Problem 1

$$-3x_1 + 5x_2 - 7x_3 = 0$$
$$-6x_1 + 7x_2 + x_3 = 0$$

+ consider to the matrix:

$$\begin{bmatrix} -3 & 5 & -7 & 0 \\ -6 & 7 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -5/3 & 7/3 & 0 \\ -6 & 7 & 1 & 0 \end{bmatrix} \cdot 6$$

$$\sim \begin{bmatrix} 1 & -5/3 & 7/3 & 0 \\ 0 & -3 & 15 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -6 & 0 \\ 0 & 1 & -5 & 0 \end{bmatrix} \cdot \frac{5}{3}$$

I have solution

$$x_1 = 6x_3$$
$$x_2 = 5x_3$$
$$x_3 = x_3 \text{ free}$$

Write this system under parametric vector form:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6x_3 \\ 5x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$$

+ Geometric description: The solution set of homogeneous equation is line through the origin and ~~parallel with~~ $\cancel{\begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}}$ in the direction of $\begin{bmatrix} 6 \\ 5 \\ 1 \end{bmatrix}$

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Problem 2

Consider to the matrix and row reduced to echelon form

$$\begin{bmatrix} 1 & -2 & 2 & 0 \\ -5 & 10 & -9 & 0 \\ -3 & 6 & h & 0 \end{bmatrix} \overset{5 \quad 3}{\sim} \begin{bmatrix} 1 & -2 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 6+h & 0 \end{bmatrix}$$

~~To set is linearly dependent if and only if $6+h=0 \Rightarrow h=-6$~~

The equation have x_2 is free variable

$\Rightarrow$ system is linearly dependent for all of value of h

Problem 3

$$A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 0 & 7 \\ -3 & -5 & -3 \end{bmatrix}$$

a) Row ~~de~~ reduced matrix A to echelon form

$$A = \begin{bmatrix} 1 & -1 & 5 \\ 2 & 0 & 7 \\ -3 & -5 & -3 \end{bmatrix} \overset{-2 \quad 3}{\sim} \begin{bmatrix} 1 & -1 & 5 \\ 0 & 2 & -3 \\ 0 & -8 & \cancel{15}\,12 \end{bmatrix} \cdot 4$$

$$\sim \begin{bmatrix} 1 & -1 & 5 \\ 0 & 2 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

equation has 2 pivot positions $\Rightarrow \dim(\text{Col } A) = 2$

$\Rightarrow$ basis for Col A is $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -5 \end{bmatrix}$

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b)

\+ We will solve system $Ax = 0$ to find null space of A

$$\begin{bmatrix} 1 & -1 & 5 & 0 \\ 2 & 0 & 7 & 0 \\ -3 & -5 & -3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 5 & 0 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x_1 - x_2 + 5x_3 = 0 \\ 2x_2 - 3x_3 = 0 \\ x_3 \text{ is free} \end{cases}$$

Since only x_3 is free variable

$\Rightarrow$ dim (Null A) = 1

\+ ~~Due to rank theorem, I have:~~

$$\cancel{n = \dim(\text{Null } A) + \dim(\text{col } A)}$$

~~(n is column of matrix A)~~

\+ Rank of A is 2 because A have 2 pivot position

Problem 4

$$A = \begin{bmatrix} 0 & -2 & -1 \\ 5 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

a) + first I calculate det A

Expanding along to second row, I have

$$\det A = -5 \begin{vmatrix} -2 & -1 \\ 1 & 1 \end{vmatrix} = -5 \cdot (-1) = 5$$

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+ second, I find adj of A

calulation of cofactors

$C_{11} = \begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = 0$, $C_{12} = -\begin{vmatrix} 5 & 0 \\ -1 & 1 \end{vmatrix} = -5$

$C_{13} = \begin{vmatrix} 5 & 0 \\ -1 & 1 \end{vmatrix} = 5$

$C_{21} = -\begin{vmatrix} -2 & -1 \\ 1 & 1 \end{vmatrix} = -1$, $C_{22} = \begin{vmatrix} 0 & -1 \\ -1 & 1 \end{vmatrix} = -1$

$C_{23} = -\begin{vmatrix} 0 & -2 \\ -1 & 1 \end{vmatrix} = +2$

$C_{31} = \begin{vmatrix} -2 & -1 \\ 0 & 0 \end{vmatrix} = 0$, $C_{32} = -\begin{vmatrix} 0 & -1 \\ 5 & 0 \end{vmatrix} = -\cancel{0}5$

$C_{33} = \begin{vmatrix} 0 & -2 \\ 5 & 0 \end{vmatrix} = +10$

$\Rightarrow \text{adj } A = \begin{vmatrix} 0 & -5 & 5 \\ -1 & -1 & +2 \\ 0 & \cancel{0}5 & +10 \end{vmatrix}$

$\Rightarrow A^{-1} = \dfrac{1}{\det A} \cdot \text{adj } A = 5 \begin{vmatrix} 0 & -5 & 5 \\ -1 & -1 & +2 \\ 0 & \cancel{0}5 & -10 \end{vmatrix}$

b) $\text{Det}(A^T)^{20} = \text{Det } A^{20} = 5^{20}$ (det A have been caculated in part a)