Simple SPH Fluid Simulator

Hieu Chau Nguyen

1. Abstract

Smooth particle hydrodynamics (SPH) is a particle-based method for simulating the behavior of fluids. The fluid contains multiple particle that carries information such as velocity, acceleration, and density. In this project, we will make a simple 2D version of SPH method described by Muller et al.[1]

2. Equations

Our first step is to compute the density of each particle $\boldsymbol{i}$ using the following formula:

$$\rho_i = \frac{4m}{\pi h^8} \sum_{j \in N_i} (h^2 - r_{ij}^2)3$$

Where **m** is the mass of particle, in which we assume that all particles have the same mass unit of one; **h** is the particle size, determine the largest radius that the particles would interact with each other; and $\boldsymbol{r}$ is the distance between these particles.

Our second step is to compute the acceleration of each particle using the rule:

$$a_i = \frac{1}{\rho_i} \sum_{j \in N_i} f_{ij} + g$$

Where:

$$f_{ij} = \frac{m_j}{\pi h^4 \rho_j} (1 - q_{ij}) C$$

And:

$$C = [15k(\rho_i + \rho_j - 2\rho_0) \frac{1 - q_{ij}}{q_{ij}} r_{ij} - 40\mu v_{ij}]$$

The parameters for these expressions are:

$$q_{ij} = \frac{\|r_{ij}\|}{h}$$

$$\rho_0 = reference\ mass\ density$$

$$k = bulk\ modulus$$

$$\mu = viscosity$$

$$g = gravitational\ vector$$

Once the acceleration is calculated, we can now calculate each particle's velocity and next position using symplectic Euler formula:

$$v^{i+1} = v^i + a^i \Delta t$$

$$r^{i+1} = r^i + v^i \Delta t$$

We also must take into account where the particles collides with the border or any inelastic platform by scaling back to the moment the particle collides the wall and reverse the position.

$$t_{bounce} = \frac{r_{ib} - b}{v}$$

$$r^{i+1} = r^i t_{bounce} (1 - d)$$

Where $\boldsymbol{b}$ is the collided wall position and $\boldsymbol{d}$ is the damping coefficient, representing the friction when collide with the wall.

3. Algorithm Implementation

Each particle of the fluid is stored in an object called Fluid, which contains the following parameters:

$Liquid\{$

$position;$

$velocity;$

$acceleration$

$\}$

By default, a liquid particle is created with no velocity and acceleration. All liquid particles are

grouped into LiquidGroup, which contains the common attributes of the liquid type the particles belongs to:

$LiquidGroup\{$

$reference\_density;$

$particle\_mass;$

$bulk\_modulus;$

$viscosity;$

$\}$

Finally, we create a Particle System that contains the LiquidGroup, and contains all attributes that are shared among the liquid type, such as gravitational force.

For each time step, The Particle System follows these instructions:

$ParticleSystem(delta\_t)\{$

$LiquidGroup.computed\_density();$

$LiquidGroup.computed\_accel(gravity);$

$LiquidGroup.symplectic\_euler(delta\_t);$

$LiquidGroup.wall\_collision(delta\_t)$

$\}$

4. Possible Improvement

There are a lot of improvement that can be done for this project. In fact, it is possible that we can simulate multiple liquid types at the same times. Furthermore, we also plan to implement a way to add extra wall, which add inelastic platform for the fluid to interact with. Lastly, we have planned for simulation of dissolution of a liquid and even solid by another liquid, such as salt dissolving in water.

5. Reference

[1] M. Muller, D CharyPar, and M. Gross. Particle-based fluid simulation for interactive applications