

Rademacher complexity bound for linear function class.

Problem : Let \mathcal{F} be the class of linear functions given by

$$\mathcal{F} = \left\{ f : \mathbb{R}^d \rightarrow \mathbb{R} \mid f(\mathbf{x}) = \mathbf{a}\mathbf{x}, \mathbf{a} \in \mathbb{R}^{1 \times d} \text{ and } \|\mathbf{a}\|_2 \leq R \right\}$$

- Prove the following bound of the Rademacher Complexity:

$$\mathcal{R}_N(\mathcal{F}) \leq \tilde{O}\left(\frac{R}{\sqrt{N}}\right)$$

- Where \tilde{O} notation hides logarithmic factors of both d and R . As a further practice, prove a tighter bound (refer to the STL notes) with no dependence on d .

Hint : Use the following lemmas from the following paper (et al. Barlett -

<https://arxiv.org/pdf/1706.08498.pdf>):

- Lemma 3.2 (Covering number of linear functions class).
- Lemma A.5 (Dudley's entropy integral).

Solution :