Sets

Power sets, Cartesian Product, Subsets,
Empty sets

Let A={ $\emptyset, \{\emptyset\}, \{0,1\}$}. Let B={2, {0}}.

- Find |A|, the cardinality of A: the elements of A are $\emptyset, \{\emptyset\}, \{0,1\}$, thus |A|=3.

- Similarly, |B|=2

- Find P(A), the set of all subsets of A. Note that the empty set ∅ is a subset of any set A, consequently ∅ ∈ $P(A)$ for any set A. The formal definit$ion\ of\ subset\ is$: $A \subseteq B \leftrightarrow \forall x(x \in A \rightarrow x \in B)$.
  If A is the empty set, hypothesis $x \in A$ is false for all x, thus the conditional $\forall x(x \in A \rightarrow x \in B)$ is true.

- Also, if |A|=n then |P(A)|=2^n.

  P(A)={∅, {∅}, {{∅}}, {{0,1}}, {∅,{∅}}, {∅,{0,1}}, {{∅},{0,1}}, {∅, $\{\emptyset\}, \{0,1\}$}}

- Find $A \times B$ and $|A \times B|$

  $A \times B$={(∅,2), (∅,{0}), ({∅}, 2), ({∅},{0}),({0,1},2), ({0,1},{0})}

  $|A \times B|$=|A|·|B| $= 3 \cdot 2 = 6$.

TRUE or FALSE? Let A=$\{ \emptyset, \{\emptyset\}, \{0,1\}\}$ and B={2, {0}}.

- $\emptyset \in P(A)$? TRUE , since the empty set is always element of the power set of any sets.
- $\emptyset \subseteq P(A)$? TRUE, since the empty set is a subset of any set.
- $\{\emptyset\} \subseteq P(A)$? TRUE, since $\emptyset \in P(A)$.
- $\{\emptyset\} \in P(A)$? TRUE.
- $\emptyset \in A$? TRUE.
- $\{0,1\} \subseteq A$? FALSE, since $0 \notin A$ and $1 \notin A$.
- $\{\{0,1\}\} \subseteq A$? TRUE, since $\{0,1\} \in A$.
- $(\emptyset,\emptyset) \in A \times A$? TRUE.
- $(\{0,1\},2) \in A \times B$? TRUE.
- $\{(\emptyset,2)\} \subseteq A \times B$? TRUE, since $(\emptyset,2) \in A \times B$.
- $\{\emptyset,\{0\}\} \subseteq P(A)$? FALSE, since $\{0\} \notin P(A)$.