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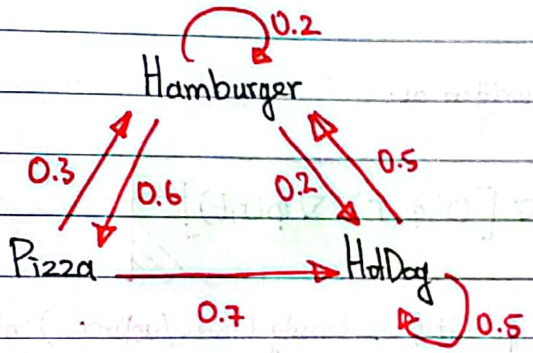
(2) Prerequisite: Markov Chains.

↳ Markov Chain (or Markov Process) is a Stochastic Model describing a sequence of possible events in which the probability of each event depends only on the state attained on the previous event. → Markov Property.

↳ Example of Markov Chains:



→ Let say there is a restaurant serving Hamburgur, Pizza and Hot dog and there is a rule: What is being served today is dependent on what is served yesterday. (Graph of prob on the left).

↳ Example: $P(\text{Hamburger} \mid \text{Pizza}) = 0.3$
$P(\text{Hotdog} \mid \text{Pizza}) = 0.7$ } $\Sigma = 1$ (Bc of probability).

↳ Random Walk along the Markov Chain:
:) Hamburger → Pizza → Hamburger → Hotdog → ...

→ Starting from a point in the Markov Chain (Hamburger), recursively generate new state based on the previous state. → Random Walk.

→ The question is, what will be the probability of each individual state if the walk goes on to infinitely many steps?

(*) ↳ $\boxed{[P(\text{Hamburger}), P(\text{Pizza}), P(\text{Hotdog})] = [\lambda_1, \lambda_2, \lambda_3]}$

↳* λ_i = Eigenvalues of adjacency matrix.

↳ Adjacency matrix

$$A = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{ccc} \text{Ham} & \text{Pizza} & \text{Hotdog} \end{array}$$
$$A = \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.3 & 0 & 0.7 \\ 0.5 & 0 & 0.5 \end{bmatrix}\begin{array}{l} \text{Ham} \\ \text{Pizza} \\ \text{Hotdog} \end{array}$$

③ Prerequisite: Diffusion Equation.

↳ Definition (Diffusion Eq): Diffusion equation is a parabolic differential equation. In physics, it is used to describe the behaviours of particles in Brownian Motion.

*) Brownian motion is the random motion of particles trapped in a medium (liquid/gas/...).

↳ The equation is usually written as:

(*) $$\boxed{\frac{\partial \phi(r,t)}{\partial t} = \nabla . \left[D(\phi, r) . \nabla \phi(r,t) \right]}$$

*Where $\phi(r,t)$ = density (mass/volume) at location r, time t.
$D(\phi, r)$ = diffusion coefficient for ϕ at location r.
$\nabla \phi(r,t)$ = gradient of density.

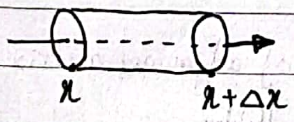
↳ If $D(\phi, r)$ depends on ϕ then the equation is non-linear.
Otherwise it's linear (D = const).

↳ How to derive the diffusion equation:

↳ A newer notation of diffusion equation:

(*) $$\boxed{\frac{\partial u(x,t)}{\partial t} = \text{div}\left[D(u,t) . \nabla u(x,t) \right]}$$

*In the GRAND paper u and x switches though.



→ Let's examine the diffusion of liquid inside ~~through~~ a very small length Δx of a pipe.

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→ We define $u(x,t)$ = concentration (density) at location x, time t.
→ The mass inside the tiny pipe can be approximated as:

$$M \approx u(x,t)\,\Delta x.$$

→ According to fick's law of diffusion:

$$(1) \quad \frac{\partial M}{\partial t} = \text{flux\_in} - \text{flux\_out}$$

Where the diffusion flx is defined as:

$$(2) \quad J(x,t) = -D\,\frac{\partial u(x,t)}{\partial x}$$

↳ (1) & (2) ⇔

$$\to \frac{\partial M}{\partial t} = D\left(\frac{\partial u(x+\Delta x,t)}{\partial x} - \frac{\partial u(x,t)}{\partial x}\right)$$

$$= D\left(u_x(x+\Delta x,t) - u_x(x,t)\right)$$

$$\to \Delta x\,\frac{\partial u(x,t)}{\partial t} = D\left(u_x(x+\Delta x,t) - u_x(x,t)\right)$$

$$\Leftrightarrow \frac{\partial u(x,t)}{\partial t} = D\,\frac{u_x(x+\Delta x,t) - u_x(x,t)}{\Delta x}$$

$$\to \text{As } \Delta x \to 0 \Rightarrow \frac{\partial u(x,t)}{\partial t} = D\,u_{xx}(x,t) \;\#$$

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④ Prerequisite: Dynamical System.

↳ Definition (Dynamical system): A dynamical system is a system whose state evolves with time over a statespace according to a fixed rule.

↳ Dynamical systems are generally described as a system of differential equations:

$$(*)\quad \boxed{\frac{dx}{dt} = \underbrace{f}_{\text{Dynamics (Vector field)}}(\overset{\text{State (vectors)}}{\bar{x}}, t, \underbrace{u}_{\text{actuation (control)}} ; \underbrace{\beta}_{\text{Other variables}})}$$

$$(*)\quad \boxed{\underbrace{y}_{\text{Measurements}} = g(x,t)}$$

(Sometimes we may not have direct control/access to the whole state).

↳ Challenges:

- ·) Unknown dynamics f. E.g: A pendulum, spread of disease,...
- ·) Non-linear dynamics.
- ·) High-dimensional, complex state space x.
- ·) Multiscale.
- ·) Chaotic dynamics.
- ·) Hidden / latent variables.
- ·) Noise / disturbance.