HA NOI UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF INFORMATION AND COMMUNICATION TECHNOLOGY

Computer Vision

Chapter 6: Motion detection and Tracking

Motion Detection and Tracking

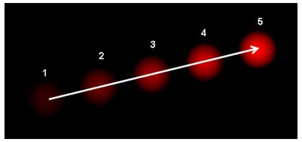
- Motion detection
- Approaches of motion detection
- Moving object tracking

Motion Detection

- Motion detection: Action of sensing physical movement in a give area
- Motion can be detected by measuring change in speed or vector of an object
- Goals of motion detection
  - Identify moving objects
  - Detection of unusual activity patterns
  - Computing trajectories of moving objects
- Applications of motion detection
  - Indoor/outdoor security
  - Real time crime detection
  - Traffic monitoring
  - Many intelligent video analysis systems are based on motion detection

Approaches to Motion Detection

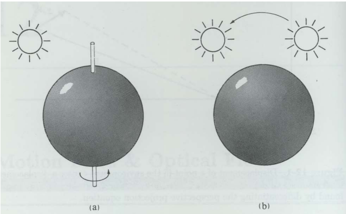
- Optical Flow
  - Compute motion within region or the frame as a whole



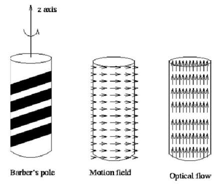
- Change detection
  - Detect objects within a scene
  - Track object across a number of frames

Optical flow vs. motion field

- Optical flow does not always correspond to motion field



Barber pole illusion



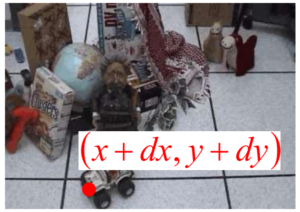
- Optical flow is an approximation of the motion field
- The error is small at points with high spatial gradient under some simplifying assumptions

Estimating optical flow

- Assume the image intensity I is constant

Time = t

Time = $t+dt$

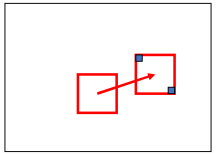


$$I_0(x, y, t) \approx I_1(x+\delta x, y+\delta y, t+\delta t)$$

Optical flow constraint

$$I(x, y, t) \approx I(x+\delta x, y+\delta y, t+\delta t)$$
$$I(x, y, t) \approx I(x, y, t) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t$$

$$\frac{\partial I}{\partial x}\frac{\delta x}{\delta t} + \frac{\partial I}{\partial y}\frac{\delta y}{\delta t} + \frac{\partial I}{\partial t} = 0, \text{ and let } \delta t \to 0$$
$$I_x u + I_y v + I_t = 0$$
$$I_x u + I_y v = -I_t,$$
$$[I_x \quad I_y]\begin{bmatrix} u \\ v \end{bmatrix} = -I_t, \nabla I^T \mathbf{u} = \mathbf{b}; A\mathbf{u} = \mathbf{b}$$
$$E(u, v) = (I_x u + I_y v + I_t)^2$$



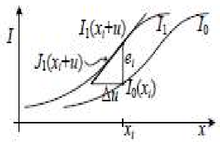
Lucas-Kanade algorithm

$$E(u, v) = \sum_{x,y \in \Omega} (I_x(x, y)u + I_y(x, y)v + I_t)^2$$

$$\|A\mathbf{u} - \mathbf{b}\|^2$$
$$\mathbf{u} = (A^T A)^{-1} A^T \mathbf{b}$$
$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$
$$\left(\sum \nabla I \cdot \nabla I^T\right) \vec{u} = -\sum \nabla I \cdot I_t$$

Matrix form

$$E(\mathbf{u}+\Delta\mathbf{u}) = \sum_i [I_1(\mathbf{x}_i+\mathbf{u}+\Delta\mathbf{u}) - I_o(\mathbf{x}_i)]^2$$
$$\approx \sum_i [I_1(\mathbf{x}_i+\mathbf{u}) + \mathbf{J}_1(\mathbf{x}_i+\mathbf{u})\Delta\mathbf{u} - I_o(\mathbf{x}_i)]^2$$
$$= \sum_i [\mathbf{J}_1(\mathbf{x}_i+\mathbf{u})\Delta\mathbf{u} + e_i]^2$$
$$\mathbf{J}_1(\mathbf{x}_i+\mathbf{u}) \approx \nabla I_1(\mathbf{x}_i+\mathbf{u}) = \left(\frac{\partial I_1}{\partial x}, \frac{\partial I_1}{\partial y}\right)(\mathbf{x}_i+\mathbf{u})$$
$$e_i = I_1(\mathbf{x}_i+\mathbf{u}) - I_o(\mathbf{x}_i)$$



$$y = f(\mathbf{x}+\Delta\mathbf{x}) \approx f(\mathbf{x}) + J(\mathbf{x})\Delta\mathbf{x} + \frac{1}{2}\Delta\mathbf{x}^{\mathrm{T}} H(\mathbf{x})\Delta\mathbf{x}$$

Matrix form

$$A\Delta\mathbf{u} = \mathbf{b}$$
$$A = \sum_i \mathbf{J}_1^T(\mathbf{x}_i+\mathbf{u})\mathbf{J}_1(\mathbf{x}_i+\mathbf{u})$$
$$\mathbf{b} = -\sum_i e_i \mathbf{J}_1(x_i+u)$$
$$A = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}, \quad \mathbf{b} = -\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$
$$\mathbf{J}_1(\mathbf{x}_i+\mathbf{u}) \approx \nabla I_1(\mathbf{x}_i+\mathbf{u}) = \left(\frac{\partial I_1}{\partial x}, \frac{\partial I_1}{\partial y}\right)(\mathbf{x}_i+\mathbf{u})$$
$$e_i = I_1(\mathbf{x}_i+\mathbf{u}) - I_o(\mathbf{x}_i)$$
$$\mathbf{J}_1(\mathbf{x}_i+\mathbf{u}) \approx \mathbf{J}_0(\mathbf{x}_i)$$

Computing gradients in X-Y-T

$$I_x = \frac{1}{4\delta x}[(I_{i+1,j,k} + I_{i+1,j,k+1} + I_{i+1,j+1,k} + I_{i+1,j+1,k+1}) - (I_{i,j,k} + I_{i,j,k+1} + I_{i,j+1,k} + I_{i,j+1,k+1})]$$

likewise for I_y and I_t

The aperture problem

$$\text{Let } A = \sum \nabla I \cdot \nabla I^T, \quad \text{and} \quad \mathbf{b} = -\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

- Algorithm: At each pixel compute u by solving $A\mathbf{u} = \mathbf{b}$
- A is singular if all gradient vectors point in the same direction
  - e.g., along an edge
  - of course, trivially singular if the summation is over a single pixel or there is no texture
  - i.e., only *normal flow* is available (aperture problem)
- Corners and textured areas are OK

Error functions

- Robust error function

$$E(\mathbf{u}) = \sum_i \rho(I(\mathbf{x}_i + \mathbf{u}) - I(\mathbf{x})), \quad \rho(x) = \frac{x^2}{1 + x^2/a^2}$$

- Spatially varying weights

$$E(\mathbf{u}) = \sum_i w_0(\mathbf{x}_i) w_1(\mathbf{x}_i + \mathbf{u}) [I(\mathbf{x}_i + \mathbf{u}) - I(\mathbf{x}_i)]^2$$

- Bias and gain: images taken with different exposure

$$I(\mathbf{x} + \mathbf{u}) = (1 + \alpha) I(\mathbf{x}) + \beta, \alpha \text{ is the gain and } \beta \text{ is the bias}$$

$$E(\mathbf{u}) = \sum_i [I(\mathbf{x}_i + \mathbf{u}) - (1 + \alpha) I(\mathbf{x}_i) - \beta]^2$$

- Correlation (and normalized cross correlation)

Horn-Schunck algorithm

- Global method with smoothness constraint to solve aperture problem
- Minimize a global energy functional with calculus of variations

$$E = \int \left((I_x u + I_y v + I_t)^2 + \alpha^2 (|\nabla u|^2 + |\nabla v|^2) \right) dx dy$$

$$\frac{\partial L}{\partial u} - \frac{\partial}{\partial x} \frac{\partial L}{\partial u_x} - \frac{\partial}{\partial y} \frac{\partial L}{\partial u_y} = 0$$

$$\frac{\partial L}{\partial v} - \frac{\partial}{\partial x} \frac{\partial L}{\partial v_x} - \frac{\partial}{\partial y} \frac{\partial L}{\partial v_y} = 0$$

where L is the integrand of the energy function

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Horn-Schunck algorithm

$$I_x(I_x u + I_y v + I_t) - \alpha^2 \Delta u = 0$$

$$I_y(I_x u + I_y v + I_t) - \alpha^2 \Delta v = 0$$

where $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace operator

$$\Delta u(x, y) = \bar{u}(x, y) - u(x, y)$$

$$(I_x^2 + \alpha^2) u + I_x I_y v = \alpha^2 \bar{u} - I_x I_t$$

$$I_x I_y u + (I_y^2 + \alpha^2) v = \alpha^2 \bar{v} - I_y I_t$$

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Horn-Schunck algorithm

- Iterative scheme

$$u^{k+1} = \bar{u}^k - \frac{I_x(I_x \bar{u}^k + I_y \bar{v}^k + I_t)}{\alpha^2 + I_x^2 + I_y^2}$$

$$v^{k+1} = \bar{v}^k - \frac{I_y(I_x \bar{u}^k + I_y \bar{v}^k + I_t)}{\alpha^2 + I_x^2 + I_y^2}$$

- Yields high density flow
- Fill in missing information in the homogenous regions
- More sensitive to noise than local methods

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Background Subtraction

- Uses a reference background image for comparison purposes.
- Current image (containing target object) is compared to reference image pixel by pixel.
- Places where there are differences are detected and classified as moving objects.

Motivation: simple difference of two images shows moving objects

a. Original scene

b. Same scene later

Subtraction of scene a from scene b

Subtracted image with threshold of 100

Static Scene Object Detection and Tracking

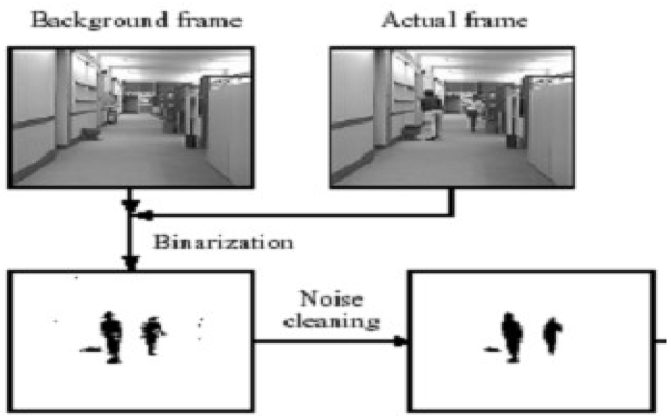
- Model the background and subtract to obtain object mask
- Filter to remove noise
- Group adjacent pixels to obtain objects
- Track objects between frames to develop trajectories

Approaches to Background Modeling

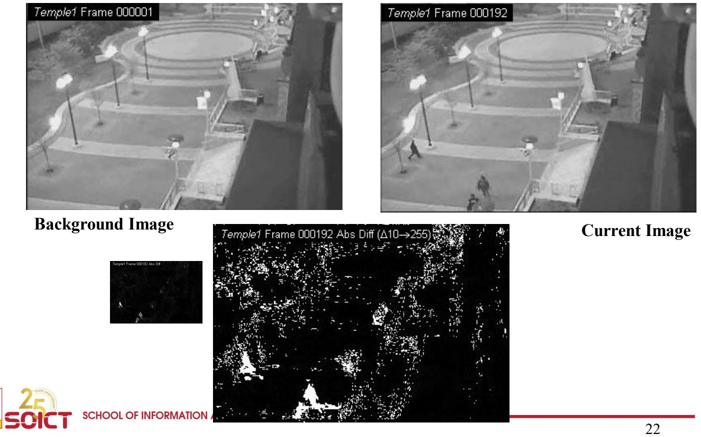
- Background Subtraction
- Statistical Methods
  (e.g., Gaussian Mixture Model, Stauffer and Grimson 2000)

Background Subtraction:

1. Construct a background image B as average of few images
2. For each actual frame I, classify individual pixels as foreground if |B-I| > T (threshold)
3. Clean noisy pixels



Background Subtraction



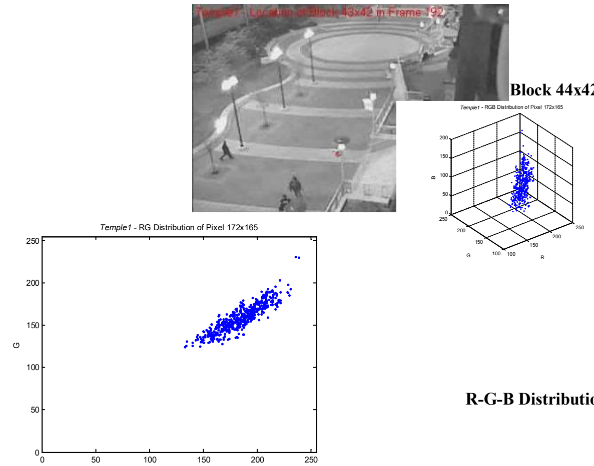
Background Image

Current Image

Statistical Methods

- Pixel statistics: average and standard deviation of color and gray level values
- Gaussian Mixture Model
  - Model the color values of a particular pixel as a mixture of Gaussians
  - Multiple adaptive Gaussians are necessary to cope with acquisition noise, lighting changes, etc.
  - Pixel values that do not fit the background distributions (Mahalanobis distance) are considered foreground

Gaussian Mixture Model



Block 44x42 Pixel 172x165

R-G-B Distribution

R-G Distribution

Detection of Moving Objects Based on Local Variation

For each block location (x,y) in the video plane

- Consider texture vectors in a symmetric window *[t-W, t+W]* at time *t*
- Compute the covariance matrix
- ***Motion measure*** is defined as the largest eigenvalue of the covariance matrix

Dynamic Distribution Learning and Outlier Detection

(1) $$\frac{f(t) - mean(t-1)}{std(t-1)} > C_1$$ **Detect Outlier**

(2) $$\frac{f(t) - mean(t-1)}{std(t-1)} < C_2$$ **Switch to a nominal state**

(3) $$mean(t) = u \cdot mean(t-1) + (1-u) \cdot f(t)$$

(4) $$std(t) = \sqrt{\sigma^2(t)}$$

Update the estimates of mean and standard deviation only when the outliers are not detected

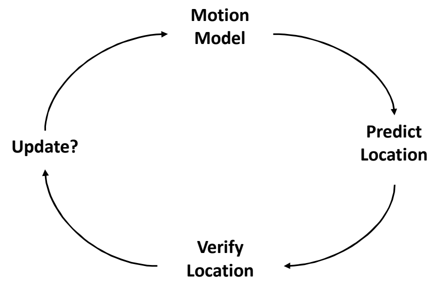
(5) $$\sigma^2(t) = u \cdot \sigma^2(t-1) + (1-u) \cdot (f(t) - mean(t-1))^2$$

Moving object tracking

- Model of object motion
- Kalman filter
- Mean Shift

Model based tracking

- Mathematical model of objects' motions:
  - position, velocity (speed, direction), acceleration
- Can predict objects' positions



Simple Motion Model

- Newton's laws

$$s(t) = s_0 + ut + \frac{1}{2} a t^2$$

- s = position
- u = velocity
- a = acceleration
  - all vector quantities
  - measured in image co-ordinates

Prediction

- Can predict position at time t knowing
  - Position
  - Velocity
  - Acceleration
- At t=0

Uncertainty

- If some error in a - Δa or u - Δu
- Then error in predicted position - Δs

$$\Delta s(t) = s_0 + \Delta u t + \frac{1}{2} \Delta a t^2$$

Verification

- Is the object at the predicted location?
  - Matching
    - How to decide if object is found
  - Search area
    - Where to look for object

Object Matching

- Compare
  - A small bitmap derived from the object vs.
  - Small regions of the image
- Matching?
  - Measure differences

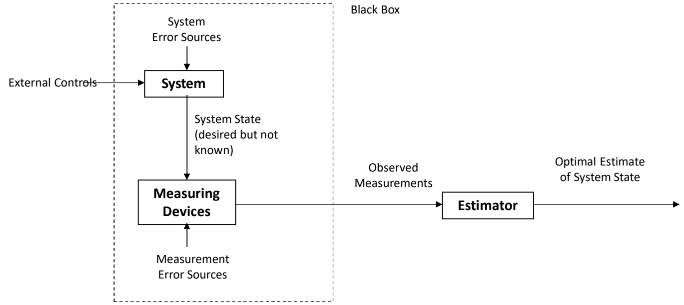
Search Area

- Uncertainty in knowledge of model parameters
  - Limited accuracy of measurement
  - Values might change between measurements
- Define an area in which object could be
  - Centred on predicted location, s ± Δs

Update the Model

- Is the object at the predicted location?
- Yes
  - No change to model
- No
  - Model needs updating
  - Kalman filter is a solution
    - Mathematically rigorous methods of using uncertain measurements to update a model

Kalman Filter



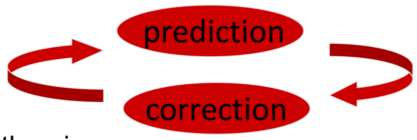
- Problem formulation
  - System state cannot be measured directly
  - Need to estimate “optimally” from measurementsh

Kalman Filter

- Recursive data processing algorithm
- Generates optimal estimate of desired quantities given the set of measurements
- Optimal?
  - For linear system and white Gaussian errors, Kalman filter is “best” estimate based on all previous measurements
  - For non-linear system optimality is ‘qualified’
- Recursive?
  - Doesn’t need to store all previous measurements and reprocess all data each time step

Kalman filter

- Matrix description of system state, model and measurement
- Progressive method



- Proper dealing with noise

Kalman filter

- Advantages of using KF in particle tracking
- Progressive method
  - No large matrices has to be inverted
- Proper dealing with system noise
- Track finding and track fitting
- Detection of outliers
- Merging track from different segments

Kalman filter assumptions

- Linear system
  - System parameters are linear function of parameters at some previous time
  - Measurements are linear function of parameters
- White Gaussian noise
  - White: uncorrelated in time
  - Gaussian: noise amplitude

$\Rightarrow$ KF is the optimal filter

Kalman filter

- Relates
  - Measurements **y**[k]
    - e.g. positions
  - System state **x**[k]
    - Position, velocity of object, etc
  - Observation matrix **H**[k]
    - Relates system state to measurements
  - Evolution matrix **A**[k]
    - Relates state of system between epochs
  - Measurement noise **n**[k]
  - Process noise **v**[k]

Mathematically

How do observations relate to model?

$$\mathbf{y}[k] = \mathbf{H}[k]\mathbf{x}[k] + \mathbf{n}[k]$$

$$\begin{bmatrix} x[k] \\ y[k] \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x[k] \\ \dot{x}[k] \\ y[k] \\ \dot{y}[k] \end{bmatrix} + \mathbf{n}[k]$$

Prediction of System State

- Relates system states at epochs k and k+1

$$\hat{\mathbf{x}}[k+1 \mid k] = \mathbf{A}[k]\mathbf{x}[k \mid k] + \mathbf{v}[k]$$

$$\begin{bmatrix} \hat{x}[k+1 \mid k] \\ \hat{\dot{x}}[k+1 \mid k] \\ \hat{y}[k+1 \mid k] \\ \hat{\dot{y}}[k+1 \mid k] \end{bmatrix} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x[k \mid k] \\ \dot{x}[k \mid k] \\ y[k \mid k] \\ \dot{y}[k \mid k] \end{bmatrix} + \mathbf{v}[k]$$

Prediction of Observation

- From predicted system state, estimate what observation should occur:

$$\hat{\mathbf{y}}[k+1 \mid k] = \mathbf{H}[k]\hat{\mathbf{x}}[k+1 \mid k] + \mathbf{n}[k]$$

$$\begin{bmatrix} \hat{x}[k+1 \mid k] \\ \hat{y}[k+1 \mid k] \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}[k+1 \mid k] \\ \hat{\dot{x}}[k+1 \mid k] \\ \hat{y}[k+1 \mid k] \\ \hat{\dot{y}}[k+1 \mid k] \end{bmatrix} + \mathbf{n}[k]$$

Updating the Model

- Predict/estimate a measurement $\hat{\mathbf{y}}[k+1|k]$
- Make a measurement $\mathbf{y}[k+1]$
- Predict state of model $\hat{\mathbf{x}}[k+1|k]$
- How does the new measurement contribute to updating the model?

Updating the Model

$$\hat{\mathbf{x}}[k+1|k+1] = \hat{\mathbf{x}}[k+1|k] + \mathbf{G}[k+1]\Delta\mathbf{y}[k+1|k]$$
$$\Delta\mathbf{y}[k+1|k] = \mathbf{y}[k+1] - \hat{\mathbf{y}}[k+1|k]$$

- G is Kalman Gain
  - Derived from A, H, v, n.

$$\mathbf{G} = \mathbf{C}[k|k]\mathbf{H}^T\left(\mathbf{H}\mathbf{C}[k|k]\mathbf{H}^T + \mathbf{n}\right)^{-1}$$
$$\mathbf{C}[k+1|k] = \mathbf{C}[k|k] - \mathbf{G}\mathbf{H}\mathbf{C}[k|k]$$
$$\mathbf{C} = \text{system covariance}$$

Example

- Tracking two corners of a minimum bounding box
- Matching using colour
- Image differencing to locate target

Condensation Tracking

- So far considered single motions
- What if movements change?
  - Bouncing ball
  - Human movements
- Use multiple models
  - plus a model selection mechanism

Selection and Tracking

- Occur simultaneously
- Maintain
  - A distribution of likely object positions plus weights
- Predict
  - Select N samples, predict locations
- Verify
  - Match predicted locations against image
  - Update distributions

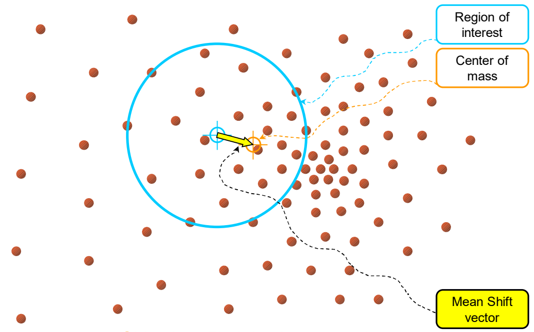
Tracking Using Hidden Markov Models

- Hidden Markov model describes
  - States occupied by a system
  - Possible transitions between states
  - Probabilities of transitions
  - How to recognise a transition

Mean-shift

- The mean-shift algorithm is an efficient approach to tracking objects whose appearance is defined by histograms. (not limited to only color)
- Motivation – to track non-rigid objects, (like a walking person), it is hard to specify an explicit 2D parametric motion model.
- Appearances of non-rigid objects can sometimes be modeled with color distributions

Intuitive Description



Objective : Find the densest region

Distribution of identical billiard balls

Stolen from: *www.wisdom.weizmann.ac.il/~deniss/vision_spring04/files/**mean_shift**/**mean_shift**.ppt*

Mean Shift Vector

- Given:

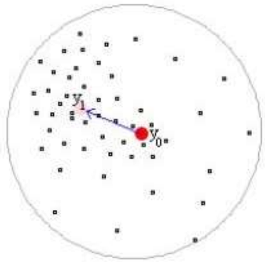
Data points and approximate location of the mean of this data:

- Task:

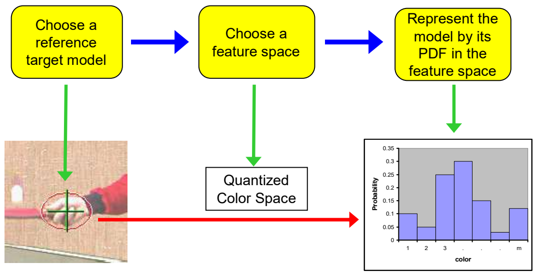
Estimate the exact location of the mean of the data by determining the shift vector from the initial mean.

Mean Shift Vector

$$M_h(\mathbf{y}) = \left[\frac{1}{n_x}\sum_{i=1}^{n_x}\mathbf{x}_i\right] - \mathbf{y}_0$$

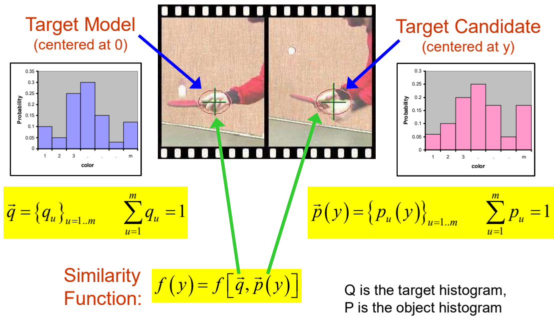


Mean-Shift Object Tracking
Target Representation



Stolen from: www.cs.wustl.edu/~pless/559/lectures/lecture22_tracking.ppt

Mean-Shift Object Tracking
PDF Representation

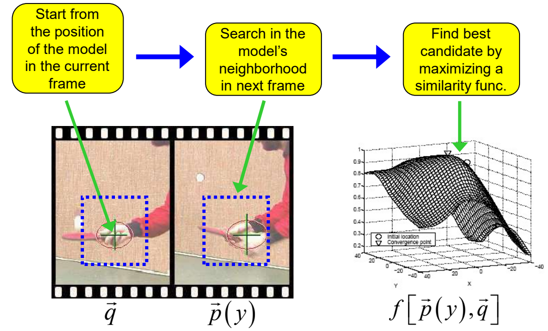


Stolen from: www.cs.wustl.edu/~pless/559/lectures/lecture22_tracking.ppt

Q is the target histogram,
P is the object histogram
(depends on location y)

Mean-Shift Object Tracking

Target Localization Algorithm



Stolen from: www.cs.wustl.edu/~pless/559/lectures/lecture22_tracking.ppt

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Mean Shift

- Mean-Shift in tracking task:
  - ➢ track the motion of a cluster of interesting features.
- 1. choose the feature distribution to represent an object (e.g., color + texture),
- 2. start the mean-shift window over the feature distribution generated by the object
- 3. finally compute the chosen feature distribution over the next video frame
  - Starting from the current window location, the mean-shift algorithm will find the new peak or mode of the feature distribution, which (presumably) is centered over the object that produced the color and texture in the first place.
  - In this way, the mean-shift window tracks the movement of the object frame by frame.

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