Mathematical Induction

Use induction to prove that $(1+x)^n \geq 1 + x \cdot n$ for all positive integers n, where x can be any real number such that $x \geq -1$.

Let $P(n)$ denote the proposition that $(1+x)^n \geq 1+n \cdot x$, where n is a positive integer and x is a fixed real number such that $x \geq -1$.
Note that $(1+x)^n \geq 0$ when $x \geq -1$.

BASIS STEP: $P(1)$ is true since $1+x \geq 1+x$.

INDUCTIVE STEP: Let us assume $P(n)$, that is $(1+x)^n \geq 1+n \cdot x$ is true for an arbitrary positive integer n. This is our inductive hypothesis.

We have to show that $P(n+1)$, $(1+x)^{n+1} \geq 1+(n+1) \cdot x$ is also true assuming the inductive hypothesis $P(n)$.

Proof:

$(\mathbf{1}+\boldsymbol{x})^{\boldsymbol{n}+\mathbf{1}} = (1+x)\cdot(\mathbf{1}+\boldsymbol{x})^{\boldsymbol{n}} \geq (1+x)\cdot(\mathbf{1}+\boldsymbol{n}\cdot\boldsymbol{x})$ using the inductive hypothesis and the fact that $(1+x)\geq 0$, since $x\geq -1$.

$(1+x)\cdot(1+n\cdot x) = 1+(n+1)x+nx^2 \geq 1+(n+1)x$
since $n\cdot x^2 \geq 0$

By the **Principle of Mathematical Induction** (Basis Step and Inductive Step together) $(1+x)^n \geq 1+x\cdot n$ for all positive integers n, where x is a fixed real number with $x\geq -1$.