CS703 - Optimization and Computing Notes

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1 Introduction

Definition 1.1 (Optimization Problem).

Generally, an optimization problem is defined as follows:

$$\begin{aligned} \text{minimize} : \; & f_0(x) \\ \text{subject to} : \; & f_i(x) \leqslant 0, \quad i = 1, \ldots, m \\ & h_i(x) = 0, \quad i = 1, \ldots, p. \end{aligned} \tag{1}$$

Where we have:

1. $x \in \mathbb{R}^n$ is the optimization variable.
2. $f_0 : \mathbb{R}^n \to \mathbb{R}$ is the opjective (cost function).
3. $f_i : \mathbb{R}^n \to \mathbb{R}$ are inequality constraints.
4. $h_i : \mathbb{R}^n \to \mathbb{R}$ are equality constraints.

Definition 1.2 (Convex Optimization Problem).

An optimization problem is a **convex optimization problem** if:

1. $f_0, f_1, \ldots, f_m$ are convex.
2. Equality constraints are affine.

The reason why we need convex optimization problems are:

1. Convex optimization problems can be solved optimally (no local minima).
2. Time required to solve convex optimization problems is polynomial (in terms of number of variables and constraints).

A List of Definitions

B Important Theorems

C Important Corollaries

D Important Propositions

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