

ENROLLMENT FORECASTING BASED ON LINGUISTIC TIME SERIES

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Abstract. Dealing with the time series forecasting problem attracts much attention from the fuzzy community. Many models and methods have been proposed in the literature since the publication of the study by Song and Chissom in 1993, in which they proposed fuzzy time series together with its fuzzy forecasting model for time series data and the fuzzy formalism to handle their uncertainty. Unfortunately, the proposed method to calculate this fuzzy model was very complex. Then, in 1996, Chen proposed an efficient method to reduce the computational complexity of the mentioned formalism. Hwang et al. in 1998 proposed a new fuzzy time series forecasting model, which deals with the variations of historical data instead of these historical data themselves. Though fuzzy sets are concepts inspired by fuzzy linguistic information, there is no formal bridge to connect the fuzzy sets and the inherent quantitative semantics of linguistic words. This study proposes the so-called linguistic time series, in which words with their own semantics are used instead of fuzzy sets. By this, forecasting linguistic logical relationships can be established based on the time series variations and this is clearly useful for human users. The effect of the proposed model is justified by applying the proposed model to forecast student enrollment historical data.

Keywords. Forecasting model; Fuzzy time series; Hedge algebras; Linguistic time series; Linguistic logical relationship.

1. INTRODUCTION

Fuzzy time series was firstly examined by Song and Chissom in 1993 [1], in which they proposed a fuzzy model of time series forecasting to deal with the uncertainty in nature of the time series data. Song and Chissom also introduced two forecasting models [2, 3] to deal, respectively, with time-invariant or time-variant fuzzy time series and applied them to forecast the enrollment time series of Alabama. However, their calculating methods were complex and incomprehensible. In 1996, to overcome this difficulty, Chen [4] proposed an arithmetic approach to the fuzzy time series forecasting model to simplify the fuzzy forecasting formalism and reduce the computational complexity. He justified that his proposed method was more efficient than Song and Chissom's and it took less computational time and offered better accuracy of forecasting results. In [5], Sullivan and Woodall proposed the

Markov model, which used linguistic labels with probability distributions to forecast student enrollment time series.

After those initial researches on fuzzy time series, many forecasting models and their calculating methods have been proposed mainly to get two aims: to improve the accuracy of the forecast results and to simplify the calculation model. In 1998, Hwang et al. [6] proposed a new fuzzy time series forecasting model based on the variations of historical data instead the time series themselves. This model pays attention to the variability of historical data which seems to be an appropriate approach to predict based on the annual variations of enrollment numbers. Fuzzy time series is an effective way to deal with uncertain and wide-range variation time series data. The calculation with fuzzy time series is mainly based on the fuzzy sets that are consistently constructed for the given historical data.

For nearly three decades, many forecasting methods on fuzzy time series have introduced. They extended the fuzzy time series forecasting with high-order models, e.g., [7, 8, 9, 10, 11, 12], and/or multi-factors models, e.g., [12, 13, 14]. To improve the performance of forecasting methods, many modern computation techniques are applied such as artificial neural network, e.g., [15, 16], evolutionary computation (genetic algorithm, particle swarm optimization), e.g., [11, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26] clustering technique, e.g., [11, 25, 27, 28, 29] and so on. However, the construction of these fuzzy sets still heavily relies on the knowledge and experience of the developers. These fuzzy sets constructed for a time series are fundamental elements to produce fuzzy logical relationships (FLRs) involved in the time series to handle the time series data.

Fuzzy sets in their nature are originated from fuzzy linguistic words of natural language which possess their own qualitative semantics. However, in the fuzzy set framework, there is no formal basis to connect fuzzy sets and their associated linguistic words whose semantics is represented by their respective fuzzy sets. It is natural and essential that one may actually deal with and immediately handle the linguistic labels with their own inherent semantics assigned to the fuzzy sets occurring in the fuzzy time series and in its FLRs. However, this requires that the word-domains of variable and the inherent semantics of their words must be mathematically formalized.

Hedge algebras (HAs) was introduced in 1990 to formalize the word-domains of variables as algebraic order-based structures and the semantics of words are formally defined in their respective structures [30]. They establish an algebraic approach to handle fuzzy linguistic information in a sound manner. In this approach, the word-domain of a variable is considered as an order-based algebraic structure, whose words are generated from its two atomic words with the opposite meaning one to the other by using linguistic hedges regarded as unary operations like very, rather, little, extremely, . . . They form a formalism sufficient to immediately handle linguistic information and to soundly construct computational objects, including fuzzy sets, to represent the inherent semantics of their words. Based on this advantage, HAs were apply to many fields such as fuzzy control, e.g., [31, 32, 33, 34, 35, 36] classification and regression problems [37, 38], computing with words [39, 40], image processing [41], and so on.

Recently, there are some studies applying the HAs theory to the fuzzy time series forecasting problem [42, 43, 44, 45] The main idea of these studies is only to apply the fuzziness intervals of words, interpreted as their interval-semantics, to decompose the universe of discourse into an interval-partition instead of determining these intervals based only on the

researchers' intuition. The authors of studies [42, 43, 44] proposed a forecasting method based on HAs using semantization and desamentization transformations, which are successfully applied in fuzzy control. They tried to determine an interval partition of historical data similarly as ordinary fuzzy time series forecasting methods and also made some modifications to improve forecasting accuracy, for instance, optimizing the selection of forecasting model parameters. Tung et al. [45] proposed a method to construct fuzzy sets for fuzzy time series forecasting method which based on HAs to establish a fuzzy partition of dataset range. The number of its fuzzy set is also limited by more or less 7. In principle, in this study, there is no limitation of the number of words used in the method.

In this study, based on the HAs formalism, we introduce the so-called linguistic time series and the linguistic model of forecasting time series data, in which words and their own qualitative semantics are taken into consideration to handle their quantitative semantics, especially, fuzzy sets are not necessary to use. Thus, it is interesting that FLRs mentioned above can be represented in terms of linguistic words, called linguistic rules, considered as linguistic knowledge for forecasting time series data, which are very useful for interacting with human users. The proposed linguistic forecasting model ensures that the linguistic knowledge formed from the constructed FLRs convey its own inherent semantics of their words similar as ordinary human knowledge. This seems to be very essential and useful for time series forecasting activities, especially, for the interface between time series data forecasting models and their human users.

The rest of this paper will be organized as follows. In Section 2, we will briefly review some concepts of fuzzy time series. In Section 3, some definitions of hedge algebras will be introduced. In Section 4, we will propose linguistic time series and its forecasting model. We also test the robustness of the proposed model and compare it with the former method. The conclusion is covered in Section 5.

2. FUZZY TIME SERIES

Fuzzy time series was introduced by Song and Chissom [1] based on the fuzzy set theory [46], where the values of historical data are presented by fuzzy sets. In the following, we briefly review some basis concepts of fuzzy time series.

Let U be the universe of discourse, $U = \{u_1, u_2, \dots, u_n\}$, where u_j 's are the expected intervals of the determined range of the values of a given data time series based on which the fuzzy sets used to produce the desired fuzzy time series constructed. These fuzzy sets aim to represent the semantics of the human words used to describe the numeric values of the time series range mentioned above, e.g., *not many*, *not too many*, *many*, *many many*, *very many*, *too many*, *too many many* [1]. Thus, a fuzzy set A on U can be defined as follows

$$A = f_A(u_1)/u_1 + f_A(u_2)/u_2 + \dots + f_A(u_n)/u_n, \quad (2.1)$$

where f_A is the membership function of A , $f_A : U \rightarrow [0, 1]$, and $f_A(u_i)$ indicates the grade of membership of u_i in A , where $f_A \in [0, 1]$ and $1 \leq i \leq n$. The concept of fuzzy time series is inspired by the observation given the authors of [1] as follows. Let us imagine a series of linguistic values describing the weather of a certain place in north America using the word vocabulary *good*, *very good*, *quite good*, *very very good*, *cool*, *very cool*, *quite cool*, *hot*, *very hot*, *cold*, *very cold*, *very very cold*,... The weather of a day in summer may be

described by *cool*, *quite good* and that of another day may be *hot*, *very bad*. However, in winter, such linguistic descriptions may be *rather cold*, *good* or *very very cold*, *very very bad*, and so on. They argued that the temperature ranges and their set of the possible words may be varied from day to day, from season to season, and the semantics of these words can be represented by fuzzy sets defined on their respective appropriate real value ranges, denoted by $Y(t)$. Thus, the weather $F(t)$ of the day ' t ' can be represented by some fuzzy sets defined on their respective ranges that can be changed in time. Therefore, they introduce the following definition.

Definition 2.1. [1] Let $Y(t)$ ($t = \dots, 0, 1, 2, \dots$), a subset of \mathbb{R} , be the universe of discourse on which fuzzy sets $f_i(t)$ ($i = 1, 2, \dots$) are defined and $F(t)$ is the collection of $f_i(t)$ ($i = 1, 2, \dots$). Then $F(t)$ is called a fuzzy time series on $Y(t)$ ($t = \dots, 0, 1, 2, \dots$).

The relationships between the fuzzy sets (and, hence, between their word-labels) are important for forecasting problem that is formalized in [1] by the following definition.

Definition 2.2. [4] Assume that there exists a fuzzy relationship $R(t-1, t)$, such that $F(t) = F(t-1) \circ R(t-1, t)$ where ' \circ ' represents a composition operator, then $F(t)$ is said to be caused by $F(t-1)$. When $F(t-1) = A_i$ and $F(t) = A_j$, the relationship between $F(t-1)$ and $F(t)$ is denoted by the fuzzy logical relationship (FLR)

$$A_i \rightarrow A_j, \quad (2.2)$$

where A_i and A_j are called the left-hand side and the right-hand side of the FLR, respectively.

In [2, 3], R is determined by a fuzzy relation, which is calculated by $R_j = [F(t-1)]^T \times F(t)$, $t = 1, 2, \dots$, $j = 1, \dots, p$. Assuming that the fuzzy time series under consideration has p FLRs in the form $A_i \rightarrow A_j$, where A_l 's are fuzzy sets defined on the set of u_k , $k = 1, \dots, n$, which are the intervals defined by a partition of the ordinary time data series, we have then p such fuzzy relations, R_j , $j = 1, \dots, p$.

Putting $R = \cup_{j=1}^p R_j$, the forecasting model is defined as

$$A_i = A_{i-1} \circ R, \quad (2.3)$$

where A_{i-1} is the enrollment of year $i-1$ and A_i is the forecasted enrollment of year i in terms of fuzzy sets and ' \circ ' is the 'max-min' operator.

Chen in [4] argued that the derivation of the fuzzy relation R is a very tedious work, and the forecasting calculation by the above forecasting model is too complex, especially when the fuzzy time series is large. Therefore, he proposed a so-called arithmetic method to compute the forecasting values based on utilizing, for a given A_i , the midpoints of the cores of the fuzzy sets of A_j 's occurring on the right-hand side of those FLRs of the form (2.2) whose left-hand side are the same A_i . Thus, he introduced fuzzy logical relationship group defined as follows.

Definition 2.3. [4] Suppose there are FLRs such that

$$A_i \rightarrow A_{j1}, A_i \rightarrow A_{j2}, \dots, A_i \rightarrow A_{jn}.$$

Then, they can be grouped into a fuzzy logical relationship group (FLRG) and denoted by

$$A_i \rightarrow A_{j1}, A_{j2}, \dots, A_{jn}. \quad (2.4)$$

Chen's method can be shortly described by the following steps:

- Step 1.* Partition the universe of discourse into equal-length intervals.
- Step 2.* Define fuzzy sets on the universe of discourse. Fuzzify the historical data and establish the fuzzy logical relationship based on fuzzified historical data.
- Step 3.* Group fuzzy logical relationship with one or more fuzzy sets on the right-hand side.
- Step 4.* Calculate the forecasted outputs.

In Step 4, Chen carried out the outputs of the experiment on enrollments by three principles:

- (1) If the fuzzified enrollment of year i is A_j , and there is only one fuzzy logical relationship in the fuzzy logical relationship groups which is show as follows $A_j \rightarrow A_k$ where A_j and A_k are fuzzy sets and the maximum membership value of A_k occurs at interval u_k , and the midpoint of u_k is m_k , then the forecasted enrollment of year $i + 1$ is m_k .
- (2) If the fuzzified enrollment of year i is A_j , and there are the following fuzzy logical relationships in the fuzzy logical relationship groups $A_j \rightarrow A_{k1}, A_{k2}, \dots, A_{kp}$ where $A_j, A_{k1}, A_{k2}, \dots, A_{kp}$ are fuzzy sets, and the maximum membership values of $A_{k1}, A_{k2}, \dots, A_{kp}$ occur at intervals u_1, u_2, \dots, u_p , respectively and the midpoints of u_1, u_2, \dots, u_p are m_1, m_2, \dots, m_p , respectively, then the forecasted enrollment of year $i + 1$ is $(m_1 + m_2 + \dots + m_p)/p$.
- (3) If the fuzzified enrollment of year i is A_j , and there do not exist any fuzzy logical relationship groups whose current state of the enrollment is A_j , where the maximum membership value of A_j occurs at interval u_j and the midpoint of u_j is m_j , then the forecasted enrollment of year $i + 1$ is m_j .

There has been a lot of researches to improve the calculation models as mentioned above. In general, the fuzzy set theory approach is very flexible, especially, for the time series modeled in terms of linguistic words or for those whose number of observations is small. However, analyzing these forecasting methods based on fuzzy time series, we observe that the fuzzy sets A_j 's are constructed based only on the researcher's intuition inspired by the semantics of human linguistic words in the aforementioned word-vocabularies. In the matter of fact, there is no formal linkage between human words and the fuzzy sets assigned to them. This motivates us to introduce the so-called linguistic time series based on hedges algebras and their quantification theory.

3. HEDGE ALGEBRAS AND SEMANTICS OF WORDS

The motivation of hedge algebras (HAs) approach is to interpret each words-set of a linguistic variable as an algebra whose order-based structure is induced by the inherent qualitative meaning of linguistic words. By this, its order relation is called semantical order relation.

In this section, we recall some basic concepts of HAs. As mentioned above, the ordering relation of linguistic values creates their semantics. We focus on fuzziness measure (fm), sign function, and semantically quantifying mappings (SQMs) of HAs. They are necessary mathematical knowledge of HAs that will be used to present our proposed forecasting model. More details can be found in [37] or [47].

Let $AX = (X, G, C, H, \leq)$ be an HAs, where $G = \{c^-, c^+\}$ is a set of generators called, respectively, the negative primary word and the positive one of X ; $C = \{0, W, 1\}$ is set of constant which are the least, the neutral and the greatest, respectively; $H = \{h^-, h^+\}$ is a set of hedges of X , regarded as unary operations, where h^- and h^+ are the negative hedge and positive one, respectively; and \leq is the semantic order relation of words in X .

Definition 3.1. Let $AX = (X, G, C, H, \leq)$ be an HAs. A function $fm : X \rightarrow [0, 1]$ is said to be fuzziness measure of words in X if

- $fm(c^-) + fm(c^+) = 1$ and $\sum_{(h \in H)} fm(hu) = fm(u)$, for $\forall u \in X$;
- For the constants 0, W and 1: $fm(0) = fm(W) = fm(1) = 0$;
- For $\forall x, y \in X$, $\forall h \in H$, $\frac{fm(hx)}{fm(x)} = \frac{fm(hy)}{fm(y)}$, this proportion does not depend on specific elements x and y and, hence, it is called fuzziness measure of the hedge h and denoted by $\mu(h)$.

Every fuzziness measure fm on X has the following properties:

- f1) $fm(hx) = \mu(h)fm(x)$ for $\forall x \in X$;
- f2) $fm(c^-) + fm(c^+) = 1$;
- f3) $\sum_{-q \leq i \leq p, i \neq 0} fm(h_i c) = fm(c)$, $c \in \{c^-, c^+\}$;
- f4) $\sum_{-q \leq i \leq p, i \neq 0} fm(h_i x) = fm(x)$;
- f5) Put $\sum_{-q \leq i \leq -1} \mu(h_i) = \alpha$, $\sum_{1 \leq i \leq p} \mu(h_i) = \beta$, we have $\alpha + \beta = 1$.

It can be seen that given the values of $fm(c^-)$, $\mu(h)$, $h \in H$, fm is completely defined and, hence, we call them the fuzziness parameters of the variable in question. It is interesting that from the given fuzziness parameters, one can define and calculate the numeric semantics of every word x , $v(x)$, which can shortly be described as follows.

Definition 3.2. A function $sign : X \rightarrow \{-1, 1\}$ is a mapping which is defined recursively as follows. For $h, h' \in H$ and $c \in \{c^-, c^+\}$:

- 1) $sign(c^-) = -1$, $sign(c^+) = +1$;
- 2) $sign(hc) = -sign(c)$ for h being negative w.r.t c , otherwise, $sign(hc) = +sign(c)$;
- 3) $sign(h'hx) = -sign(hx)$ if $h'hx \neq hx$ and h' is negative w.r.t h ;
- 4) $sign(h'hx) = +sign(hx)$ if $h'hx \neq hx$ and h' is positive w.r.t h .

Theorem 3.1. [47] For given values of the fuzziness parameter of a variable, its corresponding SQM $v : X \rightarrow [0, 1]$ is defined as follows

- 1) $v(W) = \theta = fm(c^-)$;
- 2) $v(c^-) = \theta - \alpha fm(c^-) = \beta fm(c^-)$;
- 3) $v(c^+) = \theta + \alpha fm(c^+) = 1 - \beta fm(c^+)$;
- 4) $v(h_j x) = v(x) + \text{sign}(h_j x) \left\{ \sum_{i=\text{sign}(j)}^j fm(h_i x) - \omega(h_j x) fm(h_j x) \right\}$, where

$$\omega(h_j x) = \frac{1}{2} [1 + \text{sign}(h_j x) \text{sign}(h_p h_j x) (\beta - \alpha)] \in \{\alpha, \beta\}.$$

4. LINGUISTIC TIME SERIES AND ITS FORECASTING MODEL

4.1. Linguistic time series and its forecasting model

To deal with the uncertainty of time data series forecasting, Song and Chissom in their studies [1, 2, 3] proposed a concept of fuzzy time series established based on a given ordinary data time series and a formalism to handle uncertainty represented by fuzzy sets. The main advantage of the fuzzy time series is the ability to handle the uncertainty in the nature of the time series forecasting problem. In existing approaches, however, the fuzzy sets are constructed based on the researchers' intuition in the context of the data time series in question. There is no formal basis to connect the constructed fuzzy sets to possibly intended words assigned to them. Obviously, it is very useful and beneficial when one can deal immediately with human words based on a formal formalism with sufficient reliability, say a theory developed soundly based on an axiomatic way.

As aforementioned, in this study, we deal with the so-called linguistic time series introduced to solve the time series forecasting problem which, by the fact of the matter, essentially involves uncertainty. Since human has capacities to deal with the uncertainty in terms of their own natural words, the linguistic time series and the formalism developed based on the HA-formalism to handle their uncertainty to solve the data time series forecasting problem seem to be useful and beneficial. One may find some studies using the terminology 'linguistic time series' in the literature, e.g., [48, 49]. However, these studies, in nature, are essentially based on the formalism of either the fuzzy time series forecasting methodology [49], or fuzzy recurrent neural network [48]. According to our knowledge, the linguistic time series, in which linguistic words appear as linguistic data and are handled immediately based on a strict mathematical formalism without using fuzzy sets, are, for the first time, used in this study. For this reason, we introduce the following definition of this new concept.

Definition 4.1. (Linguistic time series) Let \mathbb{X} be a set of linguistic words in natural language of a variable \mathfrak{X} defined on the universe of discourse U_x to describe its numeric quantities. Then, any series $L(t)$, $t = 0, 1, 2, \dots$, where $L(t)$ is a finite subset of \mathbb{X} , is called a linguistic time series.

For example, for a given time t , $L(t)$ is a collection of words $X(t)$'s in \mathbb{X} to describe possible data of enrollments of an university. The way to construct a linguistic time series for a given historical numeric data is simply as follows. Note that in existing fuzzy-set-based approach to the time series forecasting problem, for a given data time series, the main crucial task is to decompose the range of its possible numeric values into intended intervals u_j 's in to form a universe on which the fuzzy sets associated word-labels under consideration are

defined, refer to [1]. In our approach, we immediately start with the given possible words used to describe the values of the determined range of the given historical data.

Definition 4.2. (The linguistic logical relationship) Suppose X_i and X_j are the linguistic words representing the data at the time t and $t + 1$, respectively. Then, there exists a relationship between X_i and X_j called linguistic logical relationship (LLR) and denoted by

$$X_i \rightarrow X_j.$$

Definition 4.3. (The linguistic logical relationship group) Assume that there are LLRs such as

$$\begin{aligned} X_i &\rightarrow X_{j1}, \\ X_i &\rightarrow X_{j2}, \\ &\dots \\ X_i &\rightarrow X_{jn}. \end{aligned}$$

Then, they can be grouped into a linguistic logical relationship group (LLRG) and denoted by

$$X_i \rightarrow X_{j1}, X_{j2}, \dots, X_{jn}.$$

The proposed forecasting model based on linguistic time series comprises the following steps:

Step 1. Determine the universe of discourse. Establish hedge algebras structure, choose α, β and choose the linguistic words according to the source data.

Step 2. Calculate the quantifying semantics of words using equations 1) to 4) of Theorem 3.1.

Step 3. Mapping the quantifying semantics of words to the domain of the universe of discourse. So, we have semantic points collection.

Step 4. ‘Semantize’ the historical data. For each specified point, the semantic of this point depends on the nearest semantic point.

Step 5. Establish the linguistic logical relationships of words and group them to the linguistic logical relationship groups.

Step 6. Calculate the forecasted results based on linguistic logical relationship groups and the principles.

We applied this model to the data of enrollments of the University of Alabama from 1971 to 1992. The enrollments were observed as in Table 1.

1) *Application of the proposed model to the above numeric time series*

Based on the proposed model, the procedure to solve the linguistic time series forecasting problem of the historical enrollments of Alabama is constructed and described as follows:

Step 1. Let D_{\min} and D_{\max} be the minimum enrollment and the maximum enrollment, respectively $D_{\min} = 13055$ and $D_{\max} = 19337$.

In [4], Chen defined the universe of discourse is $[13000, 20000]$. Then, he partition the universe of discourse to seven equal length intervals and using corresponding linguistic values: *not many* (A_1), *not too many* (A_2), *many* (A_3), *many many* (A_4), *very many* (A_5), *too many* (A_6), *too many many* (A_7).

In our method, we also choose the same universe of discourse with Chen. We assume D_L, D_R be the first value and the last value of the universe of discourse, respectively. Hence,

Table 1. Historical enrollments of University of Alabama from 1971 to 1992

| Year | Actual enrollments | Label | Year | Actual enrollments | Label |
|------|--------------------|-------|------|--------------------|-------|
| 1971 | 13055 | X_1 | 1982 | 15433 | X_2 |
| 1972 | 13563 | X_1 | 1983 | 15497 | X_2 |
| 1973 | 13867 | X_1 | 1984 | 15145 | X_2 |
| 1974 | 14696 | X_2 | 1985 | 15163 | X_2 |
| 1975 | 15460 | X_2 | 1986 | 15984 | X_3 |
| 1976 | 15311 | X_2 | 1987 | 16859 | X_4 |
| 1977 | 15603 | X_3 | 1988 | 18150 | X_6 |
| 1978 | 15861 | X_3 | 1989 | 18970 | X_7 |
| 1979 | 16807 | X_4 | 1990 | 19328 | X_7 |
| 1980 | 16919 | X_4 | 1991 | 19337 | X_7 |
| 1981 | 16388 | X_3 | 1992 | 18876 | X_7 |

$D_L = 13000$ and $D_R = 20000$. We do not partition the universe of discourse into intervals. Because our model based on hedge algebras, we choose $c^- = S$ (*Small*), $c^+ = L$ (*Large*) and two hedges $h^{-1} = R$ (*Rather*), $h^{+1} = V$ (*Very*). Using two hedges R and V impact two basic elements (generators) S and L we have

$$\text{dom}(\text{Enrollments}) = \{VS, S, RS, M, RL, L, VL\}.$$

We select seven linguistic values to describe the number of enrollments: *Very Small* (X_1), *Small* (X_2), *Rather Small* (X_3), *Middle* (X_4), *Rather Large* (X_5), *Large* (X_6) and *Very Large* (X_7). Note that every linguistic value in hedge algebras has its order. We also assign seven labels to seven linguistic values as above from X_1 to X_7 .

Step 2. Apply equations from 1) to 4) of Theorem 3.1, we have quantity semantic of words as follows

$$v(X_1) = \theta - 2\theta\alpha + \theta\alpha^2; \quad (4.1)$$

$$v(X_2) = \theta - \theta\alpha; \quad (4.2)$$

$$v(X_3) = \theta - \theta\alpha^2; \quad (4.3)$$

$$v(X_4) = \theta; \quad (4.4)$$

$$v(X_5) = \theta + \alpha^2 - \theta\alpha^2; \quad (4.5)$$

$$v(X_6) = \theta - \theta\alpha + \alpha; \quad (4.6)$$

$$v(X_7) = \theta + 2\alpha - \alpha^2 - 2\theta\alpha + \theta\alpha^2. \quad (4.7)$$

Normally, the neutral values are $\theta = 0.5$ and $\alpha = 0.5$. In this study, to emphasize the meaning of the semantic of words, two parameters θ , α will be achieved by trial and error. We try to turn them with error $\epsilon = 0.01$ around the neutral values and get the choosing values of $\theta = 0.57$, $\alpha = 0.49$.

Applying above equations, we have $v(X_1) = 0.1483$, $v(X_2) = 0.2907$, $v(X_3) = 0.4331$, $v(X_4) = 0.57$, $v(X_5) = 0.6732$, $v(X_6) = 0.7807$, $v(X_7) = 0.8882$. The values of $v(X_i)$, $i = 1, \dots, 7$ will change if we choose different values of θ and α .

Step 3. Mapping $v(X_i)$, $i = 1, \dots, 7$ to the universe of discourse, we have seven real semantic points that similar with the mid-points of seven intervals in Chen's model. The equation for mapping as follows

$$v_R(i) = D_L + (D_R - D_L) \times v(X_i).$$

With the data of enrollments, $D_L = 13000$, $D_R = 20000$, we have seven real semantic points $\{14038, 15035, 16032, 16990, 17713, 18465, 19217\}$.

Seven values above corresponding to seven linguistic values: *Very Small* (X_1), *Small* (X_2), *Rather Small* (X_3), *Middle* (X_4), *Rather Large* (X_5), *Large* (X_6) and *Very Large* (X_7) where X_i , $i = 1, \dots, 7$ be the labels of linguistic values.

Step 4. 'Semantization' of the given historical data is an assignment of a linguistic value to each datum of historical data of enrollments. For the actual enrollment of specific year, we select a linguistic value in $X_1 \dots X_7$ to assign for each year depend on which semantic point is the nearest to the actual enrollment. For example, the enrollment of year 1971 is 13055, hence, the linguistic value of year 1971 is X_1 because 14038 is the nearest semantic point to the actual enrollment. Similarly, the linguistic value corresponding to 1992 is X_7 because 19217 (X_7) is the nearest semantic point to 18876.

Step 5. Scan from the beginning to the end of historical data with their linguistic values, we have the LLRs between words. If linguistic value of year k is X_i and the linguistic value of year $k + 1$ is X_j then we have the LLR: $X_i \rightarrow X_j$. In this case, we have LLRs as follows

Table 2. LLRs of the enrollments

| |
|--|
| $X_1 \rightarrow X_1; X_1 \rightarrow X_2; X_2 \rightarrow X_2; X_2 \rightarrow X_3; X_3 \rightarrow X_2; X_3 \rightarrow X_3$ |
| $X_3 \rightarrow X_4; X_4 \rightarrow X_3; X_4 \rightarrow X_4; X_4 \rightarrow X_6; X_6 \rightarrow X_7; X_7 \rightarrow X_7$ |

Establish the linguistic logical relationship groups (LLRGs) based on the LLRs that was observed above

Table 3. Linguistic logical relationship groups

| Group | LLRGs | Shorthand |
|---------|---|---------------------------------|
| Group 1 | $X_1 \rightarrow X_1, X_1 \rightarrow X_2$ | $X_1 \rightarrow X_1, X_2$ |
| Group 2 | $X_2 \rightarrow X_2, X_2 \rightarrow X_3$ | $X_2 \rightarrow X_2, X_3$ |
| Group 3 | $X_3 \rightarrow X_2, X_3 \rightarrow X_3, X_3 \rightarrow X_4$ | $X_3 \rightarrow X_2, X_3, X_4$ |
| Group 4 | $X_4 \rightarrow X_3, X_4 \rightarrow X_4, X_4 \rightarrow X_6$ | $X_4 \rightarrow X_3, X_4, X_6$ |
| Group 5 | $X_6 \rightarrow X_7$ | $X_6 \rightarrow X_7$ |
| Group 6 | $X_7 \rightarrow X_7$ | $X_7 \rightarrow X_7$ |

Step 6. Calculate the forecasted data based on LLRGs and principles as follows:

- (1) If the linguistic value of year k is X_i and there exist the LLRG: $X_i \rightarrow X_{j1}, X_{j2}, \dots, X_{jp}$, $p \geq 1$ then the forecasted value of year $k+1$ is $(s_{j1} + s_{j2} + \dots + s_{jp})/p$, where $s_{j1}, s_{j2}, \dots, s_{jp}$ is the semantic point(s) of $X_{j1}, X_{j2}, \dots, X_{jp}$, respectively.
- (2) If the linguistic value of year k is X_i and there does not exist any LLR with X_i in the right-hand side. Then, the forecasted value of year $k+1$ is s_i where s_i is the semantic point of X_i .

2) Simulation study to justify its performance

We applying this constructed procedure to the numeric time series to simulate its forecasting performance at each time of the numeric time series of to the enrollments of the University of Alabama. Performing calculations with the proposed model and comparing with Song et al.'s method and Chen's method we have results as follows.

Table 4. The comparison of forecasted results

| Year | Actual enrollments | Song et al.'s method[2] | Chen's method[4] | Proposed method |
|--------------------------|--------------------|-------------------------|------------------|-----------------|
| 1971 | 13055 | | | |
| 1972 | 13563 | 14000 | 14000 | 14537 |
| 1973 | 13867 | 14000 | 14000 | 14537 |
| 1974 | 14696 | 14000 | 14000 | 14537 |
| 1975 | 15460 | 15500 | 15500 | 15534 |
| 1976 | 15311 | 16000 | 16000 | 15534 |
| 1977 | 15603 | 16000 | 16000 | 15534 |
| 1978 | 15861 | 16000 | 16000 | 16019 |
| 1979 | 16807 | 16000 | 16000 | 16019 |
| 1980 | 16919 | 16813 | 16833 | 17162 |
| 1981 | 16388 | 16813 | 16833 | 17162 |
| 1982 | 15433 | 16709 | 16833 | 16019 |
| 1983 | 15497 | 16000 | 16000 | 15534 |
| 1984 | 15145 | 16000 | 16000 | 15534 |
| 1985 | 15163 | 16000 | 16000 | 15534 |
| 1986 | 15984 | 16000 | 16000 | 15514 |
| 1987 | 16859 | 16000 | 16000 | 16019 |
| 1988 | 18150 | 16813 | 16833 | 17162 |
| 1989 | 18970 | 19000 | 19000 | 19217 |
| 1990 | 19328 | 19000 | 19000 | 19217 |
| 1991 | 19337 | 19000 | 19000 | 19217 |
| 1992 | 18876 | - | 19000 | 19217 |
| Mean squared error (MSE) | | 412.499 | 407.507 | 262.326 |

The MSE (mean squared error) measure is defined as follows

$$MSE = \frac{1}{N} \sum_i (F_i - A_i)^2$$

where F_i and A_i are the forecasted value and actual value of year i , respectively. N is the total forecasted years. As we can see in Table 4, our proposed forecasting model has better mean squared errors (MSE) of 262.326 than Song et al.'s and Chen's are 412.499 and 407.507, respectively.

4.2. Linguistic time series based on variations of historical enrollments

In [6], Hwang et al. introduce a time-variant fuzzy time series forecasting model using variations of historical data. His study suggests that, how and in which way one can model and calculate the variations of the data is important. In this section, we will show that, based on hedge algebras, linguistic time series are very useful to linguistically describe and computationally handle the variations of historical data to solve a forecasting problem. Historical enrolments of the University of Alabama from year 1972 to 1992 and their variations given in Table 5 are used in [6], and also in this study to illustrate our forecasting process based on linguistic time series and to compare its performance with the one proposed in the study [6], where the range of the variable, denoted by ν , is defined in the interval $[-1000; +1400]$.

Table 5. Variations of historical enrollments data

| Year | Enrollments | Variations | Label | Year | Enrollments | Variations | Label |
|------|-------------|------------|-------|------|-------------|------------|-------|
| 1971 | 13055 | | | 1982 | 15433 | -955 | X_1 |
| 1972 | 13563 | +508 | X_5 | 1983 | 15497 | +64 | X_3 |
| 1973 | 13867 | +304 | X_4 | 1984 | 15145 | -352 | X_2 |
| 1974 | 14696 | +829 | X_6 | 1985 | 15163 | +18 | X_3 |
| 1975 | 15460 | +764 | X_6 | 1986 | 15984 | +821 | X_6 |
| 1976 | 15311 | -149 | X_3 | 1987 | 16859 | +875 | X_6 |
| 1977 | 15603 | +292 | X_4 | 1988 | 18150 | +1291 | X_7 |
| 1978 | 15861 | +258 | X_4 | 1989 | 18970 | +820 | X_6 |
| 1979 | 16807 | +946 | X_6 | 1990 | 19328 | +358 | X_4 |
| 1980 | 16919 | +112 | X_3 | 1991 | 19337 | +9 | X_3 |
| 1981 | 16388 | -531 | X_1 | 1992 | 18876 | -461 | X_2 |

For the method proposed by Hwang et al. [6], the above variation range is partitioned into equal intervals of the same length 400. Then, they calculate the forecasted results based on their proposed time-variant fuzzy time series forecasting model. In our method, based on hedge algebras, we choose $c^- = d(\text{decreasing})$, $c^+ = i(\text{increasing})$ and two hedges $h^{-1} = L(\text{Little})$, $h^{+1} = V(\text{Very})$. Hence, the word-domain of the variable ν can be described by the following word-set of seven linguistic values:

$$L\text{DOM}(\nu) = \{V_decre, decre, L_decre, med, L_incre, incre, V_incre\},$$

where the notations in the above braces are defined and denoted by X_i , for short, as follows: V_decre : Very decreasing (X_1), $decre$: decreasing (X_2), L_decre : Little decreasing (X_3), med : medium (X_4), L_incre : Little increasing (X_5), $incre$: increasing (X_6), V_incre : Very increasing (X_7). Then, we can transform the numeric time-variant data given in the column “Variations” into a linguistic time series given in the column “Label”, which is very comprehensive in terms of human words and is listed as follows:

$L_incre, med, incre, incre, L_decre, med, med, incre, L_decre, V_decre, V_decre, L_decre, decre, L_decre, incre, incre, V_incre, incre, med, L_decre, decre$

When the two independent fuzziness parameter values of the linguistic variable ν , $\theta = 0.55$ and $\alpha = 0.52$, are determined by trial and error, we can calculate the quantitative semantic values of the words of $LDOM(\nu)$ occurring in the above linguistic time series, using equations (4.1) to (4.7) in Section 4.1, and transform the words in the above word-domain, $LDOM(\nu)$, into their corresponding numeric values in $\{0.1267; 0.264; 0.4013; 0.55; 0.6717; 0.784; 0.8963\}$, which is a subset of the normalized numeric universe, $[-1000; +1400]$, of ν . Thus, we must transform these numeric values in $[0, 1]$ into the corresponding values in $[-1000; +1400]$, and obtain

$$\{-696; -366; -37; +320; +612; +882; +1151\}.$$

We semantize the given historical data and establish the linguistic logical relationships (LLRs) represented in Table 6, using forecasting model in Section 4.1.

Table 6. LLRs between variations

| |
|--|
| $X_1 \rightarrow X_1; X_1 \rightarrow X_3; X_2 \rightarrow X_3; X_3 \rightarrow X_1; X_3 \rightarrow X_2; X_3 \rightarrow X_4$ |
| $X_3 \rightarrow X_6; X_4 \rightarrow X_3; X_4 \rightarrow X_4; X_4 \rightarrow X_6; X_5 \rightarrow X_4; X_6 \rightarrow X_3$ |
| $X_6 \rightarrow X_4; X_6 \rightarrow X_6; X_6 \rightarrow X_7; X_7 \rightarrow X_6;$ |

Grouping the obtained linguistic logical relationships into groups, we have the following LLRGs:

Table 7. LLRGs between variations

| Group | LLRGs | Shorthand |
|---------|--|--------------------------------------|
| Group 1 | $X_1 \rightarrow X_1, X_1 \rightarrow X_3$ | $X_1 \rightarrow X_1, X_3$ |
| Group 2 | $X_2 \rightarrow X_3$ | $X_2 \rightarrow X_3$ |
| Group 3 | $X_3 \rightarrow X_1, X_3 \rightarrow X_3, X_3 \rightarrow X_4, X_3 \rightarrow X_6$ | $X_3 \rightarrow X_1, X_3, X_4, X_6$ |
| Group 4 | $X_4 \rightarrow X_3, X_4 \rightarrow X_4, X_4 \rightarrow X_6$ | $X_4 \rightarrow X_3, X_4, X_6$ |
| Group 5 | $X_5 \rightarrow X_4$ | $X_5 \rightarrow X_4$ |
| Group 6 | $X_6 \rightarrow X_3, X_6 \rightarrow X_4, X_6 \rightarrow X_6, X_6 \rightarrow X_7$ | $X_6 \rightarrow X_3, X_4, X_6, X_7$ |
| Group 7 | $X_7 \rightarrow X_6$ | $X_7 \rightarrow X_6$ |

We apply the proposed model and use above LLRGs, to solve the benchmark problem of the historical enrollments of Alabama. The obtained forecasting results of the method are represented in Table 8, whose performance measure MAPE (mean absolute percentage error) computed as follows:

$$MAPE = \frac{100\%}{N} \sum_i \left| \frac{(F_i - A_i)}{A_i} \right|,$$

where F_i and A_i are the forecasted value and actual value of year i , respectively and N is the total number of the forecasted years.

Table 8. Forecasted results of linguistic time series based on variations

| Year | Actual enrollment | Variations | Forecasting results | Errors |
|---------------------|-------------------|------------|---------------------|--------------|
| 1971 | 13055 | | | |
| 1972 | 13563 | + 508 | 133 75 | 1.39% |
| 1973 | 13867 | + 304 | 13951 | 0.61% |
| 1974 | 14696 | + 829 | 14446 | 1.70% |
| 1975 | 15460 | + 764 | 15275 | 1.20% |
| 1976 | 15311 | - 149 | 15495 | 1.20% |
| 1977 | 15603 | + 292 | 15699 | 0.62% |
| 1978 | 15861 | + 258 | 15991 | 0.82% |
| 1979 | 16807 | + 946 | 16440 | 2.18% |
| 1980 | 16919 | + 112 | 16842 | 0.46% |
| 1981 | 16388 | - 531 | 16552 | 1.00% |
| 1982 | 15433 | - 955 | 16021 | 3.81% |
| 1983 | 15497 | + 64 | 15468 | 0.19% |
| 1984 | 15145 | - 352 | 15460 | 2.08% |
| 1985 | 15163 | + 18 | 15180 | 0.11% |
| 1986 | 15984 | + 821 | 15742 | 1.51% |
| 1987 | 16859 | + 875 | 16563 | 1.76% |
| 1988 | 18150 | + 1291 | 17741 | 2.25% |
| 1989 | 18970 | + 820 | 18729 | 1.27% |
| 1990 | 19328 | + 358 | 19358 | 0.16% |
| 1991 | 19337 | + 9 | 19363 | 0.13% |
| 1992 | 18876 | - 461 | 19300 | 2.25% |
| MSE and MAPE | | | 65.029 | 1.27% |

Analyzing these results, we see that the errors vary from 0.11% to 3.81% and the average of errors is 1.27%, which is much better than the results of the method examined in [6], whose forecasting errors vary from 2.79% to 3.08%.

Table 9. The comparisons of MAPE between methods

| Methods | Song and Chissom [2] | Song and Chissom ($w = 4$) | Chen | Sullivan and Woodall [5] | Hwang ($w = 4$)[6] | Proposed |
|---------|-------------------------|---------------------------------|-------|-----------------------------|-------------------------|----------|
| MAPE | 3.2% | 4.37% | 3.22% | 2.6% | 3.12% | 1.27% |

Table 9 represents a comparison between the performance of the proposed method with the ones of some other forecasting methods discussed in [6]. It shows also that the performance of the proposed method, in general, is noticeably better than the one of the fuzzy time series forecasting methods under consideration in solving the enrollment forecasting problem.

5. CONCLUSIONS

In this study, we propose a new forecasting model based on the so-called linguistic time series, in which the linguistic words are considered as elements of a hedge algebra that models the respective word-domain of the linguistic variable. A distinguished feature of our method is to deal immediately with linguistic words instead of with fuzzy sets, whose associated words are considered merely linguistic labels. Because hedge algebras are mathematical models of the word-domains of linguistic variables, similar as the theory of the real numbers modeling the domains of their counterpart real variables, the formalism to handle linguistic time series can be developed based on the hedge algebra formalism instead of fuzzy one to deal with vague data.

In fuzzy formalism, the elementary mathematical objects are fuzzy sets. Hence, the performance of fuzzy time series forecasting method depends strongly on two main factors. The first factor is how these fuzzy sets can be soundly constructed based on the developers' subjective intuition to represent the semantics of their associated linguistic labels. The second one is which method to compute the representative (single) value can properly represent the area of its fuzzy set. It is, in general, a difficult problem in the fuzzy set framework. Moreover, words have their own objective semantics commonly understood between a human user domain community and their numeric semantics is, more or less, able commonly defined between domain experts.

Hedge algebras and their quantification theory are developed in an axiomatic way, whose axioms are justified to soundly model the semantic structures of word-domains of variables. Once the fuzziness parameter values of a variable are "properly" determined, the numeric semantics of the variable words are uniquely computed. Thus, human experts may focus their effort on proper determining only a few of these fuzziness parameter values. When they are properly determined, the numeric semantics of all words of the variable of a given time series forecasting problem can be exactly computed. Also, human words are very comprehensive, and easily, commonly understood between human users/experts of a human forecasting domain community. They can easily use their vocabulary to describe the possible variations of a time data series. This situation suggests us to introduce a concept of linguistic time (variant) time series model to solve time data series forecasting problems.

The time series forecasting method proposed in this study allows to translate a given

time variant data series into a respective linguistic time variant series, then, to determine linguistic logical relationships (LLRs) in terms of human linguistic rules, which are very informative and comprehensive for human community users. The prediction values can be easily computed in the formalism of the quantification theory of the specified hedge algebra. The experiment performed in Section 4 shows that the performance of the proposed forecasting method is noticeably better than the counterpart methods under consideration, including the one examined in [6].

As our future work, we intend to apply the proposed model to solve more complex time data series and enhance the proposed model. We can also apply the proposed model to the other data sets for illustrating its advantage.

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