# **Time Series Forecasting**

### I. Smoothing average

#### 1. Exponential moving average

• 
$$y_t = x_t + (1 - \alpha)x_{t-1} + (1 - \alpha)^2x_{t-1} + \dots + (1 - \alpha)^tx_0$$

$$ullet$$
 =  $\sum_{i=1}^t (1-lpha)^{t-i} x_t$ 

#### 2. Exponential weighted moving average

• 
$$y_t = rac{x_t + (1-lpha)x_{t-1} + (1-lpha)^2x_{t-1} + ... + (1-lpha)^tx_0}{1 + (1-lpha) + (1-lpha)^2 + ... + (1-lpha)^t}$$

$$ullet = rac{\sum\limits_{i=1}^t (1-lpha)^{t-i} x_t}{\sum\limits_{i=1}^t (1-lpha)^{t-i}}$$

#### II. Holt-Winters methods.

## 1. Single Holt-Winters Method

- $y_0 = x_0$
- $y_t = (1-\alpha)y_{t-1} + \alpha x_t$

#### 2. Double Holt-Winters Method

Take trend into account

- $ullet \ l_t = (1-lpha)l_{t-1} + lpha x_t 
  ightarrow$  (The "base line" level)
- $b_t = (1 \beta)b_{t-1} + \beta(l_t l_{t-1}) \to$ (The trend)
- ullet  $y_t = l_t + b_t 
  ightarrow ext{(The regressed value $y_t$)}$
- $\hat{y}_{t+h} = l_t + hb_t o$  (The predicted value h periods into the future)

#### 3. Triple Holt-Winters Method

Take trend and seasonality into account

- $l_t = (1 \alpha)l_{t-1} + \alpha x_t \rightarrow$  (The "base line" level)
- ullet  $b_t=(1-eta)b_{t-1}+eta(l_t-l_{t-1})
  ightarrow$  (The trend)
- $c_t=(1-\gamma)c_{t-L}+\gamma(x_t-l_{t-1}-b_{t-1}) o$  (The seasonality L is the cycle period of the season).
- ullet  $y_t = (l_t + b_t)c_t 
  ightarrow ext{(The regressed value } y_t)$
- ullet  $\hat{y}_{t+h} = (l_t + hb_t)c_{t-L+1+(m-1)modL} o$  (The predicted value h periods into the future)

## **III. Auto-regressive models**

• 
$$y_t = C_t + \sum_{i=1}^L \phi_i y_{t-i} + \epsilon_t$$

- $\circ~$  Where  $C_t$  is a base-line constant at time t.
- $\circ \ \phi_i$  are the corresponding weights for the previous values.
- $\circ \ \epsilon_t$  is some unexplainable error happening at time t.