_Z > t_N$ when the market size rises above the threshold $\chi_Z > \chi_N$*

$$\chi_Z = \frac{\phi\alpha + \frac{1-\theta}{\theta}\frac{\sigma}{\beta} + (1-\sigma)(\rho-\lambda) + \lambda + \sqrt{\left[\phi\alpha + \frac{1-\theta}{\theta}\frac{\sigma}{\beta} + (1-\sigma)(\rho-\lambda) + \lambda\right]^2 - 4\phi\alpha\frac{1-\theta}{\theta}\frac{\sigma}{\beta}}}{2\alpha\frac{1-\theta}{\theta}\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)}}. \tag{46}$$

During this phase, the market size evolves according to the dynamics

$$\dot{\chi}_t = \frac{1-\sigma}{\beta - \sigma\left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} / \chi_t} \left\{ \left[(1-\alpha)\phi - \left(\rho + \frac{\sigma\lambda}{1-\sigma} \right) \right] \left(\frac{\bar{E}}{\xi} \right)^{-1/(1-\theta)} - \left[(1-\alpha)\frac{1-\theta}{\theta} - \beta\left(\rho + \frac{\sigma\lambda}{1-\sigma} \right) \right] \chi_t \right\}. \tag{47}$$

Proof. See the Appendix.

We assume that parameters satisfy the condition that $\chi_Z > \chi_N$. Similar to phase one of industrialization, a stricter environmental policy in the form of a lower $\bar{E}$

increases the value of the threshold χ_Z, hence delaying the activation of phase two. The stricter policy lowers the demand and profit achievable by monopolistic firms for a given market size χ_t, thus undermining the incumbents' incentives to improve their products' quality. The market size must be bigger to incentivize existing firms to innovate.

We also obtain an interesting result in the following lemma:

Lemma 4. *The duration of the first phase of industrialization is independent of the stringency of environmental policy.*

Proof. See the Appendix.

This lemma simply says that a stricter environmental policy delays both industrialization phases by increasing the values of time points t_N and t_Z by the same amount. Consequently, the duration between t_N and t_Z remains constant. In other words, the economy stays for a fixed duration in the first industrial phase, irrespective of the stringency of environmental policy. This lemma is helpful later in the quantitative analysis when we consider economies that are already industrialized to various degrees.

3.4 Balanced growth path (BGP)

During the second phase of the industrial era, from eq. (47), the market size χ_t asymptotically converges to a steady-state value χ^* in the long run where

$$\chi^* = \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} \frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(1-\theta)/\theta - \beta[\rho + \sigma\lambda/(1-\sigma)]} > \chi_Z, \tag{48}$$

We can draw the dynamics of the market size χ_t through different phases of the economy as in figure 2. The figure shows how χ_t evolves from the initial state χ_0 to the steady state χ^*.

We substitute eq. (48) into eq. (43) to pin down the steady-state economic growth rate g^* as follows:

$$g^* = \alpha \left[\frac{1-\theta}{\theta} \frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(1-\theta)/\theta - \beta[\rho + \sigma\lambda/(1-\sigma)]} - \phi\right] - \rho > 0. \tag{49}$$

The environmental policy in the form of a cap for pollution permits does not impact the steady-state value of the economic growth rate. Specifically, as we have discussed above, a more stringent environmental policy reduces the overall production of the final good, which then diminishes the need for intermediate goods and reduces profits for monopolistic firms for a given market size χ_t. Consequently, this deters new firms from entering the market. This results in a negative direct effect on the economic growth rate for a given market size. On the other hand, the reduced entry of new firms leads to a higher market size of incumbent firms, which positively affects quality

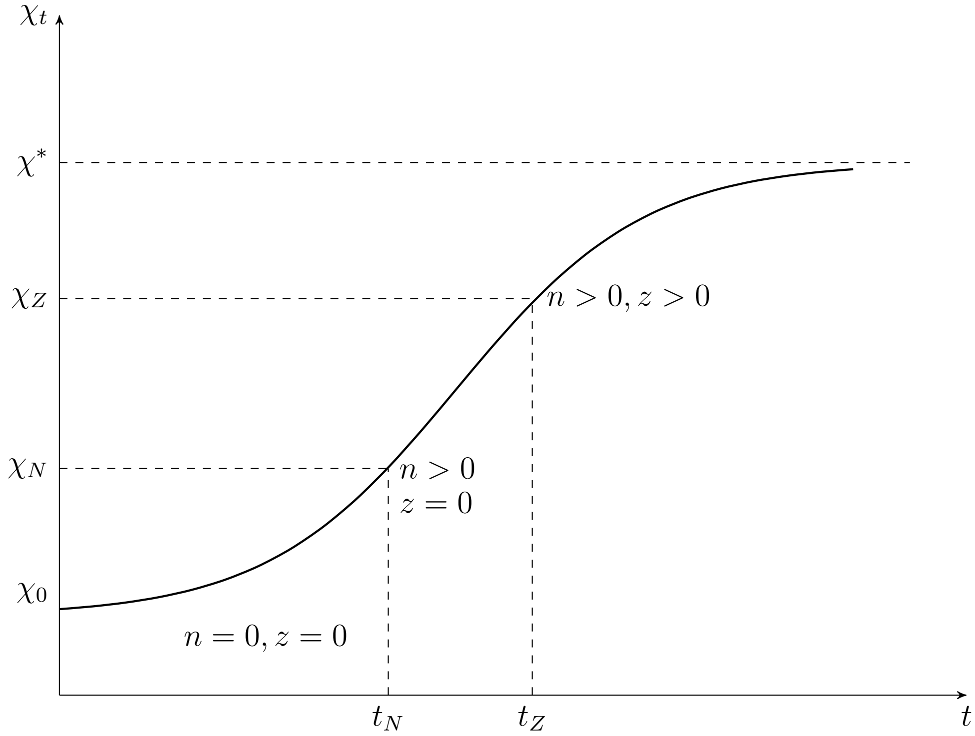

Figure 2: The dynamics of the market size χ_t. *Source: Adjusted from Peretto (2015)*

innovation and economic growth. The two opposite effects cancel out each other in the long run in eq. (43), resulting in a policy-independent economic growth rate.

Figure 3 demonstrates the dynamics of economic growth rate g_t for different stringency levels of environmental policy and shows that although a stricter environmental policy leads to a later economic takeoff, the steady-state economic growth rate remains unchanged.

Finally, we can derive the innovation growth rates on the BGP. Since the market size remains at the constant value χ^* along the BGP, we must have $\dot{\chi}_t = 0$ after it hits the steady-state value. Therefore, from eq. (32) we can obtain the steady-state variety growth rate as follows:

$$\lambda - (1-\sigma)n^* = 0 \Leftrightarrow n^* = \frac{\lambda}{1-\sigma}. \tag{50}$$

Combining eqs. (30), (49), and (50) yields the steady-state quality growth rate:

$$z^* = g^* - \sigma n^* = \alpha \left[\frac{1-\theta}{\theta} \frac{(1-\alpha)\phi - [\rho + \sigma\lambda/(1-\sigma)]}{(1-\alpha)(1-\theta)/\theta - \beta[\rho + \sigma\lambda/(1-\sigma)]} - \phi \right] - \frac{\lambda\sigma}{1-\sigma} - \rho. \tag{51}$$

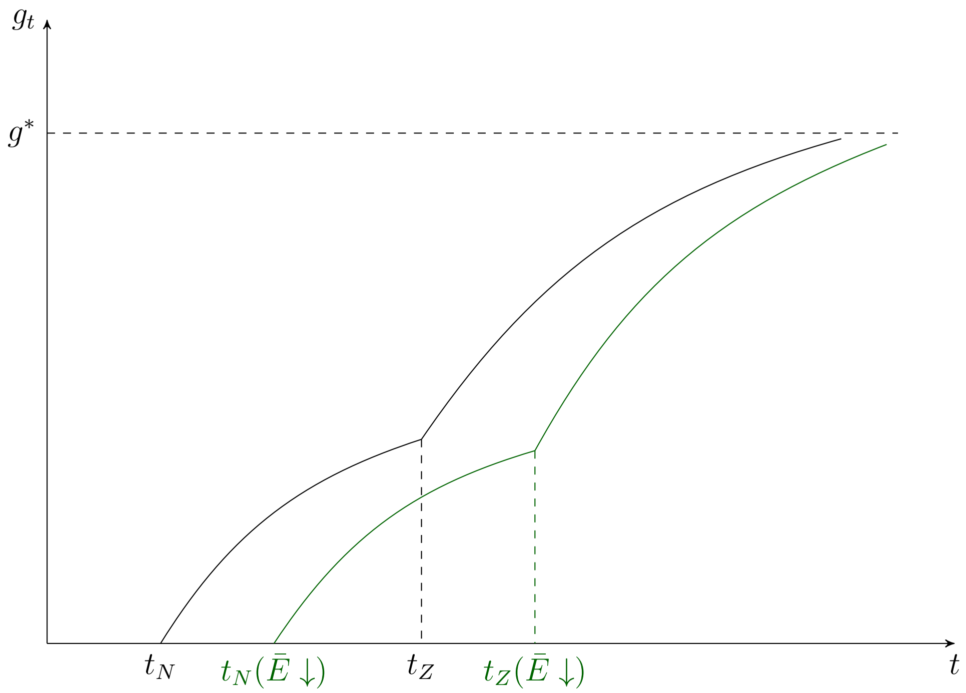

Figure 3: The dynamics of the economic growth rate g_t.

4 Quantitative analysis

We calibrate the model to quantitatively measure the potential effects of environmental policies in the form of a cap for pollution permits on the timeline of economic development in underdeveloped countries. We use the least developed countries (LDCs) classified by the United Nations (UN) to demonstrate our quantitative analysis of underdeveloped countries. The Appendix shows an exhaustive list of those 45 countries. According to UN (2023), LDCs are countries that fail to meet three criteria for income, human assets, and economic and environmental vulnerability. To graduate from the LDC list, a country must meet the graduation thresholds of at least two of the criteria for two consecutive triennial reviews.[6]

In our analysis, we focus primarily on income and economic growth within these countries. It is important to note that almost all LDCs are already industrialized to various degrees. However, we can roughly separate the LDC list into two groups: the first one with very low income and low GDP per capita growth and the second one with a relatively dynamic economy. Countries like Burundi and Sierra Leone exemplify the former group, displaying characteristics of pre-industrial economies, while the latter including countries such as Bangladesh and Cambodia can be classified as countries in the first industrialization era. We will perform our exercise for the first group to see how much later their transition from pre-industrial stagna-

[6]Exceptionally, countries that have per capita income per year constantly higher than the income threshold of \$1306, set at three times the graduation threshold, can be eligible for graduation, even though they fail to meet the other two criteria. Some countries, such as Bangladesh and Laos, are expected to leave the LDC list soon.

tion into a modern industry economy would occur if their government implemented a stricter environmental policy. The corresponding question for the second group is how much longer their economy would stay in the first industrialization phase before experiencing integrated innovation-driven growth with both variety-expanding and quality-improving innovations.[7]

Lemma 4 proves invaluable for our quantitative exercise. It establishes that the duration of the first industrialization era - specifically, the period between t_Z and t_N - remains constant and unaffected by changes in the stringency of the environmental policy. This allows us to modify our approach when examining the second group of countries. The task of answering the abovementioned question for the second group of countries might be algebra intensive due to the complex formula of the threshold χ_Z in eq. (46). Instead of determining how much longer these countries remain in the first industrial era, we can simplify the process by assessing how late they would transition to this era as if they were still in the pre-industrial stage. In other words, we apply the same analysis framework used for the first group of countries. The common question for all LDCs now is how much later the transition to the next industrialization phase would occur because of a stricter environmental policy.

The key equation for this exercise is eq. (41). Suppose the initial cap for pollution permits is $\bar{E}_1$. We want to investigate if the authorities implemented a stricter environmental policy in the form of $\bar{E}_2 < \bar{E}_1$, how would this new policy affect the starting point of industrialization, that is, the time point $t_N = \ln(\chi_N/\chi_0)/\lambda$ where χ_N is determined in eq. (41). Note that by the way we define the market size χ_t in eq. (31), the initial level χ_0 is independent of the environmental policy.[8]

We assume that a stricter environmental policy delays industrialization by Δt_N years. Using eq. (41) yields

$$\begin{aligned}\Delta t_N = t_N(\bar{E}_2) - t_N(\bar{E}_1) &= \frac{1}{\lambda}\ln\left[\frac{\chi_N(\bar{E}_2)}{\chi_N(\bar{E}_1)}\right] \\ &= \frac{1}{\lambda}\ln\left[\frac{(\bar{E}_2/\xi)^{-1/(1-\theta)}}{(\bar{E}_1/\xi)^{-1/(1-\theta)}}\right]. \qquad (52)\end{aligned}$$

For the calculation of time delay Δt_N in eq. (52), we only need to calibrate 5 parameters: $\lambda, \xi, \bar{E}_1, \bar{E}_2$ and θ. First, $1-\theta$ is the labor share of income and is easiest to calibrate. We follow the standard literature to choose a reasonable value $1-\theta = 0.6$.

Regarding the population growth parameter λ, table 1 in the Appendix shows how high the population growth rate is in LDCs. Unlike the world in the pre-industrial era in the 18th century, current underdeveloped countries are characterized by relatively high population growth rates due to scientific advancement and better medical systems compared to more than 200 years ago. Myanmar had the lowest growth rate of 0.71% among those countries in 2022 while Niger witnessed a rapid rate of 3.71% in

[7] This is the reason why emerging countries fall outside the scope of our quantitative analysis because their economies have already experienced integrated innovation-driven growth.

[8] According to eq. (31), the level of χ_0 depends only on the initial level of population L_0 and initial goods variety N_0 which are predetermined in the model.

the same year. The median population growth rate of 45 countries was 2.33% which belonged to the country of Togo.

The challenge lies in the realistic values of parameters ξ and $\bar{E}$. We can rely on the model setup to overcome this challenge. Eq. (52) says that we do not have to calibrate $\bar{E}$ and ξ separately since the expression for time delay depends only on their ratio. Let us assume that authorities and people do not care about the environment in the baseline calibration, which is somewhat true. In other words, we set $\bar{E}_1/\xi = 1$ and the aggregate abatement expenditure $D_t = 0$ in this baseline calibration.

Finally, to calibrate the ratio $\bar{E}_2/\xi$, we utilize the fact that in this model, the aggregate pollution flow is equal to the total pollution permits in each period. From eq. (29) we have

$$\frac{D_t}{Y_t} = 1 - \frac{\bar{E}}{\xi}.$$

That is, there exists a one-to-one relationship between $\bar{E}/\xi$ and D_t/Y_t. According to Eurostat (2024), the average EU national expenditure on environmental protection over the period 2018-2022 was around 2% of GDP.[9] Based on this real-life figure, we allow the abatement expenditure-output ratio D_t/Y_t to be in the range $[0.02, 0.05]$ to observe the impact of different policy regimes. The corresponding range for the ratio $\bar{E}_2/\xi$ is $[0.95, 0.98]$. We will now examine the impact of more stringent environmental policies with $\bar{E}_2/\xi \in [0.95, 0.98]$ in the following figure 4. Despite the parameter $\bar{E}_2$ showing the stringency of an environmental policy, the ratio $\bar{E}_2/\xi$ does not really have a tangible economic meaning. We use the abatement expenditure-output ratio D_t/Y_t in the figure instead of the ratio $\bar{E}_2/\xi$ for better interpretation.

The worth-mentioning point is that the population growth rate plays an important role in this quantitative analysis. The reason is that the market size χ_t relies on the population size. A higher population growth rate leads to faster market expansion, which in turn results in an earlier industrialization. Therefore, for a given environmental policy, countries with higher population growth rates would suffer shorter industrialization delays.

Figure 4 shows that the environmental costs for today's underdeveloped countries are not extremely pronounced. For an environmental policy that achieves the EU level of environmental expenditure at 2%, LDCs need to wait 1-5 years more to experience the next phase of economic development. If their government implemented a much more stringent policy that results in 5% of output devoted to abatement expenditure, the time delay would be 3-12 years with Myanmar suffering the longest delay. Given these figures, we can conclude that LDCs somewhat have the capacity to safeguard the environment while enduring a moderate delay in industrialization for a few years.

[9] Note that this figure uses GDP, not total final output as in our model. However, for simplicity, we ignore their difference.

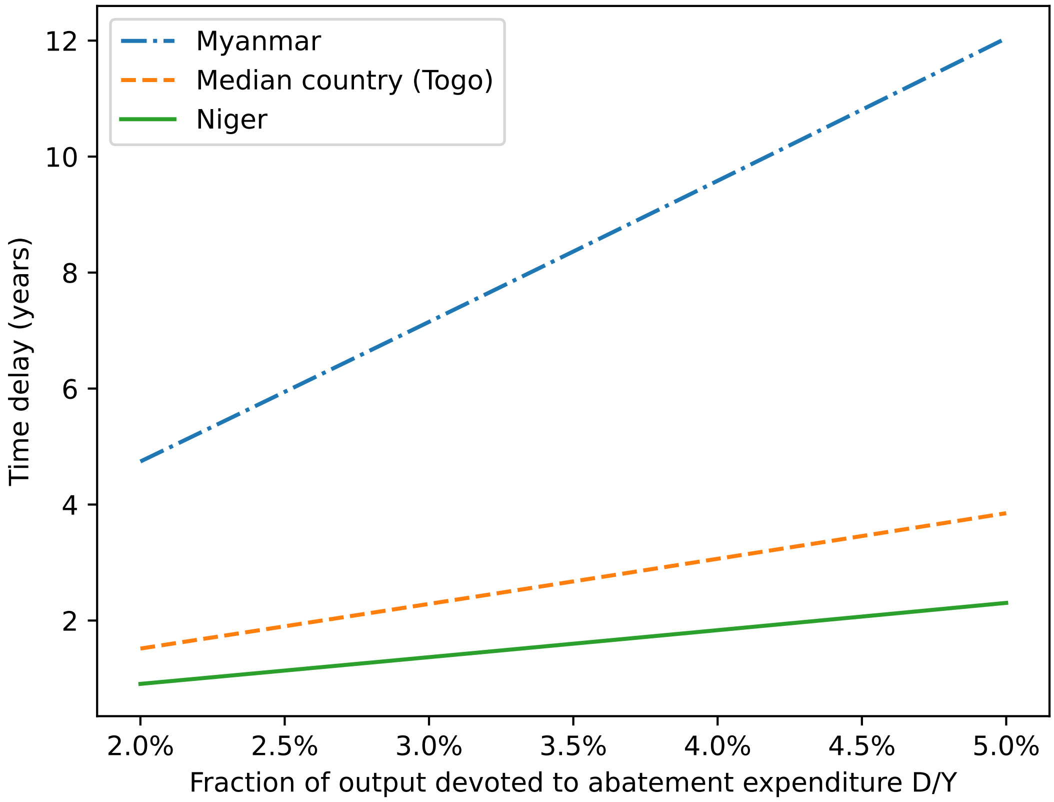

Figure 4: Industrialization delays in underdeveloped countries. *Source: Author's calculation*

5 Discussion

In this paper, we expand the Schumpeterian growth model in Peretto (2015) with endogenous economic takeoff by incorporating an environmental policy in the form of a cap-and-trade system. We assess the impact of the environmental policy in order to provide valuable information for underdeveloped countries, particularly the current least developed countries (LDCs). A significant amount of theoretical research has investigated the relationship between environmental policies and economic growth. The academic consensus remains divided. Pessimistic perspectives highlight the trade-off between economic growth and environmental protection. Conversely, a more optimistic view, supported by other researchers, suggests that stringent environmental policies, under certain conditions, can simultaneously environmental quality and economic growth, leading to a win-win scenario.

We contribute to the literature by focusing on a less explored aspect of the literature - the impact of environmental policies on the transition between economic development phases. We show that a more stringent environmental policy in the form of a lower cap for pollution permits delays industrialization and lowers economic growth along the transition path. However, a stricter policy does not affect the long-term rate of economic growth due to the endogenous market structure of the model. Specifically, the interaction between the market size, firm entry, and incentives for profit-driven innovations results in a policy-neutral economic growth rate in the long run. This result would be different if we employed a first-generation Schumpeterian growth model, which features either horizontal or vertical innovation only. In such models, a stricter environmental policy lowers demand for intermediate goods and consequently incentives to innovate. It is likely that there is a trade-off between a better environment and higher economic growth. The second-generation Schumpeterian model employed in this paper yields a different outcome due to the endogenous market structure we have previously discussed.

Our work contributes to the ongoing discussion about how to achieve economic development goals while addressing environmental issues. The quantitative analysis suggests that even with strict environmental policies, the delay in the industrialization process in underdeveloped countries is minimal, especially with countries experiencing strong population growth. This insight is encouraging for LDCs, indicating that environmental protection and economic development can go hand in hand, with only a slight delay in reaching their next industrialization phase.

While this paper provides valuable insights, it is important to recognize the limitations of our study, most notably its consideration of a closed economy framework. The world today is not like a few centuries ago when almost all countries relied exclusively on their autarkic economies. The model in this study does not capture how international trade and labor movements could influence the relationship between environmental policies and economic growth. Recognizing this limitation opens up new avenues for research. Future studies could look into how these external factors play into the environmental policy-economic growth relationship, offering a more comprehensive view of the challenges and opportunities for LDCs in achieving sustainable development.

Another promising direction for future research would be to consider the impact on economic development if underdeveloped countries had more resources to finance low-carbon innovation. These countries could access foreign aid from developed countries or international financial institutions in exchange for commitments to environmental protection. Another scenario is that once their economies reach the integrated innovation-driven growth era when there are both horizontal and vertical innovations, they can tighten the environmental policy and earn revenues from selling pollution allowances. One way to model low-carbon innovation is to integrate elements from existing models with clean-dirty sectors (e.g., Acemoglu et al., 2012) into the current framework. An alternative would be to modify the pollution function in eq. (2) and allow the aggregate technology to enter the function. In other words, we assume that better technology means cleaner technology. By adopting either approach, we envision that a stricter environmental policy in the form of a lower cap for pollution permits could allocate resources to innovations that lead to lower emissions and potentially enhance economic growth in the long run, unlike the policy-neutral scenario in the main text. This is beyond the scope of this study and is left for future research.

A Derivations

A.1 Final good sector

Final good firm i chooses labor $L_t(i)$, quantity of intermediate goods $X_t(i,j)$, and abatement expenditure $D_t(i)$ to maximize its profit $\pi_t(i)$ at each time t

$$\pi_t(i) = Y_t(i) - w_t L_t(i) - \int_0^{N_t} p_t(j) X_t(i,j) dj - D_t(i) - p_{E,t}[E_t(i) - \bar{E} Y_t(i)], \quad \text{(A.1)}$$

where w_t denotes labor wage at time t and $p_t(j)$ refers to the price of intermediate good variety j at time t. The first-order conditions of profit optimization are straightforward

$$(1-\theta)\frac{Y_t(i)}{L_t(i)} = w_t + p_{E,t}(\xi - \bar{E})(1-\theta)\frac{Y_t(i)}{L_t(i)}, \quad \text{(A.2)}$$

$$\theta\left(\frac{Z_t^{\alpha}(j) Z_t^{1-\alpha} L_t(i)}{X_t(i,j) N_t^{1-\sigma}}\right)^{1-\theta} = p_t(j) + p_{E,t}(\xi - \bar{E})\theta\left(\frac{Z_t^{\alpha}(j) Z_t^{1-\alpha} L_t(i)}{X_t(i,j) N_t^{1-\sigma}}\right)^{1-\theta}, \quad \text{(A.3)}$$

$$\xi p_{E,t} \leq 1 \text{ and } [\xi p_{E,t} - 1] D_t(i) = 0. \quad \text{(A.4)}$$

Eqs. (A.2)-(A.3) state that firms employ labor and intermediate goods until their marginal products are equal to their costs. The second term on the right-hand side of eqs. (A.2)-(A.3) captures the (net) marginal environmental cost of an additional unit of labor and intermediate goods, respectively.

Regarding abatement expenditure, eq. (A.4) says that if final good firms devote a positive amount of resources to abatement activity (i.e., $D_t(i) > 0$), the optimality condition is that the cost of one additional expenditure unit (which is equal to one) equals its marginal environmental benefit (which is equal to $\xi p_{E,t}$). In this case, we have the unit price of permits is constant, that is, $p_{E,t} = 1/\xi$ and final good firms are indifferent between different levels of abatement expenditure.

If $D_t(i) = 0$ for all final good firms i, the permit price must satisfy $p_{E,t} < 1/\xi$. In that case, no trade indeed occurs since devoting resources to abatement activities is not beneficial and every final good firm would be happy to purchase additional permits instead. The government can always intervene and set the permit price at $p_{E,t} = 1/\xi + \varepsilon$ where $\varepsilon > 0$. The intervention would encourage final good firms to engage in abatement activities. The intervention ends quickly and the free market mechanism will bring the permit price back to $p_{E,t} = 1/\xi$. At this price, trade does occur and firms are indifferent between different levels of abatement expenditure. Therefore, we can safely say that the equilibrium permit price is $p_{E,t} = 1/\xi$ and at least some final good firms trade their distributed permits.

Substituting $p_{E,t} = 1/\xi$ into eqs. (A.2)-(A.3) yields

$$(1-\theta)\frac{Y_t(i)}{L_t(i)} = w_t + \left(1 - \frac{\bar{E}}{\xi}\right)(1-\theta)\frac{Y_t(i)}{L_t(i)},$$
$$\theta\left(\frac{Z_t^{\alpha}(j)Z_t^{1-\alpha}L_t(i)}{X_t(i,j)N_t^{1-\sigma}}\right)^{1-\theta} = p_t(j) + \left(1 - \frac{\bar{E}}{\xi}\right)\theta\left(\frac{Z_t^{\alpha}(j)Z_t^{1-\alpha}L_t(i)}{X_t(i,j)N_t^{1-\sigma}}\right)^{1-\theta}.$$

Rearranging to obtain the following equations:

$$w_t = \left(\frac{\bar{E}}{\xi}\right)(1-\theta)\frac{Y_t(i)}{L_t(i)}, \tag{A.5}$$
$$X_t(i,j) = \left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)}\left(\frac{\theta}{p_t(j)}\right)^{1/(1-\theta)} Z_t^{\alpha}(j)Z_t^{1-\alpha}L_t(i)/N_t^{1-\sigma}. \tag{A.6}$$

A.2 Intermediate good sector

A.2.1 Incumbents

Proof of Lemma 1

We set up the following current-value Hamiltonian for the incumbents' optimization problem:

$$\mathscr{H}_t(j) = \Pi_t(j) - I_t(j) + \mu_t(j)\dot{Z}_t(j),$$

where $\mu_t(j)$ denotes the Hamiltonian costate variable. The first-order conditions are as follows:

$$\frac{\partial \mathscr{H}_t(j)}{\partial I_t(j)} = -1 + \mu_t(j) = 0 \Leftrightarrow \mu_t(j) = 1, \tag{A.7}$$
$$\frac{\partial \mathscr{H}_t(j)}{\partial p_t(j)} = 0, \tag{A.8}$$
$$\frac{\partial \mathscr{H}_t(j)}{\partial Z_t(j)} = \alpha\frac{\Pi_t}{Z_t} = r_t^Z\mu_t(j) - \dot{\mu}_t(j). \tag{A.9}$$

We substitute eq. (10) into eq. (A.8) and remove irrelevant factors to obtain

$$-\frac{1}{1-\theta}(p_t(j)-1)p_t(j)^{1/(1-\theta)-1} + p_t(j)^{1/(1-\theta)} = 0$$
$$p_t(j) = \frac{1}{\theta}.$$

Substituting $p_t(j) = 0, \mu_t(j) = 1$, and eq. (8) into eq. (A.9) and imposing symmetry yields

$$r_t^Z = \alpha\frac{\Pi_t}{Z_t} = \alpha\left[\frac{1-\theta}{\theta}\frac{X_t}{Z_t} - \phi\right].$$

A.2.2 Entrants

Proof of Lemma 2

Imposing the symmetric equilibrium, eqs. (9) and (10) become

$$\dot{Z}_t = I_t, \tag{A.10}$$

$$\Pi_t = (p_t - 1)X_t - \phi Z_t = \frac{1-\theta}{\theta} X_t - \phi Z_t. \tag{A.11}$$

where the second equation employs the optimal price in eq. (12).

Taking time derivatives of eq. (15) reads

$$\dot{V}_t = \beta \dot{X}_t. \tag{A.12}$$

We substitute eqs. (15), (A.10)-(A.12) into the no-arbitrage condition in eq. (16) to yield

$$\begin{aligned} r_t^N &= \frac{\Pi_t - I_t}{V_t} + \frac{\dot{V}_t}{V_t} \\ &= \frac{\frac{1-\theta}{\theta} X_t - \phi Z_t - \dot{Z}_t}{\beta X_t} + \frac{\beta \dot{X}_t}{\beta X_t} \\ &= \frac{\frac{1-\theta}{\theta}\frac{X_t}{Z_t} - \phi - \frac{\dot{Z}_t}{Z_t}}{\beta \frac{X_t}{Z_t}} + \frac{\dot{X}_t}{X_t}. \end{aligned}$$

A.3 Household

The representative household chooses consumption to maximize utility subject to the flow budget constraint. It does not have control over pollution despite the direct negative impact of pollution flow on its utility.

We set up the following current-value Hamiltonian for the household's optimization problem:

$$\mathscr{H}_{hh,t} = \ln c_t - f(E_t) + \eta_t[(r_t - \lambda)a_t + w_t - c_t],$$

where η_t denotes the Hamiltonian costate variable. The first-order conditions are as follows:

$$\frac{\partial \mathscr{H}_{hh,t}}{\partial c_t} = \frac{1}{c_t} - \eta_t = 0, \tag{A.13}$$

$$\frac{\partial \mathscr{H}_{hh,t}}{\partial a_t} = (r_t - \lambda)\eta_t = -\dot{\eta}_t + (\rho - \lambda)\eta_t. \tag{A.14}$$

Taking logs and time derivatives of eq. (A.13) yield

$$\frac{\dot{c}_t}{c_t} = -\frac{\dot{\eta}_t}{\eta_t}. \tag{A.15}$$

From eq. (A.14)

$$-\frac{\dot{\eta}_t}{\eta_t} = r_t - \rho. \tag{A.16}$$

We combine eqs. (A.15) and (A.16) to obtain the intertemporal condition, that is, the Euler equation

$$\frac{\dot{c}_t}{c_t} = r_t - \rho.$$

A.4 Consumption-output ratio

Proof of Lemma 3

Using the entry condition (15) to derive

$$a_t = \frac{V_t N_t}{L_t} = \frac{\beta X_t N_t}{L_t} = \beta\theta^2 \frac{\bar{E}}{\xi} y_t, \tag{A.17}$$

where the last equality makes use of eqs. (13) and (23). Taking time derivatives gives

$$\beta\theta^2 \frac{\bar{E}}{\xi} \dot{y}_t = \dot{a}_t = (r_t - \lambda)a_t + w_t - c_t, \tag{A.18}$$

which comes from eq. (19). We then use eqs. (7), (20), and (A.17) to rearrange eq. (A.18) as follows:

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{y}_t}{y_t} = \frac{\xi}{\beta\theta^2 \bar{E}} \frac{c_t}{y_t} - \left(\frac{1-\theta}{\beta\theta^2} + \rho - \lambda \right). \tag{A.19}$$

If $\dfrac{c_t}{y_t} > \dfrac{\bar{E}}{\xi}[1-\theta+\beta\theta^2(\rho-\lambda)]$, the growth rate of the consumption-output ratio is positive and keeps increasing without bound. This path is unfeasible since consumption cannot be larger than output even in the long run.

If, on the other hand, $\dfrac{c_t}{y_t} < \dfrac{\bar{E}}{\xi}[1-\theta+\beta\theta^2(\rho-\lambda)]$, the consumption-output ratio gradually decreases and approaches zero. This path is not optimal and the household saves too much. It can raise its utility by increasing consumption at each point in time.

Hence, eq. (A.19) says that the ratio c_t/y_t must jump to the constant value $\dfrac{\bar{E}}{\xi}[1 - \theta + \beta\theta^2(\rho - \lambda)]$.

A.5 Dynamics of the economy

A.5.1 Proof of Proposition 1

Before the industrial era, the market size χ_t evolves according to eq. (37). That is an ordinary differential equation (ODE). The solution is

$$\chi_t = \chi_0 e^{\lambda t}$$

for $\chi_t \leq \chi_N$. At the threshold χ_N defined in eq. (41)

$$\chi_N = \chi_0 e^{\lambda t_N}.$$

Hence, we can pin down the starting time point of the industrial era as follows:

$$t_N = \ln(\chi_N/\chi_0)/\lambda.$$

We substitute eq. (40) into eq. (32) to yield the following dynamics of the market size variable χ_t during the first phase of industrialization:

$$\dot{\chi}_t = \frac{1-\sigma}{\beta}\left\{\left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)}\phi - \left[\frac{1-\theta}{\theta} - \beta\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right]\chi_t\right\} > 0.$$

A.5.2 Proof of Proposition 2

Substituting eq. (45) into eq. (44) and solving for z_t yields

$$\begin{aligned}
z_t &= \alpha\left[\frac{1-\theta}{\theta}\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)}\chi_t - \phi\right] - \rho - \sigma n_t \\
z_t &= \alpha\left[\frac{1-\theta}{\theta}\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)}\chi_t - \phi\right] \\
&\quad - \sigma\frac{1}{\beta}\left[\frac{1-\theta}{\theta} - \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)}\frac{\phi + z_t}{\chi_t}\right] - (1-\sigma)\rho + \sigma\lambda \\
\left[1 - \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)}\frac{\sigma}{\beta\chi_t}\right] z_t &= \left(\frac{1-\theta}{\theta}\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)}\chi_t - \phi\right)\left(\alpha - \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)}\frac{\sigma}{\beta\chi_t}\right) \\
&\quad - (1-\sigma)(\rho-\lambda) - \lambda.
\end{aligned}$$

Hence,

$$z_t = \frac{\left(\frac{1-\theta}{\theta}\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)}\chi_t - \phi\right)\left(\alpha - \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)}\frac{\sigma}{\beta\chi_t}\right) - (1-\sigma)(\rho-\lambda) - \lambda}{1 - \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)}\frac{\sigma}{\beta\chi_t}}. \tag{A.20}$$

Substituting eq. (A.20) back into eq. (45) to obtain the expression for n_t as follows:

$$n_t = \frac{\left(\frac{1-\theta}{\theta}(1-\alpha) - \beta(\rho-\lambda)\right)\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)} \chi_t - \phi(1-\alpha) + \rho}{\beta\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)} \chi_t - \sigma}. \tag{A.21}$$

The second phase of the industrial era starts once the quality-improving innovation is activated, that is, z_t starts being positive. Hence, we can find the threshold χ_Z by setting $z_t = 0$ in eq. (A.20)

$$\chi_Z = \text{argsolve}\left\{ \left[\frac{1-\theta}{\theta}\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)} \chi_t - \phi\right] \times \left[\alpha - \left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} \frac{\sigma}{\beta\chi_t}\right] = (1-\sigma)(\rho-\lambda) + \lambda \right\}.$$

By transforming the right-hand side, we can show that χ_Z is the root of the following quadratic equation:

$$\zeta_1\chi_t^2 + \zeta_2\chi_t + \zeta_3 = 0,$$

where

$$\begin{cases} \zeta_1 = \alpha\frac{1-\theta}{\theta}\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)} \\ \zeta_2 = -\left(\phi\alpha + \frac{1-\theta}{\theta}\frac{\sigma}{\beta} + (1-\sigma)(\rho-\lambda) + \lambda\right) \\ \zeta_3 = \phi\frac{\sigma}{\beta}\left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} \end{cases}$$

This equation has the solution

$$\chi_Z = \frac{\phi\alpha + \frac{1-\theta}{\theta}\frac{\sigma}{\beta} + (1-\sigma)(\rho-\lambda) + \lambda + \sqrt{\left[\phi\alpha + \frac{1-\theta}{\theta}\frac{\sigma}{\beta} + (1-\sigma)(\rho-\lambda) + \lambda\right]^2 - 4\phi\alpha\frac{1-\theta}{\theta}\frac{\sigma}{\beta}}}{2\alpha\frac{1-\theta}{\theta}\left(\frac{\bar{E}}{\xi}\right)^{1/(1-\theta)}}.$$

We substitute eq. (A.21) into eq. (32) to obtain the dynamics of the market size during this phase

$$\dot{\chi}_t = \frac{1-\sigma}{\beta - \sigma\left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)}/\chi_t}\left\{\left[(1-\alpha)\phi - \left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right]\left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} - \left[(1-\alpha)\frac{1-\theta}{\theta} - \beta\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)\right]\chi_t\right\}.$$

A.5.3 Proof of Lemma 4

We first rewrite the dynamics of market size χ_t during the first phase of industrialization in eq. (42) as follows:

$$\dot{\chi}_t = \nu(\bar{\chi} - \chi_t), \tag{A.22}$$

where

$$\begin{cases} \nu = \dfrac{1-\sigma}{\beta}\left[\dfrac{1-\theta}{\theta} - \beta\left(\rho + \dfrac{\sigma\lambda}{1-\sigma}\right)\right] \\ \bar{\chi} = \left(\dfrac{\bar{E}}{\xi}\right)^{-1/(1-\theta)} \dfrac{\phi}{\frac{1-\theta}{\theta} - \beta\left(\rho + \frac{\sigma\lambda}{1-\sigma}\right)} \end{cases}$$

It is straightforward to show that $\bar{\chi} > \chi_N$. We assume that parameter values satisfy the condition that $\bar{\chi} > \chi_Z$. Peretto (2015) calls the situation where $\bar{\chi} < \chi_Z$ as premature saturation and the economy never reaches the threshold χ_Z.

Eq. (A.22) is a linear ODE. We solve it as follows:

$$\begin{aligned} \dot{(\chi_t - \bar{\chi})} &= \dot{\chi}_t = -\nu(\chi_t - \bar{\chi}) \\ \frac{\dot{(\chi_t - \bar{\chi})}}{\chi_t - \bar{\chi}} &= -\nu. \end{aligned}$$

Integrating from time t_N to time t yields

$$\begin{aligned} \int_{t_N}^{t} \frac{\dot{(\chi_s - \bar{\chi})}}{\chi_s - \bar{\chi}} ds &= \int_{t_N}^{t} -\nu ds \\ \log(|\chi_s - \bar{\chi}|)\Big|_{t_N}^{t} &= -\nu s\Big|_{t_N}^{t} \\ \log\left(\frac{\bar{\chi} - \chi_t}{\bar{\chi} - \chi_N}\right) &= -\nu(t - t_N) \end{aligned}$$

Hence, the solution of the ODE is

$$\chi_t = \chi_N e^{\nu(t_N - t)} + \bar{\chi}[1 - e^{\nu(t_N - t)}] \tag{A.23}$$

Substituting $t = t_Z$ yields

$$\begin{aligned} \log\left(\frac{\bar{\chi} - \chi_Z}{\bar{\chi} - \chi_N}\right) &= -\nu(t_Z - t_N) \\ t_Z - t_N &= \frac{1}{\nu}\log\left(\frac{\bar{\chi} - \chi_N}{\bar{\chi} - \chi_Z}\right). \end{aligned}$$

It is observable that the expressions for χ_N, χ_Z, and $\bar{\chi}$ have the same factor of $\left(\frac{\bar{E}}{\xi}\right)^{-1/(1-\theta)}$. Therefore, the ratio $\frac{\bar{\chi} - \chi_N}{\bar{\chi} - \chi_Z}$ is independent of the environmental policy. As a result, the policy does not impact the difference between t_Z and t_N (i.e., the duration of the first industrialization phase).

A.6 Underdeveloped countries

There are 45 countries listed as LDCs by UN (2023): Afghanistan, Angola, Bangladesh, Benin, Burkina Faso, Burundi, Cambodia, Central African Republic, Chad, Comoros, Democratic Republic of Congo, Djibouti, Eritrea, Ethiopia, Gambia, Guinea, Guinea-Bissau, Haiti, Kiribati, Laos, Lesotho, Liberia, Madagascar, Malawi, Mali, Mauritania, Mozambique, Myanmar, Nepal, Niger, Rwanda, São Tomé and Príncipe, Senegal, Sierra Leone, Solomon Islands, Somalia, South Sudan, Sudan, Tanzania, Timor-Leste, Togo, Tuvalu, Uganda, Yemen, and Zambia. Table 1 shows the latest data on the population growth rate of these countries.

Country	Population Growth Rate (%)
Afghanistan	2.53
Angola	3.10
Bangladesh	1.07
Benin	2.70
Burkina Faso	2.56
Burundi	2.66
Cambodia	1.07
Central African Republic	2.21
Chad	3.12
Comoros	1.83
Democratic Republic of Congo	3.20
Djibouti	1.37
Eritrea	1.74
Ethiopia	2.54
Gambia	2.47
Guinea	2.39
Guinea-Bissau	2.15
Haiti	1.19
Kiribati	1.81
Laos	1.40
Lesotho	1.06
Liberia	2.08
Madagascar	2.38
Malawi	2.56
Mali	3.10
Mauritania	2.59
Mozambique	2.74
Myanmar	0.71
Nepal	1.69
Niger	3.71
Rwanda	2.31
São Tomé and Príncipe	1.90
Senegal	2.57
Sierra Leone	2.17
Solomon Islands	2.29
Somalia	3.07
South Sudan	1.52
Sudan	2.63
Tanzania	2.96
Timor-Leste	1.53
Togo	2.33
Tuvalu	0.96
Uganda	3.00
Yemen	2.14
Zambia	2.76

Table 1: Population growth rate by country in 2022. *Source: World Bank (2024)*

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