

Parallelized MP2 calculation

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1 Background

Solving the Schrodinger equation for systems with more than one electron is a formidable task due to the electron-electron interaction. To obtain a rough estimation for the ground state energy, the electron-electron interaction term is often treated using a mean-field approach with a single electronic configuration (Hartree-Fock calculation). There are many approaches, such as CCSD, CI, and MP2, to calculate the remaining electron correlation starting from the mean-field solution. Each method has its own advantage, depending on the type of interactions (static vs dynamic). In computational chemistry, we want to calculate the electronic ground state energy of molecules with high accuracy to model the quantum mechanical behavior of molecules. This would aid experimental research in understanding reaction mechanisms and the design of molecules. Furthermore, the ground state energy calculation can serve as a metric/input to other data-driven methods (e.g: using machine learning to predict properties of molecules).

In this project, we aim to parallelize the calculation for MP2 energy correction starting from the mean-field solution. MP2 (Møller–Plesset) energy is derived from perturbation theory in quantum mechanics. Given the orbital energy and the two-electron integrals in the Molecular Orbitals basis, we have the expression

$$E_{MP2} = \sum_{ij} \sum_{ab} \frac{(ia|jb) [2 (ia|jb) - (ib|ja)]}{\epsilon_i + \epsilon_j - \epsilon_a - \epsilon_b} \quad (1)$$

where the electron correlation can be calculated using auxiliary basis functions to reduce the run time requirement to transform from Atomic Orbitals basis to Molecular Orbitals basis.

$$(ia|jb)^{RI} = \sum_P B_{ia}^P B_{jb}^P \quad (2)$$

2 Algorithms and Code Parallelization

The workflow for our calculation is presented in Figure 1. We will search for interesting molecules from public databases, such as CCDC. We then use QChem, a computational chemistry software, to perform the Hartree Fock calculation and get orbital energies (ϵ_p) with the corresponding B matrices. The implementation is given in Algo 1, which consists of multiple loops to do matrix multiplication. The parameters that will determine the scale of the problem is **number of occupied orbitals (N)**, **number of virtual orbitals (M)**, and **number of auxiliary basis (P)**. In a typical calculation routine, P is 4-5 times the total number of orbitals ($N + M$).

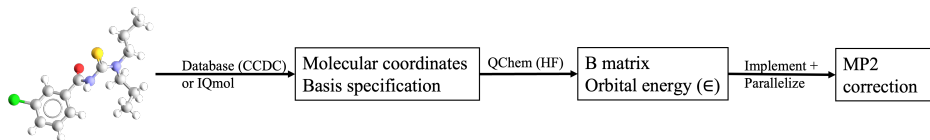


Figure 1: Workflow to obtain the MP2 energy

Code Parallelization

- We are planning on calculating the MP2 electron correlation energy on large molecular system. The number of occupied orbitals is about 200, the number of virtual orbitals is about 1000, and the number of auxiliary basis is 6000. The memory requirement is $8 \times 200 \times 1000 \times 6000 \times \frac{1}{10^9} = 9.6$ Gb.
- We are considering doing both shared memory models (OpenMP) and distributed memory models (MPI). The NUMANode size on a node on the academic cluster is 63 Gb, and we have two per node. Thus, in case we want to go with much larger basis set and many more atoms in the molecule, which would requires about 150 Gb RAM or more, then we would request multiple nodes and use MPI. We will use the single instruction multiple data (SIMD) model for the calculation since the for loop can be divided into parallel tasks.
- We plan to do calculation on multiple large molecules (from 5 to 10) and compare them to previous values. We dont think that the problem would not depend on a high volume of I/O output.

Validation, Verification

We will first do the calculation for small molecules (H₂O, CO₂, CH₄) using both sequential and parallel code with chemical accuracy (error is less than 10^{-5}). This would verify that the parallel code is working properly. I can also calculate the MP2 energy for molecules in this paper and this paper.

Algorithm 1: RI-MP2 Implementation

Data: Orbital energy(ϵ), B matrix
Result: MP2 correlation energy
 $E_{MP2} \leftarrow 0;$
for $i, j < N$ **do**
 for $N \leq a, b < N+M$ **do**
 for $P < AUX_BASIS$ **do**
 $(ia|jb) \leftarrow (ia|jb) + B_{ia}^P B_{jb}^P;$
 end
 end
end
for $i, j < N$ **do**
 for $N \leq a, b < N+M$ **do**
 $\Delta_{ij}^{ab} = \epsilon_i + \epsilon_j - \epsilon_a - \epsilon_b;$
 $E_{MP2} = E_{MP2} + \Delta_{ij}^{ab} \times [(ia|jb) [2 (ia|jb) - (ib|ja)]];$
 end
end

References