PROBLEM SET 8: DERIVATIVES

1. In part (a)–(d), use the function $y=\frac{1}{2}x^2$.

(a) Find the average rate of change of y with respect to x over the interval $[3,4]$.

(b) Find the instantaneous rate of change of y with respect to x at $x=3$.

(c) Find the instantaneous rate of change of y with respect to x at general x-value.

(d) Sketch the graph of $y=\frac{1}{2}x^2$ together with the secant line whose slope is given by the result in part (a), and indicate graphically the slope of the tangent line that corresponds to the result in part (b).

2. At time $t=0$ a car moves into the passing lane to pass a slow-moving truck. The average velocity of the car from $t=1$ to $t=1+h$ is

$$v_{ave}=\frac{3(h+1)^{2.5}+580h-3}{10h}$$

Estimate the instantaneous velocity of the car at $t=1$, where time is in seconds and distance is in feet.

3. Use the definiton of a derivative to find dy/dx, and check your answer by calculating the derivative using appropriate derivative formulas.

(a) $y=\sqrt{9-4x}$ (b) $y=\frac{x}{x+1}$.

4. Supose that $f(x)=\begin{cases} x^2-1, & x\le 1 \\ k(x-1), & x>1 \end{cases}$

For what values of k is f

(a) continuous? (b) differentiable?

5. Find the derivative of the given functions

(a) $f(x)=x^2\sin x$ (c) $f(x)=\frac{2x^2-x+5}{3x+2}$

(b) $f(x)=\sqrt{x}+\cos^2 x$ (d) $f(x)=\frac{\tan x}{1+x^2}$

6. Suppose that a function f is differentiable at $x=1$ and $\lim_{h\to 0}\frac{f(1+h)}{h}=5$. Find $f(1)$ and $f'(1)$?

7. Suppose that a function f is differentiable at $x=2$ and $\lim_{2\to 2}\frac{x^3 f(x)-24}{x-2}=28$. Find $f(2)$ and $f'(2)$?

8. Find the equation of all lines through the origin that are tangent to the curve $y=x^3-9x^2-16x$.

9. Find all values of x for which the tangent line to the curve $y=2x^3-x^2$ is perpendicular to the line $x+4y=10$.

10. In each part, evaluate the expression given that $f(1)=1, g(1)=-2, f'(1)=3,$ and $g'(1)=-1$.

(a) $\left.\dfrac{d}{dx}\left[f(x)g(x)\right]\right|_{x=1}$ (b) $\left.\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]\right|_{x=1}$

(b) $\left.\dfrac{d}{dx}\left[\sqrt{f(x)}\right]\right|_{x=1}$ (d) $\dfrac{d}{dx}\left[f(1)g'(1)\right]$

11. Find $f'(x)$.

(a) $f(x)=x^8-3\sqrt{x}+5x^{-3}$

(b) $f(x)=(2x+1)^{101}(5x^2-7)$

(c) $f(x)=\mathrm{s\,inx}+2\cos^3 x$

(d) $f(x)=(1+\sec x)(x^2-\tan x)$

(e) $f(x)=\sqrt{3\mathrm{x}+1}(x-1)^2$

(f) $f(x)=\left(\dfrac{3\mathrm{x}+1}{x^2}\right)^3$

(g) $f(x)=\dfrac{1}{2\mathrm{x}+\sin^3 x}$

12. Find the values of x at which the curve $y=f(x)$ has a horizontal tangent line.

(a) $f(x)=(2x+7)^6(x-2)^5$ (b) $f(x)=\dfrac{(x-3)^4}{\mathrm{x}^2+2x}$

13. Find all values of x for which the line that is tangent to $y=3x-\tan x$ is parallel to the line $y-x=2$.

14. Suppose that $f(x)=M\sin x+N\cos x$ for some constant M and N. If $f\left(\dfrac{\pi}{4}\right)=3$ and $f'\left(\dfrac{\pi}{4}\right)=1$, find an equation for the the tangent line to $y=f(x)$ at $x=\dfrac{3\pi}{4}$.

15. Suppose that $f(x)=M\sin x+N\cos x$ for some constants M and N. If $f\left(\dfrac{\pi}{4}\right)=3$ and $f'\left(\dfrac{\pi}{4}\right)=1$, find an equation for the the tangent line to $y=f(x)$ at $x=\dfrac{3\pi}{4}$.

16. Suppose that $f(x)=M\tan x+N\sec x$ for some constants M and N. If $f\left(\dfrac{\pi}{4}\right)=2$ and $f'\left(\dfrac{\pi}{4}\right)=0$, find an equation for the the tangent line to $y=f(x)$ at $x=0$.

17. Suppose that $f'(x)=2x.f(x)$ and $f(2)=5$.

(a) Find $g'(\pi/3)$ if $g(x)=f(\sec x)$.

(b) Find $h'(2)$ if $h(x) = \left[\dfrac{f(x)}{x-1}\right]^4$.