CS703 - Optimization and Computing Notes

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Contents

1 Introduction

Definition 1.1 (Optimization Problem).
Generally, an optimization problem is defined as follows:

$$\begin{aligned} \text{minimize}: &\ f_0(x) \\ \text{subject to}: &\ f_i(x) \leqslant 0, \quad i = 1, \dots, m \\ &\ h_i(x) = 0, \quad i = 1, \dots, p. \end{aligned} \tag{1}$$

Where we have:

1. $x \in \mathbb{R}^n$ is the optimization variable.
2. $f_0 : \mathbb{R}^n \to \mathbb{R}$ is the opjective (cost function).
3. $f_i : \mathbb{R}^n \to \mathbb{R}$ are inequality constraints.
4. $h_i : \mathbb{R}^n \to \mathbb{R}$ are equality constraints.

Definition 1.2 (Convex Optimization Problem).
An optimization problem is a **convex optimization problem** if:

1. $f_0, f_1, \dots, f_m$ are convex.
2. Equality constraints are affine.

The reason why we need convex optimization problems are:

1. Convex optimization problems can be solved optimally (no local minima).
2. Time required to solve convex optimization problems is polynomial (in terms of number of variables and constraints).

1.1 Convex Sets

Definition 1.3 (Lines).
Let $x_1, x_2 \in \mathbb{R}^n$. A line passing through x_1, x_2 is defined as:

$$L(x_1, x_2) = \Big\{ x \in \mathbb{R}^n : x = \theta x_1 + (1 - \theta) x_2, \theta \in \mathbb{R} \Big\}. \tag{2}$$

When $\theta \in (0, 1)$, we restrict the line to the points between x_1 and x_2 (exclusive).
Definition 1.4 (Affine Sets).
An affine is a set that contains the line segment between any two distinct points in it. For example,

1. An empty set is affine because there is no point.
2. A singleton is affine because there is only one point.
3. A line (extends indefinitely) is affine.
4. Any vector space is affine.
5. Linear subspaces of a vector space is affine.

Definition 1.5 (Convex Sets).

A List of Definitions

B Important Theorems

C Important Corollaries

D Important Propositions

Return to Contents

E References