3.1

解：(1) $\int_0^{1+i}[(x-y)+ix^2]dz = \int_0^1 it^2(1+i)dt = i(1+i)\frac{1}{3} = -\frac{1}{3}+\frac{i}{3}$

(2) C_1: $y=0$, $dy=0$, $dz=dx$;

C_2: $x=1$, $dx=0$, $dz=idy$;

$\int_0^{1+i}[(x-y)+ix^2]dz = \int_{C_1}+\int_{C_2}$

$= \int_0^1 (x+ix^2)dx + \int_0^1 (1-y+i)idy = -\frac{1}{2}+\frac{5}{6}i$

(3) l_1: $x=0$, $dz=idy$

l_2: $y=1$, $dz=dx$

$\int_0^{1+i}[(x-y)+ix^2]dz = \int_{l_1}+\int_{l_2}$

$= \int_0^1 (-y)idy + \int_0^1 (x-1+ix^2)dx$

$= -\frac{1}{2}-\frac{i}{6}$

3.2 解：令 $z=re^{i\theta}$.

有 $\oint_{|z|=r}\frac{\bar{z}}{|z|}dz = \int_0^{2\pi}\frac{re^{-i\theta}}{r}rie^{i\theta}d\theta = 2\pi ri$

当 $r=2$ 时，为 $4\pi i$；当 $r=4$ 时，为 $8\pi i$.

3.4 解：(1) $\oint_C \frac{1}{z^2+4z+4}dz$

积分值为0，被积函数在$|z|\leq 1$内解析．

(3) $\oint_C \frac{1}{z-\frac{1}{2}}dz$

$\oint_C \frac{1}{z-\frac{1}{2}}dz = 2\pi i$．

3.6 解：$f(z)=\frac{1}{z^2-z}=\frac{1}{z(z-1)}$ 在 $|z|=2$ 内为两奇点，为 $z=0,1$．分别作以0，1为中心的圆周 C_1, C_2，C_1与C_2不相交．

$\therefore \oint_C \frac{1}{z^2-z}dz = \oint_{C_2}\frac{1}{z-1}dz - \oint_{C_1}\frac{1}{z}dz = 2\pi i - 2\pi i = 0$

3.7 解：$\oint_{|z|=3}\frac{dz}{(z-i)(z+2)} = 0$

3.8 解：(1) $\int_0^{\pi i} \sin z\, dz$

解：$\int_0^{\pi i}\sin z\, dz = -\cos z\Big|_0^{\pi i} = 1-\cos\pi i$．

3.10

解：(1) $\oint_{|z-2|=1} \frac{e^z}{z-2} dz$

$$\oint_{|z-2|=1} \frac{e^z}{z-2} dz = 2\pi i e^z \Big|_{z=2} = 2\pi i e^2$$

3.13 解：(1) $\oint_{|z|=1} \frac{e^z}{z^{100}} dz$

解：原：$\oint_{|z|=1} \frac{e^z}{z^{100}} dz = 2\pi i \frac{1}{99!} e^z \Big|_{z=0}$

$$= \frac{2\pi i}{99!}$$

(2) $\oint_{|z|=2} \frac{\sin z}{(z-\frac{\pi}{2})} dz$

解：$\oint_{|z|=2} \frac{\sin z}{(z-\frac{\pi}{2})} dz = 2\pi i (\sin z)' \Big|_{z=\frac{\pi}{2}}$

$$= 2\pi i \cdot \cos z \Big|_{z=\frac{\pi}{2}} = 0$$

(3) $\oint_{C=C_1+C_2^-} \frac{\cos z}{z^3} dz = \oint_{C_1} \frac{\cos z}{z^3} dz + \oint_{C_2^-} \frac{\cos z}{z^3} dz$

$= 2\pi i \frac{1}{2!} (\cos z)'' \Big|_{z=0} - 2\pi i \frac{1}{2!} ((\cos z)'' \Big|_{z=0}$

$= \pi i (-1) - \pi i (-1) = 0$