PROBLEM SET 7: LIMIT

1. Find the limits.

a) $\lim_{x\to -1} \frac{x^3 - x^2}{x-1}$ b) $\lim_{x\to 1} \frac{x^3 - x^2}{x-1}$ c) $\lim_{x\to -3} \frac{3x+9}{x^2+4x+3}$	d) $\lim_{x\to 2^-} \frac{x+2}{x-2}$ e) $\lim_{x\to +\infty} \frac{(2x-1)^5}{(3x^2+2x-7)(x^3-9x)}$ f) $\lim_{x\to 0} \frac{\sqrt{x^2+4}-2}{x^2}$

2. In each part, find the horizontal asymptotes, if any.

a) $y = \frac{2x-7}{x^2-4x}$ b) $y = \frac{x^3 - x^2 + 10}{3x^2 - 4x}$ c) $y = \frac{2x^2-6}{x^2+5x}$

3. In each part, find $\lim_{x\to a} f(x)$, if it exists, where a is replaced by $0.5^+, -5^-, -5, 5, -\infty,$ and $+\infty$.

a) $f(x) = \sqrt{5-x}$

b) $f(x) = \begin{cases} \frac{x-5}{|x-5|}, & x \neq 5 \\ 0, & x = 5 \end{cases}$

4. Find the limits.

a) $\lim_{x\to 0} \frac{\sin 3x}{\tan 3x}$ b) $\lim_{x\to 0} \frac{x \sin x}{1-\cos x}$ c) $\lim_{x\to 0} \frac{3x - \sin(kx)}{x}, \quad k \neq 0$ d) $\lim_{\theta\to 0} \tan \frac{(1-\cos\theta)}{\theta}$	e) $\lim_{t\to \frac{\pi}{2}^+} e^{\tan t}$ f) $\lim_{\theta\to 0^+} \left[\ln(\sin 2\theta) - \ln(\tan\theta)\right]$ g) $\lim_{x\to +\infty} \left(1+\frac{3}{x}\right)^{-x}$ h) $\lim_{x\to +\infty} \left(1+\frac{a}{x}\right)^{bx}, \quad a,b>0$

5. The limit $\lim_{x\to 0} \frac{\sin x}{x} = 1$ ensures that there is a number δ such that $\left|\frac{\sin x}{x} - 1\right| < 0.001$ if $0 < |x-a| < \delta$. Estimate the largest such δ.

6. In each part, a positive number ε and the limit L of a function f at a are given. Find a number δ such that $|f(x) - L| < \varepsilon$ if $0 < |x-a| < \delta$.

a) $\lim_{x\to 2} (4x-7) = 1; \quad \varepsilon = 0.01$.

b) $\lim_{x \to \frac{3}{2}} \frac{4x^2 - 9}{2x - 3} = 6; \quad \varepsilon = 0.05$

c) $\lim_{x \to 4} x^2 = 16; \quad \varepsilon = 0.001$

7. Find the values of x at which the given function is not continous.

a) $f(x) = \frac{x}{x^2 - 1}$ b) $f(x) = |x^3 - 2x^2|$ c) $f(x) = \frac{x+3}{|x^2 + 3x|}$

8. Determine where f is continous.

a) $f(x) = \frac{x}{|x| - 3}$ b) $f(x) = \cos^{-1}\left(\frac{1}{x}\right)$ c) $f(x) = e^{\ln x}$

9. Suppose that

$$f(x) = \begin{cases} -x^4 + 3, & x \le 2 \\ x^2 + 9, & x > 2 \end{cases}$$

Is f continuous everywhere? Justify your conclusion.

10. Show that the conclusion of the Intermediate-Value Theorem may be false if f is not continuous on the interval $[a, b]$.

11. Suppose that f is continuous on the interval $[0, 1]$ that $f(0) = 2$, and that f has no zeros in the interval. Prove that $f(x) > 0$ for all x in $[0, 1]$.

12. Show that the equation $x^4 + 5x^3 + 5x - 1 = 0$ has at least two real solutions in the interval $[-6, 2]$.