Linear Algebra Notes

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Contents

1 Vector Spaces

1.1 Definition of Vector Space

Definition 1.1 (Vector Space)**.** *A **vector space** (over a field $\mathbb{F}$) consists of a set V with two operations "$+$" and "$\cdot$" subject to the conditions that for all $\vec{v}, \vec{w}, \vec{u} \in V$ and scalars $r, s \in \mathbb{F}$:*

1. ***Closure under**:*
   - *Vector addition: $\vec{v} + \vec{w} \in V$.*
   - *Scalar multiplication: $r \cdot \vec{v} \in V$.*
2. ***Properties of vector addition**:*
   - *Commutativity: $\vec{v} + \vec{w} = \vec{w} + \vec{v}$.*
   - *Associativity: $(\vec{v} + \vec{w}) + \vec{u} = \vec{v} + (\vec{w} + \vec{u})$.*
3. ***Properties of scalar multiplication**:*
   - *Distributivity over scalar addition: $(r + s) \cdot \vec{v} = r \cdot \vec{v} + s \cdot \vec{v}$.*
   - *Distributivity over vector addition: $r \cdot (\vec{v} + \vec{w}) = r \cdot \vec{v} + r \cdot \vec{w}$.*
4. ***Inverse elements**:*
   - *Additive inverse: $\forall \vec{v} \in V, \exists -\vec{v} \in V : \vec{v} + (-\vec{v}) = \vec{0}$.*
5. ***Identity elements**:*
   - *Additive identity: $\exists \vec{0} \in V : \vec{0} + \vec{v} = \vec{v}, \quad \forall \vec{v} \in V$.*
   - *Multiplicative identity: $\exists 1 \in \mathbb{F} : 1 \cdot \vec{v} = \vec{v}, \quad \forall \vec{v} \in V$.*

1.2 Linear Independence

1.3 Basis and Dimension

A List of Definitions

B Important Theorems

C Important Corollaries

D Important Propositions

E References

References

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