Một danh sách các phần tử được lưu trữ trong một danh sách đặc, có các phần tử sau:

40, 70, 20, 60, 90, 10, 50, 30

**2.5: Dùng phương pháp sắp xếp thứ tự QuickSort, mô tả từng bước quá trình xếp thứ tự dãy số trên. Tính độ phức tạp của quá trình xếp thứ tự danh sách trên.**

**\****Danh sách chưa được sắp xếp*

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Vị trí | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Phần tử | a[0] | a[1] | a[2] | a[3] | a[4] | a[5] | a[6] | a[7] |
| Giá trị | 40 | 70 | 20 | 60 | 90 | 10 | 50 | 30 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 40 |  | 70 |  | 20 |  | 60 |  | 90 |  | 10 |  | 50 |  | 30 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |

Phân hoạch danh sách: i = left = 0, j = right = 7

Chọn x = a[(left + right)/2] = a[3] = 60

* **x = a[3] = 60**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | 70 |  | 20 |  | **60** |  | 90 |  | 10 |  | 50 |  | 30 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |
|  | i |  |  |  |  |  |  |  |  |  |  |  |  |  | j |

a[i] = a[0] = 40 < x, i++ (I tăng lên 1 giá trị)

a[i] = a[1] = 70 > x, dừng i (với i = 1)

a[j] = a[7] = 30 < x, dừng j (với j = 7)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | 70 |  | 20 |  | **60** |  | 90 |  | 10 |  | 50 |  | 30 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |
|  |  |  | **i = 1** |  |  |  | **Swap a[1], a[7]** | | | | |  |  |  | **j = 7** |
|  |  |  |  | | | | | | | | | | | | |

Hoán vị giá trị giữa a[i] = 70 và a[j] = 30

Tăng i lên 1 giá trị (i = 2)

Giảm j xuống 1 giá trị (j = 6)

Ta được:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 40 |  | 30 |  | 20 |  | **60** |  | 90 |  | 10 |  | 50 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |

* **x = a[3] = 60**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | 30 |  | 20 |  | **60** |  | 90 |  | 10 |  | 50 |  | 70 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | i =2 |  |  |  |  |  |  |  | j=6 |  |  |

a[i] = a[2] = 20 < x, i++ (I tăng lên 1 giá trị)

a[i] = a[3] = 60 = x, dừng i (với i = 3)

a[j] = a[6] = 50 < x, dừng j (với j = 6)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | 30 |  | 20 |  | **60** |  | 90 |  | 10 |  | 50 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |
|  |  |  |  |  |  |  | **i=3** | **Swap a[3], a[6]** | | | |  | **j=6** |  |  |
|  |  |  |  |  |  |  |  | | | | | | |  |  |

Hoán vị giá trị giữa a[i] = 60 và a[j] = 50

Tăng i lên 1 giá trị (i = 4)

Giảm j xuống 1 giá trị (j = 5)

Ta được:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 40 |  | 30 |  | 20 |  | 50 |  | 90 |  | 10 |  | 60 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |

* **x = a[3] = 60**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | 30 |  | 20 |  | 50 |  | 90 |  | 10 |  | 60 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | i=4 |  | j=5 |  |  |  |  |

a[i] = a[4] = 90 > x, dừng i (với i = 4)

a[j] = a[5] = 10 < x, dừng j (với j = 5)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | 30 |  | 20 |  | **50** |  | 90 |  | 10 |  | 60 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |
|  |  |  |  |  |  |  |  |  | **i=4** |  | **j=5** |  |  |  |  |
|  |  |  |  |  |  |  |  |  | **Swap a[4], a[5]** | | |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  | | |  |  |  |  |

Hoán vị giá trị giữa a[i] = 90 và a[j] = 10

Tăng i lên 1 giá trị (i = 5)

Giảm j xuống 1 giá trị (j = 4)

Ta được:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 40 |  | 30 |  | 20 |  | 50 |  | 10 |  | 90 |  | 60 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | 30 |  | 20 |  | 50 |  | 10 |  |  | 90 |  | 60 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 | 5 |  | 6 |  | 7 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | j=4 | i=5 |  |  |  |  |

* **i > j dừng**
* Phân hoạch lại x = a[(left + right)/2] = a[3] = 50

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | 30 |  | 20 |  | **50** |  | 10 |  | 90 |  | 60 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |
|  | i |  |  |  |  |  |  |  |  |  |  |  |  |  | j |

a[i] = a[0] = 40 < x, i++ (i tăng lên 1 giá trị)

a[i] = a[1] = 30 < x, i++ (i tăng lên 1 giá trị)

a[i] = a[2] = 20 < x, i++ (i tăng lên 1 giá trị)

a[i] = a[3] = 50 = x, I dừng(với i = 3)

a[j] = a[7] = 70 > x, j++ (giảm j xuống 1 giá trị)

a[j] = a[6] = 60 > x, j++ (giảm j xuống 1 giá trị)

a[j] = a[5] = 90 > x, j++ (giảm j xuống 1 giá trị)

a[j] = a[4] = 10 < x, j dừng (với j = 4)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | 30 |  | 20 |  | **50** |  | 10 |  | | 90 |  | | 60 | |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | | 5 |  | | 6 | |  | 7 |
|  |  |  |  |  |  |  | **i = 3** |  | **j = 4** |  | | |  | |  | |  |  |
|  |  |  |  | |  | |  | | |  | | |  | | |  | | |
|  |  |  |  | | | | **Swap a[3], a[4]** | | | |  | | |  | | | | |

Hoán vị giá trị giữa a[i] = 50 và a[j] = 10

Tăng i lên 1 giá trị (i = 4)

Giảm j xuống 1 giá trị (j = 3)

Ta được:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 40 |  | 30 |  | 20 |  | 10 |  | 50 |  | 90 |  | 60 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  | 6 |  | 7 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | 30 |  | 20 |  | 10 |  |  | 50 |  | 90 |  | 60 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 | 4 |  | 5 |  | 6 |  | 7 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | j=3 | j=4 |  |  |  |  |  |  |

* ***Phân hoạch thành 2 dãy con như sau:***

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Dãy con 1** |  | 40 |  | 30 |  | 20 |  | 10 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Dãy con 2** |  | 50 |  | 90 |  | 60 |  | 70 |

\*Tiến hành QuickSort trên dãy con 1:

Chọn x = a[(left + right)/2] = a[1] = 30

* **x = a[1] = 30**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | **30** |  | 20 |  | 10 |
|  | 0 |  | 1 |  | 2 |  | 3 |
|  | i |  |  |  |  |  | j |

a[i] = a[0] = 40 > x, dừng i (với i = 0)

a[j] = a[3] = 10 < x, dừng j (với j = 3)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 40 |  | **30** |  | 20 |  | 10 |
|  | 0 |  | 1 |  | 2 |  | 3 |
|  | **i=0** |  | **Swap a[0], a[3]** | | |  | **j=3** |
|  |  | | | | | | |

Hoán vị giá trị giữa a[i] = 40 và a[j] = 10

Tăng i lên 1 giá trị (i = 1)

Giảm j xuống 1 giá trị (j = 2)

Ta được:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 10 |  | 30 |  | 20 |  | 40 |
| 0 |  | 1 |  | 2 |  | 3 |

* **x = a[1] = 30**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 10 |  | **30** |  | 20 |  | 40 |
|  | 0 |  | 1 |  | 2 |  | 3 |
|  |  |  |  |  |  |  |  |

a[i] = a[1] = 30 = x, dừng i (với i = 1)

a[j] = a[3] = 20 < x, dừng j (với j = 2)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 10 |  | **30** | |  | 20 |  | 40 |
|  | 0 |  | 1 | |  | 2 |  | 3 |
|  |  |  | **Swap a[0], a[3]** | | | |  |  |
|  |  |  | **i=1** |  | | **j=2** |  |  |

Hoán vị giá trị giữa a[i] = 30 và a[j] = 20

Tăng i lên 1 giá trị (i = 2)

Giảm j xuống 1 giá trị (j = 1)

Ta được:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 10 |  | 20 |  | 30 |  | 40 |
| 0 |  | 1 |  | 2 |  | 3 |

* **x = a[1] = 30**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 10 |  |  | | 20 | |  | | 30 | |  | | 40 | |
|  | 0 | 1 | |  | | 2 | |  | | 3 | |
|  |  |  | | j | |  | | i | |  | |  | |

* **i > j dừng**

|  |  |
| --- | --- |
| ***Ta sắp xếp được phần tử đầu tiên:*** | 10 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Dãy sau khi Sort: |  | 20 |  | 30 |  | 40 |
|  |  | 0 |  | 1 |  | 2 |

* Làm tương tự các bước trên:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Ta sắp xếp được dãy:*** | 10 | 20 | 30 | 40 |

\*Tiến hành QuickSort trên dãy con 2:

Chọn x = a[(left + right)/2] = a[1] = 90

* **x = a[1] = 90**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **b** | 50 |  | **90** |  | 60 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |
|  | i |  |  |  |  |  | j |

a[i] = a[0] = 50 < x, i++ (tăng i lên 1 giá trị)

a[i] = a[0] = 90 = x, dừng i (với i = 1)

a[j] = a[3] = 70 < x, dừng j (với j = 3)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **b** | 50 |  | **90** |  | 60 |  | 70 |
|  | 0 |  | 1 |  | 2 |  | 3 |
|  |  |  | **i=1** | **Swap a[1],a[3]** | | | **j=3** |
|  |  |  |  | | | | |

Hoán vị giá trị giữa a[i] = 90 và a[j] = 70

Tăng i lên 1 giá trị (i = 2)

Giảm j xuống 1 giá trị (j = 2)

Ta được:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 50 |  | 70 |  | 60 |  | 90 |
| 0 |  | 1 |  | 2 |  | 3 |

* **x = a[1] = 90**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **b** | 50 |  | 70 |  | 60 |  | 90 |
|  | 0 |  | 1 |  | 2 |  | 3 |
|  |  |  |  |  |  |  |  |

a[i] = a[2] = 60 < x, dừng i(với i = 2)

a[j] = a[2] = 60 < x, dừng j (với j = 2)

Tăng i lên 1 giá trị (i = 3)

Giảm j xuống 1 giá trị (j = 1)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 50 |  |  | | 70 | |  | | 60 | |  | | 90 | |
|  | 0 | 1 | |  | | 2 | |  | | 3 | |
|  |  |  | | j | |  | |  | |  | | i | |

* **i > j dừng**

|  |  |
| --- | --- |
| ***Ta sắp xếp được phần tử đầu tiên:*** | 50 |

* Làm tương tự các bước trên:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Ta sắp xếp được dãy:*** | 50 | 60 | 70 | 90 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **a** | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 90 |
|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

* ĐỘ PHỨC TẠP: **O(n log n)**