# Linear Algebra Notes

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## Contents

# 1 Vector Spaces

## 1.1 Definition of Vector Space

**Definition 1.1** (Vector Space)**.**
*A* ***vector space*** *(over a field $\mathbb{F}$) consists of a set $V$ with two operations "$+$" and "$\cdot$" subject to the conditions that for all $\vec{v}, \vec{w}, \vec{u} \in V$ and scalars $r, s \in \mathbb{F}$:*

1. ***Closure under****:*
   - *Vector addition:* $\vec{v} + \vec{w} \in V$.
   - *Scalar multiplication:* $r \cdot \vec{v} \in V$.
2. ***Properties of vector addition****:*
   - *Commutativity:* $\vec{v} + \vec{w} = \vec{w} + \vec{v}$.
   - *Associativity:* $(\vec{v} + \vec{w}) + \vec{u} = \vec{v} + (\vec{w} + \vec{u})$.
3. ***Properties of scalar multiplication****:*
   - *Distributivity over scalar addition:* $(r + s) \cdot \vec{v} = r \cdot \vec{v} + s \cdot \vec{v}$.
   - *Distributivity over vector addition:* $r \cdot (\vec{v} + \vec{w}) = r \cdot \vec{v} + r \cdot \vec{w}$.
4. ***Inverse elements****:*
   - *Additive inverse:* $\forall \vec{v} \in V, \exists - \vec{v} \in V : \vec{v} + (-\vec{v}) = \vec{0}$.
5. ***Identity elements****:*
   - *Additive identity:* $\exists \vec{0} \in V : \vec{0} + \vec{v} = \vec{v}, \quad \forall \vec{v} \in V$.
   - *Multiplicative identity:* $\exists 1 \in \mathbb{F} : 1 \cdot \vec{v} = \vec{v}, \quad \forall \vec{v} \in V$.

**Remark 1.1** (Trivial Space)**.** *A vector space with one element is called a* ***trivial space****.*

**Example 1.1.** *The following is a vector space over $\mathbb{R}^2$:*

$$\mathrm{L} = \left\{ \begin{pmatrix} x & y \end{pmatrix}^\top : y = 3x \right\}$$

## 1.2 Linear Independence

## 1.3 Basis and Dimension

# A List of Definitions



# B Important Theorems

# C Important Corollaries

# D Important Propositions

# E References

## References


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