Projection matrices

The vector $\mathbf{v}'$ is a result of multiplying the vector $\mathbf{v}$ with transformation matrix $\mathbf{M}$ ("postmultiplying"):

$$\mathbf{v}' = \mathbf{M}\mathbf{v}$$

For a three-dimensional vector $\mathbf{v} = \begin{pmatrix} x & y & z \end{pmatrix}^{\mathbf{T}}$ we use the four-dimensional homogenous coordinates $\mathbf{v} = \begin{pmatrix} x & y & z & w \end{pmatrix}^{\mathbf{T}}$.

Now

$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$$

where

$$\begin{aligned} x' &= xm_{11} + ym_{21} + zm_{31} + wm_{41} \\ y' &= xm_{12} + ym_{22} + zm_{32} + wm_{42} \\ z' &= xm_{13} + ym_{23} + zm_{33} + wm_{43} \\ w' &= xm_{14} + ym_{24} + zm_{34} + wm_{44} \end{aligned}$$

...

Our goal is the perspective projection matrix $\mathbf{M}_{\text{per}}$ defined as:

$$\mathbf{M}_{\text{per}} = \begin{pmatrix} \frac{2n}{r-l} & 0 & \frac{l+r}{l-r} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{f+n}{n-f} & \frac{2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Here n is the *near* clipping plane, f the *far* clipping plane, r the *right* clipping plane, l the *left* clipping plane, t the *top* clipping plane, and b the *bottom* clipping plane: $\textit{left} \le x \le \textit{right}$, $\textit{bottom} \le y \le \textit{top}$, $\textit{far} \le z \le \textit{near}$.