CS336

Homework Assignment ♯7 Solutions

1. 6.2, 4.
   A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.
   a) How many balls must she select to be sure of having at least 3 balls of the same color?
   The colors are the pigeonholes, and the balls are the pigeons. We want to know the minimum value N such that $\lceil \frac{N}{2} \rceil = 3$. The smallest such N is 5.
   b) How many balls must she select to be sure of having at least three blue balls?
   In the worst case, she may choose all 10 red first, and then 3 blue balls. So she must select 13 balls.

2. 6.2, 14
   a) Show that if seven integers are selected from the first 10 positive integers there must be at least two pairs of these integers with the sum 11.
   Let the pigeonholes be the sets $\{1, 10\}, \{2, 9\}, \{3, 8\}, \{4, 7\}, \{5, 6\}$. So there are 5 pigeonholes. Let the seven pigeons be the sever integers that are chosen from $\{1, 2, ..., 10\}$, and place the pigeon/integer in the pigeonhole corresponding to the set that contains the integer. E.g., if 3 is chosen, it goes in the pigeonhole $\{3, 8\}$. Since $7 > 5$, there are 2 pigeons in one of the pigeonholes, and so 2 values that sum to 11 were chosen.
   b) Is the conclusion in (a) true if six integers are selected rather than 7?
   Yes, since $6 > 5$.

3. 6.2, 36
   There are 5 pigeonholes. For each of the six computers, put an x in pigeonhole i if that computer is connected to i of the other computers in the system. Since $6 > 5$, some pigeonhole must contain 2 x's, and so at least 2 computers are connected to the same number of computers.

4. 6.3, 22
   How many permutations of ABCDEFGH contain
   a) the string ED? Think of ED as a single symbol. The number of arrangements of A, B, C, ED, F, G, H is 7!.
   b) the string CDE? The number of arrangements of A, B, CDE, F, G, H is 6!
   c) the strings BA and FGH? The number of arrangements of BA, C, D, E, FGH is 5!
   d) the strings AB, DE, and GH? The number of arrangements of AB, DE, C, F, GH is 5!
   e) the strings CAB and BED? The number of arrangements of CABED, F, G, H is 4!
   f) the strings BCA and ABF? There are 0 permutations of these letters that contain both BCA and ABF, since 'A' comes before 'B' in ABF, and after 'B' in BCA.

5. 6.3, 24
   How many ways are there for 10 women and six men to stand in a line so that no two

men stand next to each other?
There are 10! ways to position the women. There are then 11 possible positions for the men. So using the product rule: $10!P(11,6)$.

6. 6.3, 26
Thirteen people on a softball team show up for a game.
a) How many ways are there to choose 10 players to take the field?
From the wording, I assume position assignment is not being considered. So $C(13,10)$.
b) How many ways are there to assign 10 positions by selecting players from the 13 who showed up?
There are 13 ways to choosen position 1, 12 ways to choose position 2, etc. So $P(13,10)$.
c) Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?
There is only one way to choose a team of all men, so $C(13,10)-1$.

7. 6.3, 32
How many strings of six lowercase letters from the English alphabet contain
a) the letter a? There are six ways to choose a position for the required 'a', and then 26^5 ways to choose the remaining letters. So $6*26^5$.
b) the letters a and b? 6 ways to choose the a's position, 5 ways to choosen the b's position, and 26^4 ways to choose the remaining letters. So: $6*5*26^4$.
c) the letters a and b in consecutive positions with a preceding b, with all the letters distinct? So there are 5 ways to choose the position of ba, and then $24*23*22*21$ ways to choose the letters for the remaining positions. So $5*24*23*22*21$.
d) the letters a and b, where a is somewhere to the left of b in the string, with all the letters distinct?
There are $C(6,2)$ ways to pick the positions of the a and b, assuming that the a is always put in the leftmost position. Then there are $24*23*22*21$ ways to pick the remaining letters. So: $C(6,2)*24*23*22*21$

8. 6.4, 8
What is the coefficient of x^8y^9 i $(3x+2y)^{17}$?
The term in the expansion is $C(17,8)(3x)^8(2y)^9$. So the coefficient is $C(17,8)3^82^9$.

9. 6.5, 10
A croissant shop has plain, cherry, chocolate, almond, apple and broccoli. How many ways are there to choose
a) A dozen croissants? Let the 6 croissant flavors be the bins, and distribute 12 balls in these bins. So there are $C(6+12-1,12)=C(17,12)$ ways to choose.
b) three dozen croissants? Same idea: $C(6+36-1,36)$
c) two dozen croissants with at least two of each kind? So once we discount the 12 croissants (2 of each kind) that we must buy, we need to select $24-12$ croissants. Same answer as (a).
d) two dozen croissants with no more than 2 broccoli?

Ways to select with 0 broccoli: $C(24+5-1,4)$
Ways to select with 1 broccoli: $C(23+5-1,4)$
Ways to select with 2 broccoli: $C(22+5-1,4)$
So the total number of ways to choose is the sum of these 3 values.
e) two dozen croissants with at least 5 chocolate and at least 3 almond? So we need to select $24-5-3=16$ croissants. So $C(16+6-1,16)$ ways.

10. 6.5, 14
How many solutions are there to the equation $x_1+x_2+x_3+x_4=17$ in $\mathbb{N}$?
Let the variables correspond to the bins, so we have 4 bins. Now distribute 17 balls in the bins - the number of balls in bin i is the value of x_i in a solution. So there are $C(17+4-1,17)$ solutions.

11. 6.5, 18
How many strings of 20-decimal digits are there that contain two 0s, four 1s, three 2s, one 3, two 4s, three 5s, two 7s, three 9s?
The number of arrangements of 00111122234455577999 is $\frac{20!}{2!4!3!2!3!2!3!}$.