Linear Algebra Notes

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Contents

1 Vector Spaces

1.1 Definition of Vector Space

Definition 1.1 (Vector Space)**.**
*A **vector space** (over a field $\mathbb{F}$) consists of a set V with two operations "$+$" and "$\cdot$" subject to the conditions that for all $\vec{v}, \vec{w}, \vec{u} \in V$ and scalars $r, s \in \mathbb{F}$:*

1. ***Closure under**:*
   - *Vector addition:* $\vec{v} + \vec{w} \in V$.
   - *Scalar multiplication:* $r \cdot \vec{v} \in V$.
2. ***Properties of vector addition**:*
   - *Commutativity:* $\vec{v} + \vec{w} = \vec{w} + \vec{v}$.
   - *Associativity:* $(\vec{v} + \vec{w}) + \vec{u} = \vec{v} + (\vec{w} + \vec{u})$.
3. ***Properties of scalar multiplication**:*
   - *Distributivity over scalar addition:* $(r + s) \cdot \vec{v} = r \cdot \vec{v} + s \cdot \vec{v}$.
   - *Distributivity over vector addition:* $r \cdot (\vec{v} + \vec{w}) = r \cdot \vec{v} + r \cdot \vec{w}$.
4. ***Inverse elements**:*
   - *Additive inverse:* $\forall \vec{v} \in V, \exists -\vec{v} \in V : \vec{v} + (-\vec{v}) = \vec{0}$.
5. ***Identity elements**:*
   - *Additive identity:* $\exists \vec{0} \in V : \vec{0} + \vec{v} = \vec{v}, \quad \forall \vec{v} \in V$.
   - *Multiplicative identity:* $\exists 1 \in \mathbb{F} : 1 \cdot \vec{v} = \vec{v}, \quad \forall \vec{v} \in V$.

For brevity, we will denote vectors as bold face letters instead of overhead arrows after this definition. For example, $\mathbf{u}, \mathbf{v}$ *and* $\mathbf{w}$

Remark 1.1 (Trivial Space)**.** *A vector space with one element is called a **trivial space**.*

Example 1.1. *The following is a vector space over* $\mathbb{R}^2$*:*

$$\mathrm{L} = \left\{ \begin{pmatrix} x & y \end{pmatrix}^\top : y = 3x \right\}.$$

This is easy to verify. Let us go through each condition one by one. Suppose that we have two vectors $\mathbf{u}_1, \mathbf{u}_2 \in \mathrm{L}$ *defined as follows:*

1.2 Linear Independence

1.3 Basis and Dimension

A List of Definitions

B Important Theorems

C Important Corollaries

D Important Propositions

E References

References

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