PROBLEM SET 11: PRINCIPLES OF INTEGRAL EVALUATION

1) Evaluate the given integral with the aid of an appropriate u-substitution.

a) $\int \sqrt{4+9x}\,dx$ b) $\int \frac{1}{\sec \pi x}\,dx$	c) $\int \sqrt{\cos x}\,\sin x\,dx$ d) $\int \frac{1}{x \ln x}\,dx$	e) $\int x\tan^2(x^2)\sec^2(x^2)\,dx$ f) $\int_0^9 \frac{\sqrt{x}}{x+9}\,dx$

2) (a) Evaluate the integral $\int \frac{1}{\sqrt{2x-x^2}}\,dx$ three ways: using the substitution $u=\sqrt{x}$, using the substitution $u=\sqrt{2-x}$, and completing the square.
(b) Show that the answers in part (a) are equivalent.

3) Evaluate the integral $\int_0^1 \frac{x^3}{\sqrt{x^2+1}}\,dx$

a) Using integration by parts
b) Using the substitution $u=\sqrt{x^2+1}$.

4) Using integration by parts

a) $\int xe^{-x}\,dx$ b) $\int x\sin 2x\,dx$	c) $\int \ln(2x+3)\,dx$ d) $\int_0^{1/2} \tan^{-1}(2x)\,dx$

5) Evaluate $\int 8x^4\cos 2x\,dx$ using tabular integration by parts.

6) Evaluate the integral.

a) $\int \sin^2 5\theta\,d\theta$ b) $\int \sin^3 2x\cos^2 2x\,dx$	c) $\int \sin x\cos 2x\,dx$ d) $\int_0^{\pi/6} \sin 2x\cos 4x\,dx$	e) $\int \sin^4 2x\,dx$ f) $\int x\cos^5(x^2)\,dx$

7) Evaluate the integral by making an appropriate trigonometric substitution.

a) $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$ b) $\int \frac{dx}{x^2\sqrt{16-x^2}}$	c) $\int \frac{dx}{\sqrt{x^2-1}}$ d) $\int \frac{x^2}{\sqrt{x^2-25}}\,dx$	e) $\int \frac{x^2}{\sqrt{9+x^2}}\,dx$ f) $\int \frac{\sqrt{1+4x^2}}{x}\,dx$

8) Evaluate the integral using the method of partial fractions.

a) $\int \frac{dx}{x^2+3x-4}$ b) $\int \frac{dx}{x^2+8x+7}$	c) $\int \frac{x^2+2}{x+2}dx$ d) $\int \frac{x^2+x-16}{(x-1)(x-3)^2}dx$	e) $\int \frac{x^2}{(x+2)^3}dx$ f) $\int \frac{dx}{x^3+x}$

9) Find the area of the region that is enclosed by the curves $y=\frac{x-3}{x^3+x^2}, y=0, x=1,$ and $x=2$.

10) Evaluate the integral if it converges.

a) $\int_0^{+\infty} e^{-x}dx$	b) $\int_{-\infty}^{2} \frac{dx}{x^2+4}$	c) $\int_0^9 \frac{dx}{\sqrt{9-x}}$	d) $\int_0 \frac{1}{2x-1}dx$

11) Find the area that is enclosed between the x-axis and the curve $y=\frac{\ln x-1}{x^2}$ for $x \geq e$.

12) Consider the following method for evaluating integrals: u-substitutions, integration by parts, patial fractions, reduction formulas, and trigonometric substitutions. In each part, state the approach that you would try first to evaluate the integral. If none of them seems appropriate, then say so. You need not evaluate the integral.

a) $\int x\sin x\, dx$ b) $\int \cos x \sin x\, dx$ c) $\int \tan^7 x\, dx$	d) $\int \tan^7 x \sec^2 x\, dx$ e) $\int \frac{3x^2}{x^3+1}dx$ f) $\int \frac{3x^2}{(x+1)^3}dx$	g) $\int \tan^{-1} x\, dx$ h) $\int \sqrt{4-x^2}\, dx$ i) $\int x\sqrt{4-x^2}\, dx$

13) Evaluate the integral.

a) $\int \frac{dx}{(3+x^2)^{3/2}}$ b) $\int x\cos 3x\, dx$ c) $\int_0^{\pi/4} \tan^7 \theta\, d\theta$ d) $\int \frac{\cos\theta}{\sin^2\theta-6\sin\theta+12}d\theta$	i) $\int \frac{dx}{(x-1)(x+2)(x-3)}$ j) $\int_0^{1/3} \frac{dx}{(4-9x^2)^2}$ k) $\int_4^8 \frac{\sqrt{x-4}}{x}dx$	o) $\int_0^{1/2} \sin^{-1} x\, dx$ p) $\int \tan^5 4x \sec^4 4x\, dx$ q) $\int \frac{x+3}{\sqrt{x^2+2x+2}}dx$ r) $\int \frac{\sec^2\theta}{\tan^3\theta-\tan^2\theta}d\theta$

e) $\int \sin^2 2x \cos^3 2x \, dx$ f) $\int \frac{1}{(x-3)^2} dx$ g) $\int e^{2x} \cos 3x \ \ dx$ h) $\int_{-1/\sqrt{2}}^{1/\sqrt{2}} \left(1-2x^2\right)^{3/2} dx$	l) $\int_0^{\ln 2} \sqrt{e^x - 1} \, dx$ m) $\int \frac{1}{\sqrt{e^x+1}} dx$ n) $\int \frac{dx}{x\left(x^2+x+1\right)}$	s) $\int_a^{+\infty} \frac{x}{\left(x^2+1\right)^2} dx$ t) $\int_0^{+\infty} \frac{dx}{a^2+b^2x^2}, \qquad a,b>0$