Section 1.1:

Exercise 7

$\begin{pmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$ Interchange R3, R4 → $\begin{pmatrix} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ R1 → -3R3 + R1 ; R2→ R3+ R1

→ $\begin{pmatrix} 1 & 7 & 0 & -10 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 2 & 1 \end{pmatrix}$ R1 → -7R2+R1 ; R4 → -2R3 +R4

→ $\begin{pmatrix} 1 & 0 & 0 & -31 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & -9 \end{pmatrix}$

In R4 have 0=-9 → System is inconsistent

Exercise 11

$\begin{pmatrix} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 7 & 6 \end{pmatrix}$ Interchange R1,R2 → $\begin{pmatrix} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 3 & 7 & 7 & 6 \end{pmatrix}$ R3 → -3R1 +R3 →

$\begin{pmatrix} 1 & 3 & 5 & 2 \\ 0 & 1 & 4 & -5 \\ 0 & -2 & -8 & 0 \end{pmatrix}$ R3 → 2R2 + R3; R1 → -3R2 + R1 → $\begin{pmatrix} 1 & 0 & -6 & 17 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & -10 \end{pmatrix}$

In R3 have 0= -10 → system is inconsistent

Exercise 19

$\begin{pmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{pmatrix}$ R2 → -3R1 + R2 → $\begin{pmatrix} 1 & h & 4 \\ 0 & -3h+6 & -4 \end{pmatrix}$

Tim dieu kien chi ma tran -3h+6 khac 0

Solve -3h + 6 = -4 → 10= 3h → h= 3/10

Exercise 25

$\begin{pmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{pmatrix}$ R3 → 2R1 + R3 →$\begin{pmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & 2g+k \end{pmatrix}$ R3 → R3+ R2

→

$$\begin{pmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & 2g+k+h \end{pmatrix}$$

→ System is consistent ↔2g+k +h =0

Exercise 29

Transforms the first matrix into the second: Swap row 1 and row 2

Transforms the second matrix into the first: Swap row 1 and row 2

Section 1.2

Exercise 7

$\begin{pmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{pmatrix}$

X1= -5-3X2, X2 is free , X3= 3

Ex11

$$\begin{pmatrix} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -4/3 & 2/3 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -4/3 & 2/3 & 0 \\ 0 & 0 & 18 & 0 \\ 0 & 4 & 16 & 0 \end{pmatrix}$$

X1= 4/3 X2 – 2/3X2, X2 : free , X3: free

Ex 13

$$\begin{pmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & -3 & 5 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

X1= (x5-5)/3 , X2= 4-9X1 , X3: free , X4: 1+ 4X1, x5: free

Ex 17

$$\begin{pmatrix} 2 & 3 & h \\ 4 & 6 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3/2 & h/2 \\ 4 & 6 & 7 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3/2 & h/2 \\ 0 & 0 & -2h+7 \end{pmatrix}$$

Solve -2h+7=0 → h= 7/2

Ex23

Yes. The system is consistent because with three pivots, there must be a pivot in the third (bottom) row of the coefficient matrix. The reduced echelon form cannot contain a row of the form(0 0 0 0 0)

Ex 25

If the coefficient matrix has a pivot position in every row, then there is a pivot position in the bottom row, and there is no room for a pivot in the augmented column. So, the system is consistent, by Theorem 2.

Section 1.3

Ex 11

$$x_1 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

$$\Leftrightarrow \begin{cases} x_1 + 5x_3 = 2 \\ -2x_1 + x_2 + -6x_3 = -1 \\ 2x_2 + 8x_3 = 6 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix} \times 2$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{bmatrix} \times -2$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ $x_1 + 5x_3 = 2$

$x_2 + 4x_3 = 3$

x_3 : free

$\Rightarrow$ $x_1 = 2 - 5x_3$

$x_2 = 3 - 4x_3$

x_3 : free

$\Rightarrow$ b is a linear combination of a_1, a_2, a_3

Ex 13

$$\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{bmatrix} \xrightarrow{\times 2} \begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

In R3 have 0 = 3 => System is consistent
-> b is not a linear combination of the vector formed from the columns of the matrix A

Ex 15

$$v_1 = \begin{bmatrix} 7 \\ 1 \\ -6 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$$

$$1 v_1 + 1 v_2 = \begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix},$$

$$1 . v_1 - 1 . v_2 = \begin{bmatrix} 12 \\ -2 \\ 6 \end{bmatrix}$$

=> weights on v_1 and v_2 : 0

Ex ~~16~~ 17

Assume value of h spanned by a_1 and a_2

=> $b = v_1 a_1 + v_2 a_2$

$$v_1 \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} + v_2 \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{bmatrix} \xrightarrow{\times -4, \times 2} \begin{bmatrix} 1 & -2 & 4 \\ 0 & 5 & -15 \\ 0 & 3 & 8+h \end{bmatrix}.$$

$$\rightarrow \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 17+h \end{bmatrix} \times -3$$

$\rightarrow$ $17 + h = 0$ $\qquad$ $h = -17$

$v_2 = -3$ $\qquad \rightarrow \qquad$ $v_2 = -3$

$v_1 - 2v_2 = 4$ $\qquad$ $v_1 = -2$

Ex 19

span $\{v_1, v_2\}$ is the set of points on the line through v_1 and 0

Ex 22

b is not spanned by the columns of A if the system of three linear equations

$$A(x, y, z)^T = b^T$$

has no solution (is inconsistent and cannot be reduced to row echelon form)

$\rightarrow$ when row reduction is completed, at least one of the rows in the transformed augmented

Ex 23

a. False

b. True

c. False

d. True

e. False

Ex 25

a)
$$\begin{bmatrix}1\\0\\-2\end{bmatrix}\begin{bmatrix}0\\3\\6\end{bmatrix}\begin{bmatrix}-1\\-2\\3\end{bmatrix}\begin{bmatrix}1\\1\\-4\end{bmatrix}$$
a_1 a_2 a_3 b

a) $\begin{bmatrix}1 & 0 & -1 & -1\\0 & 3 & -2 & 1\\-2 & 6 & 3 & -1\end{bmatrix} \Rightarrow \{a_1, a_2, a_3\} = b$

$\Rightarrow \begin{bmatrix}1 & 0 & -4 & 4\\0 & 3 & -2 & 1\\0 & 0 & -1 & 2\end{bmatrix}$

$W = \text{span}\{a_1, a_2, a_3\}$

b) $b \in W$, there are 3 vector in W

c) $a_1 = a_1 + 0a_2 + 0a_3 = a_1$
$\Rightarrow a_1 \in W$

Section 1.4

Ex 6

$v_1x_1 + v_2x_2 = b$

$$-2\begin{bmatrix}7\\2\\9\\-3\end{bmatrix} + (-5)\begin{bmatrix}-3\\1\\-6\\2\end{bmatrix} = \begin{bmatrix}1\\-9\\12\\-4\end{bmatrix}$$

$$\Rightarrow x_1\begin{bmatrix}7\\2\\9\\-3\end{bmatrix} + x_2\begin{bmatrix}-3\\1\\-6\\2\end{bmatrix} = \begin{bmatrix}1\\-9\\12\\-4\end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 7 & -3 \\ 2 & 1 \\ 9 & -6 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix}$$

Ex 8

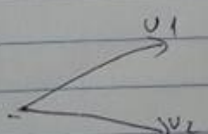
$$z_1 . v_1 + z_2 . v_2 + z_3 . v_3 + z_4 . v_4 = b$$

$$\rightarrow z_1 \begin{bmatrix} 4 \\ -2 \end{bmatrix} + z_2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} + z_3 \begin{bmatrix} -5 \\ 4 \end{bmatrix} + z_4 \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 4 & -4 & -5 & 3 \\ -2 & 5 & 4 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$$

Ex 13

$$u = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}, \quad A = \begin{bmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{bmatrix} = [v_1, v_2]$$



$$Ax = u$$

$$\begin{bmatrix} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{bmatrix} \xrightarrow{\times 2, \times 3} \begin{bmatrix} 1 & 1 & 4 \\ 0 & 8 & 12 \\ 0 & -8 & -12 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{array}{l} x_1 + x_2 = 4 \Rightarrow x_1 = 4 - x_2 \\ x_2 : \text{free} \end{array}$$