PROBLEM SET 10 : INTEGRATION

1. Evalute the integral

a) $\int\left[\frac{1}{2x^3}+4\sqrt{x}\right]dx$

b) $\int\left[u^3-2u+7\right]du$

c) $\int[4\sin x+2\cos x]dx$

d) $\int\sec x(\tan x+\cos x)dx$

e) $\int\left[x^{-2/3}-5e^x\right]dx$

f) $\int\left[\frac{3}{4x}-\sec^2 x\right]dx$

g) $\int\left[\frac{1}{1+x^2}+\frac{2}{\sqrt{1-x^2}}\right]dx$

h) $\int\left[\frac{12}{x\sqrt{x^2-1}}+\frac{1-x^4}{1+x^2}\right]dx$

2. Solve the initial problems

a) $\frac{dy}{dx}=\frac{1-x}{\sqrt{x}}, y(1)=0$

b) $\frac{dy}{dx}=\cos x-5e^x, y(0)=0$

c) $\frac{dy}{dx}=\sqrt[3]{x}, y(1)=2$

d) $\frac{dy}{dx}=xe^{x^2}, y(0)=0$

3. a) Show that the substitution $u=\sec x$ and $u=\tan x$ produce different values for the integral

$$\int\sec^2 x\tan x dx$$

b) Explain why both are correct.

4. Use the two substitution in Exercise 3 to evaluate the definite integral

$$\int_0^{\pi/4}\sec^2 x\tan x\,dx$$

And confirm that they produce the same result.

5. Evaluate the integral

$$\int\frac{x}{(x^2-1)\sqrt{x^4-2x^2}}dx$$

by making the substitution $u=x^2-1$.

6. Evaluate the integral

$$\int\sqrt{1+x^{-2/3}}dx$$

by making the substitution $u=1+x^{2/3}$.

7. Find the are under the graph of $f(x)=4x-x^2$ over the interval $[0;4]$ using Definition 5.4.3 with x_k^* as the right endpoint of each subinterval.

8. Find the are under the graph of $f(x)=5x-x^2$ over the interval $[0;5]$ using Definition 5.4.3 with x_k^* as the left endpoint of each subinterval.

9. Use the geometric argument to evaluate $\int_0^1|2x-1|dx$.

10. Suppose that

$$\int_0^1 f(x)\,dx = \frac{1}{2}, \qquad \int_1^2 f(x)\,dx = \frac{1}{4},$$
$$\int_0^3 f(x)\,dx = -1, \qquad \int_0^1 g(x)\,dx = 2$$

In each part use this information to evaluate the given integral, if possible. If there is not enough information to evaluate the integral, then say so.

a) $\int_0^2 f(x)\,dx$ b) $\int_1^3 f(x)\,dx$ c) $\int_2^3 5f(x)\,dx$

d) $\int_1^0 g(x)\,dx$ e) $\int_0^1 g(2x)\,dx$ f) $\int_0^1 [g(x)]^2\,dx$

g) $\int_0^1 [f(x)+g(x)]\,dx$ h) $\int_0^1 [f(x)g(x)]\,dx$ i) $\int_0^1 \frac{f(x)}{g(x)}\,dx$

k) $\int_0^1 [4g(x)-3f(x)]\,dx$

11. Evaluate the integals using the Fundamental Theorem of Calculus and (if necessary) properties of the definite integral.

a) $\int_{-3}^0 (x^2-4x+7)\,dx$ e) $\int_0^1 (x-\sec x\tan x)\,dx$

b) $\int_{-1}^2 x(1+x^3)\,dx$ f) $\int_1^4 \left(\frac{3}{\sqrt{t}}-5\sqrt{t}-t^{-3/2}\right)dt$

c) $\int_1^3 \frac{1}{x^2}\,dx$ g) $\int_0^2 |2x-3|\,dx$

d) $\int_1^8 (5x^{2/3}-4x^{-2})\,dx$ h) $\int_0^{\pi/2} \left|\frac{1}{2}-\sin x\right|dx$

12. Find the area under the curve $y=f(x)$ over the stated interval

a) $f(x)=\sqrt{x}$; $[1,9]$

b) $f(x)=e^x$; $[1,3]$

13. Find the area that is above the x-axis but below the curve $y=(1-x)(x-2)$. Make a sketch of the region.

14. Sketch the curve and find the total area between the curve and the given interval on the x-axis.

a) $y=x^2-1$; $[0,3]$

b) $y=\sqrt{x+1}-1$; $[-1,1]$

15. Evaluate the integral by making an appropriate substitution.

a) $\int_0^1 (2x+1)^4 \, dx$

b) $\int_{-5}^0 x\sqrt{4-x}\,dx$

c) $\int_0^1 \frac{dx}{\sqrt{3x+1}}$

d) $\int_0^{\sqrt{\pi}} x \sin x^2 \, dx$

e) $\int_0^1 \sin^2(\pi x)\cos(\pi x)\,dx$

f) $\int_e^{e^2} \frac{dx}{x \ln x}$

g) $\int_0^1 \frac{dx}{\sqrt{e^x}}$

h) $\int_0^{2/\sqrt{3}} \frac{1}{4+9x^2}\,dx$