

Time Series Forecasting

I. Smoothing average

1. Exponential moving average

- $y_t = x_t + (1 - \alpha)x_{t-1} + (1 - \alpha)^2x_{t-1} + \dots + (1 - \alpha)^tx_0$
- $= \sum_{i=1}^t (1 - \alpha)^{t-i} x_t$

2. Exponential *weighted* moving average

- $y_t = \frac{x_t + (1-\alpha)x_{t-1} + (1-\alpha)^2x_{t-1} + \dots + (1-\alpha)^tx_0}{1 + (1-\alpha) + (1-\alpha)^2 + \dots + (1-\alpha)^t}$
- $= \frac{\sum_{i=1}^t (1-\alpha)^{t-i} x_t}{\sum_{i=1}^t (1-\alpha)^{t-i}}$

II. Holt-Winters methods.

1. Single Holt-Winters Method

- $y_0 = x_0$
- $y_t = (1 - \alpha)y_{t-1} + \alpha x_t$

2. Double Holt-Winters Method

Take trend into account

- $l_t = (1 - \alpha)l_{t-1} + \alpha x_t \rightarrow$ (The "base line" level)
- $b_t = (1 - \beta)b_{t-1} + \beta(l_t - l_{t-1}) \rightarrow$ (The trend)
- $y_t = l_t + b_t \rightarrow$ (The regressed value y_t)
- $\hat{y}_{t+h} = l_t + hb_t \rightarrow$ (The predicted value h periods into the future)

3. Triple Holt-Winters Method

Take trend and seasonality into account

- $l_t = (1 - \alpha)l_{t-1} + \alpha x_t \rightarrow$ (The "base line" level)
- $b_t = (1 - \beta)b_{t-1} + \beta(l_t - l_{t-1}) \rightarrow$ (The trend)
- $c_t = (1 - \gamma)c_{t-L} + \gamma(x_t - l_{t-1} - b_{t-1}) \rightarrow$ (The seasonality - L is the cycle period of the season).
- $y_t = (l_t + b_t)c_t \rightarrow$ (The regressed value y_t)
- $\hat{y}_{t+h} = (l_t + hb_t)c_{t-L+1+(m-1)modL} \rightarrow$ (The predicted value h periods into the future)

III. Auto-regressive models

- $y_t = C_t + \sum_{i=1}^L \phi_i y_{t-i} + \epsilon_t$
 - Where C_t is a base-line constant at time t .
 - ϕ_i are the corresponding weights for the previous values.
 - ϵ_t is some unexplainable error happening at time t .