Training 6 - Graph Algorithms - 20201

A. CONNECTED COMPONENTS

1 second, 256 megabytes

Given a undirected graph $G = (V, E)$ where $V = \{1, \ldots, N\}$ is the number of nodes and the set E has M edges. Compute number of connected components of G.

Input

- Line 1: two positive integers N and M ($1 \leq N \leq 10^5, 1 \leq M \leq 10^5$)
- Lline $i + 1$ ($i = 1, \ldots, M$): contains two positive integers u and v which are endpoints of i^{th} edge

Output

Write the number of connected components of G.

input
8 8 1 2 1 7 1 8 2 7 4 5 4 6 5 6 7 8
output
3

B. BIPARTIE GRAPH

1 second, 256 megabytes

Given a undirected graph $G = (V, E)$ in which $V = \{1, \ldots, N\}$ is the set of nodes and $|E| = M$. You are required to write a program to check if G is a bipartie graph.

Input

- Line 1 N and M ($1 \leq N, M \leq 10^5$)
- Line $i + 1$ ($i = 1, \ldots, M$): u and v which are endpoints of the i^{th} edge

Output

Write 1 if G is a bipartie graph and 0, otherwise.

input
6 6 1 2 1 3 2 5 2 6 4 5 4 6
output
1

C. ARTICULATION_POINTS_N_BRIDGES

1 second, 256 megabytes

Given an undirected graph containing N vertices and M edges, find all the articulation points and the bridges in the graph.

Input

The first line consists of two space-separated integers denoting N and M. M lines follow, each containing two space-separated integers X and Y denoting there is an edge between X and Y.

Output
One line consists of two integers denoting the number of articulation points and the number of bridges.

input
10 12 1 10 10 2 10 3 2 4 4 5 5 2 3 6 6 7 7 3 7 8 8 9 9 7
output
4 3

Constraints: $1 \le N, M \le 10^5$, $1 \le X, Y \le N$

D. STRONGLY CONNECTED COMPONENTS

1 second, 256 megabytes

Given a directed graph $G = (V, E)$ where $V = \{1, \ldots, N\}$ is the number of nodes and the set E has M arcs. Compute number of strongly connected components of G

Input

- Line 1: two positive integers N and M ($1 \le N \le 10^5, 1 \le M \le 10^6$)
- Lline $i + 1$ ($i = 1, \ldots, M$): contains two positive integers u and v which are endpoints of i^{th} arc

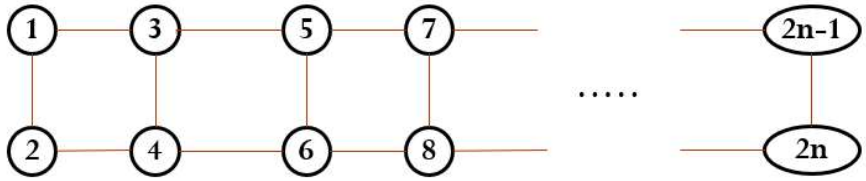
Output
Write the number of strongly connected components of G

input
8 13 1 2 1 8 2 3 2 6 3 6 4 3 4 6 5 4 6 5 7 1 7 2 7 6 8 7
output
3

E. GRID SPANNING TREE

1 second, 512 megabytes

Given a undirected grid graph $G_n = (V, E)$ having $2n$ nodes and $3n - 2$ edges (see example in the figure here). Counting the number Q of spanning trees of G.



Input
The input consists of following lines:

- Line 1 contains an integer n ($1 \le n \le 10^3$).

Output
Write the value Q modulo $10^9 + 7$.

input
2
output
4

F. K-MST

1 second, 256 megabytes

Given a undirected graph $G = (V, E)$, in which $V = \{1, \ldots, N\}$ is the set of nodes and G has M edges, $w(e)$ is the weight of the edge e ($e \in E$). Given a positive integer K, find the subgraph of G which is a tree containing exactly K edges having minimal weight.

Input

- Line 1 contains N, M, and K ($1 \le N \le 20, 1 \le M \le 30, 1 \le K \le 15$)
- Line i+1 ($i = 1, \ldots, M$) contains u, v, and w in which w is the weight of the edge (u, v)

Output

Write the weight of the tree found, or write -1 if no such tree exists.

input
5 7 3 1 2 4 1 3 3 1 4 1 2 3 2 2 5 6 3 4 5 3 5 5
output
6

G. DIAMETER BOUNDED MST

3 seconds, 256 megabytes

The diameter of a tree is defined to be the length of the longest path (in term of number of edges of the path) on that tree. Given a undirected graph $G = (V, E)$, $c(e)$ is the weight of the edge e ($e \in E$). Given a positive integer K, find the minimum spanning tree T of G such that the diameter of T is less than or equal to K.

Input

- Line 1: N, M, and K ($1 \le N \le 15, 1 \le M \le 25, 1 \le K \le 10$)
- Line $i + 1$ ($i = 1, \ldots, M$): u, v, and w in which w is the weight of edge (u, v)

Output

Write the weight of the MST found, or write -1 if no such MST exists.

input
6 9 3 1 2 4 1 3 3 1 4 1 2 3 7 2 5 1 3 4 5 3 5 5 3 6 5 5 6 2
output
16

H. K-CYCLE

1 second, 256 megabytes

Given an undirected graph $G = (V, E)$ in which $V = \{1, \ldots, N\}$ is the set of nodes. Given an integer K. Compute the number Q of elementary cycles of G having exactly K edges.

Input

- Line 1 N and M and K ($1 \le N \le 30, 1 \le M \le 50, 3 \le K \le 15$)
- Line $i+1$ ($i = 1, \ldots, M$): u and v which are endpoints of the i^{th} edge

Output
Write the value of Q.

input
5 7 4 1 2 1 3 2 3 2 4 2 5 3 4 4 5
output
2

I. COUNT SPANNING TREES

2 seconds, 256 megabytes

Given a undirected connected graph $G = (V, E)$ in which $V = \{1, \ldots, N\}$ is the set of nodes and E is the set of M edges. Count the number of spanning trees of G.

Input

- Line 1: contains positive integers N and M ($1 \le N \le 20, 1 \le M \le 25$)
- Line $i+1$ ($i = 1, \ldots, M$): contains u and v which are endpoints of the i^{th} edge of G

Output
Write the number of spanning trees of G

input
4 5 1 2 1 3 1 4 2 3 3 4
output
8

J. MAX CLIQUE

1 second, 256 megabytes

Given an undirected graph $G = (V, E)$. A graph $G' = (V', E')$ is called a subgraph of G if $V' \subseteq V$ and $E' \subseteq E$. Find the subgraph C of G which is a complete graph and has maximal number of nodes.

Input

- Line 1 N and M ($1 \le N \le 12, 1 \le M \le 60$)
- Line $i+1$ ($i = 1, \ldots, M$): u and v which are endpoints of the i^{th} edge

Output
Write the number of nodes of the complete subgraph C found.

input
5 8 1 2 1 3 2 3 2 4 2 5 3 4 3 5 4 5
output
4

K. LONGEST PATH ON A TREE

1 second, 256 megabytes

Given a undirected tree $G = (V, E)$ in which $V = \{1, \ldots, N\}$ is the set of nodes. Each edge $(u, v) \in E$ has weight $w(u, v)$. The length of a path is defined to be the sum of weights of edges of this path. Find the longest elementary path on G.

Input

- Line 1: positive integer N ($1 \le N \le 10^5$)
- Line $i + 1$ ($i = 1, \ldots, N - 1$): positive integers u, v, w in which w is the weight of edge (u, v) (1 ≤ w ≤ 100)

Output

The weight of the longest path on the given tree

input
6 1 3 3 1 6 2 2 6 5 4 5 2 4 6 1
output
10

L. SHORTEST PATH PRIORITY QUEUE

2 seconds, 256 megabytes

Given a directed graph $G = (V, A)$ where $V = \{1, \ldots, n\}$ is the set of nodes. Each arc $(u, v) \in A$ has weight $w(u, v)$. Given $s, t \in V$, compute the shortest path from s to t.

Input

- Line 1: n and m ($1 \le n \le 10^5, 1 \le m \le 10^6$)
- Line $i + 1$ ($i = 1, \ldots, m$): positive integers u, v, and w in which w is the weight of arc (u, v)
- Line m+2: s, t

Output

Write the weight of the shortest path found or -1 if no path from s to t exists.

input
5 7 2 5 87 1 2 97 4 5 78 3 1 72 1 4 19 2 3 63 5 1 18 1 5
output
97

M. MINIMUM SPANNING TREE

1 second, 256 megabytes

Given a undirected connected graph $G = (V, E)$ where $V = \{1, \ldots, N\}$. Each edge $(u, v) \in E$ has weight $w(u, v)$. Compute minimum spanning tree of G.

Input

- Line 1: N and M ($1 \le N, M \le 10^5$) in which N is the number of nodes and M is the number of edges.
- Line $i + 1$ ($i = 1, \ldots, M$): contains 3 positive integers u, v, and w where w is the weight of edge (u, v)

Output

Write the weight of the minimum spanning tree found.

input

```
5 8
1 2 1
1 3 4
1 5 1
2 4 2
2 5 1
3 4 3
3 5 3
4 5 2
```

output

```
7
```

N. FINDPARENT

1 second, 256 megabytes

Given a tree T (T is given by the list of edges). Let 1 be the root of T. For each node x of T such that $x \neq 1$, find the parent of x. Parent of a node x is the node next to x in the simple path from x to the root of tree.

Input

- First line contains n: number of nodes of T
- $n-1$ next line, each line contains an edge of T: $u\ v$

Output

Contains $n-1$ numbers, i-th number is the parent of node $i+1$

Scoring

- $1 \leq n \leq 10^5$ for all tests
- $n \leq 1000$ for at least 50% tests

input

```
7
4 7
2 6
3 4
1 6
4 1
7 5
```

output

```
6 4 1 7 1 4
```

O. LCOUNT

1 second, 256 megabytes

Given a tree T (T is given by the list of edges). Let 1 be the root of T. You have to count the number of leaves of T

Input

- First line contains n: number of nodes of T
- $n-1$ next line, each line contains an edge of T: $u\ v$

Output

Contains number of leaves of T

Scoring

- $1 \leq n \leq 10^5$ for all tests
- $n \leq 1000$ for at least 50% tests

input

```
7
4 7
2 6
3 4
1 6
4 1
7 5
```

output
3

P. 06.ICBUS

1 second, 256 megabytes

Country SS consists of N towns indexed from 1 to N, and each town i has its own inter city bus (IC Bus for short) system i. There is K roads between towns, each road connects two different towns. The bus can move freely in both directions on the road.

Quang is living in the town 1 in the country, and decided to go to the grandmother's house in the town N by some inter city buses. There are some special rules in this country:

- If the passenger want to use the IC Bus of the town i, he has to only ride at the town i.
- The bus fares of the IC Bus system i is C_i regardless of the distance that the passenger used.
- The IC Bus system i allows to pass maximum D_i towns per trip. If the trip has to pass more than D_i towns, the passenger has to change to another IC Bus system.
- The passenger will not be able ride to or down from the bus at a middle point different than the town.

Your task is to to find the minimum value of the sum of the fare needed for Quang to reach the town N from the town 1.

Input

The input consists of $1 + N + K$ lines.

The first line contains two positive integers N and K ($2 \le N \le 5000; N - 1 \le K \le 10000$).

i-th line in the N following lines contains 2 positive integers C_i and D_i ($1 \le C_i \le 10000; 1 \le D_i \le N$) which are the taxi fare and the maximum number of passing towns of the IC Bus system i.

Each line in the K following lines contains two positive integers i and j ($1 \le i < j \le N$) which means these two towns has a direct road connecting them.

Output

You should output on a single line an unique integer that is the minimum value of the sum of the fare necessary for Quang to go to the town N from the town 1.

input
6 6 400 2 200 1 500 3 900 1 400 4 200 5 1 2 1 5 2 3 2 4 3 6 4 6
output
800

Quang uses the IC Bus of the town 1 and then of the town 5.

Q. 06.ADDEDGE

3 seconds, 256 megabytes

An is obseving an undirected graph. He wonders if there exits in this graph 2 vertices that connecting these 2 vertices will create exactly one more simple cycle. Recall that a simple cycle may be defined either as a sequence of vertices with no repetitions of vertices and edges allowed, other than the repetition of the starting and ending vertex, with each two consecutive vertices in the sequence adjacent to each other in the graph.

Your task is to help An count the number of pairs of vertices satisfying the conditions above.

Input
The first line consist of two positive integers n, m ($n, m \leq 10^5$) which are the number of vertices and the number of edges of the given graph.

Each line in the m following lines contains two positive integers u, v ($u, v \leq n$) which are two vertices connected by an edge.

Output
You should output on a single line an unique integer that is the number of pairs you found.

input
5 4 1 2 2 3 3 4 4 5
output
6

R. 06.ELEVTRBL

1 second, 256 megabytes

You are on your way to your first job interview as a program tester, and you are already late. The interview is in a skyscraper and you are currently in floor s, where you see an elevator. Upon entering the elvator, you learn that it has only two buttons, marked "UP u" and "DOWN d". You conclude that the UP-button takes the elevator u floors up (if there aren't enough floors, pressing the UP-botton does nothing, or at least so you assume), whereas the DOWN-button takes you d stories down (or none if there aren't enough). Knowing that the interview is at floor g, and that there are only f floors in the building, you quickly decide to write a program that gives you the amount of button pushes you need to perform. If you simply cannot reach the correct floor, your program halts with the message "use the stairs".

Given input f, s, g, u and d (floors, start, goal, up, down), fi nd the shortest sequence of button presses you must press in order to get from s to g, given a building of floors, or output "use the stairs" if you cannot get from s to g by the given elevator.

Input
The input will consist of one line, namely $f\ s\ g\ u\ d$, where $1 \leq s, g \leq f \leq 1000000$ and $0 \leq u, d \leq 1000000$. The floors are one-indexed, i.e. if there are 10 stories, s and g be in [1; 10].

Output
You must reply with the minimum numbers of pushes you must make in order to get from s to g, or output "use the stairs" if it is impossible given the con guration of the elevator.

input
10 1 10 2 1
output
6

S. 06.BUGLIFE

1 second, 256 megabytes

Professor Hopper is researching the sexual behavior of a rare species of bugs. He assumes that they feature two different genders and that they only interact with bugs of the opposite gender. In his experiment, individual bugs and their interactions were easy to identify, because numbers were printed on their backs.

Given a list of bug interactions, decide whether the experiment supports his assumption of two genders with no homosexual bugs or if it contains some bug interactions that falsify it.

Input

The first line of the input contains the number of scenarios. Each scenario starts with one line giving the number of bugs (at least one, and up to 2000) and the number of interactions (up to 1000000) separated by a single space. In the following lines, each interaction is given in the form of two distinct bug numbers separated by a single space. Bugs are numbered consecutively starting from one.

Output

The output for every scenario is a line containing "Scenario #i:", where i is the number of the scenario starting at 1, followed by one line saying either "No suspicious bugs found!" if the experiment is consistent with his assumption about the bugs' sexual behavior, or "Suspicious bugs found!" if Professor Hopper's assumption is definitely wrong.

input

```
2
3 3
1 2
2 3
1 3
4 2
1 2
3 4
```

output

```
Scenario #1:
Suspicious bugs found!
Scenario #2:
No suspicious bugs found!
```

T. MAKESPAN SCHEDULE

1 second, 256 megabytes

A project has n tasks $1, \ldots, n$. Task i has duration $d(i)$ to be completed ($i = 1, \ldots, n$). There are precedence constraints between tasks represented by a set Q of pairs: for each $(i, j) \in Q$, task j cannot be started before the completion of task i. Compute the earliest completion time of the project.

Input

- Line 1: contains n and m $(1 \leq n \leq 10^4, 1 \leq m \leq 200000)$
- Line 2: contains $d(1), \ldots, d(n)$ $(1 \leq d(i) \leq 1000)$
- $i + 3$ $(i = 1, \ldots, m)$: contains i and j : task j cannot be started to execute before the completion of task i

Output

Write the earliest completion time of the project.

input

```
9 13
5 3 1 2 6 4 3 1 4
1 3
1 5
1 6
2 1
2 3
3 5
4 1
4 2
4 6
5 8
7 9
9 5
9 8
```

output

```
18
```