Chapter 9 Linear Momentum and Collisions

P9.1 $m = 3.00 \text{ kg}$, $\vec{\mathbf{v}} = \left(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right) \text{ m/s}$

(a) $\vec{\mathbf{p}} = m\vec{\mathbf{v}} = \left(9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}}\right) \text{ kg}\cdot\text{m/s}$

Thus, $\boxed{p_x = 9.00 \text{ kg}\cdot\text{m/s}}$

and $\boxed{p_y = -12.0 \text{ kg}\cdot\text{m/s}}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0 \text{ kg}\cdot\text{m/s}}$

$\theta = \tan^{-1}\left(\dfrac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$

***P9.4** (a) For the system of two blocks $\Delta p = 0$,

or $p_i = p_f$

Therefore, $0 = Mv_m + (3M)(2.00 \text{ m/s})$

Solving gives $v_m = \boxed{-6.00 \text{ m/s}}$ (motion toward the left).

(b) $\dfrac{1}{2}kx^2 = \dfrac{1}{2}Mv_M^2 + \dfrac{1}{2}(3M)v_{3M}^2 = \boxed{8.40 \text{ J}}$

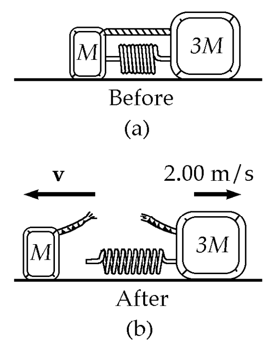

FIG. P9.4

(c) The original energy is in the spring. A force had to be exerted over a distance to compress the spring, transferring energy into it by work. The cord exerts force, but over no distance.

(d) System momentum is conserved with the value zero. The forces on the two blocks are of equal magnitude in opposite directions. Their impulses add to zero. The final momenta of the two blocks are of equal magnitude in opposite directions.

***P9.6** From the impulse-momentum theorem, $F(\Delta t) = \Delta p = mv_f - mv_i$, the average force required to hold onto the child is

$$F = \frac{m\left(v_f - v_i\right)}{(\Delta t)} = \frac{(12 \text{ kg})(0 - 60 \text{ mi/h})}{0.050 \text{ s} - 0}\left(\frac{1 \text{ m/s}}{2.237 \text{ mi/h}}\right) = -6.44 \times 10^3 \text{ N}$$

In trying to hang onto the child, he would have to exert a force of 6.44 kN (over 1400 lb) toward the back of the car, to slow down the child's forward motion. He is not strong enough to exert so large a force. If he were belted in and his arms were firmly tied around the child, the child would exert this size force on him toward the front of the car. A person cannot safely exert or feel a force of this magnitude and a safety device should be used.

P9.7 (a) $I = \int F dt = \text{area under curve}$

$$I = \frac{1}{2}\left(1.50 \times 10^{-3} \text{ s}\right)\left(18\,000 \text{ N}\right) = \boxed{13.5 \text{ N} \cdot \text{s}}$$

(b) $$F = \frac{13.5 \text{ N} \cdot \text{s}}{1.50 \times 10^{-3} \text{ s}} = \boxed{9.00 \text{ kN}}$$

(c) From the graph, we see that $F_{\max} = \boxed{18.0 \text{ kN}}$

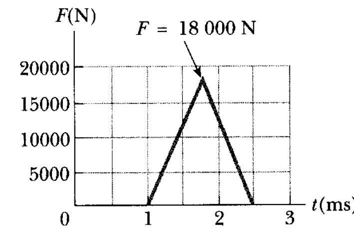

FIG. P9.7

P9.9

$$\Delta \vec{\mathbf{p}} = \vec{\mathbf{F}} \Delta t$$

$$\Delta p_y = m\left(v_{fy} - v_{iy}\right) = m\left(v \cos 60.0°\right) - mv \cos 60.0° = 0$$

$$\Delta p_x = m\left(-v \sin 60.0° - v \sin 60.0°\right) = -2mv \sin 60.0°$$

$$= -2\left(3.00 \text{ kg}\right)\left(10.0 \text{ m/s}\right)\left(0.866\right)$$

$$= -52.0 \text{ kg} \cdot \text{m/s}$$

$$F_{\text{avg}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0 \text{ kg} \cdot \text{m/s}}{0.200 \text{ s}} = \boxed{-260 \text{ N}}$$

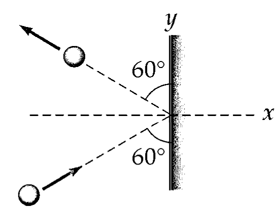

FIG. P9.9

P9.10 Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse, $\vec{\mathbf{I}} = \Delta \vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = (0.060\,0)(40.0)\hat{\mathbf{i}} - (0.060\,0)(50.0)\left(-\hat{\mathbf{i}}\right) = \boxed{5.40\hat{\mathbf{i}} \text{ N} \cdot \text{s}}$

(b) Work $= K_f - K_i = \frac{1}{2}(0.060\,0)\left[(40.0)^2 - (50.0)^2\right] = \boxed{-27.0 \text{ J}}$

P9.12 A graph of the expression for force shows a parabola opening down, with the value zero at the beginning and end of the 0.8 s interval.

$$I = \int_0^{0.8\text{s}} F dt = \int_0^{0.8\text{s}} \left(9200\ t \text{ N/ s } - 11500\ t^2 \text{ N/ s}^2\right) dt$$

$$= \left[(9200 \text{ N/ s})t^2/\ 2 - \left(11500 \text{ N/ s}^2\right)t^3/\ 3\right]_0^{0.8\text{s}}$$

$$= (9200 \text{ N/ s})(0.8 \text{ s})^2/\ 2 - \left(11500 \text{ N/ s}^2\right)(0.8 \text{ s})^3/\ 3$$

$$= 2944 \text{ N} \cdot \text{s} - 1963 \text{ N} \cdot \text{s} = 981 \text{ N} \cdot \text{s}$$

The athlete imparts downward impulse to the platform, so the platform imparts 981 N · s of upward impulse to her.

(b) We could find her impact speed as a free-fall calculation, but we choose to write it as a conservation-of energy calculation: $mgy_{top} = (1/2)mv_{impact}^2$

$$v_{impact} = (2\ g\ y_{top})^{1/2} = [2(9.8 \text{ m/s}^2)0.6 \text{ m}]^{1/2} = \boxed{3.43 \text{ m/s down}}$$

(c) Gravity, as well as the platform, imparts impulse to her during the interaction with the platform.

$$mv_i + I_{platform} + mgt = mv_f$$

$$(65 \text{ kg})(-3.43 \text{ m/s}) + 981 \text{ N·s} - (65 \text{ kg})(9.8 \text{ m/s}^2)(0.8 \text{ s}) = 65 \text{ kg } v_f$$

$-223\ \text{N·s} + 981\ \text{N·s} - 510\ \text{N·s} = 65\ \text{kg}\ v_f$ $\qquad v_f = 249\ \text{N·s}/65\ \text{kg} = \boxed{3.83\ \text{m/s up}}$

Note that the athlete is putting a lot of effort into jumping and does not exert any force "on herself." The usefulness of the force platform is to measure her effort by showing the force she exerts on the floor.

(d) Again energy is conserved in upward flight. $(1/2)mv_{takeoff}^2 = mgy_{top}$

$y_{top} = v_{takeoff}^2/2g = (3.83\ \text{m/s})^2/2(9.8\ \text{m/s}^2) = \boxed{0.748\ \text{m}}$

P9.18 Energy is conserved for the bob-Earth system between bottom and top of swing. At the top the stiff rod is in compression and the bob nearly at rest.

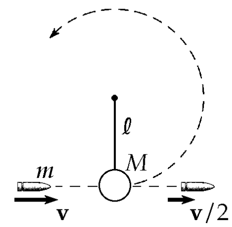

FIG. P9.18

$$K_i + U_i = K_f + U_f: \qquad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$$

$$v_b^2 = g4\ell \text{ so } v_b = 2\sqrt{g\ell}$$

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \qquad \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

P9.19 First we find v_1, the speed of m_1 at B before collision.

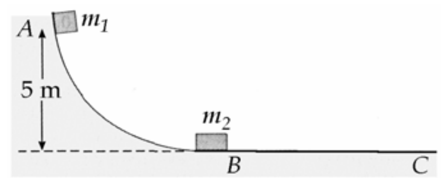

FIG. P9.19

$$\frac{1}{2}m_1v_1^2 = m_1gh$$

$$v_1 = \sqrt{2(9.80)(5.00)} = 9.90\ \text{m/s}$$

Now we use the text's analysis of one-dimensional elastic collisions to find v_{1f}, the speed of m_1 at B just after collision.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_1 = -\frac{1}{3}(9.90)\ \text{m/s} = -3.30\ \text{m/s}$$

Now the 5-kg block bounces back up to its highest point after collision according to

$$m_1gh_{\max} = \frac{1}{2}m_1(-3.30)^2 \qquad h_{\max} = \frac{(-3.30\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{0.556\ \text{m}}$$

***P9.20** (a) We assume that energy is conserved in the fall of the basketball and the tennis ball. Each reaches its lowest point with a speed given by

$$\left(K + U_g\right)_{\text{release}} = \left(K + U_g\right)_{\text{bottom}}$$

$$0 + mgy_i = \frac{1}{2}mv_b^2 + 0$$

$$v_b = \sqrt{2gy_i} = \sqrt{2\left(9.8\ \text{m/s}^2\right)(1.20\ \text{m})} = \boxed{4.85\ \text{m/s}}$$

(b) The two balls exert no forces on each other as they move down. They collide with each other after the basketball has its velocity reversed by the floor. We choose upward as positive. Momentum conservation:

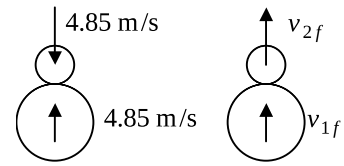

FIG. P9.20

$$(57\text{ g})(-4.85\text{ m/s})+(590\text{ g})(4.85\text{ m/s})=(57\text{ g})v_{2f}+(590\text{ g})v_{1f}$$

To describe the elastic character of the collision, we use the relative velocity equation

$$4.85\text{ m/s}-(-4.85\text{ m/s})=v_{2f}-v_{1f}$$

we solve by substitution

$$v_{1f}=v_{2f}-9.70\text{ m/s s}$$

$$2\,580\text{ gm/s}=(57\text{ g})v_{2f}+(590\text{ g})(v_{2f}-9.70\text{ m/s})$$

$$=(57\text{ g})v_{2f}+(590\text{ g})v_{2f}-5\,720\text{ gm/s}$$

$$v_{2f}=\frac{8\,310\text{ m/s}}{647}=12.8\text{ m/s}$$

Now the tennis ball-Earth system keeps constant energy as the ball rises:

$$\frac{1}{2}(57\text{ g})(12.8\text{ m/s})^2=(57\text{ g})(9.8\text{ m/s}^2)y_f$$

$$y_f=\frac{165\text{ m}^2/\text{s}^2}{2(9.8\text{ m/s}^2)}=\boxed{8.41\text{ m}}$$

P9.21 (a), (b) Let v_g and v_p be the x-components of velocity of the girl and the plank relative to the ice surface. Then we may say that v_g-v_p is the velocity of the girl relative to the plank, so that

$$v_g-v_p=1.50 \qquad (1)$$

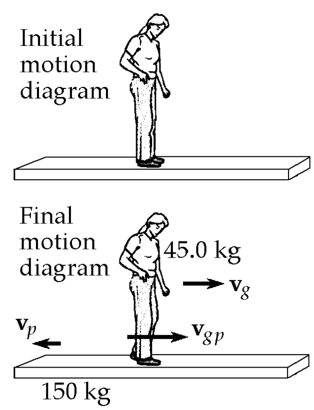

FIG. P9.21

But also we must have $m_gv_g+m_pv_p=0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0v_g+150v_p=0\text{, or }v_g=-3.33v_p$$

Putting this into the equation (1) above gives

$$-3.33v_p-v_p=1.50\text{ or }v_p=\boxed{-0.346\,\hat{\mathbf{i}}\text{ m/s}}\quad\text{(answer b)}$$

$$\text{Then }v_g=-3.33(-0.346)=\boxed{1.15\,\hat{\mathbf{i}}\text{ m/s}}\quad\text{(answer a)}$$

P9.24 (a) Using conservation of momentum, $\left(\sum \vec{\mathbf{p}}\right)_{\text{before}} = \left(\sum \vec{\mathbf{p}}\right)_{\text{after}}$, gives

$$(4.0\text{ kg})(5.0\text{ m/s})+(10\text{ kg})(3.0\text{ m/s})+(3.0\text{ kg})(-4.0\text{ m/s})=[(4.0+10+3.0)\text{ kg}]v$$

Therefore, $v=+2.24\text{ m/s}$, or $\boxed{2.24\text{ m/s toward the right}}$

(b) $\boxed{\text{No.}}$ For example, if the 10-kg and 3.0-kg mass were to stick together first, they would move with a speed given by solving

$$(13\text{ kg})v_1=(10\text{ kg})(3.0\text{ m/s})+(3.0\text{ kg})(-4.0\text{ m/s}),\text{ or } v_1=+1.38\text{ m/s}$$

Then when this 13 kg combined mass collides with the 4.0 kg mass, we have

$$(17\text{ kg})v=(13\text{ kg})(1.38\text{ m/s})+(4.0\text{ kg})(5.0\text{ m/s}),\text{ and } v=+2.24\text{ m/s}$$

just as in part (a). $\boxed{\text{Coupling order makes no difference to the final velocity.}}$

***P9.26** (a) Over a very short time interval, outside forces have no time to impart significant impulse—thus the interaction is a collision. The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction—thus the collision is completely inelastic.

(b) First, we conserve momentum for the system of two football players in the *x* direction (the direction of travel of the fullback).

$$(90.0\text{ kg})(5.00\text{ m/s})+0=(185\text{ kg})V\cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V\cos\theta=2.43\text{ m/s} \qquad (1)$$

Now consider conservation of momentum of the system in the y direction (the direction of travel of the opponent).

$$(95.0\text{ kg})(3.00\text{ m/s})+0=(185\text{ kg})(V\sin\theta)$$

which gives $V\sin\theta=1.54\text{ m/s}$ (2)

Divide equation (2) by (1) $\tan\theta=\dfrac{1.54}{2.43}=0.633$

From which $\boxed{\theta=32.3°}$

Then, either (1) or (2) gives $V=\boxed{2.88\text{ m/s}}$

(c) $$K_i=\frac{1}{2}(90.0\text{ kg})(5.00\text{ m/s})^2+\frac{1}{2}(95.0\text{ kg})(3.00\text{ m/s})^2=1.55\times10^3\text{ J}$$

$$K_f=\frac{1}{2}(185\text{ kg})(2.88\text{ m/s})^2=7.67\times10^2\text{ J}$$

Thus, the kinetic energy lost is $\boxed{786\text{ J into internal energy}}$.

P9.31 $m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = (m_1 + m_2)\vec{\mathbf{v}}_f$: $3.00(5.00)\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}} = 5.00\vec{\mathbf{v}}$

$$\vec{\mathbf{v}} = \boxed{\left(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}}\right)\text{ m/s}}$$

P9.32 x-component of momentum for the system of the two objects:

$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$: $-mv_i + 3mv_i = 0 + 3mv_{2x}$

y-component of momentum of the system: $0 + 0 = -mv_{1y} + 3mv_{2y}$

by conservation of energy of the system:

$$+\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$$

we have $v_{2x} = \dfrac{2v_i}{3}$

also $v_{1y} = 3v_{2y}$

So the energy equation becomes $4v_i^2 = 9v_{2y}^2 + \dfrac{4v_i^2}{3} + 3v_{2y}^2$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $v_{2y} = \dfrac{\sqrt{2}v_i}{3}$

(a) The object of mass m has final speed $v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$

and the object of mass 3 m moves at $\sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\dfrac{4v_i^2}{9} + \dfrac{2v_i^2}{9}}$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta = \tan^{-1}\left(\dfrac{v_{2y}}{v_{2x}}\right)$ $\theta = \tan^{-1}\left(\dfrac{\sqrt{2}v_i}{3}\dfrac{3}{2v_i}\right) = \boxed{35.3°}$

P9.39 This object can be made by wrapping tape around a light stiff uniform rod.

(a) $M = \displaystyle\int_0^{0.300\text{ m}} \lambda dx = \int_0^{0.300\text{ m}} \left[50.0\text{ g/m} + 20.0x\text{ g/m}^2\right]dx$

$$M = \left[50.0x\text{ g/m} + 10.0x^2\text{ g/m}^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

(b) $x_{\text{CM}} = \dfrac{\displaystyle\int_{\text{all mass}} x\,dm}{M} = \dfrac{1}{M}\displaystyle\int_0^{0.300\text{ m}} \lambda x\,dx = \dfrac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x\text{ g/m} + 20.0x^2\text{ g/m}^2\right]dx$

$$x_{\text{CM}} = \frac{1}{15.9\text{ g}}\left[25.0x^2\text{ g/m} + \frac{20x^3\text{ g/m}^2}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

P9.40 Take the origin at the center of curvature. We have $L = \frac{1}{4} 2\pi r$, $r = \frac{2L}{\pi}$. An incremental bit of the rod at angle θ from the x axis has mass given by $\frac{dm}{r d\theta} = \frac{M}{L}$, $dm = \frac{Mr}{L} d\theta$ where we have used the definition of radian measure. Now

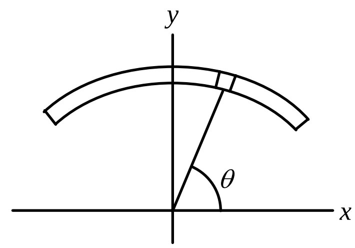

FIG. P9.40

$$y_{\text{CM}} = \frac{1}{M} \int_{\text{all mass}} y\, dm = \frac{1}{M} \int_{\theta=45°}^{135°} r \sin\theta \frac{Mr}{L} d\theta = \frac{r^2}{L} \int_{45°}^{135°} \sin\theta\, d\theta$$

$$= \left(\frac{2L}{\pi}\right)^2 \frac{1}{L} \left(-\cos\theta\right)\Big|_{45°}^{135°} = \frac{4L}{\pi^2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{4\sqrt{2}L}{\pi^2}$$

The top of the bar is above the origin by $r = \frac{2L}{\pi}$, so the center of mass is below the middle of the bar by $\frac{2L}{\pi} - \frac{4\sqrt{2}L}{\pi^2} = \frac{2}{\pi}\left(1 - \frac{2\sqrt{2}}{\pi}\right)L = \boxed{0.063\,5L}$.

P9.51 (a) Thrust $= \left| v_e \frac{dM}{dt} \right|$ Thrust

$$= \left(2.60 \times 10^3 \text{ m/s}\right)\left(1.50 \times 10^4 \text{ kg/s}\right) = \boxed{3.90 \times 10^7 \text{ N}}$$

(b) $\sum F_y = \text{Thrust} - Mg = Ma$: $3.90 \times 10^7 - \left(3.00 \times 10^6\right)(9.80) = \left(3.00 \times 10^6\right) a$

$$a = \boxed{3.20 \text{ m/s}^2}$$

P9.52 (a) From the equation for rocket propulsion in the text,

$$v - 0 = v_e \ln\left(\frac{M_i}{M_f}\right) = -v_e \ln\left(\frac{M_f}{M_i}\right)$$

Now, $M_f = M_i - kt$, so $v = -v_e \ln\left(\frac{M_i - kt}{M_i}\right) = -v_e \ln\left(1 - \frac{k}{M_i} t\right)$

With the definition $T_p \equiv \frac{M_i}{k}$, this becomes

$$v(t) = \boxed{-v_e \ln\left(1 - \frac{t}{T_p}\right)}$$

(b) With $v_e = 1\,500$ m/s, and $T_p = 144$ s, $v = -\left(1\,500 \text{ m/s}\right) \ln\left(1 - \frac{t}{144 \text{ s}}\right)$

$t\,(s)$	$v\,(\text{m/s})$
0	0
20	224
40	488
60	808
80	1220
100	1780
120	2690
132	3730

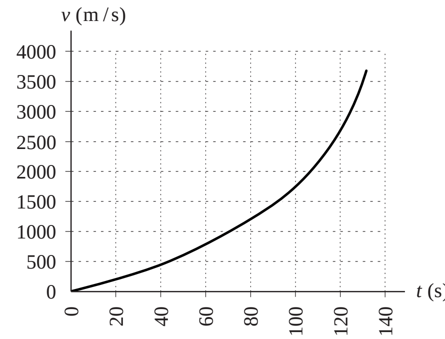

FIG. P9.52(b)

(c) $$a(t)=\frac{dv}{dt}=\frac{d\left[-v_e\ln\left(1-\frac{t}{T_p}\right)\right]}{dt}=-v_e\left(\frac{1}{1-\frac{t}{T_p}}\right)\left(-\frac{1}{T_p}\right)=\left(\frac{v_e}{T_p}\right)\left(\frac{1}{1-\frac{t}{T_p}}\right),\text{ or}$$

$$a(t)=\boxed{\frac{v_e}{T_p-t}}$$

(d) With $v_e=1\,500\text{ m/s}$, and $T_p=144\text{ s}$, $a=\dfrac{1\,500\text{ m/s}}{144\text{ s}-t}$

$t\,(s)$	$a\,(\text{m/s}^2)$
0	10.4
20	12.1
40	14.4
60	17.9
80	23.4
100	34.1
120	62.5
132	125

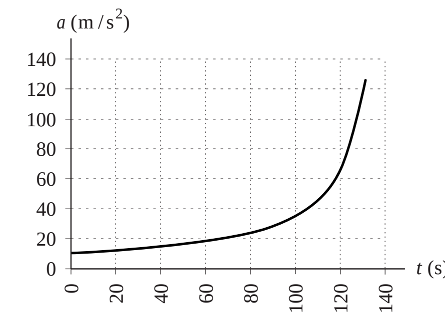

FIG. P9.52(d)

(e) $$x(t)=0+\int_0^t v\,dt=\int_0^t\left[-v_e\ln\left(1-\frac{t}{T_p}\right)\right]dt=v_eT_p\int_0^t\ln\left[1-\frac{t}{T_p}\right]\left(-\frac{dt}{T_p}\right)$$

$$x(t)=v_eT_p\left[\left(1-\frac{t}{T_p}\right)\ln\left(1-\frac{t}{T_p}\right)-\left(1-\frac{t}{T_p}\right)\right]_0^t$$

$$x(t)=\boxed{v_e\left(T_p-t\right)\ln\left(1-\frac{t}{T_p}\right)+v_et}$$

(f) With $v_e=1\,500\text{ m/s}=1.50\text{ km/s}$, and $T_p=144\text{ s}$,

$$x=1.50(144-t)\ln\left(1-\frac{t}{144}\right)+1.50t$$

$t(\text{s})$	$x(\text{km})$
0	0
20	2.19
40	9.23
60	22.1
80	42.2
100	71.7
120	115
132	153

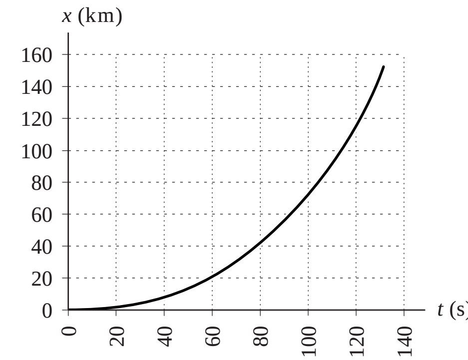

FIG. P9.52(f)

P9.61 (a) Each primate swings down according to

$$mgR = \frac{1}{2}mv_1^2 \qquad MgR = \frac{1}{2}Mv_1^2 \qquad v_1 = \sqrt{2gR}$$

The collision: $-mv_1 + Mv_1 = +(m+M)v_2$

$$v_2 = \frac{M-m}{M+m}v_1$$

Swinging up: $\frac{1}{2}(M+m)v_2^2 = (M+m)gR(1-\cos 35°)$

$$v_2 = \sqrt{2gR(1-\cos 35°)}$$

$$\sqrt{2gR(1-\cos 35°)}(M+m) = (M-m)\sqrt{2gR}$$

$$0.425M + 0.425m = M - m$$

$$1.425m = 0.575M$$

$$\boxed{\frac{m}{M} = 0.403}$$

(b) No change is required if the force is different. The nature of the forces within the system of colliding objects does not affect the total momentum of the system. With strong magnetic attraction, the heavier object will be moving somewhat faster and the lighter object faster still. Their extra kinetic energy will all be immediately converted into extra internal energy when the objects latch together. Momentum conservation guarantees that none of the extra kinetic energy remains after the objects join to make them swing higher.

P9.67 (a) Find the speed when the bullet emerges from the block by using momentum conservation:

$$mv_i = MV_i + mv$$

The block moves a distance of 5.00 cm. Assume for an approximation that the block quickly reaches its maximum velocity, V_i, and the bullet kept going with a constant velocity, v. The block then compresses the spring and stops.

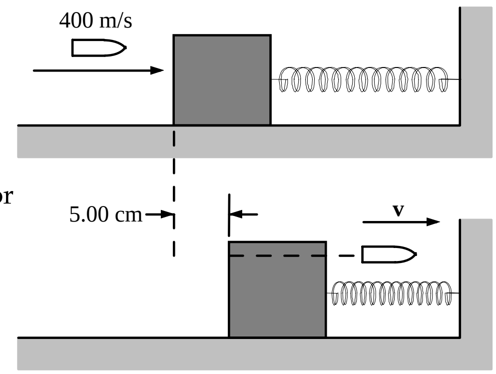

FIG. P9.67

$$\frac{1}{2}MV_i^2 = \frac{1}{2}kx^2$$

$$V_i = \sqrt{\frac{(900\ \text{N/m})(5.00\times10^{-2}\ \text{m})^2}{1.00\ \text{kg}}} = 1.50\ \text{m/s}$$

$$v = \frac{mv_i - MV_i}{m} = \frac{(5.00\times10^{-3}\ \text{kg})(400\ \text{m/s}) - (1.00\ \text{kg})(1.50\ \text{m/s})}{5.00\times10^{-3}\ \text{kg}}$$

$$v = \boxed{100\ \text{m/s}}$$

(b) $$\Delta E = \Delta K + \Delta U = \frac{1}{2}(5.00\times10^{-3}\ \text{kg})(100\ \text{m/s})^2 - \frac{1}{2}(5.00\times10^{-3}\ \text{kg})(400\ \text{m/s})^2 + \frac{1}{2}(900\ \text{N/m})(5.00\times10^{-2}\ \text{m})^2$$

$\Delta E = -374$ J, or there is a mechanical energy loss of $\boxed{374\ \text{J}}$.

P9.69 The force exerted by the table is equal to the change in momentum of each of the links in the chain.

By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}$$

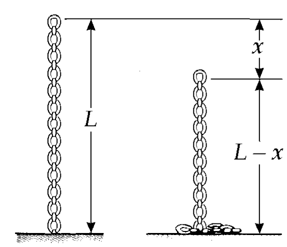

FIG. P9.69

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0$$

Since the mass per unit length is uniform, we can express each link of length dx as having a mass dm:

$$dm = \frac{M}{L}dx$$

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements *dm*.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance *x*, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}$$

The links already on the table have a total length *x*, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}$$

That is, *the total force is three times the weight of the chain on the table at that instant.*