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# Linear mixed-effects models

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# Chapter 1

## Linear mixed-effects models

### 1.1 Introduction

In Chapter ??, we used linear models to estimate ‘fixed’ effects, which consist of specific and repeatable categories/variables that are representative of an entire population (e.g., species, age). In longitudinal studies (repeated measures) and in studies using hierarchical (nested) sampling, it is also possible to estimate effects associated with individuals sampled at random from the population of interest. These are ‘random’ effects and convey information about the degree that individuals in a population differ but not how or why they differ.

One way to differentiate fixed and random effects is that fixed effects contain levels that are informative beyond the current analysis (e.g., a species of tree or a specific management type) while random effects contain levels that are not informative beyond the current analysis (e.g., a group of trees within an observation plot or a field under specific management). Another way to understand the difference is that fixed effects influence the *mean* of the response while random effects influence the *variance* of the response.

Mixed-effects models estimate both fixed and random effects and are particularly useful when dealing with potential pseudoreplication and unbalanced designs. Including random effects can also account for variation that could mask patterns in an analysis considering only fixed effects.

Two commonly used packages for fitting mixed-effects models are `nlme` and `lme4`. In this chapter, we will use the newer `lme4` package, but note that more complex correlation structures are only possible with the `nlme` package.

Rather than to present the theory underlying mixed-effects models, which is very complex, we will treat this topic by example, and thus aim at a practical application.

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**Further reading** We encourage you to study other text books on more general descriptions of mixed-effects models, as well as many other examples and extensions. The list below contains a few recommended books and articles.

- Pinheiro, J. and Bates, D., 2006. Mixed-effects models in S and S-PLUS. Springer Science & Business Media. - *Seminal text on mixed-effects models in R, using the nlme package but still important reading when using the lme4 package*
- Crawley, M.J. 2007. The R Book. Wiley. - *The chapter on mixed-effects models includes many practical examples and readable explanation of the output*
- Bolker, B. M., et al. 2009, Generalized linear mixed models: a practical guide for ecology and evolution., Trends in ecology & evolution 24: 127-135. - *A seminal, practical description of the use of the lme4 package for generalized linear mixed models, by one of the developers of the lme4 package*
- Bates, D. et al. 2015. Fitting Linear Mixed-Effects Models Using lme4. J. Stat. Software 67(1):1-48. - *The main publication describing the lme4 package, although largely very technical, includes practical tips and a nice table to understand random effects specifications*
- Faraway, J. J. Extending the linear model with R: generalized linear, mixed effects and nonparametric regression models. Vol. 124. CRC press. - *Rigorous detail on mixed-effects models, useful examples and extensions, as well as many other more advanced linear modelling approaches*

### 1.1.1 A note about p-values

The `lme4` package does not report p-values. The developers made this decision because p-values require calculating degrees of freedom. Random effects don't necessarily have to expend the same degrees of freedom as treating them as fixed effects, so the package developers have decided not to fudge this by calculating them for you.

In this chapter, we use two approaches to calculate p-values of fixed effects. To obtain the p-values of all fixed effects in a model, we use the Kenward-Roger approximation to the degrees of freedom, as recommended by Halekoh and Højsgaard (2014)<sup>1</sup>. This approximation is most conveniently implemented in the `Anova` function from the `car` package (not `anova!`), and used like this (if you have a fitted model `mymodel` returned by `lmer`),

```
library(car)
Anova(mymodel, test="F")
```

Note that we cannot use `test="F"` for generalized linear mixed-effects models (i.e. fit with `glmer`), and some caution needs to be applied with the `Anova` default use of the chi-squared test statistic.

The second approach is to use likelihood-ratio tests to test the significance of a single fixed effect in a model. To do this, we fit two models (one with, and one without the fixed effect of interest), and use the `KRmodcomp` function from the `pbkrtest` package (again using the Kenward-Roger approximation as mentioned above). Note that the results are usually slightly different, and sometimes very different, if you use the standard likelihood-ratio test with the `anova` function.

Finally, you may wish to inspect p-values for individual coefficients in the `summary` statement of the fitted model. With the `lmerTest` package loaded, fit the model with `lmer`, which will now show p-values in the `summary`. This even works for `glmer`, but the z-score approximation is probably quite poor. Use with caution.

We will demonstrate the use of each of these functions in the examples that follow.

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<sup>1</sup>Ulrich Halekoh and Søren Højsgaard, 2014, A Kenward-Roger Approximation and Parametric Bootstrap Methods for Tests in Linear Mixed Models – The R Package `pbkrtest`, J. Stat. Software 59, 1-30

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**Further reading** See the help page `?pvalues` for references to several options to calculate p-values with `lme4`.

## 1.2 Example: individual-level variation in tree canopy gradients

The following example uses the `pref` data (not yet described in Appendix A, download the file ‘`prefdata.csv`’). The dataset contains measurements of leaf mass per area (LMA), and distance from the top of the tree (`dfromtop`) on 35 trees of two species. We want to know whether LMA decreases with `dfromtop`, as expected, and whether this decrease in LMA with distance from top differs by species.

```
# Read the data and inspect the first few rows, and the species factor variable.
pref <- read.csv("prefdata.csv")
head(pref)

##      ID      species dfromtop totheight height      LMA      narea
## 1 FP11 Pinus ponderosa    8.88    22.40  13.52 319.4472 2.779190
## 2 FP11 Pinus ponderosa    0.62    22.40  21.78 342.7948 4.010700
## 3 FP11 Pinus ponderosa    4.72    22.40  17.68 329.5399 3.365579
## 4 FP15 Pinus ponderosa    2.74    27.69  24.95 312.4467 3.682907
## 5 FP15 Pinus ponderosa    5.48    27.69  22.21 278.4037 2.524224
## 6 FP15 Pinus ponderosa    8.40    27.69  19.29 255.9716 2.351546

levels(pref$species)

## [1] "Pinus monticola" "Pinus ponderosa"
```

Before we fit mixed-effects models, let's start with a linear regression that includes `dfromtop` and `species` as the predictor variables to observe the general patterns. We use `visreg` to quickly visualize the fitted linear model. The following code produces Fig. 1.1.

```
# Fit a linear regression by species (ignoring individual-level variation)
lm1 <- lm(LMA ~ species + dfromtop + species:dfromtop, data=pref)

# Plot predictions
library(visreg)
visreg(lm1, "dfromtop", by="species", overlay=TRUE)
```

As we can see in Fig. 1.1, there is a strong effect of species, but it appears that LMA and `dfromtop` are not significantly correlated.

**Try this yourself** Look at the `anova` table for the fitted linear regression model from the example above to confirm that LMA does not significantly change with `dfromtop`.

To see whether the relationship between LMA and `dfromtop` potentially varies from tree to tree, we fit a linear regression separately for each tree using the `lmList` function in the `lme4` package and then plot the outcome (in Fig. 1.2).

```
# For the lmList function (Note: the nlme package also includes the lmList function)
library(lme4)

# fit linear regression by tree ('ID')
lm1s1 <- lmList(LMA ~ dfromtop | ID, data=pref)
```

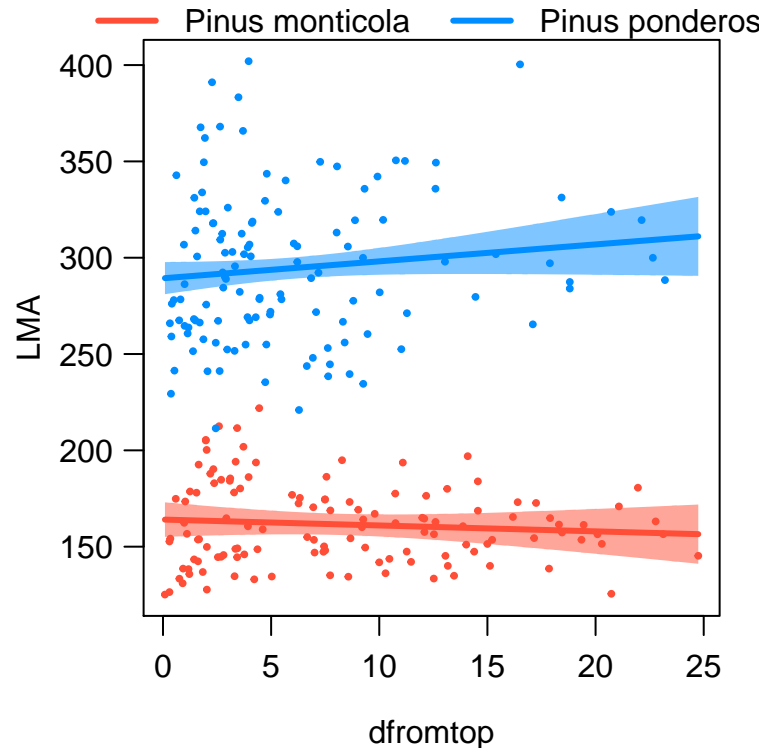


Figure 1.1: Leaf mass per area as a function of tree species (two colours) and the distance from the top of each tree, as fitted with a simple linear model and visualized with visreg.

```
# Extract coefficients (intercepts and slopes) for each tree
liscoef <- coef(lmlis1)

# load plottix for the 'ablineclip' function, which clips lines within the range of x
library(plotrix)

# split pref by tree (prefsp is a list)
prefsp <- split(pref, pref$ID)

# Plot
palette(c("red", "blue"))
with(pref, plot(dfromtop, LMA, col=species, pch=16, cex=0.8))
for(i in 1:length(prefsp)){

  # Find min and max values of dfromtop, to send to ablineclip
  xmin <- min(prefsp[[i]]$dfromtop)
  xmax <- max(prefsp[[i]]$dfromtop)

  # add regression lines
  ablineclip(liscoef[i,1], liscoef[i,2], x1=xmin, x2=xmax,
            col=prefsp[[i]]$species)
}
```

From the figure we can conclude (informally) that:

1. Intercepts vary a lot between trees

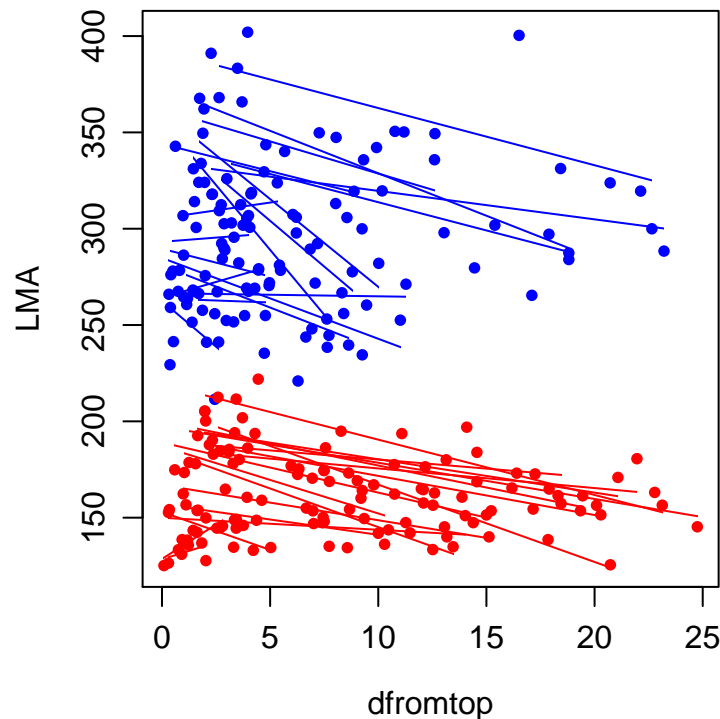


Figure 1.2: Leaf mass per area as a function of tree species (two colours) and the distance from the top of each tree. The solid lines represent the slope of the relationship for each individual tree.

2. There seems to be a negative relationship for many trees
3. It seems there is less variation between slopes than intercepts

We know the individual data points are not independent, as they are nested within trees (that is, multiple samples were collected for each tree). To properly account for this non-independence, we have to use a mixed-effects model. In the example below, we will fit two models: one with a random intercept only, and one with a random intercept and slope.

### 1.2.1 A simple equation for a mixed-effects model

Before we fit a mixed-effects model, let's write down an equation that illustrates the roles of the fixed effects and random effects in the model.

For a simple linear regression of  $Y$  versus  $X$  (where  $X$  is numeric), we fit the model:

$$Y = \beta_0 + \beta_1 \cdot X$$

where the  $\beta$ 's are the intercept and the slope of the fitted regression line.

A mixed-effects model additionally fits two random parameters, and we can write the model as:

$$Y = (\beta_0 + b_0) + (\beta_1 + b_1) \cdot X$$

where  $b_0$  is the random intercept (which is normally distributed with mean zero, and some standard deviation), and  $b_1$  is the random slope (also assumed to be normally distributed with mean of zero and some standard deviation).

---

When we fit a mixed-effects model, not only do we get an estimate of the variance (or standard deviation) of the random effects, we also get estimates of this random effect for each individual. These estimates are known as the BLUPs (best linear unbiased predictors).

## 1.2.2 Fitting the mixed-effects model

To specify random effects with `lmer`, we add it to the formula in the right-hand side. For example, a random intercept for 'ID' (that is, the intercept will vary randomly among ID's) is coded as `(1|ID)`. If we also allow the slope of the relationship to vary, we specify it as `(dfromtop|ID)` so that the slope and intercept of the relationship between LMA and `dfromtop` will vary randomly between tree ID's.

```
# Random intercept only
pref_m1 <- lmer(LMA ~ species + dfromtop + species:dfromtop + (1|ID), data=pref)

# Random intercept and slope
pref_m2 <- lmer(LMA ~ species + dfromtop + species:dfromtop + (dfromtop|ID), data=pref)

# The AIC and a likelihood-ratio test tell us that we don't need a random slope.
# lower AIC indicates that model fit is better (more efficient)
AIC(pref_m1, pref_m2)

##           df          AIC
## pref_m1    6 2251.997
## pref_m2    8 2255.735

# Likelihood ratio test : the more complex model is not supported by the data.
# Note: the models will be re-fitted with ML instead of REML; this is necessary
# when performing likelihood-ratio tests.
anova(pref_m1, pref_m2)

## Data: pref
## Models:
## pref_m1: LMA ~ species + dfromtop + species:dfromtop + (1 | ID)
## pref_m2: LMA ~ species + dfromtop + species:dfromtop + (dfromtop | ID)
##           Df      AIC      BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## pref_m1    6 2263.2 2284.3 -1125.6  2251.2
## pref_m2    8 2267.1 2295.2 -1125.5  2251.1 0.1274      2    0.9383

# Output from the random intercept model - not shown (inspect yourself!)
# summary(pref_m1)

# Using Anova from car, we get p-values for the main effects.
library(car)
Anova(pref_m1)

## Analysis of Deviance Table (Type II Wald chisquare tests)
##
## Response: LMA
##           Chisq Df Pr(>Chisq)
## species      147.475 1 <2e-16 ***
## dfromtop       86.832 1 <2e-16 ***
## species:dfromtop  2.531 1  0.1116
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Note that in the above, we use the familiar `anova` function to test between models with different ran-



dom effects structure. Since we use `anova` on an object returned by `lmer`, this command invokes the `anova.merMod` function, from the `lme4` package.

**Try this yourself** Inspect the help file `?anova.merMod`, and find out that you can name the two models, simplifying the interpretation of the command `anova(lme1, lme2)` from the above example.

We now conclude that LMA decreases significantly with `dfromtop`. Compare this with the fixed-effects model we started with:

```
summary(lm1)

##
## Call:
## lm(formula = LMA ~ species + dfromtop + species:dfromtop, data = pref)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -80.047  -21.737   -2.908   17.231  109.207
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      164.0749     4.5800   35.824  <2e-16 ***
## speciesPinus ponderosa      125.2444     6.2581   20.013  <2e-16 ***
## dfromtop           -0.3062     0.4406   -0.695    0.4878
## speciesPinus ponderosa:dfromtop    1.1855     0.6923    1.712    0.0881 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 31.34 on 245 degrees of freedom
## Multiple R-squared:  0.8211, Adjusted R-squared:  0.8189
## F-statistic: 374.9 on 3 and 245 DF, p-value: < 2.2e-16
```

To visualise these differences, also compare predictions from the mixed-effects model, in Fig. 1.3, to those of the fixed-effects model depicted in Fig. 1.1.

```
# Plot predictions
visreg(pref_m1, "dfromtop", by="species", overlay=TRUE)
```

Ignoring the tree-to-tree variance thus resulted in drawing the wrong conclusion from our data. When we accounted for this variation with a mixed-effects model, we did find a significant overall relationship between LMA and `dfromtop`. The reason for this discrepancy is that the large variation in intercepts between the individual trees (between-subject variation) masked the relationship between the two variables within individuals (within-subject variation).

Finally, we may be interested in quantifying the variation between individuals in terms of the intercept and slope. These are the standard deviations of the random effects, and can be extracted with the `VarCorr` function (it is also shown in the `summary` statement of the mixed-effects model).

```
# Standard deviation of the intercept and slope ('dfromtop') between individuals,
# as estimated from the random effects.
VarCorr(pref_m1)

## Groups   Name                Std.Dev.
## ID      (Intercept)  30.714
## Residual                                18.088
```

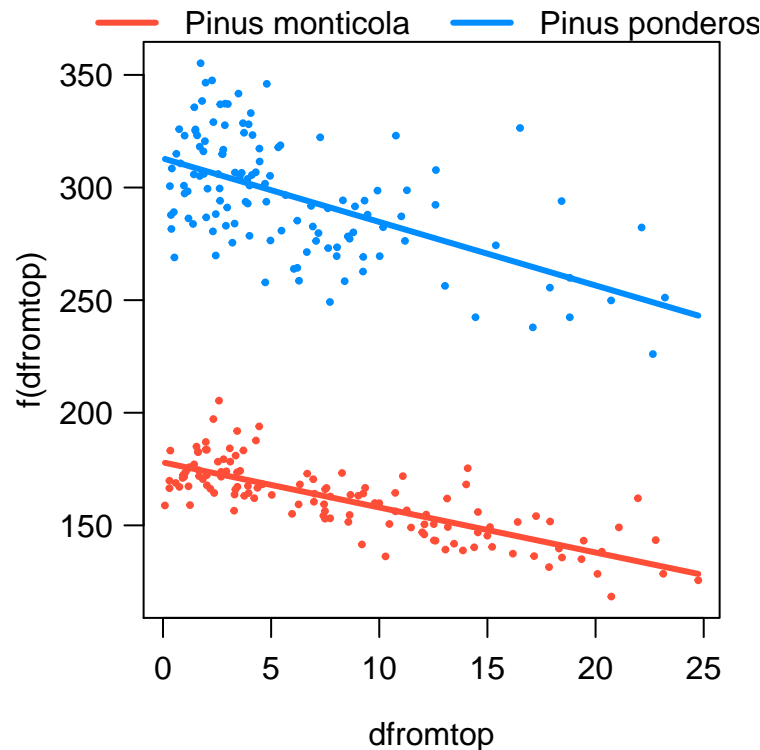


Figure 1.3: Leaf mass per area as a function of tree species (two colours) and the distance from the top of each tree, as fitted with a linear mixed-effects model and visualized with visreg.

```
# For comparison, output from a model with random slope and intercept.
```

```
VarCorr(pref_m2)
```

```
## Groups      Name      Std.Dev. Corr
## ID          (Intercept) 31.36023
##            dfromtop    0.47658 -0.304
## Residual                        17.96337
```

**Try this yourself** Try using `ranef` on `lme1` and `lme2`; the first will show intercepts for each of the random effects (trees), while the second will show both estimated intercepts and slopes for the random effects.

To this point, we have put a lot of effort into evaluating the significance of different terms in the model, but we have not run any model diagnostics. The following code makes a plot (Fig. ??) of the residuals versus fitted, and a quantile-quantile plot of the residuals. Unfortunately, these diagnostics suggest that our assumptions regarding normality and/or heteroscedasticity are violated.

```
# plot residuals against fitted values
# done by calling 'plot' on object returned by 'lmer'
plot(pref_m1)
```

```
# quantile-quantile plot
library(car)
qqPlot(residuals(pref_m1))
```

```
## [1] 100 57
```

The positive scaling of the residuals with increasing fitted values suggest that either a log- or square

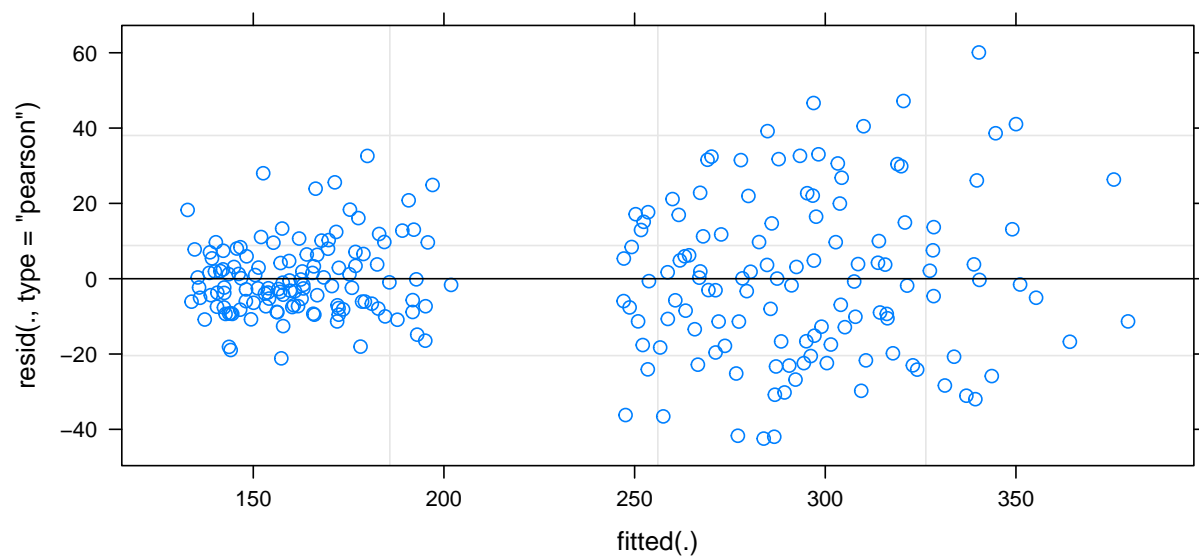


Figure 1.4: Diagnostic plots for the `pref_m2` model.

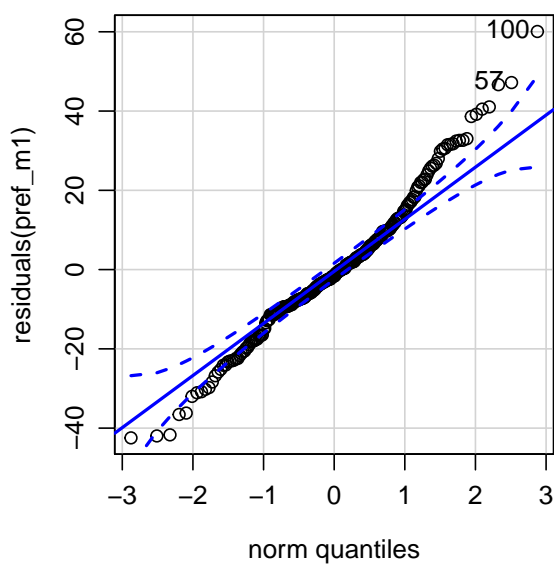


Figure 1.5: Diagnostic plots for the `pref_m2` model.

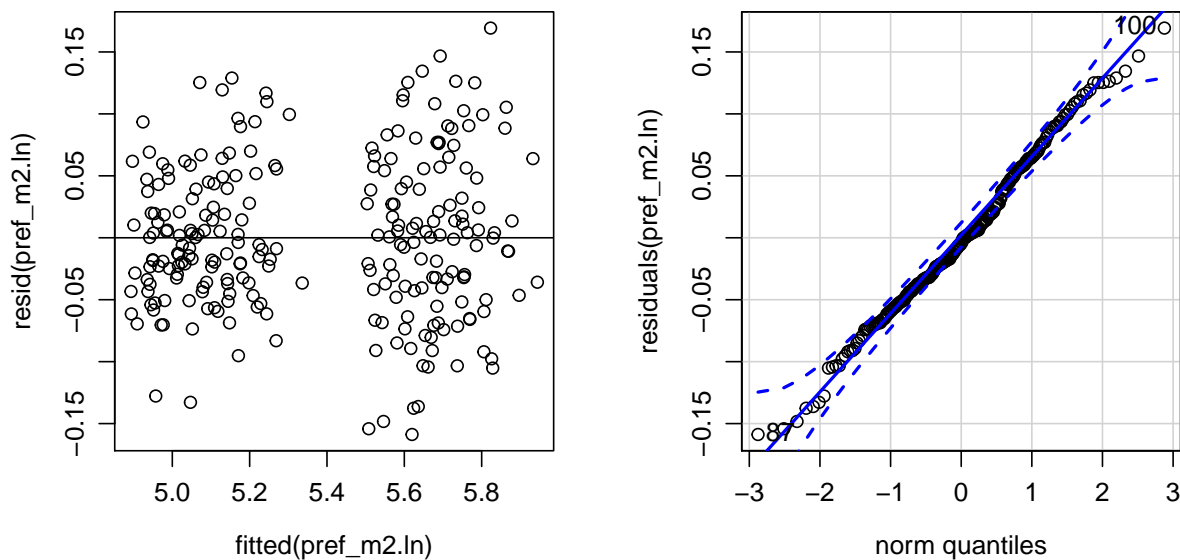


Figure 1.6: Diagnostic plots for the model following log-transformation.

root-transformation might be appropriate. Here we try a log-transformation; Fig. 1.6 suggests that this transformation worked.

```
# fit model with log-transformed response
pref_m2.ln <- lmer(log(LMA) ~ species + dfromtop + species:dfromtop +
  (dfromtop|ID), data=pref)

# plot residuals against fitted values
plot(resid(pref_m2.ln) ~ fitted(pref_m2.ln))
abline(h=0)

# quantile-quantile plot
library(car)
qqPlot(residuals(pref_m2.ln))

## [1] 100 87
```

### Try this yourself

Check the significance of the fixed and random effects for the log-transformed data.

Check the significance of the fixed and random effects for the log-transformed data.

## 1.3 Example: individual-level variation in metabolic rate of mice

In this example we will look at a dataset containing metabolic rate measured on mice at three different temperatures. Measurements on every mouse were repeated three times. We are interested in estimating the change in metabolic rate with temperature, and whether this relationship changes with body mass, sex (male or female), whether the mice were fed on the day of measurements, and whether they were using a wheel.

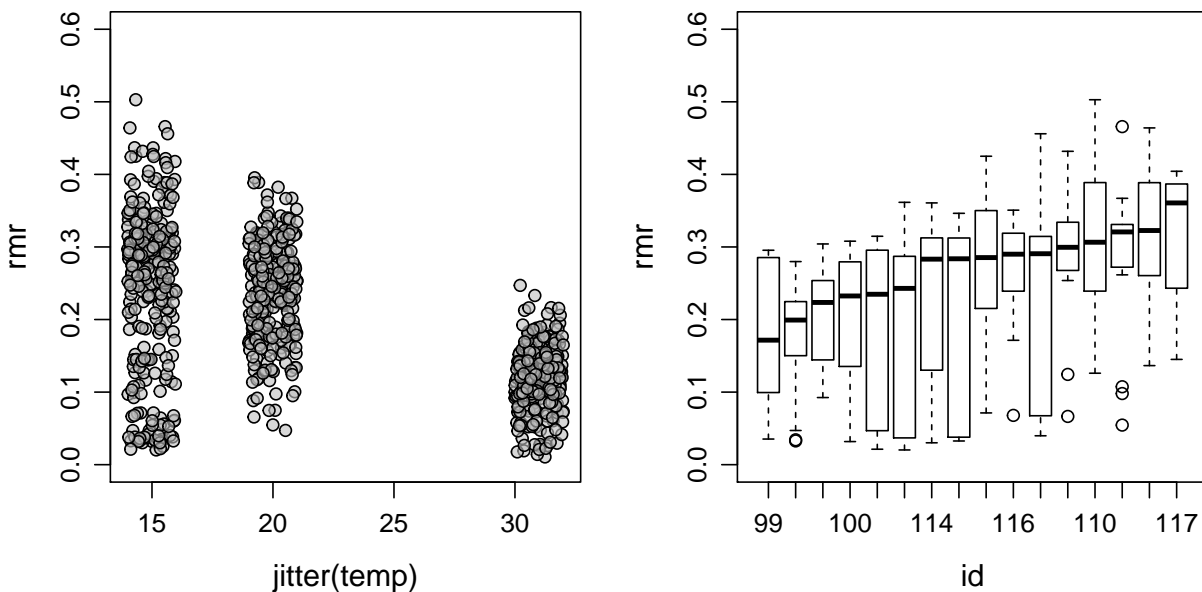


Figure 1.7: Resting metabolic rate (rmr) for individual mice. Left panel: raw data against temperature, demonstrating increase in rmr with decreasing temperature, and larger variance at lower temperature. Right panel: rmr at a measurement temperature of 15C, by individual (id).

First we read and inspect the dataset (data not yet described in the Appendix).

```
mouse <- read.csv("wildmousemetabolism.csv")

# Make sure the individual label ('id') is a factor variable
mouse$id <- as.factor(mouse$id)
```

As a first step let's look at the data. We would like to visualize whether temperature has an effect on metabolic rate, and whether individuals were very different in terms of metabolic rate (rmr) at a fixed temperature. The following code makes Fig. 1.7.

```
# Raw data of rmr versus temperature (temp). Note use of jitter()
# to minimize overlap between data.
with(mouse, plot(jitter(temp),rmr, pch=21, bg="#BEBEBE99", ylim=c(0,0.6)))

# Take a subset, and reorder the 'id' factor levels by rmr.
mouse15 <- subset(mouse, temp == 15)
mouse15$id <- with(mouse15, reorder(id, rmr, median, na.rm=TRUE))

# A simple boxplot showing variation in rmr across and within individuals
boxplot(rmr ~ id, data=mouse15, xlab="id", ylab="rmr",ylim=c(0,0.6))
```

A practical way to test for multiple variables is to start with a simple model that includes the most important variables, and gradually increase model complexity. Every time we add one variable to a model, the significance can be tested with a likelihood ratio test. As discussed, we can use the built-in `anova` function, but probably more precise is the use of `KRmodcomp` from the `pbkrtest` package. We will demonstrate its use in this example but note that all conclusions will be the same when using `anova`.

We know for sure that resting metabolic rate should increase when the temperature decreases, so let's start with a model that only includes `temp` and appropriate random effects. Since each individual was measured multiple times (indicated by the `run` variable), we specify a `run` random effect nested within individual (`id`).

---

```
mouse_m0 <- lmer(rmr ~ temp + (1|id/run), data=mouse )
```

In this first model we only allow the intercept to vary between individuals (`1|id/run`). A side-effect of the nested random effect is that we can quickly quantify the variation between individuals, as well as 'runs' within individuals, using the `VarCorr` function:

```
VarCorr(mouse_m0)
```

```
## Groups   Name                Std.Dev.
## run:id   (Intercept) 0.029825
## id       (Intercept) 0.027776
## Residual                    0.071433
```

This demonstrates that the variation between individuals (`id`) is in fact similar to variation between runs within individuals (`run:id`). It is quite obvious that the fixed effect `temp` is significant in this model (inspect the `Anova` table yourself). We could add a random slope effect by `id`, but since there are few data points for every individual, this causes some problem that we would like to avoid (but feel free to try this for yourself).

Note that the intercept of the model is defined as the value of the response variable (`rmr`) when the predictor (`temp`) is zero. This is not very informative since we did not measure metabolic rate at zero C. In a case like this, it is advisable to recenter the data so that the intercept is more meaningful. We will subtract 31C from `temp`, so that the intercept can now be interpreted as the metabolic rate at 31C. This is no choice of convenience, but rather has a biological justification: 31C approximates the lower critical limit of the thermal neutral zone (the range of temperature where metabolic rate does not vary). Below this temperature, metabolic rate increases to counter heat loss.

We will make a new variable and add it to the dataframe, and refit the model.

```
mouse$temp31 <- mouse$temp - 31
```

```
mouse_m1 <- lmer(rmr ~ temp31 + (1|id/run), data=mouse )
```

Compare the summary statement from this model with `mouse_m0`, and also note that the two models are identical in goodness of fit (compare the AIC, for example). We are only expressing the intercept differently, allowing biological interpretation of the coefficients.

Next we include body mass as a predictor, as well as the interaction with temperature. It is well known that body mass is an important determinant of metabolic rate, so we should not be surprised by a strong significant effect:

```
mouse_m2 <- lmer(rmr ~ temp31*bm + (1|id/run), data=mouse )
```

```
library(pbkrtest)
```

```
KRmodcomp(mouse_m2, mouse_m1)
```

```
## F-test with Kenward-Roger approximation; computing time: 0.59 sec.
## large : rmr ~ temp31 * bm + (1 | id/run)
## small : rmr ~ temp31 + (1 | id/run)
##          stat      ndf      ddf F.scaling  p.value
## Ftest 13.424   2.000 57.769   0.98861 1.629e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The F-test shows that body mass is highly significant overall. The `Anova` shows that both the main effect and interaction are significant:

```
Anova(mouse_m2, test="F")
```

```
## Analysis of Deviance Table (Type II Wald F tests with Kenward-Roger df)
##
## Response: rmr
##           F Df Df.res    Pr(>F)
## temp31    465.240  1 784.03 < 2.2e-16 ***
## bm        17.137  1  22.55  0.000411 ***
## temp31:bm  10.021  1 784.03  0.001608 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

An interesting side effect of including body mass as a fixed effect is that the estimated variance of the `id` is now much smaller. This makes sense because body mass varies between individuals, and thus including it in the model reduces the individual-level variation *after having accounted for body mass*.

Compare the standard deviation for `id` for the `mouse_m2` model (see below) with that estimated for the `mouse_m0` model above.

```
# Random effects for the mixed-effects model including body mass
VarCorr(mouse_m2)

## Groups      Name                Std.Dev.
## run:id      (Intercept) 0.029872
## id          (Intercept) 0.012096
## Residual                                0.071025
```

Next we test whether individuals vary significantly in terms of their response to temperature, by adding a random slope effect. The models above only allowed the intercept (i.e. `rmr` at 31C) to vary between individuals. To test whether a more complex random effects structure is supported by the data, we fit two models and compare them with a likelihood ratio test (with `anova`), like so:

```
# Like mouse_m2, but with random slope (temp31)
mouse_m3 <- lmer(rmr ~ temp31*bm + (temp31|id/run), data=mouse )

anova(mouse_m3, mouse_m2)

## Data: mouse
## Models:
## mouse_m2: rmr ~ temp31 * bm + (1 | id/run)
## mouse_m3: rmr ~ temp31 * bm + (temp31 | id/run)
##           Df      AIC      BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## mouse_m2  7 -1962.4 -1929.3 988.20  -1976.4
## mouse_m3 11 -1971.0 -1919.0 996.52  -1993.0 16.634    4  0.002276 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The test shows that the more complex model is better, providing evidence that individuals differ substantially in their response to temperature.

As always, when we have a significant main effect and interaction, it is not easy to see *how* they affect the response variable. As always, it is most convenient to plot the model predictions with the `visreg` package. The following code makes Fig. 1.8. The figure shows that `rmr` increases with body mass (`bm`) and temperature (`temp`), and the temperature effect increases with body mass.

```
# Model predictions as a function of body mass, for the three temperatures.
# The argument 'partial=FALSE' turns off the partial residuals, producing a cleaner plot.
visreg(mouse_m3, "bm", by="temp31", overlay=TRUE, partial=FALSE, ylim=c(0,0.4))
```

Next, we test for the effect of three additional fixed effects on metabolic rate. As before, we test these

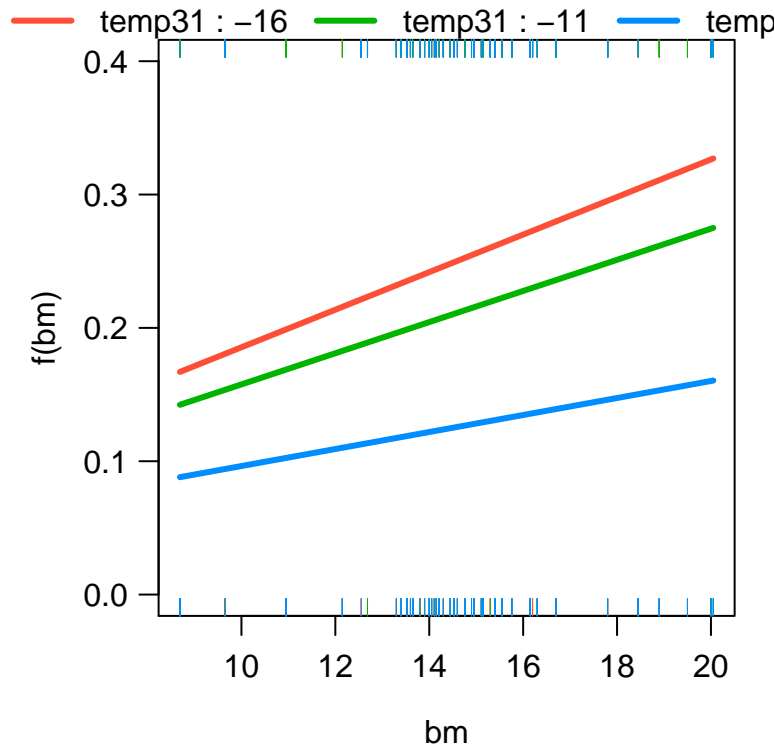


Figure 1.8: Plot fitted effects of the mouse m2 model, demonstrating significant effects of body mass and temperature on resting metabolic rate (rmr). Note that temp31 is temperature - 31, bm is body mass.

effects one by one. The following code shows tests of `sex` and `wheel` on the `rmr` response variable.

```
# No effect of sex
mouse_m4 <- lmer(rmr ~ bm*temp31 + sex + (temp31|id/run), data=mouse)
KRmodcomp(mouse_m4, mouse_m3)

## F-test with Kenward-Roger approximation; computing time: 0.67 sec.
## large : rmr ~ bm * temp31 + sex + (temp31 | id/run)
## small : rmr ~ temp31 * bm + (temp31 | id/run)
##          stat      ndf      ddf F.scaling p.value
## Ftest    0.2275   1.0000 16.1828         1  0.6398

# We add 'wheel' only as an additive effect. The interaction cannot be estimated because
# the only cases where 'wheel=No' were at a temperature of 31C:
with(mouse, table(temp,wheel))

##          wheel
## temp  No Yes
##   15    0 288
##   20    0 288
##   31 144 144

mouse_m5 <- lmer(rmr ~ bm*temp31 + wheel + (temp31|id/run), data=mouse)
KRmodcomp(mouse_m5, mouse_m3)

## F-test with Kenward-Roger approximation; computing time: 0.68 sec.
## large : rmr ~ bm * temp31 + wheel + (temp31 | id/run)
## small : rmr ~ temp31 * bm + (temp31 | id/run)
```



```
##          stat      ndf      ddf F.scaling    p.value
## Ftest   46.516    1.000 740.194         1 1.889e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Note: the first test will print a note to the screen which you may ignore.

We finish with the standard anova table, showing p-values for the main effects.

```
Anova(mouse_m5, test="F")
## Analysis of Deviance Table (Type II Wald F tests with Kenward-Roger df)
##
## Response: rmr
##          F Df Df.res    Pr(>F)
## bm          14.2522  1  19.73 0.001212 **
## temp31       104.6149  1  22.07 7.738e-10 ***
## wheel        46.5156  1 740.19 1.889e-11 ***
## bm:temp31     4.4861  1  22.96 0.045211 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Finally we visualize the individual-level variation in resting metabolic rate and its response to temperature, as estimated by the mixed-effects model. We use the `mouse_m2` model to visualize the predicted effects for every individual, ignoring effects of `wheel` and `food`. One interesting aspect of the data is that the variance in `rmr` was clearly higher for lower `temp`. The mixed-effects model is able to partially account for this as a result of a) the variation in body mass between individuals (which, as you recall, interacted with the temperature response), and b) the random variation in intercepts and slopes between individuals. This is a good result because it is otherwise very difficult to model changes in variance with response variables.

The following, more advanced, code produces Fig. 1.9.

```
# Make a dataframe with all combinations of temp and id, for run 1 only
pred_dfr <- expand.grid(temp31=c(-16,-11,0),
                        id=levels(mouse$id), run=1)

# Get average body mass by individual, merge onto pred_dfr
library(doby)
bmid <- summaryBy(bm ~ id, FUN=mean, data=mouse, keep.names=TRUE)
pred_dfr <- merge(pred_dfr, bmid)

# Predict rmr for every id and temp, from the mouse_m3 model
# The default behaviour is to make predictions including the random
# effects (i.e. id and run:id)
pred_dfr$rmr_pred <- predict(mouse_m3, pred_dfr)

# Plot the data for run 1
with(subset(mouse, run==1), plot(jitter(temp),rmr, pch=21, bg="#BEBEBE99", ylim=c(0,0.6)))

# Add a prediction line for every individual. This is an alternative implementation,
# avoiding a for loop. The use of invisible() avoids lapply from printing output.
invisible(lapply(split(pred_dfr, pred_dfr$id), function(x)lines(x$temp31 + 31, x$rmr_pred)))
```

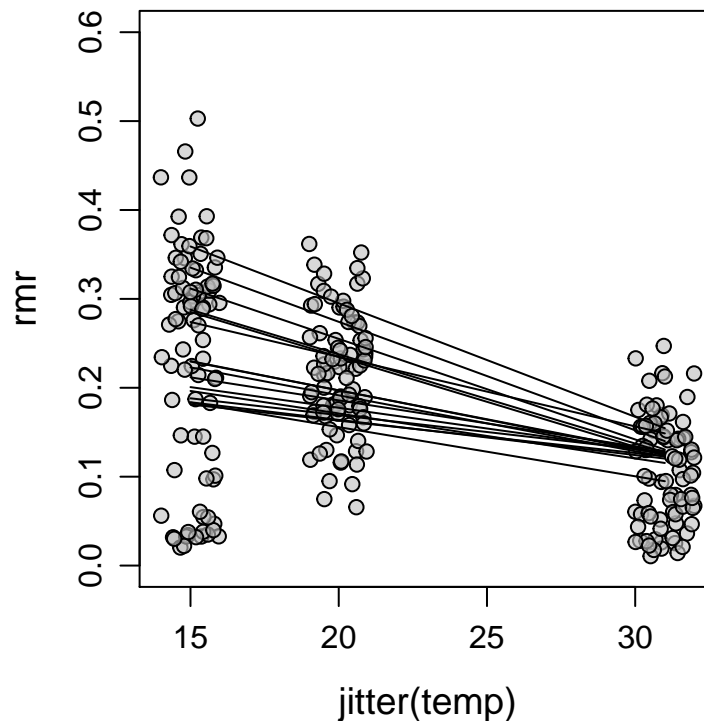


Figure 1.9: Measured (symbols) and predicted (lines) resting metabolic rate for the mouse dataset. Lines are predictions from the mouse m3 model for the individuals in the dataset, using the random variation in intercepts and slopes as well as the individual level variation in body mass.

## 1.4 Example: blocked designs in the litter decomposition data

Blocked designs are often used in field experiments to account for known or suspected environmental gradients at the study site. By blocking the experimental design, the effect of the environmental gradient can be separated from the effect of the treatment(s) of interest increasing the ability to detect significant effects. Let's look at the effects of herbicide and profile on soybean litter decomposition as a function of agricultural management (herbicide usage), microenvironment, and time. In this experiment, herbicide treatments were applied at the level of whole plots, with both treatments represented within each of four blocks. Both levels of variety and profile were each represented within each plot, with six replicates of each treatment added to each plot. The data description can be found in Section ??.

The following code prepares the dataset for analysis, and produces Fig. 1.10.

```
# Read data
litter <- read.csv("masslost.csv")

# Make sure the intended random effects (plot and block) are factors
litter$plot <- as.factor(litter$plot)
litter$block <- as.factor(litter$block)

# Represent date as number of days since the start of the experiment
library(lubridate)
```

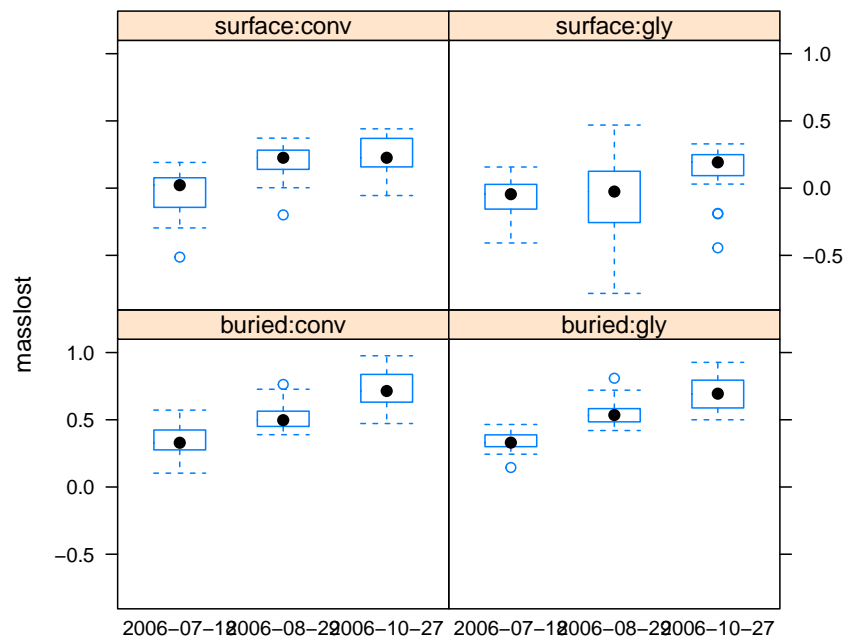


Figure 1.10: Proportion of litter mass lost from bags during field incubation as a function of microenvironment and herbicide program.

```
litter$date <- mdy(litter$date)
litter$date2 <- litter$date - ymd("2006-05-23")

# Quickly visualize the data to look for treatment effects
library(lattice)
bwplot(masslost ~ factor(date) | profile:herbicide, data=litter)
```

From inspecting Fig. 1.10, the buried litter appears to be decomposing faster than the surface litter (masslost is higher for buried compared to surface). If there are effects of herbicide (gly vs. conv), they are not immediately clear from the figure.

The blocking can be treated as a fixed effect or random effect. However, the design is unbalanced because some litter bags were lost, resulting in a variable number of litter bags recovered from each treatment. This affects the calculation of sums of squares, which vary depending on the order the terms are introduced to the model. It is therefore more appropriate to treat the blocking factor as a random effect, and use a mixed-effects model.

We first fit a simple linear model which ignores some details of the experimental design, and use block as a fixed effect. It is often very useful to start with a linear model, perhaps on subsets of the data, to gradually try to make sense of the data.

```
# Count the data to confirm that the design is unbalanced (ignore blocks for brevity)
ftable(xtabs(~ date2 + profile + herbicide, data=litter))

##           herbicide conv gly
## date2 profile
## 56    buried           22  22
##      surface           21  21
## 98    buried           23  23
```

```
##          surface          20  20
## 157      buried          19  16
##          surface          18  21

# Simple linear model with 'herbicide' as the first predictor in the model,
m1fix <- lm(masslost ~ date2 + herbicide * profile + block, data = litter)
anova(m1fix)

## Analysis of Variance Table
##
## Response: masslost
##              Df   Sum Sq Mean Sq  F value    Pr(>F)
## date2          1   3.1696   3.1696  122.0249 < 2.2e-16 ***
## herbicide       1   0.4327   0.4327   16.6579 6.113e-05 ***
## profile         1  13.7225  13.7225  528.3018 < 2.2e-16 ***
## block           3   0.4674   0.1558    5.9987 0.0005891 ***
## herbicide:profile 1   0.3207   0.3207   12.3475 0.0005284 ***
## Residuals      238   6.1820   0.0260
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# ... or listing 'profile' first in the model.
m2fix <- lm(masslost ~ date2 + profile * herbicide + block, data = litter)
anova(m2fix)

## Analysis of Variance Table
##
## Response: masslost
##              Df   Sum Sq Mean Sq  F value    Pr(>F)
## date2          1   3.1696   3.1696  122.0249 < 2.2e-16 ***
## profile         1  13.8335  13.8335  532.5749 < 2.2e-16 ***
## herbicide       1   0.3217   0.3217   12.3848 0.0005184 ***
## block           3   0.4674   0.1558    5.9987 0.0005891 ***
## profile:herbicide 1   0.3207   0.3207   12.3475 0.0005284 ***
## Residuals      238   6.1820   0.0260
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The sums of squares and p-values differ for profile and herbicide across the two fits (although they are still highly significant). Note that the order of the variables entered in the model matters because each next term is tested against a model that includes *all terms preceding it* (so-called Type-I tests). These standard tests with `anova` are sequential tests, which is perhaps not the most intuitive behaviour.

In many cases it is more intuitive to use so-called Type-II tests, in which each main effect is tested against a model that includes *all other terms*. We can use `Anova` (from the `car` package) to do this.

```
library(car)
Anova(m1fix, test="F")

## Anova Table (Type II tests)
##
## Response: masslost
##              Sum Sq Df  F value    Pr(>F)
## date2          3.7056  1 142.6601 < 2.2e-16 ***
## herbicide       0.3330  1  12.8198 0.0004157 ***
## profile        13.6594  1 525.8732 < 2.2e-16 ***
## block           0.4933  3   6.3311 0.0003793 ***
```

```
## herbicide:profile 0.3207 1 12.3475 0.0005284 ***
## Residuals        6.1820 238
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Anova(m2fix, test="F")

## Anova Table (Type II tests)
##
## Response: masslost
##
##      Sum Sq Df F value    Pr(>F)
## date2      3.7056 1 142.6601 < 2.2e-16 ***
## profile    13.6594 1 525.8732 < 2.2e-16 ***
## herbicide    0.3330 1 12.8198 0.0004157 ***
## block        0.4933 3  6.3311 0.0003793 ***
## profile:herbicide 0.3207 1 12.3475 0.0005284 ***
## Residuals    6.1820 238
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

If the data were in fact balanced, the sequential (Type-I) and Type-II tests would be identical.

We also have not yet accounted for the fact that multiple litter bags were placed within single plots and that the 'herbicide' treatment was applied at the level of the plots, not the individual bags, which further complicates the analysis. Treating `block` and `plot` as random effects addresses both the imbalance and the hierarchical nature of the design.

In this example, we specify the nested nature of the data (plots within blocks) in the formula for the random effects as `(1|block/plot)`.

```
# fit model with random effects, plots nested within blocks
litter_m1 <- lmer(masslost ~ date2 + herbicide * profile + (1|block/plot),
  data = litter)
# examine random effects, 'plot' explains essentially zero variance
VarCorr(litter_m1)

## Groups      Name      Std.Dev.
## plot:block (Intercept) 7.4201e-09
## block      (Intercept) 4.7726e-02
## Residual                    1.6117e-01

# refit model without 'plot'
litter_m2 <- lmer(masslost ~ date2 + herbicide * profile + (1|block),
  data = litter)
anova(litter_m1, litter_m2)

## refitting model(s) with ML (instead of REML)

## Data: litter
## Models:
## litter_m2: masslost ~ date2 + herbicide * profile + (1 | block)
## litter_m1: masslost ~ date2 + herbicide * profile + (1 | block/plot)
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## litter_m2  7 -183.7 -159.16 98.848   -197.7
## litter_m1  8 -181.7 -153.65 98.848   -197.7    0    1      1

# look at significance of main effects and interactions
Anova(litter_m2, test="F")
```

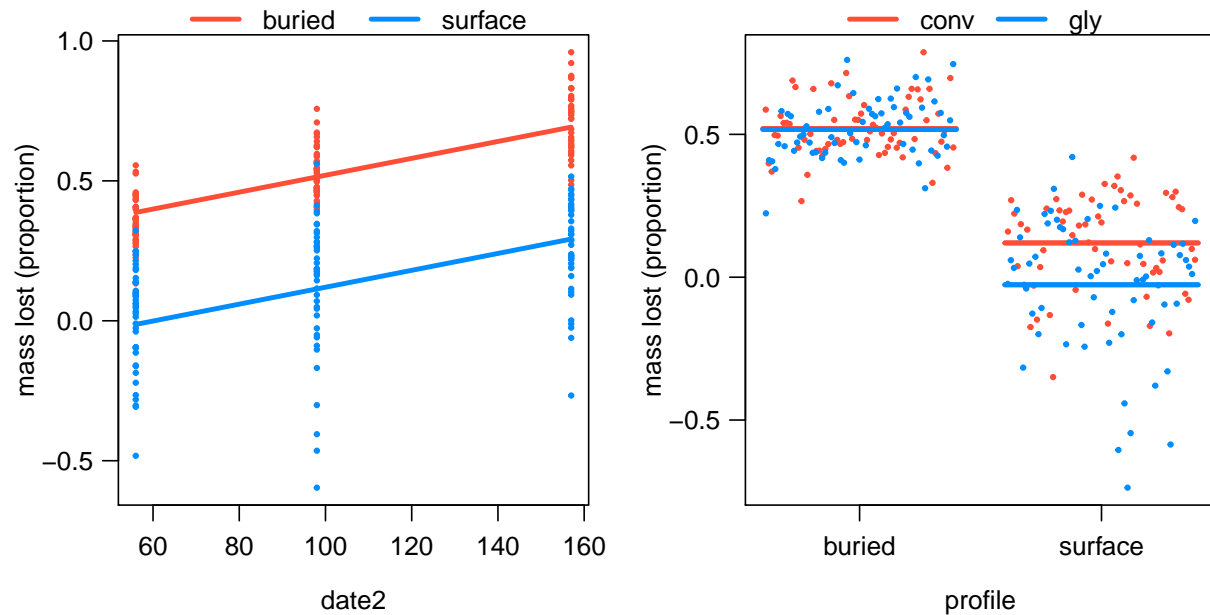


Figure 1.11: Visualization of the fixed effects of the mixed-effects model fit to the litter decomposition data.

```
## Analysis of Deviance Table (Type II Wald F tests with Kenward-Roger df)
##
## Response: masslost
##
```

|                   | F       | Df | Df.res | Pr(>F)        |
|-------------------|---------|----|--------|---------------|
| date2             | 142.509 | 1  | 238.04 | < 2.2e-16 *** |
| herbicide         | 12.746  | 1  | 238.03 | 0.0004315 *** |
| profile           | 526.217 | 1  | 238.03 | < 2.2e-16 *** |
| herbicide:profile | 12.184  | 1  | 238.09 | 0.0005743 *** |

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Note that in the model, we used `date2` as a numeric variable, which assumes that the relationship between `masslost` and `date2` is more or less linear. Figure 1.11 shows that this is a reasonable assumption. However, in the case where you have a timeseries where no transformation exists to linearize the relationship, you will have to represent your time variable as a factor. We return to this issue in Section 1.5.1, after we treat another repeated measures example where time was continuous in Section 1.5.1.

Finally we visualize the fit, to make sense of the fitted coefficients, and to make sure we draw the right conclusions as to the direction of the significant effects. As we saw in Chapter ??, we can use `visreg` to quickly visualize the fitted model.

```
library(visreg)

# Because we have three fixed effects, we can make two plots to visualize
# certain pairs of combinations.
visreg(litter_m2, "date2", by="profile",
       ylab='mass lost (proportion)', overlay=TRUE)
visreg(litter_m2, "profile", by="herbicide", cond=list(date2=100),
       ylab='mass lost (proportion)', overlay=TRUE)
```

---

## 1.4.1 Likelihood ratio tests

Another approach is to evaluate the importance of a term by comparing models that contain or do not contain that term using likelihood ratio tests (as already mentioned in Section 1.1.1). The recommended method for this is the `KRmodcomp` function (from the `pbkrtest` package), but the familiar `anova` can also be used (though p-values are approximate since likelihood ratios don't quite fit a chi-square distribution). You could also rely on model selection based on AIC (the lower the better). The following code shows all three approaches, and shows, as is often the case, that all methods show similar results.

```
# remove the interaction term from the model
litter_m2.int <- lmer(masslost ~ date2 + herbicide + profile + (1|block), data = litter)

# 1. anova
# Note that anova() will refit the models with ML (not REML) automatically,
# this is necessary when comparing models with different fixed or random effects terms.
anova(litter_m2, litter_m2.int)

## refitting model(s) with ML (instead of REML)

## Data: litter
## Models:
## litter_m2.int: masslost ~ date2 + herbicide + profile + (1 | block)
## litter_m2: masslost ~ date2 + herbicide * profile + (1 | block)
##           Df      AIC      BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## litter_m2.int  6 -173.68 -152.64 92.838  -185.68
## litter_m2      7 -183.70 -159.16 98.848  -197.70 12.02      1 0.0005263
##
## litter_m2.int
## litter_m2      ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# 2. KRmodcomp
library(pbkrtest)
KRmodcomp(litter_m2, litter_m2.int)

## F-test with Kenward-Roger approximation; computing time: 0.04 sec.
## large : masslost ~ date2 + herbicide * profile + (1 | block)
## small : masslost ~ date2 + herbicide + profile + (1 | block)
##           stat      ndf      ddf F.scaling  p.value
## Ftest  12.184      1.000 238.087          1 0.0005743 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# 3. AIC
AIC(litter_m2, litter_m2.int)

##           df      AIC
## litter_m2      7 -146.9159
## litter_m2.int  6 -141.5371
```

The model that includes the interaction provides the much better model fit. We can tell this by the significant p-value from `anova` and `KRmodcomp` result and by the lower AIC score for the model that includes an interaction. When an interaction is significant, the automatic follow-up question is 'what is the source of this interaction?'. Again inspecting Fig. 1.11, it appears that the herbicide treatments affected decomposition differently, but only on the surface of the soil. To further understand the nature of the interactions, it is useful to combine variables into a single variable, as the following example illustrates.

---

```

# Create a new variable containing the combinations of herbicide and profile.
# This new variable will have 4 levels
litter$combtrt <- paste(litter$herbicide, litter$profile, sep='-')

# Lump all observations for which the bags were buried into a single level,
# we now have just three levels in the new combined variable.
litter$combtrt[litter$profile == 'buried'] <- 'buried'

# Make this new variable into a factor
litter$combtrt <- as.factor(litter$combtrt)

# Fit a models using this new factor (3 levels) and a model without herbicide (2 levels)
litter_m3 <- lmer(masslost ~ date2 + combtrt + (1|block), data = litter, REML=FALSE)
litter_m3.herb <- lmer(masslost ~ date2 + profile + (1|block), data = litter, REML=FALSE)

# Compare the models by AIC (lower is 'better')
AIC(litter_m2, litter_m3, litter_m3.herb)

##           df      AIC
## litter_m2    7 -146.9159
## litter_m3    6 -185.6860
## litter_m3.herb 5 -163.6908

```

The model with the lowest AIC is m3, which is the model describing the relationship where herbicide affected decomposition rates only at the soil surface (because for that model, we combined all 'buried' litter samples into one level, regardless of the herbicide application).

## 1.5 Example: repeated measures in tree measurements

This example shows a very common use of mixed-effects models in repeated measurements. The basic idea is that when you have measurements on the same individuals (or plots, or some other unit) over time, you cannot treat the measurements as independent because that would be pseudo-replication, inflation of your sample size, and anti-conservative conclusions about significant effects.

We use data from the Hawkesbury Forest Experiment irrigation by fertilisation experiment (HFEIF, see Section ?? for description of the data). In this experiment, sixteen plots of 72 *Eucalyptus saligna* trees were remeasured 20 times for height and diameter (although on a number of dates, not all trees were measured). Four treatments were applied (control, irrigated, fertilised, irrigated + fertilised). We ask in the following example whether tree height differs by treatment.

It is important that you recognize that the experimental unit in this example is the plot, not the tree, because the treatments were applied at a plot level. We therefore have to take into account the fact that trees are nested in plots, to avoid pseudoreplication.

```

# Read data, make proper date and make sure the intended factor variables are factors.
hfeif <- read.csv("HFEIFbytree.csv")
hfeif$Date <- as.Date(hfeif$Date)

# Make sure plot number (plotnr) is a factor; it is read in as a numeric variable.
hfeif$plotnr <- as.factor(hfeif$plotnr)

# Days since start of experiment
# The as.numeric statement converts this into a simple numeric variable
hfeif$Time <- as.numeric(with(hfeif, Date - min(Date)))

```



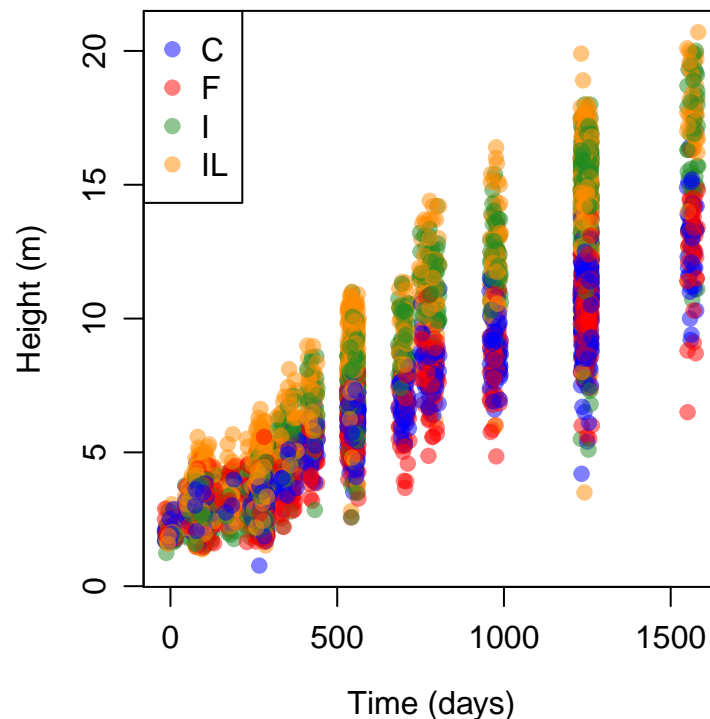


Figure 1.12: Plot tree height over time for the HFE irrigation x fertilisation experiment.

Before we do anything, always explore the data with a few simple figures. Here we show tree height over time, colored by treatment (Fig. 1.12). The data show some separation between at least some of the treatments over time. Also note that the increase in height over time is perhaps not exactly linear, but we will ignore this in the remainder of the example (and further note that no straightforward transformation exists in this case).

Note the use of `jitter` in the example below, this adds some random noise to the Time variable to avoid excessive overlap of data points on each Date. We also use `sample` to randomly reorder the rows of the dataset to avoid the final treatment in the dataset to be plotted on top (this way, it is easier to see treatment differences).

The following code produces Fig. 1.12.

```
# For the alpha() function (transparency)
library(scales)

# Set colours, transparent
palette(alpha(c("blue","red","forestgreen","darkorange"), 0.5))
with(hfeif[sample(nrow(hfeif)),],
     plot(jitter(Time,3), height, col=treat, pch=19,
          xlab="Time (days)", ylab="Height (m)"))
legend("topleft", levels(hfeif$treat), pch=19, col=palette())
```

Again, the reason we want to use mixed-effects models in this case is because we want to use the correct number of degrees of freedom to test for the treatment effect. If you are not sure what that should be, let's start with a linear model on the data from just one date, when we have averaged the data by plot (our experimental unit). In this case we can simply use `lm`, as follows.

The following example produces Fig. 1.13.

---

```

# Take subset of data at last Date
hfeif_last <- subset(hfeif, Date == max(Date))

# Average all variables by plot (and include the 'treat' factor variable in the result)
library(doby)
hfeif_last_plot <- summaryBy(. ~ Date + plotnr, data=hfeif_last,
                             FUN=mean, na.rm=TRUE,
                             id=~treat, keep.names=TRUE)

# Linear model with treatment only.
lm_last <- lm(height ~ treat, data=hfeif_last_plot)

# Note that height is highly significant, and that we use 3 numerator df
# to test for treatment effects
anova(lm_last)

## Analysis of Variance Table
##
## Response: height
##           Df Sum Sq Mean Sq F value    Pr(>F)
## treat       3  74.160  24.7201   19.137 7.226e-05 ***
## Residuals  12  15.501   1.2917
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# A quick visualization of the fitted model shows much taller trees
# in I and IL,
library(visreg)
visreg(lm_last, "treat", xlab="Treatment", ylab="Tree height (m)")

```

The above shows 3 numerator degrees of freedom in the F-test, which makes sense because we have 4 levels of our treatment (thus  $df = 4 - 1$ ). You can do no such simple check for the denominator degrees of freedom, but it's a useful check nonetheless.

To account for the repeated measures nature of the data as well as the fact that the experimental unit is the plot, not the tree, all we need is to specify the plot as the random effect. We will fit two models, one without and one with the interaction between Time and treat, and again use Anova (from the car package) to test for significant effects.

Note that we specify Time as a numeric variable, which assumes that the relationship between height and Time is more or less linear, which according to Figure 1.12 is a reasonable assumption. We return to this issue in Section 1.5.1.

```

# Effect of treatment on intercept only.
lmeif1 <- lmer(height ~ treat + Time + (1|plotnr), data=hfeif)
Anova(lmeif1)

## Analysis of Deviance Table (Type II Wald chisquare tests)
##
## Response: height
##           Chisq Df Pr(>Chisq)
## treat    27.752  3  4.095e-06 ***
## Time  48339.942  1  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# Effect of treatment on intercept and slope (i.e. main effect + interaction)

```

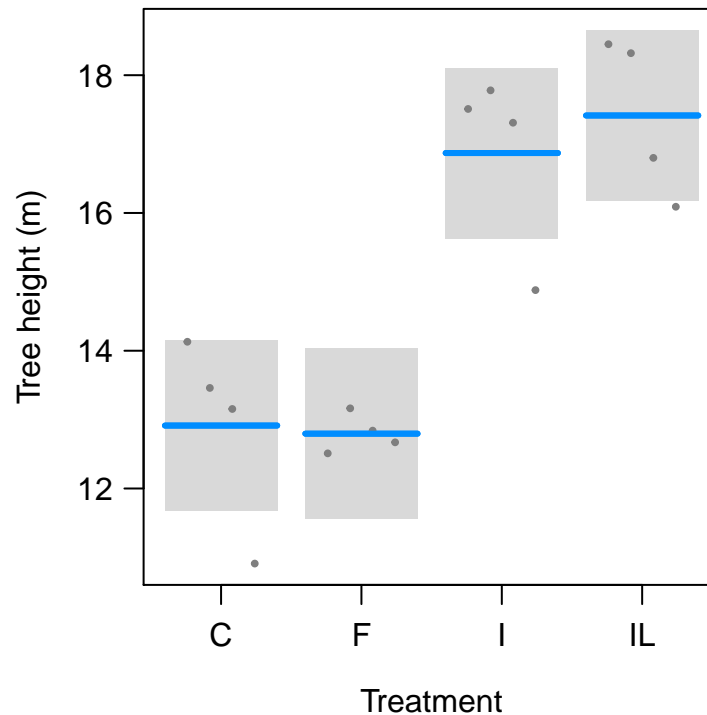


Figure 1.13: Simple visualization of fitted linear model (with `lm`) of tree height on the last date of the HFE IF data.

```
lmeif2 <- lmer(height ~ treat*Time + (1|plotnr), data=hfeif)
Anova(lmeif2)

## Analysis of Deviance Table (Type II Wald chisquare tests)
##
## Response: height
##               Chisq Df Pr(>Chisq)
## treat         27.389  3  4.879e-06 ***
## Time        72342.647  1 < 2.2e-16 ***
## treat:Time   3208.442  3 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# A likelihood ratio tests shows the interaction is highly significant
anova(lmeif1, lmeif2)

## refitting model(s) with ML (instead of REML)

## Data: hfeif
## Models:
## lmeif1: height ~ treat + Time + (1 | plotnr)
## lmeif2: height ~ treat * Time + (1 | plotnr)
##           Df   AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## lmeif1    7 22654 22702 -11320   22640
## lmeif2   10 20054 20122 -10017   20034 2605.9    3 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

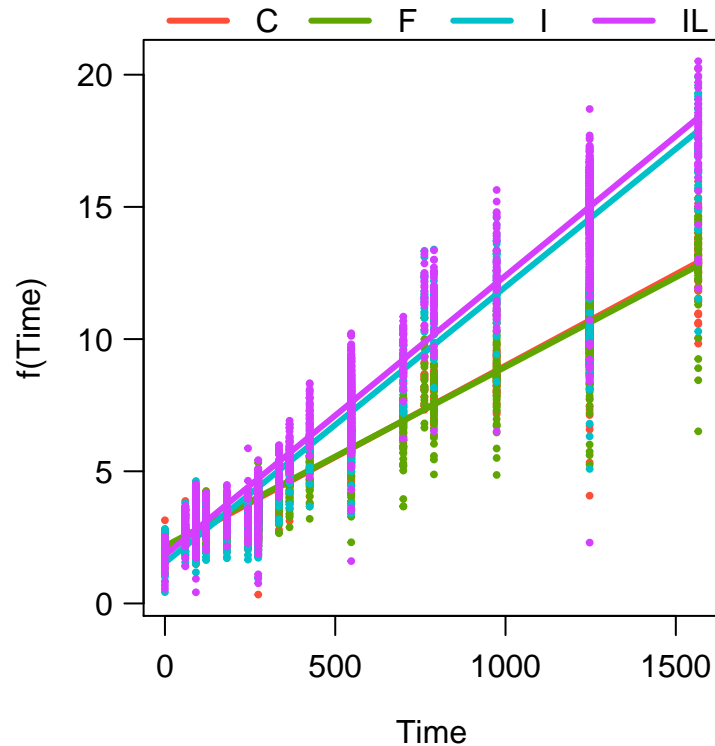


Figure 1.14: Visualized effect of Time on height, by treatment for the HFE IxF dataset, fitted with a linear mixed-effects model.

**Try this yourself** Repeat the likelihood-ratio test with the more accurate `KRmodcomp` function from the `pbkrtest` package.

Note that in the above, we have correctly used 3 numerator degrees of freedom to test for the effect of treatment on height. The individual `Anova` statements summarize the significance of the fixed effects in each model, whereas the `anova` of the two models uses a likelihood-ratio test on the two models. In this case, it effectively tests for significance of the interaction (because the only difference between the two models was the inclusion of the `treat` by `Time` interaction in `lmeif2`). The interaction is overwhelmingly significant.

The final step is to try to understand this interaction, how large is the effect size, and which direction does it point? The time by treatment interaction is significant, but in which way? It is never sufficient in an analysis to state that an interaction was 'significant', we must make more sense of it. One simple approach is to use the `visreg` package to visualize the fit (see Fig. 1.14).

```
library(visreg)
visreg(lmeif2, "Time", by="treat", overlay=TRUE)
```

In this case it is abundantly clear that irrigated (I) and irrigated + fertilised (IL) have a steeper slope of height with Time (that is, they have a faster height growth), compared to control (C) and fertilised (F). If there was no significant interaction (or a small effect size), the lines would be parallel to each other.

We can further look at the p-values for the individual effects (slopes and intercepts by treatment). Note that p-values in the `summary` statement are only computed if we have loaded the `lmerTest` package before fitting the model. Consider this example,

---

```

# Loading this package first affects both summary and anova methods
library(lmerTest)

# ... we must refit the model after loading lmerTest
lmeif2 <- lmer(height ~ treat*Time + (1|plotnr), data=hfeif)

# Print just the coefficients table from the summary
summary(lmeif2)$coefficients

##              Estimate   Std. Error      df    t value
## (Intercept)  2.073764669 2.992535e-01  12.32194   6.9297919
## treatF       0.093074081 4.233121e-01  12.33402   0.2198711
## treatI      -0.535359455 4.232604e-01  12.32800  -1.2648466
## treatIL     -0.246322580 4.232390e-01  12.32551  -0.5819940
## Time         0.006921244 6.399533e-05 6453.01159 108.1523211
## treatF:Time  -0.000141759 9.111223e-05 6453.02355  -1.5558723
## treatI:Time   0.003514427 9.095049e-05 6453.00909  38.6411001
## treatIL:Time  0.003645960 9.085136e-05 6453.02674  40.1310445
##              Pr(>|t|)
## (Intercept)  1.377535e-05
## treatF       8.295686e-01
## treatI       2.293139e-01
## treatIL      5.710629e-01
## Time         0.000000e+00
## treatF:Time  1.197875e-01
## treatI:Time  5.042934e-294
## treatIL:Time 1.392970e-314

```

**Try this yourself** The `lmerTest` package also modifies the `anova` function, so that it calculates p-values for a fitted model with `lmer`. Compare `anova(lmeif2)` with `Anova(lmeif2)`, these will rarely be exactly the same as they use different methods to approximate the degrees of freedom of the random effects.

Looking at the interaction terms (`treat:Time`), the summary table shows that `treatF:Time` is not significantly different from the first level (`treatC:Time`), that is, there is no difference between fertilized and control in terms of the interaction with Time. But both irrigated (I) and irrigated + fertilised (IF) are highly significant, again, this comparison is in relation to the first level of the factor (control, C).

Although there is a significant main effect of treatment, none of the levels are actually different from the first (the control). This shows that the intercept itself is different from zero, but the treatments are not actually different in terms of the intercept. This makes sense, because seedlings were planted at time zero before any treatment was applied.

## 1.5.1 Repeated measures: is time numeric or factor?

In both examples in this chapter where we used time as a predictor in our models, we treated time as a continuous (numeric) variable. This was appropriate in both cases because the relationship between the dependent variable (`masslost` or `height`) showed a nearly linear relationship with time, allowing us to estimate and interpret an intercept and a slope of the variable with time. In the example with the tree height measurements, the slope of `height` with time can actually be interpreted as the height growth rate.

But there are many cases in which it would be more appropriate to use time as a factor variable. These include cases where the relationship is highly non-linear and cannot be transformed, or you only have

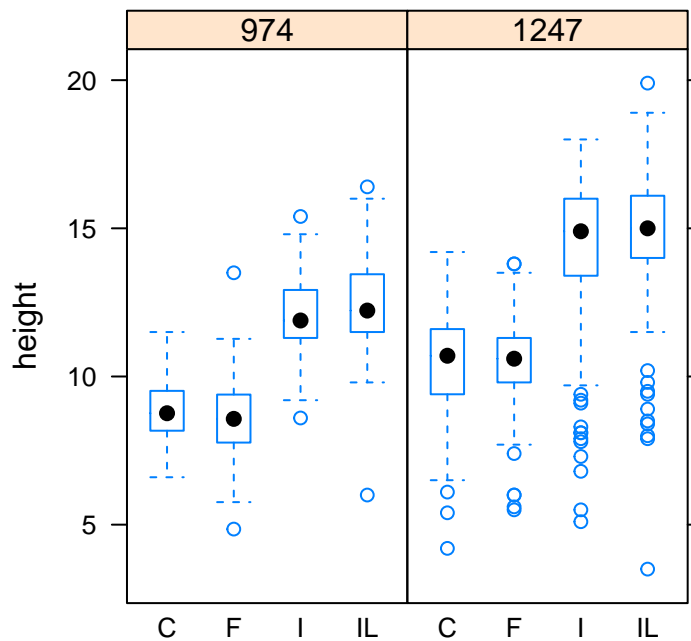


Figure 1.15: Boxplots of height vs. treatment and time (days since start of experiment, shown in the panel label) for a subset of the HFE IF data.

two or three dates of measurements. The example below shows a simple example for the latter case. (see also Fig. 1.15).

```
# A repeated measures example with only two dates of measurement.
# Though it is possible to have time as a continuous variable, it is much
# more useful to code it as a factor.

# We take a subset of hfeif.
hfeif2 <- subset(hfeif, Date %in% as.Date(c("2010-09-01", "2011-06-01")))

# Convert Time to a factor
hfeif2$Time_fac <- as.factor(hfeif2$Time)

# As before, we can quickly use bwplot to inspect the data
library(lattice)
bwplot(height ~ treat | Time_fac, data=hfeif2)
```

**Try this yourself** Repeat the above example, but using the entire dataset (rather than a subset of the data for two dates). Inspect the model with `Anova`, and also look at the `summary` statement.

Let's fit the mixed-effects model on this small subset of the data to test whether a) treatment affects height, b) there is an effect of time on height, c) there is an interaction (i.e. height response to treatment depends on time).

```
# Fit the model
lmeif4 <- lmer(height ~ treat*Time_fac + (1|plotnr), data=hfeif2)
```

```
# Overall significance shows no interaction
Anova(lmeif4, test="F")

## Analysis of Deviance Table (Type II Wald F tests with Kenward-Roger df)
##
## Response: height
##
##          F Df Df.res    Pr(>F)
## treat      23.0111  3     12 2.89e-05 ***
## Time_fac    359.1285  1    1333 < 2.2e-16 ***
## treat:Time_fac  2.8048  3    1333  0.03859 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

**Try this yourself** Use `visreg` to visualize the fit, and compare it to the box plots produced above.

## 1.6 Example: repeated measures in leaf photosynthesis

In this example we look at another case of repeated measurements, this time for measurements of leaf photosynthesis at the EucFACE. Measurements were repeated during four campaigns in 2013 (labelled simply as A-D), in all six rings at the EucFACE (three of which have an ambient CO<sub>2</sub> concentration, and three an elevated CO<sub>2</sub> concentration), on a few trees per ring. The random effects naturally follow from the experimental design, as measurements were performed on trees nested within rings.

In this example we are particularly interested in comparing the four dates with each other - we would like to know not only if leaf photosynthesis was enhanced by elevated CO<sub>2</sub>, but also whether this enhancement differed between the campaigns.

We start out with a simple barplot showing averages by campaign and treatment. The following code produces Fig. 1.16. Note that the error bars cannot be used for judging differences between treatments, as these are simply calculated across all the individual data points.

```
eucgas <- read.csv("eucface_gasexchange.csv")

library(sciplot)
bargraph.CI(Date, Photo, CO2, data=eucgas, legend=TRUE,
            ylab="Photo")
```

Next, we fit the mixed-effects model. We have just two fixed effects in the model: `Date` and `CO2`, and the random effects structure is `tree` within `ring` (`1|Ring/Tree`). We can test whether we need to include a `Date x CO2` interaction by fitting two models and comparing them with a likelihood ratio test, as before.

We also load the `lmerTest` package here because we would like to inspect the p-values for the individual coefficients printed in the `summary` statement, in what follows.

```
library(lmerTest)

eucgas_m0 <- lmer(Photo ~ Date + CO2 + (1|Ring/Tree), data=eucgas)
eucgas_m1 <- lmer(Photo ~ Date * CO2 + (1|Ring/Tree), data=eucgas)

# Compare the two models - this tests for the significance of Date:CO2
library(pbkrtest)
KRmodcomp(eucgas_m0, eucgas_m1)
```

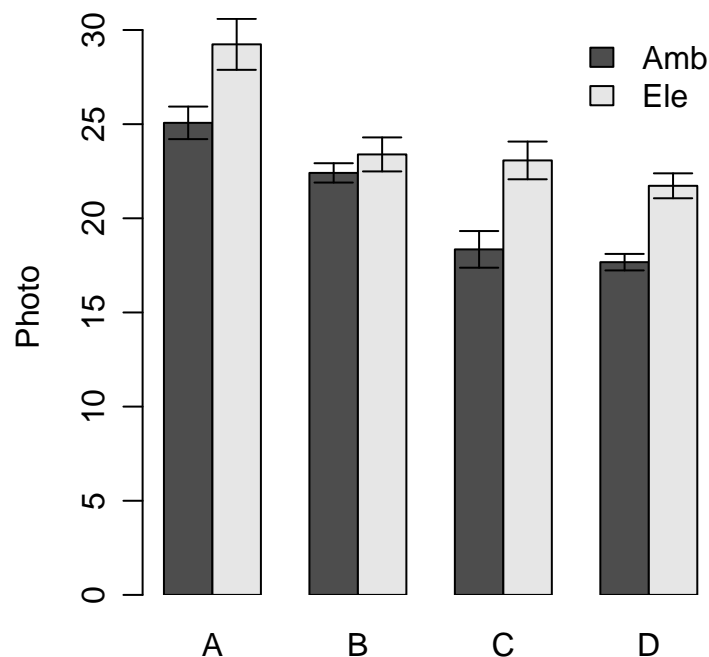


Figure 1.16: Average net leaf photosynthesis (Photo) by Date and CO<sub>2</sub> treatment for the `eucgas` dataset.

```
## F-test with Kenward-Roger approximation; computing time: 0.05 sec.
## large : Photo ~ Date * CO2 + (1 | Ring/Tree)
## small : Photo ~ Date + CO2 + (1 | Ring/Tree)
##          stat      ndf      ddf F.scaling p.value
## Ftest    2.0218   3.0000 56.7654   0.99995  0.1211
```

The F-test shows that the interaction is not significant ( $p = 0.12$ ). Make sure that you understand the difference between these two models: the interaction allows for a different CO<sub>2</sub> response at different dates. The model fits are visualized in Fig. 1.17, produced by the following code.

```
visreg(eucgas_m0, "Date", by="CO2", overlay=TRUE)
visreg(eucgas_m1, "Date", by="CO2", overlay=TRUE)
```

*Note:* if you are using an older version of `visreg`, you will see some warnings printed that you can ignore.

Comparing the two panels in Fig. 1.17, we are not quite yet convinced that the CO<sub>2</sub> response was the same for all dates. For Date B, the response seems particularly small. Also, although the F-test above did not give us evidence for a significant interaction, the model with the interaction does have a lower AIC, suggesting it does improve the model somewhat:

```
AIC(eucgas_m0, eucgas_m1)
##          df      AIC
## eucgas_m0   8 413.7925
## eucgas_m1  11 406.0069
```

Before we continue, it is a good idea as always to check the model diagnostics. The following code makes a plot (Fig. ??) of the residuals versus fitted, and a quantile-quantile plot of the residuals. These diagnostics look quite good, so we are happy to continue.



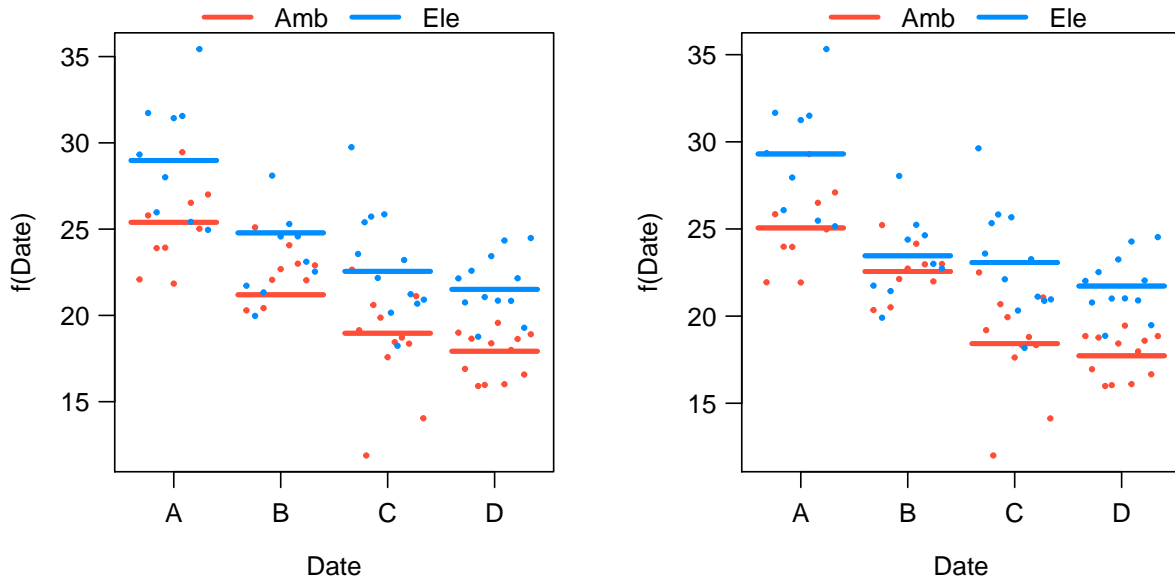


Figure 1.17: A model for net leaf photosynthesis without (left) and with (right) an interaction between Date and CO<sub>2</sub> treatment.

```
plot(eucgas_m1)

library(car)
qqPlot(residuals(eucgas_m1))

## [1] 61 67
```

To address our question, we are interested to find out whether there was a significant CO<sub>2</sub> response on each of the four dates. With the standard way that **R** codes the factor levels, this is not straightforward to address. Recall that each factor level is tested against the first level, as we can see in the coefficients printed as part of the `summary(eucgas_m1)`:

| ##              | Estimate   | Std. Error | df       | t value    | Pr(> t )   |     |
|-----------------|------------|------------|----------|------------|------------|-----|
| ## (Intercept)  | 25.0602675 | 0.9256916  | 74.12448 | 27.0719398 | < 2.22e-16 | *** |
| ## DateB        | -2.4997186 | 1.2432792  | 57.94872 | -2.0105851 | 0.0490306  | *   |
| ## DateC        | -6.6395257 | 1.1843577  | 57.67173 | -5.6060141 | 6.1091e-07 | *** |
| ## DateD        | -7.3377824 | 1.1647215  | 59.04853 | -6.3000316 | 4.0758e-08 | *** |
| ## C02Ele       | 4.2444231  | 1.3099306  | 73.91283 | 3.2401892  | 0.0017917  | **  |
| ## DateB:C02Ele | -3.3455189 | 1.7475408  | 54.84093 | -1.9144153 | 0.0607901  | .   |
| ## DateC:C02Ele | 0.4082904  | 1.6564477  | 56.76677 | 0.2464855  | 0.8061956  |     |
| ## DateD:C02Ele | -0.2414369 | 1.6424652  | 57.44633 | -0.1469967 | 0.8836493  |     |

In this table, (Intercept) is the estimated Photo for the first Date (A), for the first level of CO<sub>2</sub> (Amb). All other values are expressed relative to this value. Thus, C02Ele tests the CO<sub>2</sub> effect for the first Date. It shows that for Date A, Photo was 4.24 units higher in elevated compared to the ambient treatment. The next three parameters (DateB:C02Ele and so on) are expressed relative to the first date, so that DateB:C02Ele tests the difference in CO<sub>2</sub> response between Date B and Date A.

This table therefore does not address our question: was CO<sub>2</sub> significant on each date? A straightforward way to compute the right tests is to refit the model without an intercept, so that the individual coefficients are not tested against the first level, but against zero.

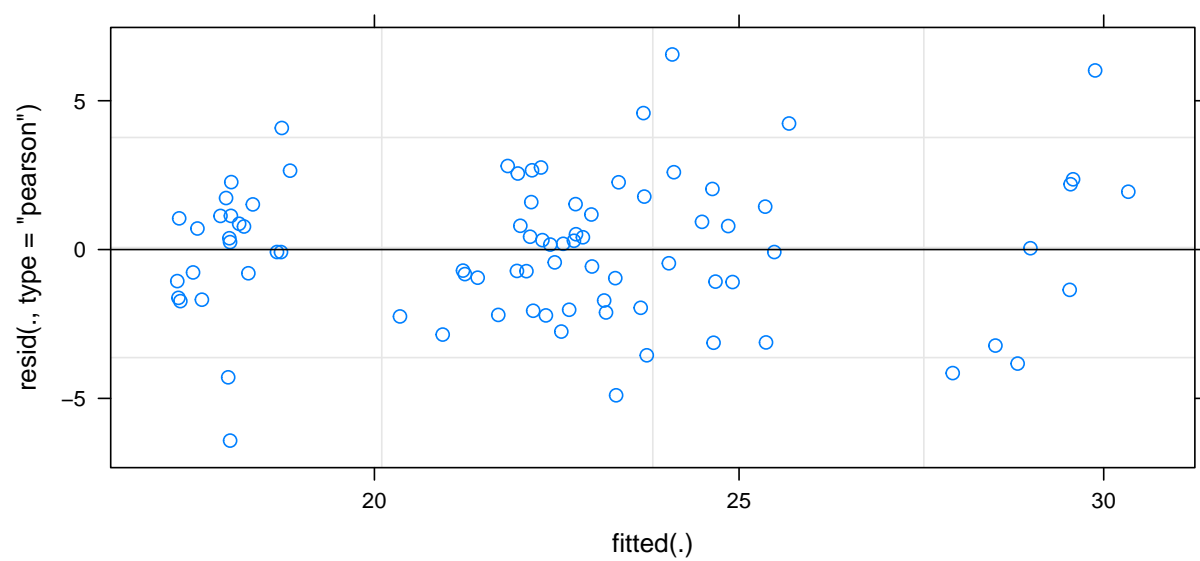


Figure 1.18: Diagnostic plots for the eucgas\_m1 model.

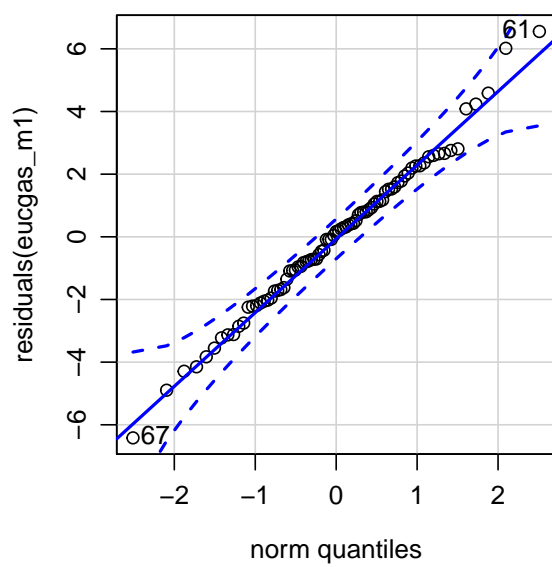


Figure 1.19: Diagnostic plots for the eucgas\_m1 model.

---

```
# Refit the model, without an intercept ("-1"), and including CO2 only as
# the interaction
eucgas_m2 <- lmer(Photo ~ Date + Date:CO2 -1 + (1|Ring/Tree), data=eucgas)

# Note that this model is exactly the same as before - see the AIC for example
AIC(eucgas_m1, eucgas_m2)

##           df          AIC
## eucgas_m1 11 406.0069
## eucgas_m2 11 406.0069
```

The new model expresses the coefficients very differently, as we can see from this extract from `summary(eucgas_m2)`:

```
##           Estimate Std. Error      df    t value    Pr(>|t|)
## DateA          25.0602675   0.9256916  74.12448  27.0719398 < 2.22e-16 ***
## DateB          22.5605489   0.9262188  73.94295  24.3576877 < 2.22e-16 ***
## DateC          18.4207419   0.8403054  72.50729  21.9214852 < 2.22e-16 ***
## DateD          17.7224852   0.8051202  72.03373  22.0122233 < 2.22e-16 ***
## DateA:CO2Ele    4.2444231   1.3099306  73.91283   3.2401892 0.00179168 **
## DateB:CO2Ele    0.8989041   1.3103033  73.81486   0.6860276 0.49484494
## DateC:CO2Ele    4.6527134   1.1640994  72.09041   3.9968350 0.00015330 ***
## DateD:CO2Ele    4.0029861   1.1389613  71.82238   3.5145936 0.00076672 ***
```

This time, the first four rows give the estimates for the four dates in the ambient CO<sub>2</sub> treatment, together with the p-values of a test against zero (not very meaningful in this case). The next four rows give the actual CO<sub>2</sub> response, that is, the difference between elevated and ambient CO<sub>2</sub> for each of the four dates. Since we loaded the `lmerTest` package before fitting this model, the p-values are computed for each of these effects. The results show that, as we suspected, there was no significant CO<sub>2</sub> effect on Date B ( $p = 0.49$ ).

Finally we show another method to test individual coefficients against zero, using the `glht` function from the `multcomp` package. We have met this package before for performing Tukey tests on fitted models (in the introductory, week-long workshop).

To test coefficients against zero, we can do:

```
library(multcomp)
summary(glht(eucgas_m2, linfct=c("DateA:CO2Ele = 0",
                                "DateB:CO2Ele = 0",
                                "DateC:CO2Ele = 0",
                                "DateD:CO2Ele = 0"))))

##
## Simultaneous Tests for General Linear Hypotheses
##
## Fit: lmer(formula = Photo ~ Date + Date:CO2 - 1 + (1 | Ring/Tree),
## data = eucgas)
##
## Linear Hypotheses:
##           Estimate Std. Error z value Pr(>|z|)
## DateA:CO2Ele == 0   4.2444    1.3099   3.240 0.004757 **
## DateB:CO2Ele == 0   0.8989    1.3103   0.686 0.932107
## DateC:CO2Ele == 0   4.6527    1.1641   3.997 0.000257 ***
## DateD:CO2Ele == 0   4.0030    1.1390   3.515 0.001762 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)
```

---

Note that the p-values will be different from the ones computed above because a) `lmerTest` approximates the degrees of freedom and uses a t-test, whereas `glht` relies on a normal approximation (the two will be closer together for larger sample sizes), and b) `glht` adjusts the p-values for multiple comparisons. This time, though, the conclusions are no different using either method.

Using the `glht` function, we can also test various parameter combinations quite easily. This final example shows how we can test whether the CO<sub>2</sub> response was different on Date C from Date D:

```
summary(glht(eucgas_m2, linfct="DateC:CO2Ele - DateD:CO2Ele = 0"))

##
##   Simultaneous Tests for General Linear Hypotheses
##
## Fit: lmer(formula = Photo ~ Date + Date:CO2 - 1 + (1 | Ring/Tree),
##   data = eucgas)
##
## Linear Hypotheses:
##
##              Estimate Std. Error z value Pr(>|z|)
## DateC:CO2Ele - DateD:CO2Ele == 0  0.6497      1.5232   0.427    0.67
## (Adjusted p values reported -- single-step method)
```

From this test we can conclude that there is no evidence to suggest that CO<sub>2</sub> increased photosynthesis more on Date C compared to Date D.

## 1.7 Example: analysis of count data with the ground-cover data (glmer)

The EucFACE ground cover dataset (see Section ??) contains estimates of plant and litter cover within the rings of the EucFACE experiment, evaluating forest ecosystem responses to elevated CO<sub>2</sub>, on two dates. There are six rings (Ring), three for each treatment (Trt; ambient and elevated CO<sub>2</sub>). Within each ring are four plots (Plot) and within each plot are four 1m by 1m subplots (Sub). Here we will test for an interaction between Trt and Date on ground cover measurements of Forbes (these are count data).

The following code produces Fig. 1.20, showing a plot of the raw groundcover data.

```
# read data and convert random effects to factors
eucface <- read.csv("eucfaceGC.csv")
eucface$Ring <- as.factor(paste(eucface$Ring, eucface$Trt, sep='-'))
eucface$Plot <- as.factor(eucface$Plot)
eucface$Sub <- as.factor(eucface$Sub)

# load packages
library(lme4)
library(lattice)

# A quick plot to visualize ground cover by Date and Ring.
# Colours represent 'Plot'.
xyplot(Forbes~Date|Ring, groups=Plot, data=eucface, pch=16, jitter.x=T)
```

Since the data are count data, it is usually appropriate to use the Poisson distribution. The following code fits a `glmer` with the `poisson` error family, and produces diagnostic plots in Fig. 1.21.

To usual way to decide on the appropriate family is to inspect the diagnostic plots, especially a plot of residuals versus the fitted values. In the following, we fit the model three times, first with normal

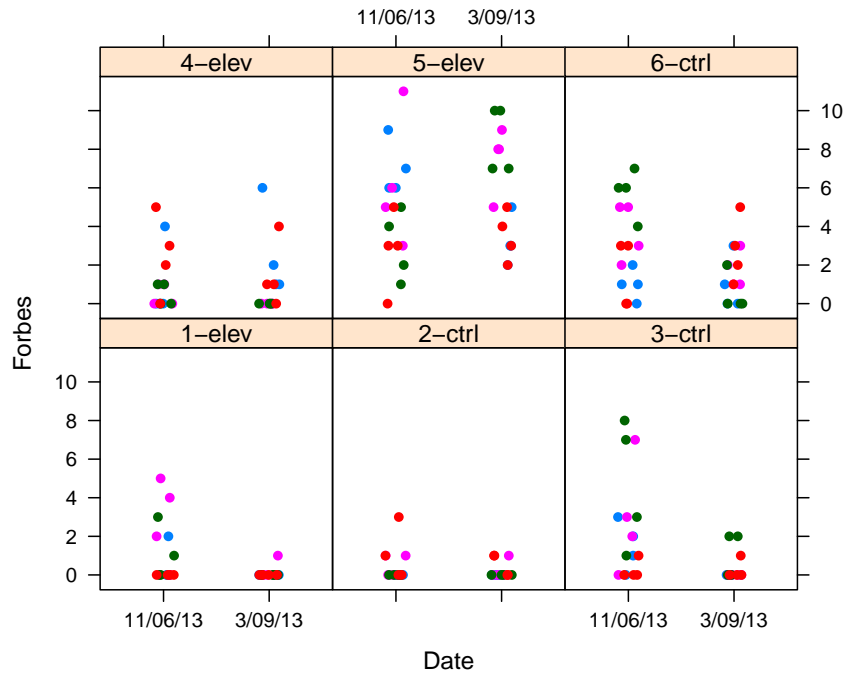


Figure 1.20: Counts of forbs within plant communities exposed to ambient or elevated carbon dioxide concentrations at two dates. Points represent estimates within subplots, subplots common to the same plot within each ring have a common colour.

errors (using `lmer`), then with Poisson errors (the distribution we expect to be appropriate), and a Poisson distribution with square-root link function.

The following code produces three diagnostic plots (Fig. 1.21).

```
forb.norm <- lmer(Forbes~Date*Trt+(1|Ring/Plot/Sub),
                 data=eucface)

forb.pois <- glmer(Forbes~Date*Trt+(1|Ring/Plot/Sub),
                  family=poisson, data=eucface)

forb.pois.sqrt <- glmer(Forbes~Date*Trt+(1|Ring/Plot/Sub),
                       family=poisson('sqrt'), data=eucface)

# The standard plot() on an (g)lmer object makes a 'grid-based'
# plot. To arrange plots side-by-side we have to use gridExtra, like so.
library(gridExtra)
p1 <- plot(forb.norm, main="Normal")
p2 <- plot(forb.pois, main="Poisson (log-link)")
p3 <- plot(forb.pois.sqrt, main="Poisson (sqrt-link)")
grid.arrange(p1,p2,p3,ncol=3)
```

Note that the syntax of `glmer` is identical to that of `lmer`, with the exception of the `family` argument.

Inspecting Fig. 1.21, the residuals look better for the models fit with the Poisson family compared to the gaussian error (as assumed by `lmer`). The residuals improve a bit more when using the `sqrt` link function in the Poisson family.

It is however difficult to assess the model goodness of fit with these plots. A more dependable ap-

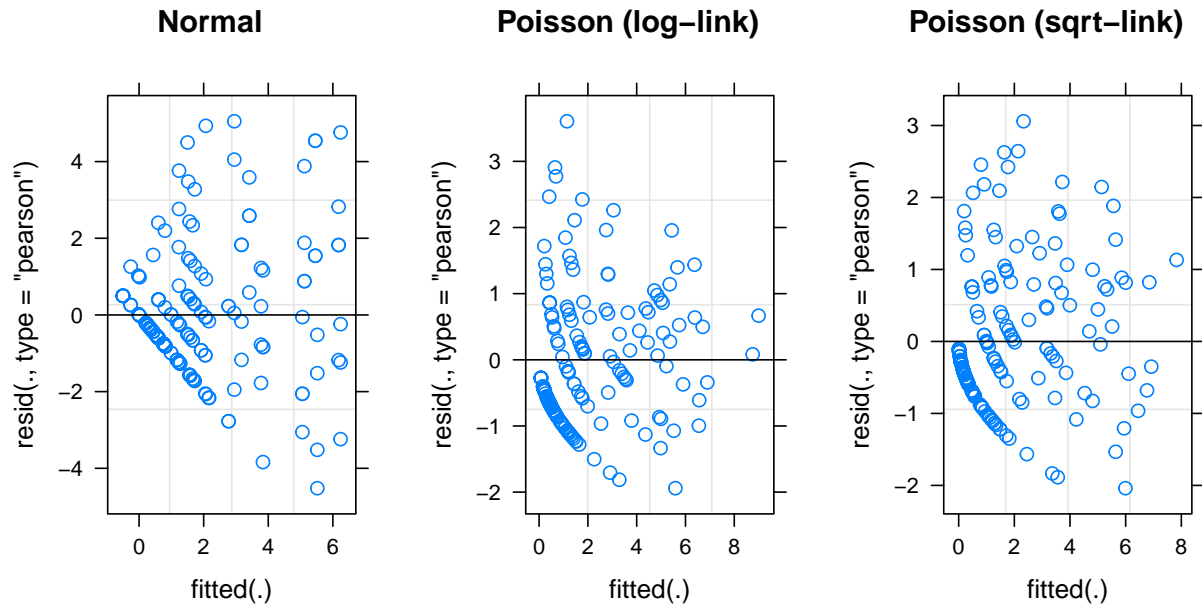


Figure 1.21: Diagnostic plots for three model fits of the EucFACE groundcover data.

proach is provided by the `DHARMa` package. The function `simulateResiduals` simulates a number of residuals by drawing randomly from the specified error distribution (e.g. binomial or normal distribution), and then plotting the distribution of these simulated residuals against the distribution of the actual residuals. If the error distribution was specified correctly, a quantile-quantile plot of the two distributions will show a straight line.

To use this, try the following. This produces two plots, a QQ plot of the simulated vs. observed residuals, and residuals versus fitted with quantiles shown. The following example makes Fig. 1.22.

```
library(DHARMa)
r <- simulateResiduals(forb.pois.sqrt)
plotSimulatedResiduals(r)
```

Note that the fit of the model may be improved further by using one of the other plant cover variables as a covariate or by incorporating other data from the site, but for the purpose of this chapter the fit is good enough.

Something else to consider when it comes to poisson errors is that overdispersion can result in the underestimation of error terms for the model coefficients. Overdispersion occurs when you have a large number of zeros in the data and/or an important predictor is not accounted for. Testing hypotheses from models where overdispersion is evident is dangerous as the probability of Type I error is increased. A good model fit should result in the ratio of residual deviance to degrees of freedom being close to one.

```
# Calculate the residual deviance
sum(resid(forb.pois.sqrt, type='pearson')^2)
## [1] 171.408

# The degrees of freedom for the residual,
df.residual(forb.pois.sqrt)
## [1] 185
```

Overdispersion does not appear to be a problem here as the ratio (171.4 / 185) is less than one. We can also perform a statistical test using functions in the `DHARMa` package to confirm. For example:

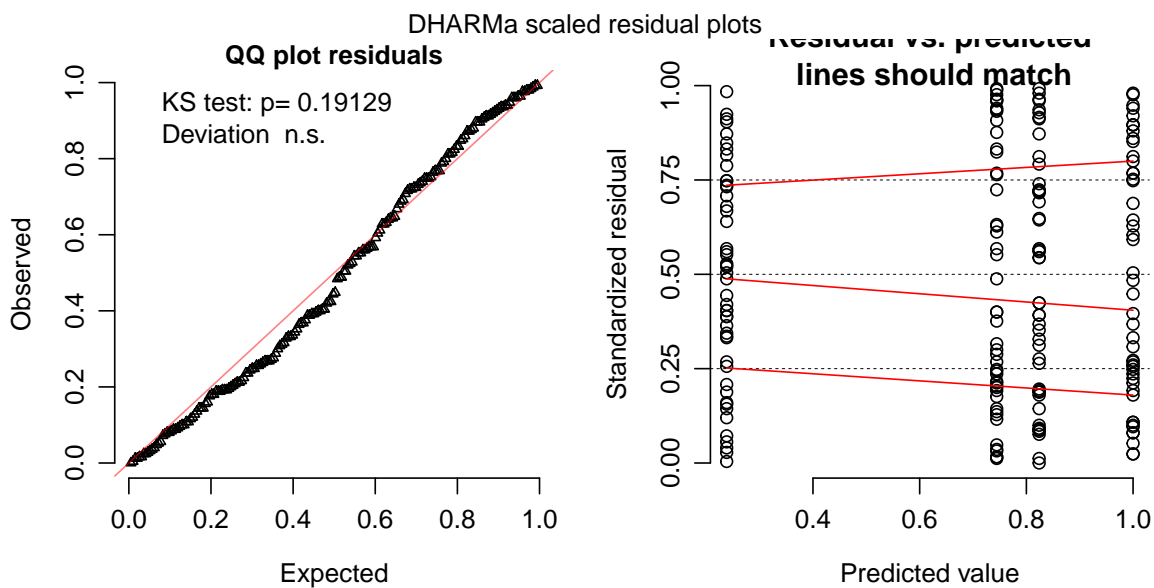


Figure 1.22: Model diagnostics with the DHARMA package for the groundcover data.

```
# Evaluate whether overdispersion evident
testOverdispersionParametric(forb.pois.sqrt)

##
## Chisq test for overdispersion in GLMMs
##
## data:  poissonsqrt
## dispersion = 0.92653, pearSS = 171.41000, rdf = 185.00000, p-value
## = 0.7548
## alternative hypothesis: true dispersion greater 1
```

The p-value is very high so we conclude that overdispersion is not an issue here. If overdispersion was a problem, we could use a quasipoisson family as for GLMs (but unfortunately `glmer` does not support that family). It has also been suggested that including individual-level random effects in the model could alleviate the problem of overdispersion. We try this in the following example.

```
# Create a variable for individual-level random effects
eucface$Ind <- as.factor(1:nrow(eucface))

# fit the model and look at the model summary
forb.pois.ind <- glmer(Forbes~Date*Trt+(1|Ring/Plot/Sub/Ind),
                      family=poisson(sqrt), data=eucface)

# The estimated variances of the random effects
VarCorr(forb.pois.ind)

## Groups          Name          Std.Dev.
## Ind:(Sub:(Plot:Ring)) (Intercept) 0.26215
## Sub:(Plot:Ring)      (Intercept) 0.10949
## Plot:Ring            (Intercept) 0.35224
## Ring                 (Intercept) 0.57035

# Inspect the entire summary yourself:
# summary(forb.pois.ind)
```

The random effects block indicates that a small amount of variance is accounted for by the individual level random effects. This is expected as we did not observe overdispersion.

We can use `Anova` function from the `car` package to calculate significance associated with the main effects and interaction. Note that is is only one way of many to calculate p-values, as we discussed above linear mixed models.

```
library(car)
Anova(forb.pois.sqrt)

## Analysis of Deviance Table (Type II Wald chisquare tests)
##
## Response: Forbes
##           Chisq Df Pr(>Chisq)
## Date      21.2014  1  4.135e-06 ***
## Trt        0.3565  1   0.55043
## Date:Trt   5.4055  1   0.02007 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There appears to be a significant interaction between `Date` and `Trt`. Again, when an interaction is significant we must dig deeper to understand the source of the interaction. Looking at Fig. 1.23, it appears that :

1. Forbs decreased in frequency between the two dates in the control but not in the elevated CO<sub>2</sub> treatment
2. Forb abundance was lower in the control than in the elevated CO<sub>2</sub> treatment on the second date, but not the first date.

```
# plot model predictions and data
visreg(forb.pois.sqrt, 'Date', 'Trt', overlay=TRUE)
visreg(forb.pois.sqrt, 'Trt', 'Date', overlay=TRUE)
```

We can evaluate each of these hypotheses using the approach described in section 1.6, with multiple comparisons based on specified contrasts. First we refit the model without an intercept term so that we can interpret the model coefficients in a more straightforward way. We are also going to generate a new factor representing `Date` for the new model because mathematical symbols in character strings result in errors when passed to the `linfct` argument.

```
# new factor for specifying dates without mathematical symbols
eucface$Date.fac <- factor(with(eucface, paste('date', as.numeric(Date), sep='')))
```

```
# fit intercept-free model
forb.pois.int <- glmer(Forbes~Date.fac*Trt-1+(1|Ring/Plot/Sub),
                      family=poisson('sqrt'), data=eucface)
```

```
##           Estimate Std. Error   z value    Pr(>|z|)
## Date.facdate1      1.1985466  0.3555951  3.3705379 0.00075022 ***
## Date.facdate2      0.6444388  0.3563787  1.8082974 0.07056022 .
## Trtelelev          0.1217973  0.5021998  0.2425276 0.80837138
## Date.facdate2:Trtelelev 0.3729185  0.1603933  2.3250246 0.02007064 *
```

```
library(multcomp)
```

```
# Test for an elevated CO2 effect on each of the two dates
summary(glht(forb.pois.int, linfct=c("Trtelelev - Date.facdate1 = 0",
```



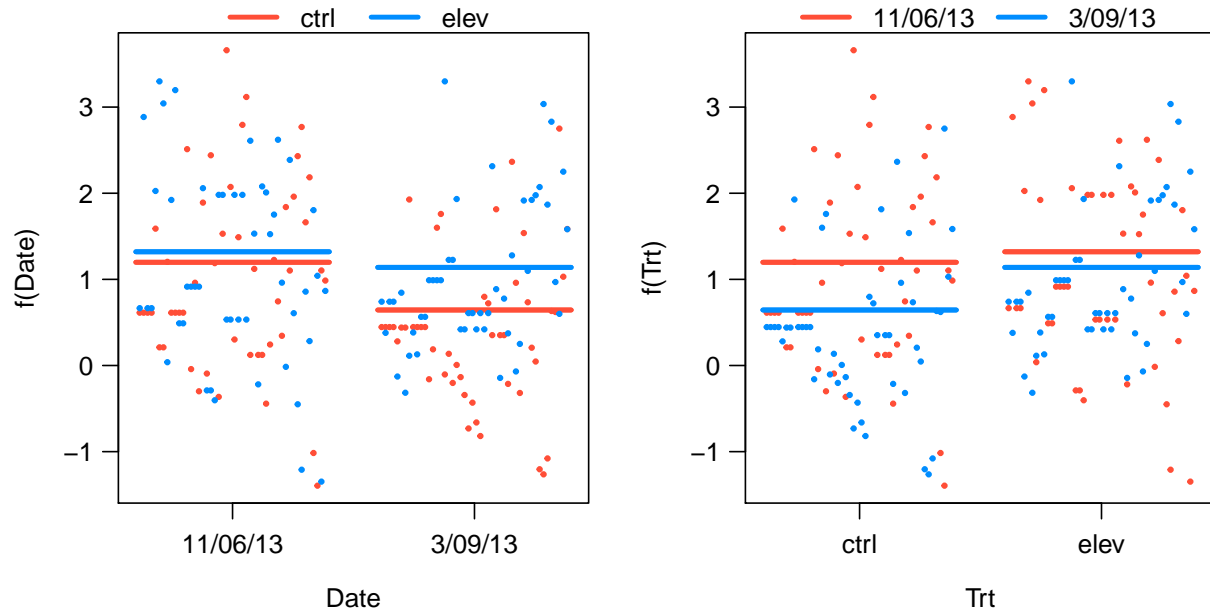


Figure 1.23: Model predictions, looking at treatment effects by date (left) and date effects by treatment (right).

```

                                "Date.facdate2:Trtelev - Date.facdate2 = 0"))))

##
##   Simultaneous Tests for General Linear Hypotheses
##
## Fit: glmer(formula = Forbes ~ Date.fac * Trt - 1 + (1 | Ring/Plot/Sub),
##   data = eucface, family = poisson("sqrt"))
##
## Linear Hypotheses:
##
##               Estimate Std. Error z value
## Trtelev - Date.facdate1 == 0      -1.0767    0.7945  -1.355
## Date.facdate2:Trtelev - Date.facdate2 == 0  -0.2715    0.4080  -0.666
##               Pr(>|z|)
## Trtelev - Date.facdate1 == 0              0.273
## Date.facdate2:Trtelev - Date.facdate2 == 0    0.698
## (Adjusted p values reported -- single-step method)

# Test for a change through time within each CO2 treatment
summary(glht(forb.pois.int, linfct=c("Date.facdate2 - Date.facdate1 = 0",
                                "Date.facdate2:Trtelev - Trtelev = 0"))))

##
##   Simultaneous Tests for General Linear Hypotheses
##
## Fit: glmer(formula = Forbes ~ Date.fac * Trt - 1 + (1 | Ring/Plot/Sub),
##   data = eucface, family = poisson("sqrt"))
##
## Linear Hypotheses:
##
##               Estimate Std. Error z value Pr(>|z|)
## Date.facdate2 - Date.facdate1 == 0   -0.5541    0.1132  -4.894 1.97e-06
## Date.facdate2:Trtelev - Trtelev == 0    0.2511    0.5496   0.457   0.87
##

```

```
## Date.facdate2 - Date.facdate1 == 0    ***
## Date.facdate2:Trtelev - Trtelev == 0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)
```

The only significant contrast is between the two dates within the ambient CO<sub>2</sub> treatment, which supports hypothesis 1 above (forbs decreased in frequency between the two dates in the control but not in the elevated CO<sub>2</sub> treatment).

## 1.8 Example: logistic regression with the ground cover data (glmer)

Something that wasn't mentioned regarding this EucFACE ground cover dataset is that vegetation was assessed at a maximum of 16 points within each subplot and, therefore, the maximum number of observations per plot is constrained. This does not affect our analysis of forb abundances because these are generally low (less than ten in almost all plots) and so interpreting these as count data is appropriate. This is not the case for other response variables. As an example, let's look at grass cover in Fig. 1.24.

```
# A quick plot to visualize ground cover by Date and Ring.
# Colours represent 'Plot'.
xyplot(Grass~Date|Ring, groups=Plot, data=eucface, pch=16, jitter.x=T)
```

The data are clearly bounded at both the lower and upper range. We can treat these as binomial distributed, with the presence or absence of grass assessed at each of the sixteen points within each subplot. To do this, we use `cbind` to create a two-column response matrix indicating the number of presences and absences within each subplot, as we did for logistic regression in Section ???. We also specify the random effects as we did for the analysis of forb abundance above.

```
# fit model with binomial error distribution
grass.binom <- glmer(cbind(Grass, 16-Grass)~Date*Trt+(1|Ring/Plot/Sub),
                    family=binomial, data=eucface)

# test significance of main effects and interaction
Anova(grass.binom)

## Analysis of Deviance Table (Type II Wald chisquare tests)
##
## Response: cbind(Grass, 16 - Grass)
##           Chisq Df Pr(>Chisq)
## Date      145.3658 1 < 2.2e-16 ***
## Trt         0.0000 1  0.997729
## Date:Trt    8.3183 1  0.003925 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The results suggest a highly significant effect of date and a significant date by treatment interaction. The model predictions are shown in Fig. 1.25. A similar approach could be used to tease apart the source of the interaction as was used for forb abundances.

```
# plot model predictions and data
visreg(grass.binom, 'Date', 'Trt', overlay=TRUE)
```

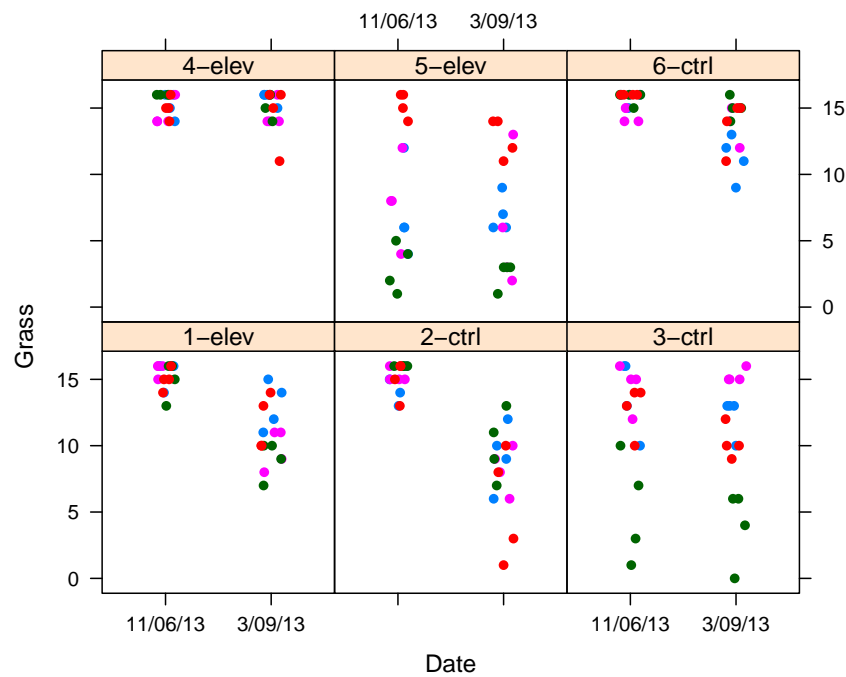


Figure 1.24: Grass cover within plant communities exposed to ambient or elevated carbon dioxide concentrations at two dates. Cover was assessed based on presence at each of sixteen locations within a subplot. Points represent estimates within subplots, subplots common to the same plot within each ring have a common colour.

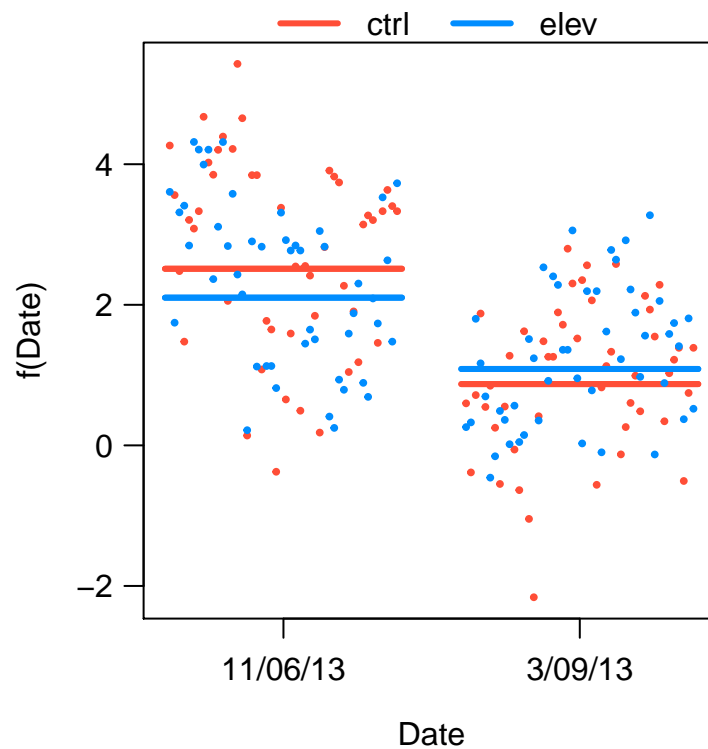


Figure 1.25: Model predictions, looking at treatment effects by date.

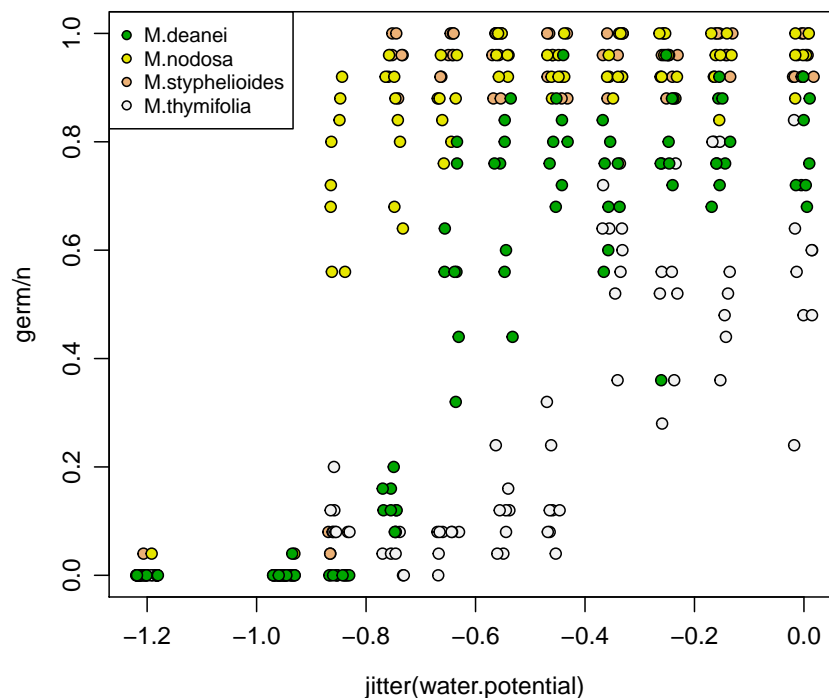


Figure 1.26: Germination success by species and as a function of water potential for the seedwater data.

## 1.9 Example: logistic regression with seed germination data (glmer)

In this example we will take another look at logistic regression when we have random effects, using a generalized linear mixed effects model. We have two related datasets on germination success of seeds. Seeds of four Tea Tree (*Melaleuca* sp.) species were collected at 9 sites, and subjected to either a 'fire cue' treatment (the `seedfire` dataset), or a dehydration treatment (`seedwater` data). Multiple cabinets were used for the experiment in the `seedfire` data, but not for the `seedwater` data. Each replicate contains ca. 20 seeds (given by the `n` variable), for which the germinated seeds were counted after some time. We thus have tabulated data, which we can use in a generalized linear model with a simple trick (we encountered this already in Section ??).

First we read the data, and make a simple plot of germination success (number of germinated seeds divided by sample size) for the `seedwater` data, producing Fig. 1.26.

```
seedfire <- read.csv("germination_fire.csv")

# Make sure temperature treatment is a factor
seedfire$temp <- as.factor(seedfire$temp)

seedwater <- read.csv("germination_water.csv")

palette(terrain.colors(4))
with(seedwater, plot(jitter(water.potential), germ/n, pch=21, bg=species))
legend("topleft", levels(seedwater$species), pch=21, pt.bg=palette(), cex=0.8)
```

We will first work with the `seedfire` dataset to test for temperature effects, and whether the fire cue

treatment had any effect on germination success. We choose to treat the `site` variable as a random effect, since we are at this point not interested in specific site-to-site comparisons, but we do wish to account for the source of variation. It is also possible to treat `site` as a fixed effect, demonstrating that a variable can be either fixed or random depending on the question.

As mentioned above, we use a trick to allow the use of tabulated data in the `glmer` specification, using `cbind(germ, n - germ)`, it represents a matrix with number of 'successes' and 'failures' tabulated in the two columns. We here treat the temperature treatment (`temp`) as a factor since the response is very non-linear, as we will see.

```
# A simple first fit with species and temperature
firefit1 <- glmer(cbind(germ, n-germ) ~ species + temp +
                 (1|site), data=seedfire, family=binomial)
Anova(firefit1)

## Analysis of Deviance Table (Type II Wald chisquare tests)
##
## Response: cbind(germ, n - germ)
##           Chisq Df Pr(>Chisq)
## species    271.18  3 < 2.2e-16 ***
## temp       2368.74  5 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Clearly both `species` and `temp` are highly significant. Next we test whether the interaction between `species` and `temp` is significant, as well as the `fire` treatment. We prefer to test additional variables in steps, as follows.

```
# Fire cue is not significant
firefit2 <- glmer(cbind(germ, n-germ) ~ species + temp + fire.cues +
                 (1|site), data=seedfire, family=binomial)

# fire.cues is not significant
anova(firefit1, firefit2)

## Data: seedfire
## Models:
## firefit1: cbind(germ, n - germ) ~ species + temp + (1 | site)
## firefit2: cbind(germ, n - germ) ~ species + temp + fire.cues + (1 | site)
##           Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## firefit1 10 2712.6 2756.1 -1346.3  2692.6
## firefit2 11 2714.5 2762.4 -1346.3  2692.5 0.0367      1    0.8481

# Include the interaction between species and temperature.
firefit3 <- glmer(cbind(germ, n-germ) ~ species * temp +
                 (1|site), data=seedfire, family=binomial)

# The interaction is very significant (also note large decrease in AIC)
anova(firefit1, firefit3)

## Data: seedfire
## Models:
## firefit1: cbind(germ, n - germ) ~ species + temp + (1 | site)
## firefit3: cbind(germ, n - germ) ~ species * temp + (1 | site)
##           Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## firefit1 10 2712.6 2756.1 -1346.3  2692.6
## firefit3 25 2266.2 2375.1 -1108.1  2216.2 476.36     15 < 2.2e-16 ***
## ---
```

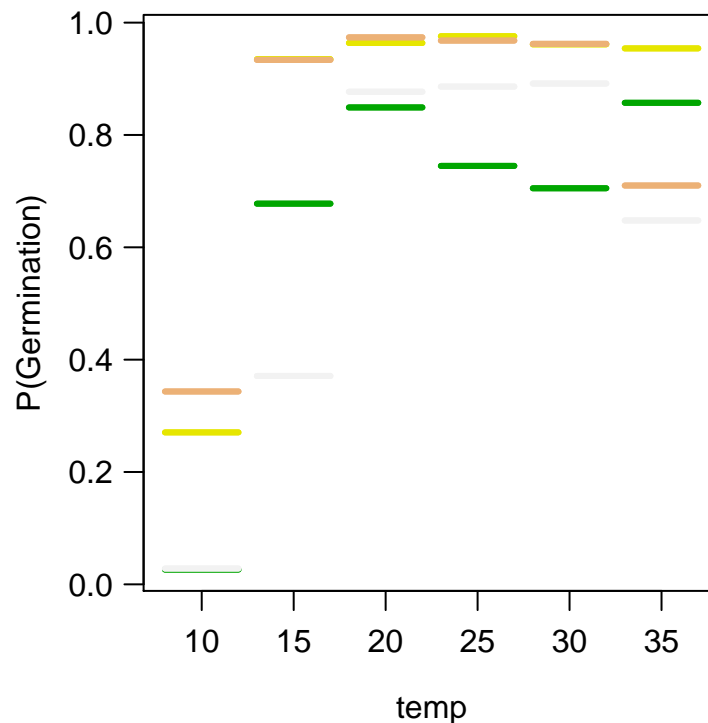


Figure 1.27: The firefit3 model visualized.

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

### Try this yourself

*Note:* When fitting the `firefit3` model, you will get a warning that says **Model failed to converge**. This is a fairly common problem when fitting `glmer` models, and it is generally not possible to say whether it is a serious problem or not. Read the help page `?convergence` for more information. In this case (and many others), using a different optimizer fixes the problem (but in this case all coefficients, random effects, and F values did not change). Try the following :

```
firefit3 <- glmer(cbind(germ, n-germ) ~ species * temp +
                  (1|site), data=seedfire, family=binomial,
                  control = glmerControl(optimizer = "bobyqa"))
```

We can now conclude that there is ample evidence that the different species respond very differently to temperature, but that there is no evidence for the fire cue to have any effect on germination success.

Finally we visualize the effects estimated with the `visreg` package. When visualizing a fitted `glmer` model, we have to specify `scale="response"` to convert the link function back to the original scale. The following code makes Fig. 1.27. Note the various settings to simplify the plot, and the use of `line.par` to change the colour of the lines.

```
visreg(firefit3, "temp", by="species", overlay=TRUE,
       partial=FALSE, scale="response",
       ylab="P(Germination)",
       line.par=list(col=terrain.colors(4)),
       legend=FALSE, rug=FALSE)
```

Next, we will use the `seedwater` dataset to test for effects of dehydration on germination success. The degree of dehydration is measured as the water potential, where more negative values indicate drier

---

seed. The model fitted will be similar as for the seedfire data, except our main predictor (in addition to species) is a numeric variable, not a factor.

```
fitwater1 <- glmer(cbind(germ, n-germ) ~ species + water.potential +
  (1|site), data=seedwater, family=binomial)
fitwater2 <- glmer(cbind(germ, n-germ) ~ species * water.potential +
  (1|site), data=seedwater, family=binomial)

anova(fitwater1, fitwater2)

## Data: seedwater
## Models:
## fitwater1: cbind(germ, n - germ) ~ species + water.potential + (1 | site)
## fitwater2: cbind(germ, n - germ) ~ species * water.potential + (1 | site)
##           Df      AIC      BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## fitwater1  6 2623.9 2647.1 -1306.0   2611.9
## fitwater2  9 2482.8 2517.6 -1232.4   2464.8 147.14     3 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The following code makes Fig. 1.28, and shows a simple way to include the raw data alongside the fitted model. Note that the model does not fit equally well for all species - the response is steeper than expected for at least two of the species. This suggests some transformation of water potential would be appropriate, but we have not included further analyses here.

```
visreg(fitwater2, "water.potential", by="species",
  overlay=TRUE, rug=FALSE, legend=FALSE,
  line.par=list(col=terrain.colors(4)),
  scale="response", ylab="P(Germination)")
with(seedwater, points(jitter(water.potential), germ/n, pch=19, col=species))
```

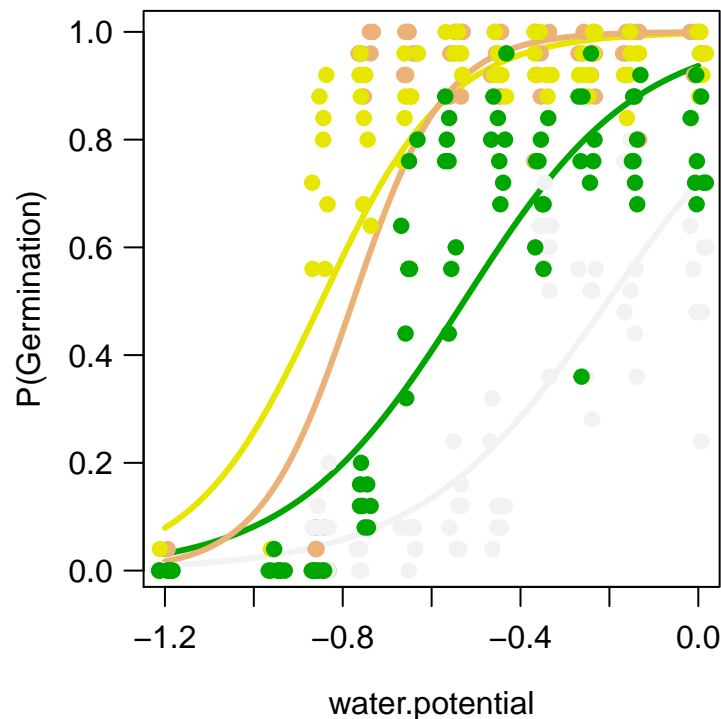


Figure 1.28: The fitwater2 model visualized, with the raw data superimposed

## 1.10 Exercises

In these exercises, we use the following colour codes:

- **Easy:** make sure you complete some of these before moving on. These exercises will follow examples in the text very closely.
- ◆ **Intermediate:** a bit harder. You will often have to combine functions to solve the exercise in two steps.
- ▲ **Hard:** difficult exercises! These exercises will require multiple steps, and significant departure from examples in the text.

We suggest you complete these exercises in an **R** markdown file. This will allow you to combine code chunks, graphical output, and written answers in a single, easy-to-read file.

### 1.10.1 PREF Canopy data

1. ■ Fit and compare mixed-effects models testing the fixed effects of `species` and `dfromtop` on `narea` (nitrogen concentration in grams per square metre). Evaluate whether the model fit is improved when including a random slope for each individual. Evaluate the significance of the fixed effects and their interaction.
2. ■ Generate diagnostic plots for the output of the previous analysis to evaluate whether assumptions of normality and heteroscedasticity are violated. Does a `sqrt`- or `log`-transformation improve model fit? Repeat the analysis on the transformed data, using the most appropriate transformation.



---

## 1.10.2 Mouse metabolism

In this exercise you will practice with the mouse metabolism data ('wildmousemetabolism.csv').

1. ■ For the `mouse_m5` model, it was determined that `wheel` (whether or not the mice were using the wheel for exercise) was a significant predictor. What was the effect size? Use both `summary` and `visreg`.
2. ♦ Inspect diagnostics for the best of the mouse models (use AIC on all models to choose the best one), including QQ plot, and residuals versus fitted. Does a logarithmic transformation (either predictor or response) improve the diagnostics?
3. ♦ The `mouse` dataframe also contains a variable indicating the day on which the measurements took place. Each mouse was exposed to each temperature once on each of six sequential days. Include day of measurement as another random effect, check whether the model fit is improved, and evaluate the significance of each of the fixed effects.
4. ♦ Take a subset of the mouse data where temperature (`temp`) is 31. Run a mixed-effects model of resting metabolic rate with only `sex` (Male or Female) as predictor. Choose appropriate random effects. Does `rmr` vary with sex of the mouse?
5. ♦ Now add body mass (`bm`) to the model with `sex`, as a main effect and interaction. Is `sex` still significant? Use `visreg` to understand how body mass, `sex`, and resting metabolic rate are related. If it helps, also test whether body mass varies with `sex`.

## 1.10.3 Litter decomposition data

1. ■ Generate some diagnostic plots for the `litter_m1` model described in Section 1.4.
2. ♦ The `litter` data contain a factor (`variety`) describing whether the litter is derived from a genetically modified (`gm`) or conventional (`nongm`) soy variety. Plot the data to observe the effect of `variety`. Use `lmer` to test the effect of `variety`, in addition to the other significant variables, on litter decomposition.

## 1.10.4 Repeated measures in leaf photosynthesis

For this exercise you will use the EucFACE leaf gas exchange dataset. First walk through Section 1.6 before starting this exercise.

1. Read the data, and add a new variable called `WUE` (water-use efficiency), calculated as `Photo` divided by `Trmmol`. This new variable represents the amount of CO<sub>2</sub> taken up per unit water lost.
2. ■ Make a barplot of `WUE` by `Date` and CO<sub>2</sub> treatment, as shown in Section 1.6.
3. ■ Fit a linear mixed-effects model with `Tree` within `Ring` as the random effects structure, and `Date` and `CO2` as fixed effect. Fit a second model with additionally the interaction between `Date` and `CO2`; is the interaction significant (use both `anova` on the two models, and `Anova` on the model with the interaction).
4. ♦ Inspect the standard deviation of the random effects with `VarCorr`. Notice that the standard deviation for the term `Tree:Ring` (trees within rings) is very small compared to the `Ring` random effect. A naive user might remove this random effect since it explains little variation. Why is that a bad idea?
5. ▲ Look at the summary statement of the model for `WUE` that does not contain an interaction. Notice that `Date D` is significantly lower than the intercept (`Date A`). Refit the model without an

---

intercept, and use `glht` to test whether Date D was lower than all of Date A, B and C. *Hint: this follows closely the example shown at the end of Section ??.*

6. ♦ A colleague tells you that water-use efficiency may be lower on Date D because air temperature was higher, resulting in a lower air humidity. This is indicated by the variable `vpdL` in the dataset, which represents the 'vapour pressure deficit' - higher values indicate drier air. Refit the model for `WUE` with the `CO2*Date` interaction, and now add `vpdL` as a fixed effect to the model (but no interactions between `vpdL` and Date or `CO2`). Is VPD significant (use multiple methods to test this)?
7. ▲ Use `visreg` to visualize a) the `vpdL` effect (make a `visreg` plot with VPD as the predictor, showing predictions for both `CO2` treatment), b) The Date effect. For this latter plot, use Date as the `xvar` argument to `visreg`, and `CO2` as the `by` argument (with `overlay=TRUE`). For this last plot, `visreg` will show the Date effect *at a median value of vpdL*, that is, the effect of `vpdL` has been removed first. Is Date D still different from the other Dates? Can you explain this in terms of the `vpdL` covariate?

### 1.10.5 EucFACE ground cover data

The file `eucfaceGC.csv` contains estimates of plant and litter cover within the rings of the EucFACE experiment, evaluating forest ecosystem responses to elevated `CO2`, on two dates; the data description can be found in Section ?? (p. ??).

1. ♦ Convert the variables indicating the nested sampling design to factors, then use `glmer` in `lme4` to test for an interaction between `Trt` and `Date` on `Litter` cover. Litter cover represents counts (use the `poisson` family).
2. Make model diagnostics plot for the model of `Grass` with treatment and Date as fixed effects. Use the `DHARMA` package as shown in Fig. 1.7. Now refit this model with the `poisson` family - which is a wrong model specification because Grass count is clearly bounded by a maximum (16 quadrants in each plot). Make the model diagnostic plot again, and note how it has changed.

### 1.10.6 Logistic regression

For this exercise, we will use the seed germination data (fire experiment). First walk through Section 1.9.

1. For the seedfire dataset, the examples in the text use `temp` as a factor variable. Attempt to instead use it as an (untransformed) continuous variable in the model. Compare the model with `temp` as a factor and `temp` as a continuous variable, in terms of AIC. Now plot diagnostic plots for both models (see Section 1.7). Ignore the warning.
2. Try to understand why `temp` as a continuous variable does so badly, using `visreg` (set `temp` as the `xvar`, and also use `scale="response"`). Compare the fitted response to the data (i.e., also make a plot of germination success versus temperature, and compare to the `visreg` plot).

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