PROBLEM SET 9 – TOPIC IN DEIFFERENTIATION

1. (i) Find dy/dx by differentiating implicitly.
(ii) Solve the equation for y as a function of x, and find dy/dx from that equation.
(iii) Confirm that the two results are consistent by expressing the derivative in part (a) as a funtion of x alone.
(a) $x^3 + xy - 2x = 1$
(b) $xy = x - y$

2. Find dy/dx by implicit differentiation.
(a) $\frac{1}{y} + \frac{1}{x} = 1$
(b) $x^3 - y^3 = 6xy$
(c) $\sec(xy) = y$
(d) $x^2 = \frac{\cot y}{1 + \csc y}$

3. Find d^2y/dx^2 by implicit differentiation.
(a) $3x^2 - 4y^2 = 7$
(b) $2\mathrm{x}y - y^2 = 3$

4. Use implicit differentiation to find the slope of the tangent line to the curve $y = \tan(\pi y/2), x > 0, y > 0$ at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.

5. At what point(s) is the tangent line to the curve $y^2 = 2x^3$ perpendicular to the line $4\mathrm{x} - 3y + 1 = 0$?

6. Find the coordinate of the point in the first quadrant at which the tangent line to the curve $x^3 - xy + y^3 = 0$ is parallel to the x-axis.

7. Find the coordinate of the point in the first quadrant at which the tangent line to the curve $x^3 - xy + y^3 = 0$ is parallel to the y-axis.

8. Use implicit differentiation to show that the equation of the tangent line to the curve $y^2 = k\mathrm{x}$ at (x_0, y_0) is $yy_0 = \frac{1}{2}k(x + x_0)$

9. Find $\frac{dy}{d\mathrm{x}}$ by first using algebraic properties of the natural logarithm function.
(a) $\ln\left(\frac{(x+1)(x+2)^2}{(x+3)^3(x+4)^4}\right)$
(b) $\ln\left(\frac{\sqrt{x}\sqrt[3]{x+1}}{\sin \mathrm{x} \sec \mathrm{x}}\right)$

10. Find $\frac{dy}{d\mathrm{x}}$

(a) $y = \ln 2\mathrm{x}$

(b) $y = \sqrt[3]{\ln \mathrm{x} + 1}$

(c) $y = \log(\ln \mathrm{x})$

(d) $y = \ln\left(x^{3/2}\sqrt{1+x^4}\right)$

(e) $y = e^{\ln\left(x^2+1\right)}$

(f) $y = 2\mathrm{x}e^{\sqrt{x}}$

(g) $y = x^{\left(e^x\right)}$

(h) $y = (\ln \mathrm{x})^2$

(i) $y = \ln\left(\sqrt[3]{x+1}\right)$

(j) $y = \frac{1+\log \mathrm{x}}{1-\log \mathrm{x}}$

(k) $y = \ln\left(\frac{\sqrt{x}\mathrm{cosx}}{1+x^2}\right)$

(l) $y = \ln\left(\frac{1+e^x+e^{2\mathrm{x}}}{1-e^{3\mathrm{x}}}\right)$

(m)

(n) $y = (1+x)^{\frac{1}{x}}$

11. Find $\frac{dy}{d\mathrm{x}}$ using logarithmic differentiation

(a) $y = \frac{x^3}{\sqrt{x^2+1}}$

b) $y = \sqrt[3]{\frac{x^2-1}{x^2+1}}$

12. Find a point on the graph of $y = e^{3\mathrm{x}}$ at which the tangent line passes through the origin.

13. Show that the function $y = e^{\mathrm{ax}}\sin bx$ satisfies $y'' - 2\mathrm{a}y' + \left(a^2+b^2\right)y = 0$ for any real constant a and b.

14. In each part, find each limit by interpreting the expression as an appropriate derivative

(a) $\lim\limits_{h\to 0}\frac{(1+h)^{\pi}-1}{h}$

b) $\lim\limits_{x\to e}\frac{1-\ln \mathrm{x}}{(x-e)\ln \mathrm{x}}$

15. Evaluate the given limit

(a) $\lim\limits_{x\to+\infty}\left(e^x - x^2\right)$

b) $\lim\limits_{x\to 1}\sqrt{\frac{\ln \mathrm{x}}{x^4-1}}$

(c) $\lim\limits_{x\to 0}\frac{\mathrm{x}^2e^x}{\sin^2 3\mathrm{x}}$

d) $\lim\limits_{x\to e}\frac{a^x-1}{\mathrm{x}},\ a>0$

16. The hypotenuse of a right triangle is growing at a constant rate of a centimetters per second and one leg is decreasing at a constant rate b centimetters per second. How fast is the acute angle between the hypotenuse and the other leg changing at the instant whn both legs are 1 cm?

17. In each part, use the given information to find $\Delta x, \Delta y$ and dy.

(a) $y = \frac{1}{x-1}$; x decreases from 2 to 1.5.

(b) $y = \tan x$; x increases from - $\pi/4$ to 0.

(c) $y = \sqrt{25 - x^2}$; x increases from 0 to 3.

18. Use an appropriate local linear approximation to estimate the value of $\cot 46^o$, and compare your answer to the value obtained with a calculating device.