



Computer Vision

Chapter 3: Image Processing

Computer Vision

Chapter 3. Image Processing



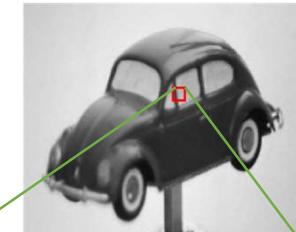
Content

- Remind: digital image representation
- Point Operators
- Convolution and spatial filtering
- More neighborhood operators
- Image transforms



Digital images ?

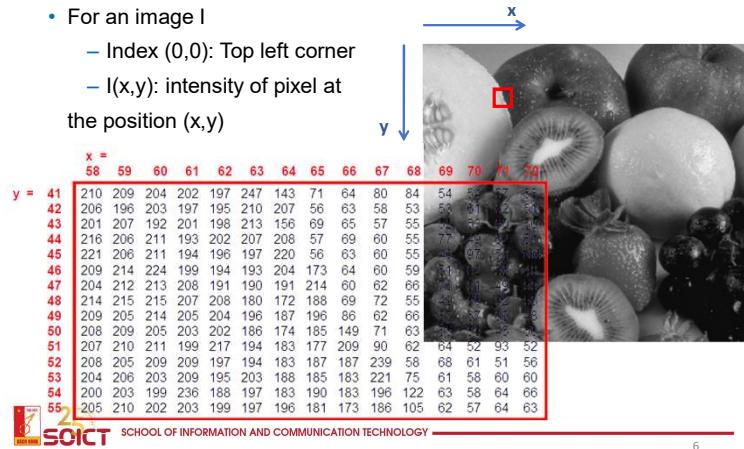
- What we can see on the picture?
— A car?
- What does the machine see?
— Image is a matrix of pixels
— Image $N \times M$: $N \times M$ matrix
— 1 pixel (gray levels):
 - A intensity value: 0-255
 - Black: 0
 - White: 255



64	60	69	100	149	151	176	182	179
65	62	68	97	145	148	175	183	181
65	66	70	95	142	146	176	185	184
66	66	68	90	135	140	172	184	184
66	64	64	84	129	134	168	181	182
59	63	62	88	130	128	166	185	180
60	62	60	85	127	125	163	183	178
62	62	58	81	122	120	160	181	176
63	64	58	78	118	117	159	180	176

Digital images ?

- For an image I
 - Index (0,0): Top left corner
 - $I(x,y)$: intensity of pixel at the position (x,y)



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Digital images ?

- Principal type of images

- Binary image:
 - $I(x,y) \in \{0, 1\}$
 - 1 pixel: 1 bit



- Gray image:

- $I(x,y) \in [0..255]$
- 1 pixel: 8 bits (1 byte)



- Color image

- $I_R(x,y), I_G(x,y), I_B(x,y) \in [0..255]$
- 1 pixel: 24 bits (3 bytes)



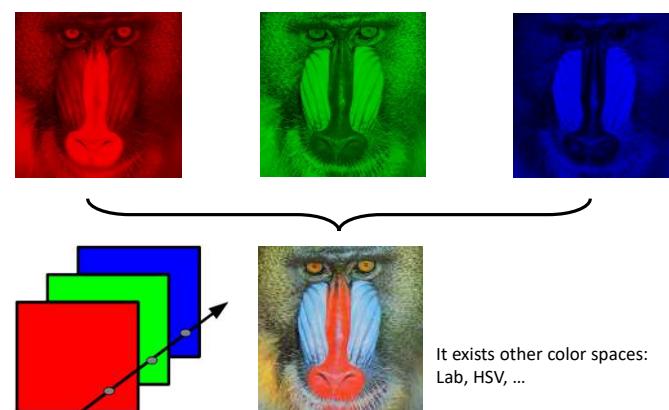
- Other : multi-spectre, depth image,...



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Color image in RGB space



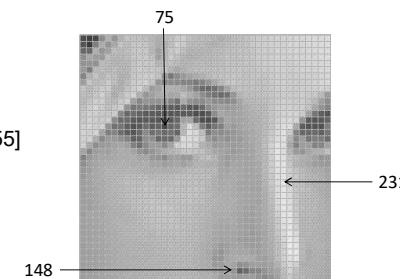
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Images are sampled and quantized

- An image contains discrete number of pixels
- A simple example
- Pixel value:

- “grayscale”
(or “intensity”): $[0,255]$



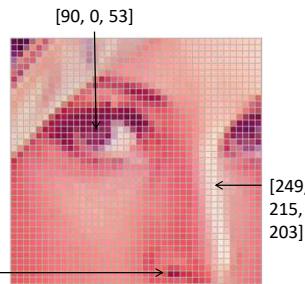
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Images are sampled and quantized

- An image contains discrete number of pixels
- A simple example
- Pixel value:

- “grayscale”
(or “intensity”): [0,255]
- “color”
 - RGB: [R, G, B]
 - Lab: [L, a, b]
 - HSV: [H, S, V]



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Modification of images values

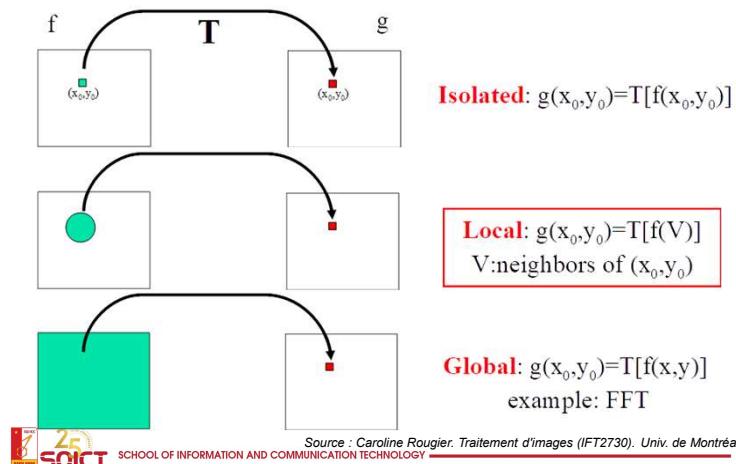
- **Isolated transformations (point operators/point processing)** of the pixels in an image
 - *Read the value of a pixel → replace it by another*
 - Ex.: contrast enhancement, histogram equalization
- There are also **local transformations**
 - *Read the values of few neighboring pixels → compute a new value for a pixel*
 - *Convolutions, correlations, ...*
- ...and **global transformations**
 - *Read the values of all pixels in the image → compute a new value for one pixel*
 - *FFT, ...*



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Pixel transformation



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Content

- Rappel: digital image representation
- **Point Processing**
 - Point operators
 - Histogram and histogram equalization
- Convolution and spatial filtering
- More neighborhood operators
- Image transforms



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Point processing

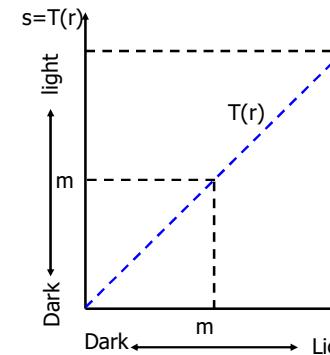
- The output at one point depends only on that point but not other neighborhoods

$$s = T(r)$$

- $r = f(x, y)$
- $s = g(x, y)$
- T : transformation function



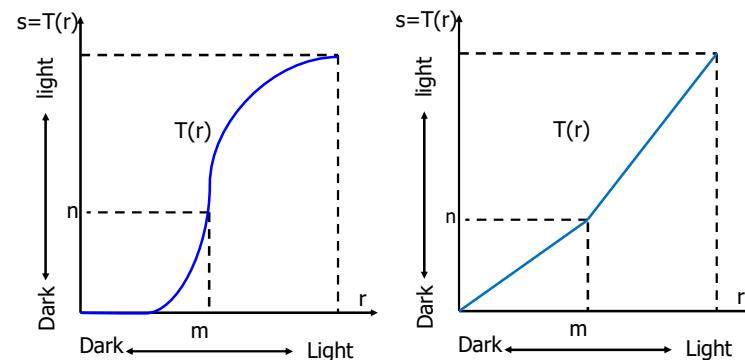
Homogenous transformation



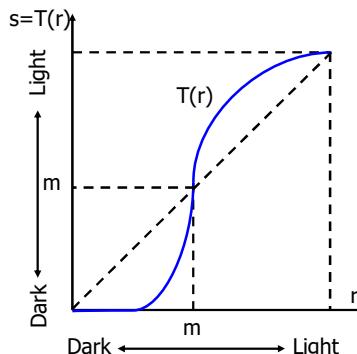
The output is identical as the input : $s = r$



Linear / Non-linear transformation



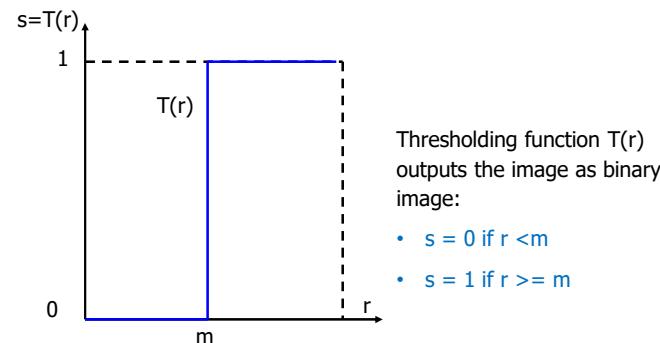
Contrast stretching



- The values of r below m are compressed by T into a narrow range of s , toward black.
- The opposite effect takes place for values of r above m .



Thresholding function



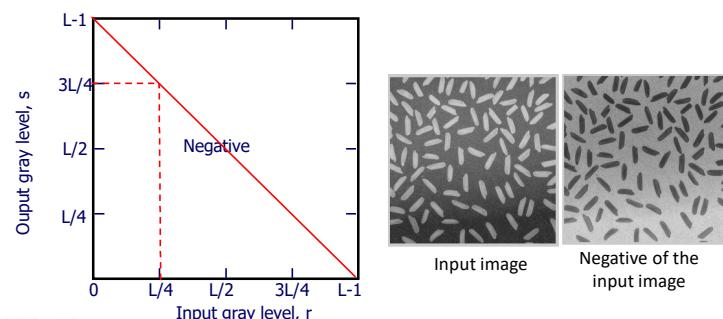
Some Basic Gray Level Transformations

- Image Negatives
- Log Transformations
- Power-Law Transformations
- Piecewise-Linear Transformation Functions
- Histogram Processing



Image Negatives

- The negative of an image with gray levels in the range $[0, L-1]$ is obtained by $s = L - 1 - r$

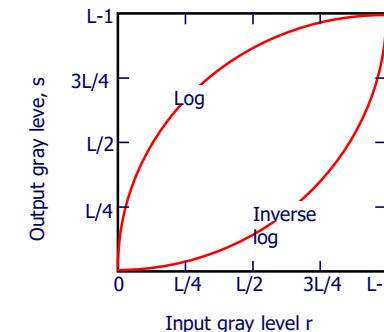


Log Transformations

- The general form of the log transformation:

$$s = c \times \log(1 + r)$$

- c : constant
- $r \geq 0$



Log Transformations

- This transformation maps
 - a **narrow range** of low gray-level values in the input image **into a wider range** of output levels.
 - The opposite is true of higher values of input levels.
- Use this Log to **expand the values of dark pixels** in an image while compressing the higher-level values.



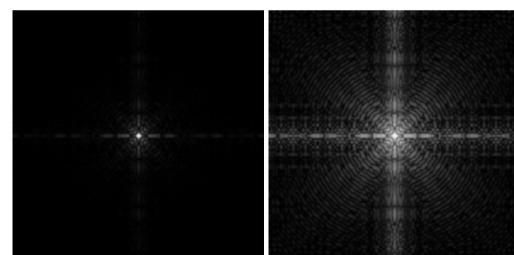
Inverse Log Transformations

- This transformation maps
 - a **wide range** of low gray-level values in the input image **into a more narrow range** of output levels.
 - The opposite is true of higher values of input levels.
- Use this Inverse Log if you want to **compress the values of dark pixels** in an image while expanding the higher-level values.



Log Transformations

- Expand the values of dark pixels in an image while compressing the higher-level values

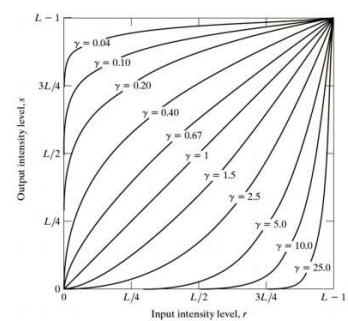


Power-Law (Gamma) Transformations

- The general form of power-law transformation is:

$$s = c \cdot r^\gamma$$

- $\gamma > 1$: **compress values in dark area**, while expanding values in light area
- $\gamma < 1$: **expand values in dark area**, while compressing values in light area
- r : normalized values to $[0, 1]$
- c : scaling constant c corresponding to the bit size used



Power-Law (Gamma) Transformations

- $\gamma_1 = 1; \gamma_1 = 3; \gamma_2 = 4; \gamma_3 = 5;$



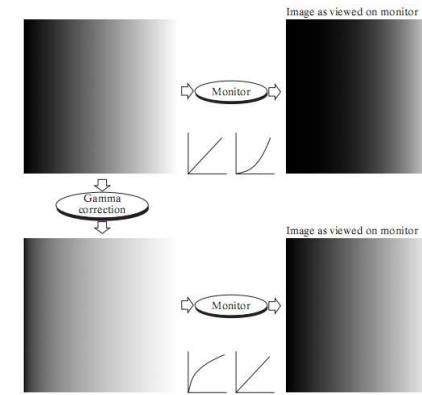
Increasing the dynamic ranges of high intensities



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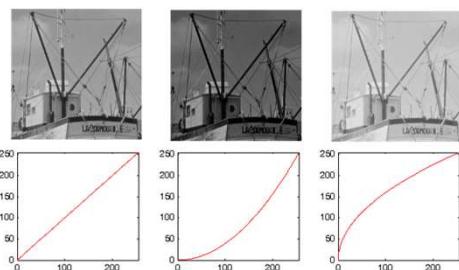
Gama correction



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Gama correction



Ex: Gamma correction (g)

$$\begin{aligned} I_{\text{emitted}} &= k \cdot n^{\gamma} \\ &= n' = n^{\gamma} \\ \gamma_{\text{screen}} &\in [1.3, 3.0] \\ \gamma_{\text{eye}} &\approx \frac{1}{2} \text{ to } \frac{1}{3} \end{aligned}$$

In Practice with grey level image :

$$k = 255$$

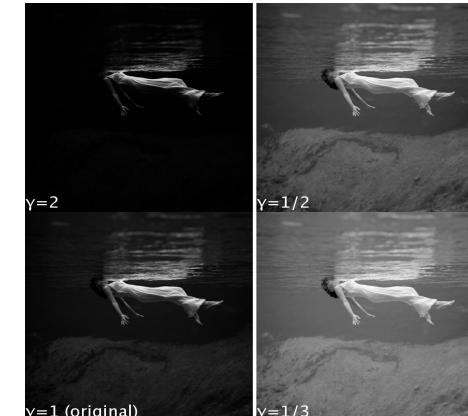
$$n = r/255$$



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Gama correction



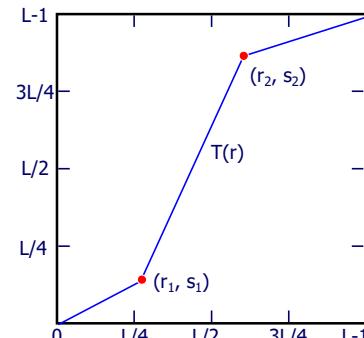
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Image: Wikipedia

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Piecewise-Linear Transformation

- Contrast stretching:
 - the idea behind contrast stretching is to **increase the dynamic range** of the gray levels in the image being processed
- Output:
 - Output values of input ones from (0, 0) to (r_1, s_1) decrease
 - Output values of input ones from (r_1, s_1) đến (r_2, s_2) increase.



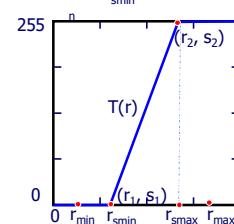
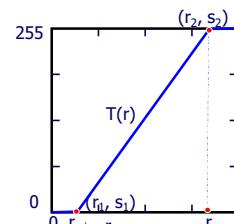
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Piecewise-Linear Transformation

- If $(r_1, s_1) = (r_{\min}, 0)$; $(r_2, s_2) = (r_{\max}, 255)$
 - Linear stretching
- If $(r_1, s_1) = (r_{\min}, 0)$, $r_{\min} > r_{\min}$ $(r_2, s_2) = (r_{\max}, 255)$ $r_{\max} < r_{\max}$
 - Linear stretching with saturation



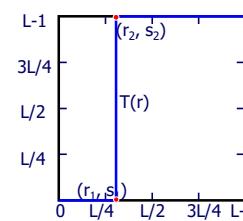
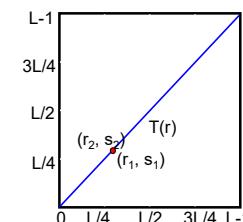
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Piecewise-Linear Transformation

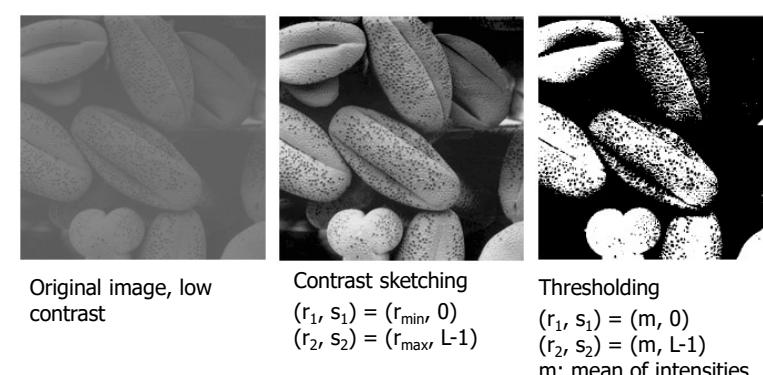
- If $r_1 = s_1$, $r_2 = s_2$,
 - the function becomes Homogenous transformation
- If $s_1 = 0$, $s_2 = L-1$,
 - the function become thresholding function
 - Output is a binary image



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Piecewise-Linear Transformation



Original image, low contrast

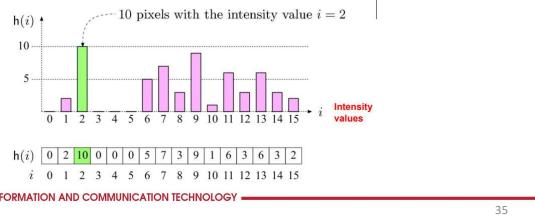
Contrast stretching
 $(r_1, s_1) = (r_{\min}, 0)$
 $(r_2, s_2) = (r_{\max}, L-1)$ Thresholding
 $(r_1, s_1) = (m, 0)$
 $(r_2, s_2) = (m, L-1)$
m: mean of intensities

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Image histogram

- What is histogram?

- Histogram of an image with gray level range $[0, L-1]$ is a discrete function: $h(r_k) = n_k$
 - r_k is k^{th} gray level
 - n_k : number of pixels that have level r_k
- Histogram is a graphical representation of the repartition of colours among the pixels of a digital image



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Image histogram

- Normalized histogram?

- The histogram should be normalized by dividing all elements to total number of pixels in the image (n) : $h(r_k) = \frac{n_k}{n}$

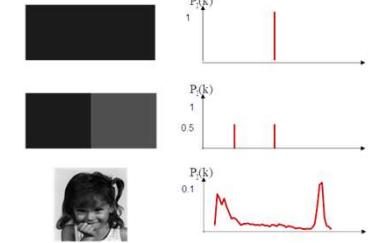


Image dynamic range = [min intensity, max intensity]



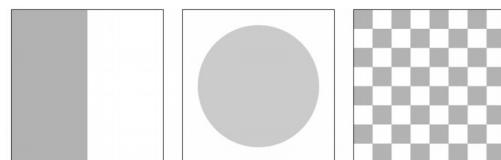
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Image histogram

- Properties?

- Only statistic information
- No indication about the location of pixel (no spatial information)
- Different images can have the same histogram



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Image Brightness

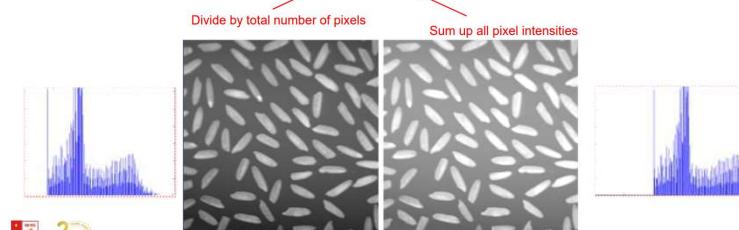
- Brightness of a grayscale image is the average intensity of all pixels in an image

- refers to the overall lightness or darkness of the image

$$B(I) = \frac{1}{wh} \sum_{v=1}^h \sum_{u=1}^w I(u, v)$$

Divide by total number of pixels

Sum up all pixel intensities



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Contrast

- The contrast of a grayscale image indicate how easily object in the image can be distinguished
- Many different equations for contrast exist
 - Standard deviation of intensity values of pixels in the image
 - Difference between intensity value maximum et minimum

$$C = \sqrt{\frac{1}{wh} \sum_{u=1}^h \sum_{v=1}^w (I(u, v) - \text{mean})^2}$$

– Difference between intensity value maximum et minimum

$$C = \frac{\max(I(u, v)) - \min(I(u, v))}{\max(I(u, v)) + \min(I(u, v))}$$

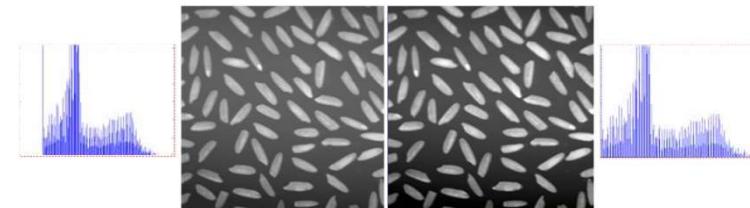


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Contrast

- Contrast vs histogram

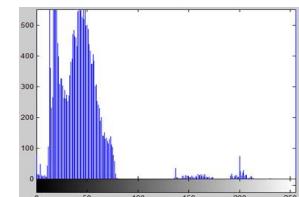
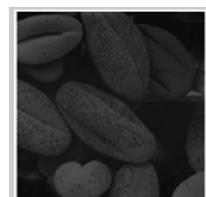


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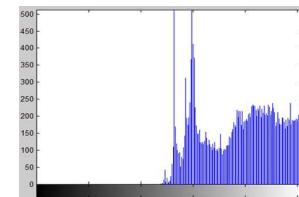
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Histogram and Brightness

Dark image



Light image

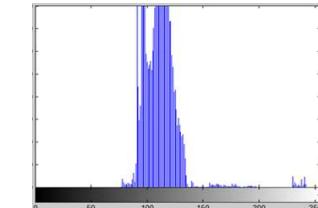
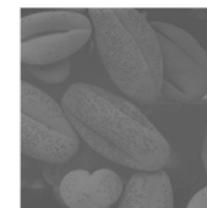


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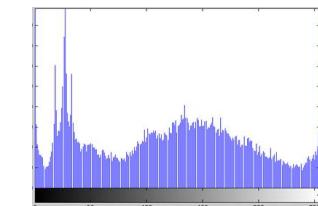
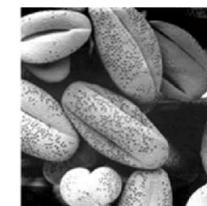
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Histogram and Contrast

Low- contrast image



High- contrast image

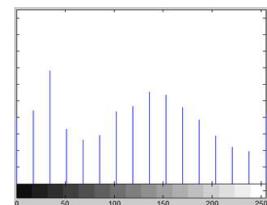


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Histogram

- If we can somehow equalize the histogram of an image → we can make it higher contrast.

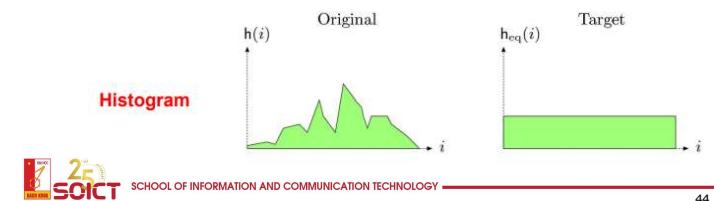


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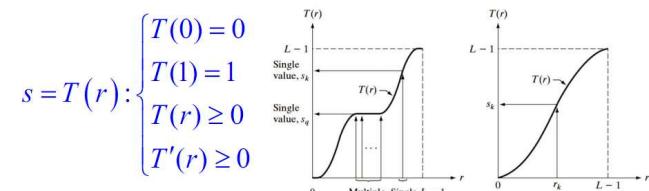
Histogram equalization

- Input:**
 - Image I which have L gray levels $r_k \in [0, L-1]$, $k=0, 1, \dots, L-1$. Total pixels is n .
 - Assume that the input image is low contrast
- Output:**
 - Image J with gray level $s_k \in [0, L-1]$, $k=0, 1, \dots, L-1$
 - Histogram of modified image: **uniform distribution**
 - Each gray level in the image occurs with the same frequency



Histogram equalization

- First assume continuous case
 - r : random variable represent value of gray level in input image, $0 \leq r \leq 1$
 - s : random variable represent value of gray level in output image → We have to find out T where: $s = T(r)$



→ The transform should remain the characteristic of image



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Histogram equalization

- Matching function:**
 - If T is the **cumulative distribution function** of r multiplied by $(L-1)$, then s is uniformly distributed on $[0, L-1]$
 - $s = T(r) = (L-1) \text{ CDF}(r)$



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Histogram equalization

- Discrete domain (apply to image)

$$p_r(r_k) = \frac{n_k}{MN} = \frac{n_k}{n}, \quad k = 0, 1, 2, \dots, L-1$$

$$S_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j) = \frac{L-1}{MN} \sum_{j=0}^k n_j, \quad k = 0, 1, \dots, L-1$$



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Histogram equalization

- Step-by-step

— s1. count the number of pixels which have gray level k in image, let's say n_k ($k=0, 1, \dots, L-1$)

— s2. compute $p(k)$ like follows
(normalized histogram):

$$p(k) = \frac{n_k}{n}$$

$k = 0, 1, 2, \dots, L-1$.

Histogram equalization

- s3. Compute $T(k)$ (cumulative histogram)

$$T(k) = \sum_{j=0}^k p(j) = \frac{1}{n} \sum_{j=0}^k n_j \quad k = 0, 1, 2, \dots, L-1.$$

- s4. Gray level s_k of output image, J , responsible to gray level k of input image is calculated by:

$$s_k = \text{round}[(L-1)T(k)] \quad k = 0, 1, 2, \dots, L-1.$$

- s5. Build the output image by replacing the gray level k by s_k .



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Histogram equalization

Ex. 1. Input image I has 10 levels.

Equalize the histogram of I

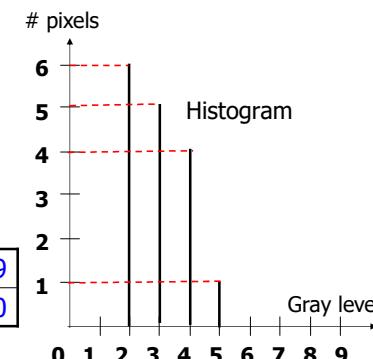
2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

k	0	1	2	3	4	5	6	7	8	9
n_k	0	0	6	5	4	1	0	0	0	0



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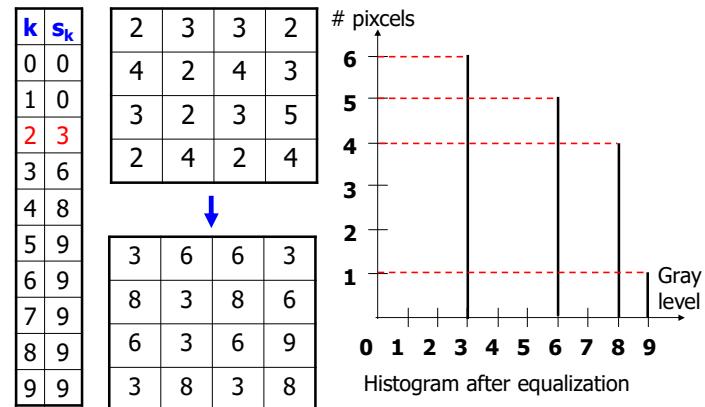


Histogram equalization

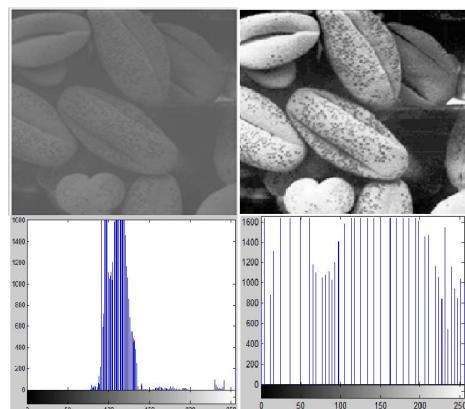
k	0	1	2	3	4	5	6	7	8	9
n_k	0	0	6	5	4	1	0	0	0	0
$\sum_{j=0}^k n_j$	0	0	6	11	15	16	16	16	16	16
$T(k)$	0	0	6/16	11/16	15/16	16/16	16/16	16/16	16/16	16/16
$s_k = 9T(k)$	0	0	3.3 ≈3	6.1 ≈6	8.4 ≈8	9	9	9	9	9



Histogram equalization

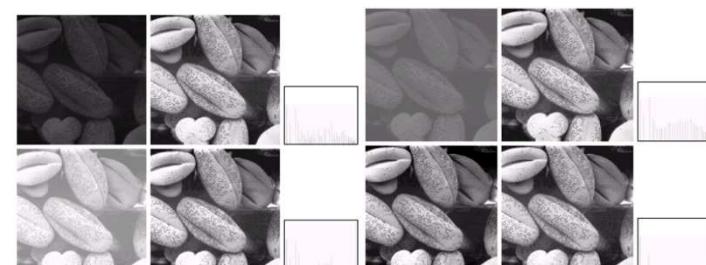


Histogram equalization



Histogram equalization

If we take the **same image** with **different contrasts**, histogram equalization will give the **same results** for all images



Histogram equalization

- Discussion
 - Compare between histogram equalization and Piecewise-Linear Transformation Functions
 - Is histogram equalization always good?

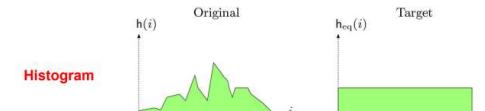


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Histogram equalization

- Change histogram of modified image into uniform distribution



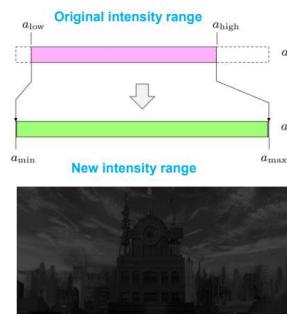
- No parameters. OpenCV: `cv2.equalizeHist(img)`



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Linear stretching



$$f_{ac}(a) = a_{\min} + (a - a_{\min}) \cdot \frac{a_{\max} - a_{\min}}{a_{\max} - a_{\min}}$$

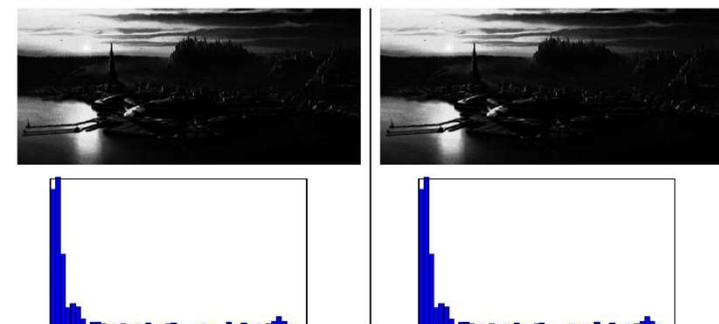
If $a_{\min} = 0$ and $a_{\max} = 255$

$$f_{ac}(a) = (a - a_{\min}) \cdot \frac{255}{a_{\max} - a_{\min}}$$



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Linear stretching



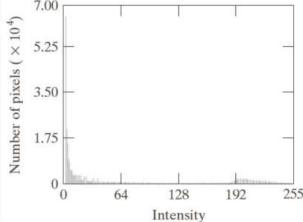
No efficace?



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Histogram equalization



a

b

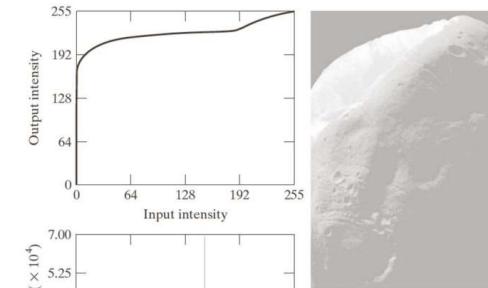
FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*.
(b) Histogram. (Original image courtesy of NASA.)



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Histogram equalization



a

b

c

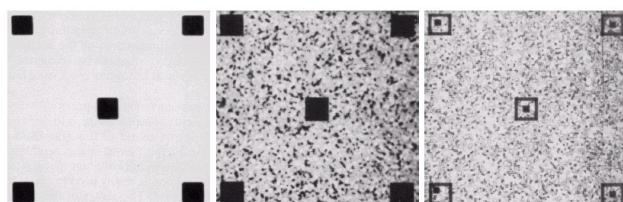
FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).



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Local Histogram equalization



a

b

c

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7×7 neighborhood about each pixel.



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Histogram matching/specification

- In some cases: uniform histogram does not have a best result
 - Specify a good histogram
 - Transform the image to have the specific histogram

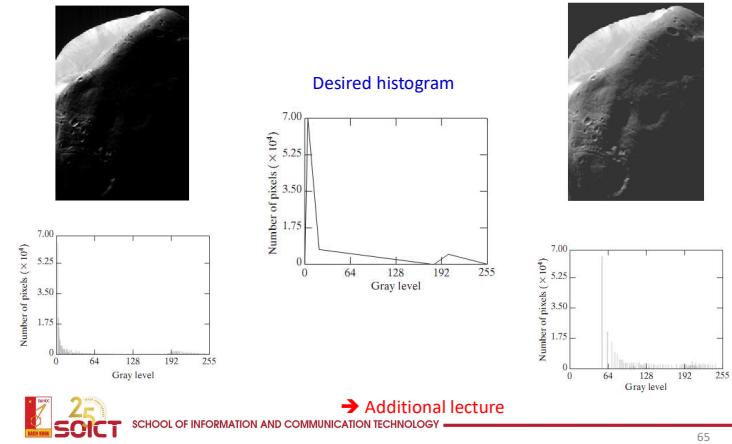
So, we have **histogram matching** or **histogram specification** problem



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Histogram matching



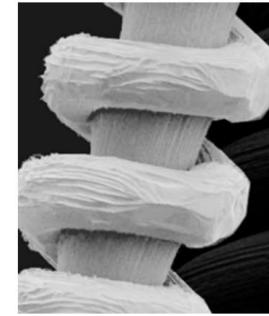
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Local histogram enhancement

- Problem: how to light up the bottom right part in bellow image?

FIGURE 3.24 SEM image of a tungsten filament and support, magnified approximately 130 \times . (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).

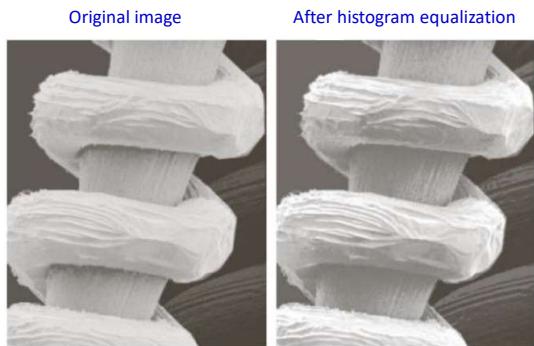


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Histogram statistic



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Histogram statistic

- Compute global statistic

$$\begin{aligned}\mu_n(r) &= \sum_{i=0}^{L-1} (r_i - m)^n p(r_i) \approx \frac{1}{MN} \sum_{x=1}^M \sum_{y=1}^N [f(x, y) - m]^n \\ m &= \sum_{i=0}^{L-1} r_i p(r_i) \approx \frac{1}{MN} \sum_{x=1}^M \sum_{y=1}^N f(x, y)\end{aligned}$$

- Local mean and local variance:

$$\begin{aligned}m_{S_{xy}} &= \sum_{(s, t) \in S_{xy}} r_{s, t} p(r_{s, t}) \\ \sigma_{S_{xy}}^2 &= \sum_{(s, t) \in S_{xy}} [r_{s, t} - m_{S_{xy}}]^2 p(r_{s, t})\end{aligned}$$



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Histogram statistic

- Problem:
 - how to enhance the contrast of one part in the image but not affect other parts
- Idea:
 - only increase or decrease desired parts while remaining the others

$$g(x, y) = \begin{cases} E.f(x, y) & m_s(x, y) \leq k_0 m_G \text{ and } k_1 \sigma_G \leq \sigma_s(x, y) \leq k_2 \sigma_G \\ f(x, y) & \text{OW} \end{cases}$$

- In which: k_0, k_1, k_2 are constant coefficients
- $S(x, y)$: neighborhood of 3×3

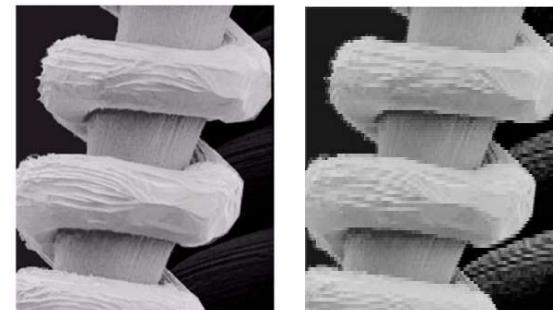


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Histogram statistic

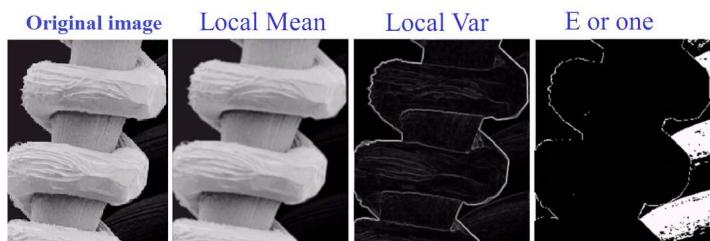
$$E = 4.0, k_0 = 0.4, k_1 = 0.02, k_2 = 0.4$$



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Histogram statistic



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Implement Histogram statistic

- step 1. read the input image
- step 2. compute the mean and the variance of image.
- step 3. use a window of 3×3 slide for every point of the input image
 - At each point: compute the local mean and local variance

$$g(x, y) = \begin{cases} E.f(x, y) & m_s(x, y) \leq k_0 m_G \text{ and } k_1 \sigma_G \leq \sigma_s(x, y) \leq k_2 \sigma_G \\ f(x, y) & \text{OW} \end{cases}$$



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Contrast enhancement for color images ?

- Histogram equalization of color images ?
 - Apply separately the procedure to **each of the three channels**
 - will yield in a dramatic change in the color balance
 - **NOT RECOMMENDED** unless you have a good reason for doing that.
 - Recommended method
 - Convert the image into **another color space** such as the Lab color space or HSL/HSV color space
 - Apply the **histogram equalization on the Luminance channel**
 - Convert **back into the RGB color space**



Content

- Rappel: digital image representation
- Point Processing
- Convolution and spatial filtering
 - Spatial convolution
 - Correlation
 - Filters
- More neighborhood operators
- Image transforms



Image filtering

- Image filtering:
 - **Image filters in spatial domain (called spatial filtering)**
 - Filter is a mathematical operation of a grid of numbers
 - Smoothing, sharpening, measuring texture, ...
 - Image filters in the frequency domain (**later**)
 - Filtering is a way to modify the frequencies of images
 - Denoising, sampling, image compression,...
- Really important!
 - Enhance images: Denoise, smooth, increase contrast, etc.
 - Extract information from images:
 - Texture, edges, distinctive points, etc.
 - Detect patterns
 - Template matching



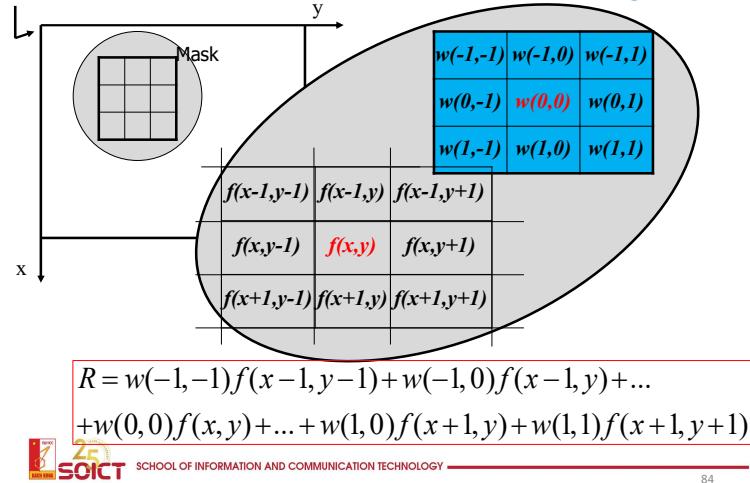
Convolution and spatial filtering

- Spatial filtering
 - affects directly to pixels in image
 - Local transformation in the spatial domain can be represented as: $g(x,y) = T(f(x,y))$
 - T may affect the **point (x,y) and its neighborhood (K) and output a new value**
(K : Filter/Kernel/Window/Template Processing)
 - **Same function** applied at each position
 - Output and input image are typically **the same size**
- Convolution: Linear filtering, function is a weighted sum/difference of pixel values

$$I' = I * K$$



Fundamental of spatial filtering



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Spatial Convolution

- Convolution: Linear filtering, function is a weighted sum/difference of pixel values

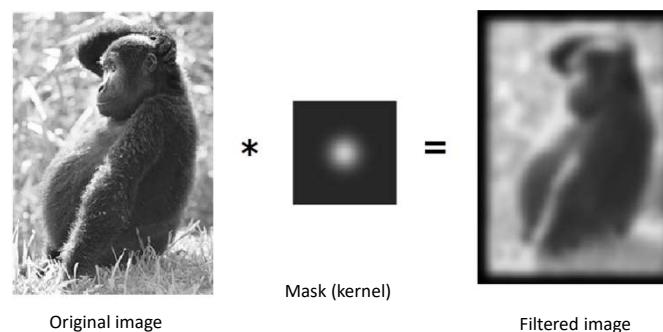
$$f(n, m) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k, l] \times h[n - k, m - l]$$



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Spatial convolution

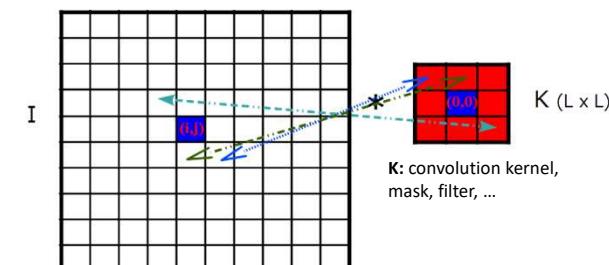


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Spatial convolution

- New value of a pixel (i,j) is a weighted sum of its neighbors



$$I'(i, j) = \sum_{u=-\frac{L-1}{2}}^{\frac{L-1}{2}} \sum_{v=-\frac{L-1}{2}}^{\frac{L-1}{2}} I(i-u, j-v) K(u, v)$$

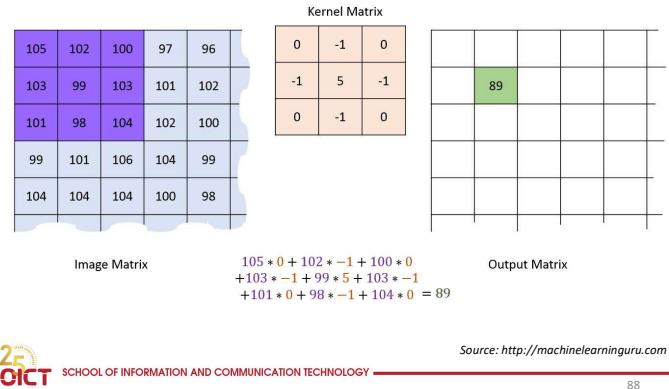


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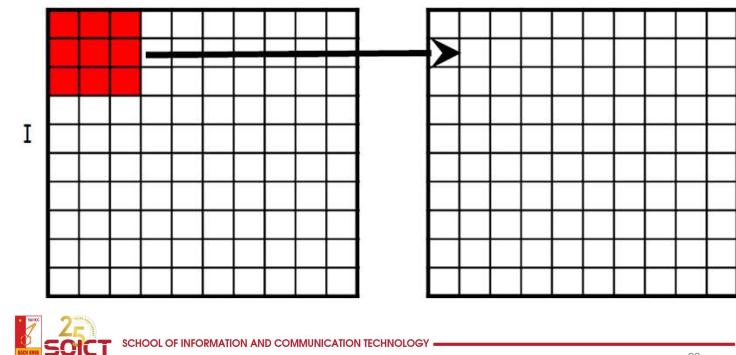
Spatial convolution

- New value of a pixel(i,j) is a weighted sum of its neighbors

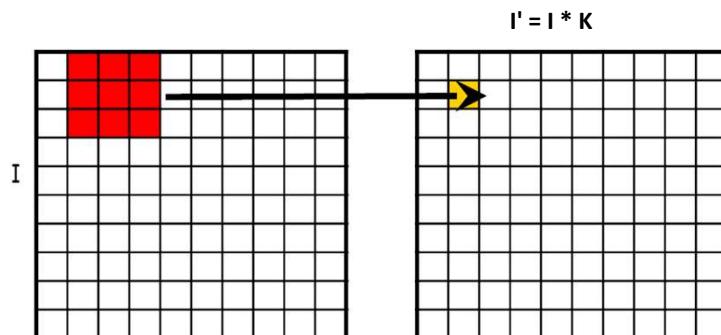


Spatial convolution

$$I' = I * K$$

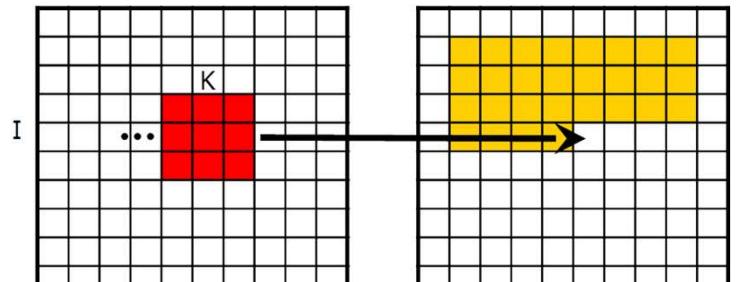


Spatial convolution



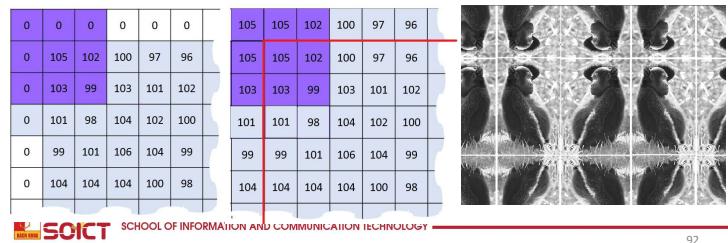
Spatial convolution

$$I' = I * K$$



Spatial convolution

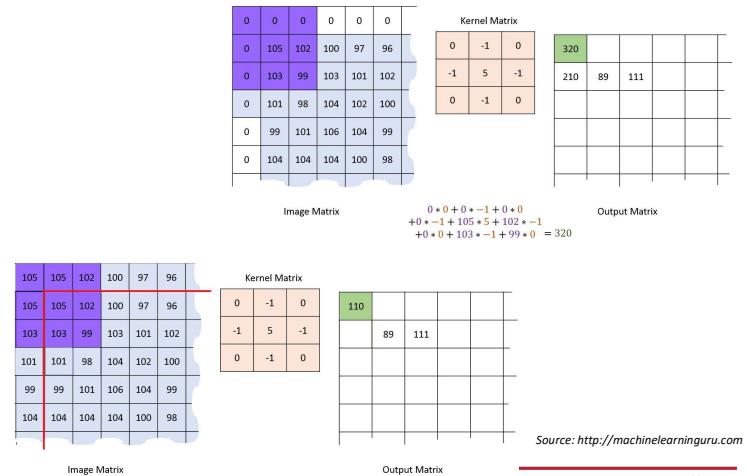
- Border problem?
 - Ignore, set to 0
 - Zero padding to the input matrix
 - reflect across edge:
 - $f(-x,y) = f(x,y)$
 - $f(-x,-y) = f(x,y)$



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Spatial convolution



Source: <http://machinelearningguru.com>

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Spatial Correlation vs Convolution

Spatial Correlation (\star) and Convolution (\star)

$$w(x, y) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x-s, y-t)$$

$$w(x, y) \star f(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x+s, y+t)$$

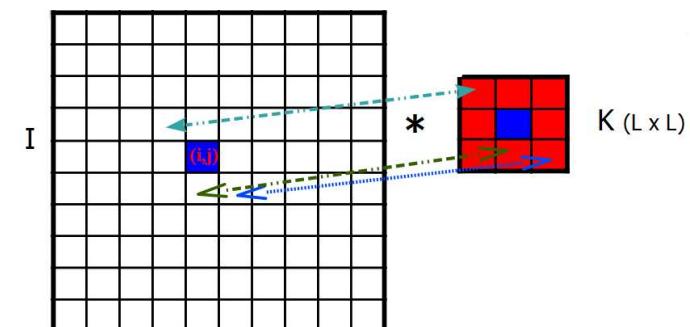
$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \Rightarrow \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

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Spatial Correlation vs Convolution

- Correlation



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Spatial Correlation vs Convolution

- If the **mask is symmetric** these two operations are identical
- **Correlation:**
 - Use to find which part in image match with a certain “template”
 - Not associative \rightarrow if you are doing **template matching**, i. e. looking for a single template, correlation is sufficient
- **Convolution:**
 - Use for filtering image (noise removing, enhancement, ..)
 - Associative: allows you to “pre-convolve” the filters \rightarrow useful when you need to use multiple filters in succession:
$$I * h * g = I * (h * g)$$

In Matlab: correlation: `filter2`, convolution: `conv2`

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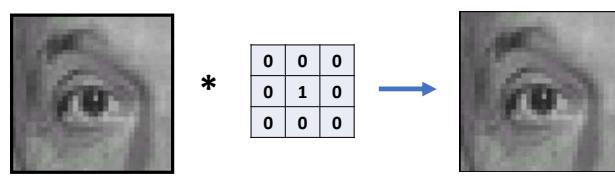
Some kernels

- 2D spatial convolution
 - is mostly used in image processing for feature extraction
 - And is also the core block of Convolutional Neural Networks (CNNs)
- Each kernel has its own effect and is useful for a specific task such as
 - blurring (noise removing),
 - sharpening,
 - edge detection,
 -

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Smooth filtering

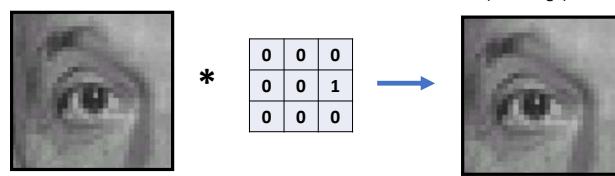


Original image

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

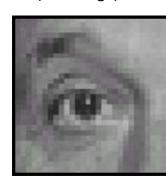


Filtered image (no change)



Original image

Source: David Lowe



Filtered image (shifted left by 1 pixel)

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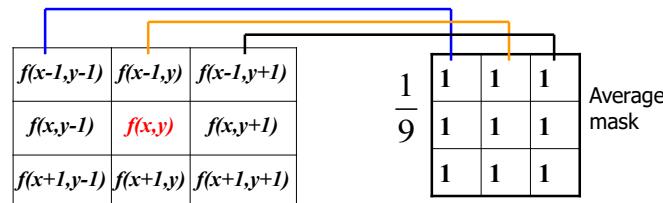
Smooth filtering

- **Application**
 - Noise remove
 - Image blurring
- Average filtering is the simplest of smooth filtering

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Average (mean) filtering



$$g(x, y) = \frac{1}{9} [f(x - 1, y - 1) + f(x - 1, y) + f(x - 1, y + 1) + f(x, y - 1) + f(x, y) + f(x, y + 1) + f(x + 1, y - 1) + f(x + 1, y) + f(x + 1, y + 1)]$$



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Average (mean) filtering

- Mask:

- All elements of the mask are equal

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

- Results:

- Replacing each pixel with an average of its neighborhood
- Achieve smoothing effect



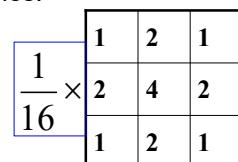
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with box size 5x5

with box size 11x11

Weighted Average filtering

- The pixel corresponding to the center of the mask is more important than the other ones.



$$g(x, y) = \frac{1}{16} [f(x - 1, y - 1) + 2f(x - 1, y) + f(x - 1, y + 1) + 2f(x, y - 1) + 4f(x, y) + 2f(x, y + 1) + f(x + 1, y - 1) + 2f(x + 1, y) + f(x + 1, y + 1)]$$



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Gaussian filtering



Gaussian function in 3D

0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

Gaussian filter with size 5x5, sigma = 1

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$



Gaussian image

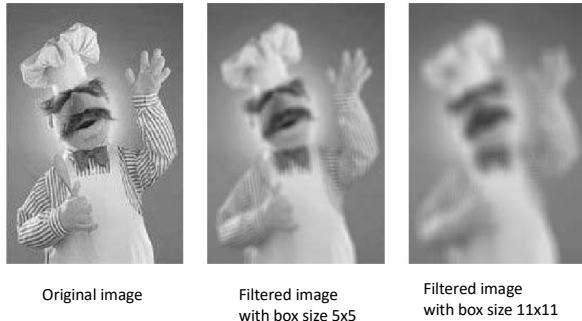
Rule for Gaussian filter:
set filter width to about 6σ or 8σ [+1]



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Gaussian filtering



Smooth filtering

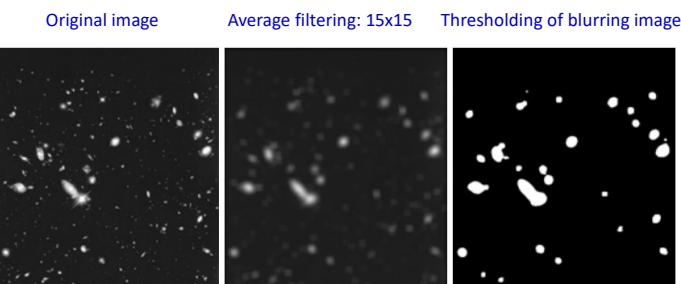
- General formulation

$$g(x, y) = \frac{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)}{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t)}$$

- To avoid modifying the overall luminance of the image, **the sum of the coefficients must be equal to 1**

Smooth filtering

- Blurring usage: delete unwanted (small) subjects



Sharpening filter

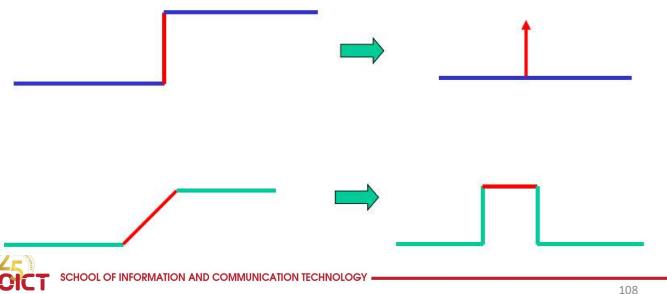
- Highlights Intensity Transitions
- Base on first and second order derivatives

$$\frac{\partial f}{\partial x} \approx \begin{cases} f(x+1, y) - f(x, y) \\ f(x, y) - f(x-1, y) \\ 0.5(f(x+1, y) - f(x-1, y)) \end{cases}$$

$$\frac{\partial^2 f}{\partial x^2} \approx f(x+1, y) - 2f(x, y) + f(x-1, y)$$

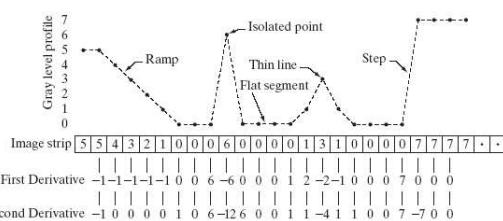
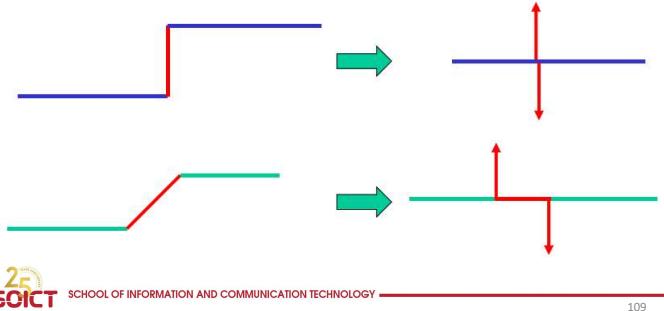
First order derivative

- Zero in flat region
 - Non-zero at start of step/ramp region
 - Non-zero along ramp



Second order derivative

- Zero in flat region
 - Non-zero at start/end of step/ramp region
 - Zero along ramp



Derivative

- 1st and 2nd Order Derivative Comparison:
 - First Derivative:
 - Thicker Edge;
 - Strong Response for step changes;
 - Second Derivative:
 - Strong response for fine details and isolated points;
 - Double response at step changes

First derivatives

- Filters used to compute the first derivatives of image

- Robert

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- Prewitt

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

- less sensitive to noise

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

- Smoothing with mean filter,

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

- then compute 1st derivative

- Sobel:

- less sensitive to noise

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

- Smoothing with gaussian,

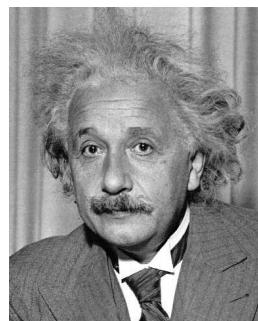
$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

- then computing 1st derivative

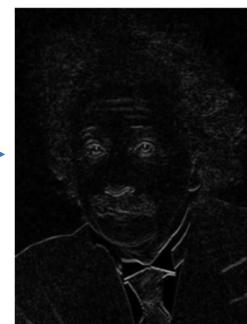


First derivatives

- Sobel



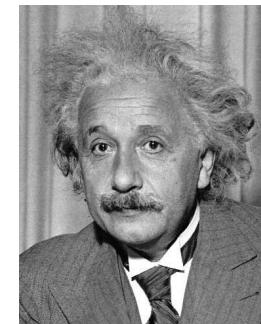
$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$



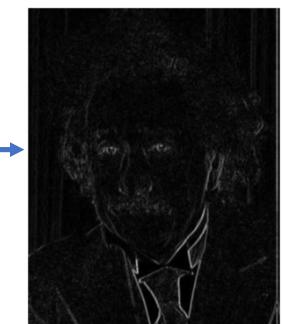
Horizontal Edge
(absolute value)

First derivatives

- Sobel



$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$



Vertical Edge
(absolute value)

2nd derivatives - Laplacian filtering

- The Laplacian operator is defined as:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$



2nd derivatives - Laplacian filtering

- Can be computed as

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$$

- Or

$$\nabla^2 f = 4f(x,y) - [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)]$$

- 90° isotropic filter

0	1	0
1	-4	1
0	1	0

0	-1	0
-1	4	-1
0	-1	0



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2nd derivatives - Laplacian filtering

- Can be computed as

$$\nabla^2 f = [f(x+1,y+1) + f(x+1,y) + f(x+1,y-1) + f(x-1,y+1) + f(x-1,y) + f(x-1,y-1) + f(x,y+1) + f(x,y-1)] - 8f(x,y)$$

- Or

$$\nabla^2 f = 8f(x,y) - [f(x+1,y+1) + f(x+1,y) + f(x+1,y-1) + f(x-1,y+1) + f(x-1,y) + f(x-1,y-1) + f(x,y+1) + f(x,y-1)]$$

- 45° isotropic filter

1	1	1
1	-8	1
1	1	1

-1	-1	-1
-1	8	-1
-1	-1	-1



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Sharpening filter using laplacian

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & -\text{sign} \\ f(x,y) + \nabla^2 f(x,y) & +\text{sign} \end{cases}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & +5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad 90^\circ \text{ isotropic} \quad \begin{bmatrix} -1 & -1 & -1 \\ -1 & +9 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad 45^\circ \text{ isotropic}$$



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Note

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) \\ f(x,y) + \nabla^2 f(x,y) \end{cases}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$



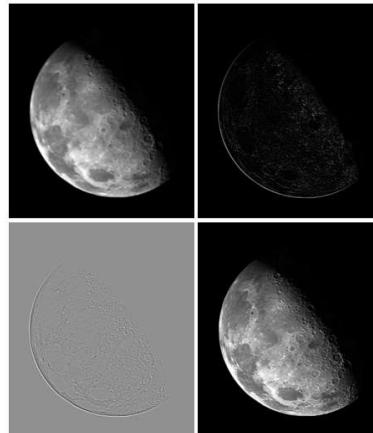
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Sharpening filter

- a) Original image
- b) Laplacian filtering
- c) Laplacian with scaling
- d) Adding original with Laplacian image

FIGURE 3.40
(a) Image of the North Pole of the moon.
(b) Laplacian-filtered image.
(c) Laplacian image scaled for display purposes.
(d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)



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Sharpening filter

0	-1	0
-1	5	-1
0	-1	0

-1	-1	-1
-1	9	-1
-1	-1	-1

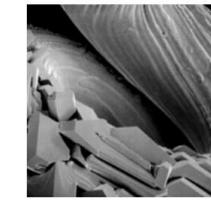
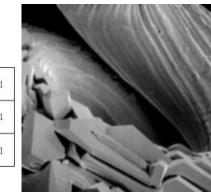


FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)



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Content

- Rappel: digital image representation
- Point Processing
 - Point operators
 - Histogram and histogram equalization
- Convolution and Linear filtering
- More neighborhood operators
- Image transforms

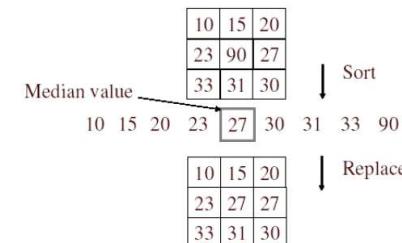


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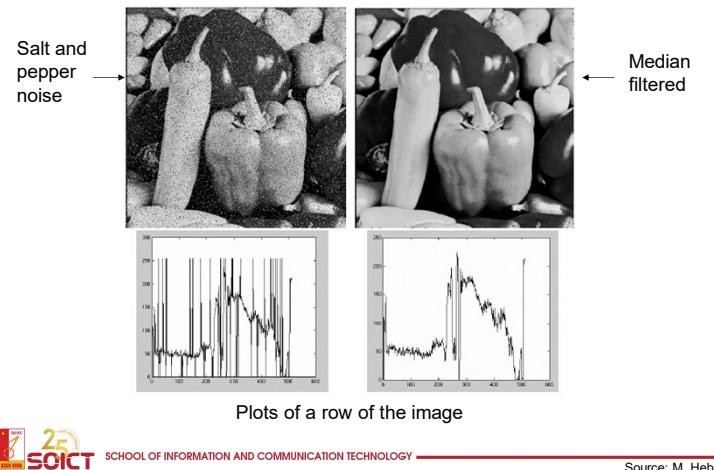
Median filter

- No new pixel values introduced
- Removes spikes:
 - good for impulse, salt & pepper noise
- Non-linear filter



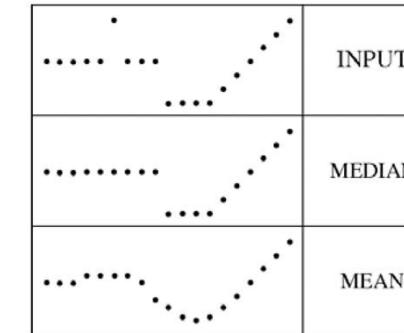
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Median filter



Median filter

- Median filter is edge preserving



Max/ Min filters

- The maximum and minimum filters are shift-invariant
 - The Minimum Filter: replaces the central pixel with **the darkest one** in the running window



- The Maximum Filter: replaces the central pixel with **the lightest one**

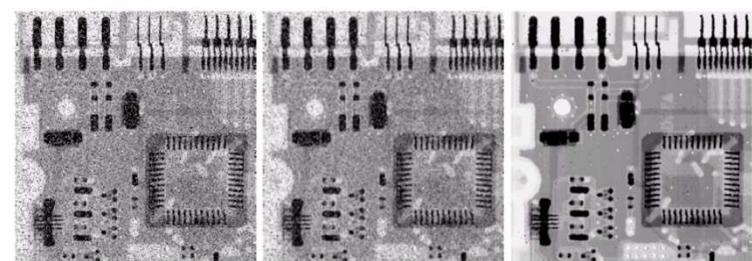
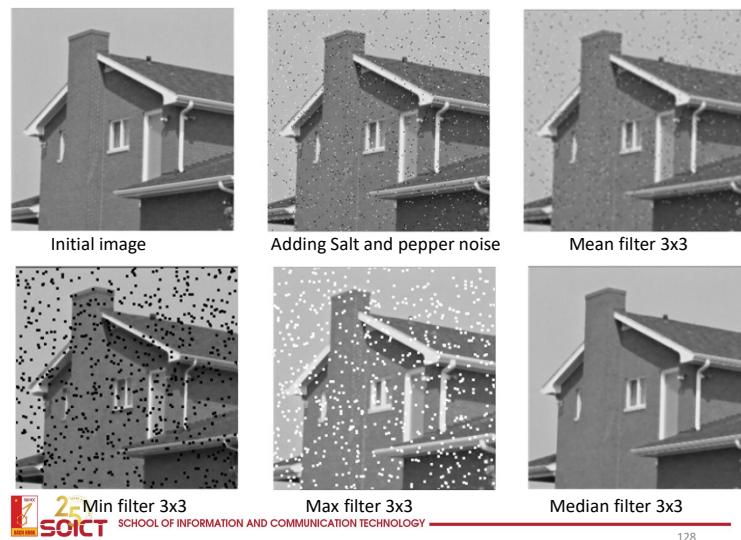


FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



Exercise

- Given an 8-bit image – 8 x 8

52	55	61	66	70	61	64	73
63	59	55	90	109	85	69	72
62	59	68	113	144	104	66	73
63	58	71	122	154	106	70	69
67	61	68	104	126	88	68	70
79	65	60	70	77	68	58	75
85	71	64	59	55	61	65	83
87	79	69	68	65	76	78	94

- Compute and show the histogram: 8 bins, 16 bins, 32 bins
- Comment about the contrast of the image
- Equalize the histogram for above image with 8-bins

Continue....

- Next lesson

