Homework 1-1

Due: 11:57pm on Tuesday, October 21, 2014

To understand how points are awarded, read the [Grading Policy] for this assignment.

Tactics Box 1.1 Vector Addition

Learning Goal:

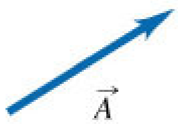
To practice Tactics Box 1.1 Vector Addition.

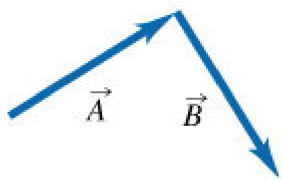
Vector addition obeys rules that are different from those for the addition of two scalar quantities. When you add two vectors, their directions, as well as their magnitudes, must be taken into account. This Tactics Box explains how to add vectors graphically.

TACTICS BOX 1.1 Vector addition

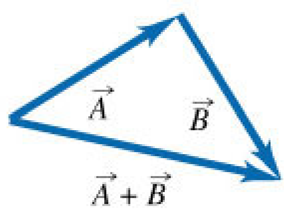
To add $\vec{B}$ to $\vec{A}$, perform these steps:

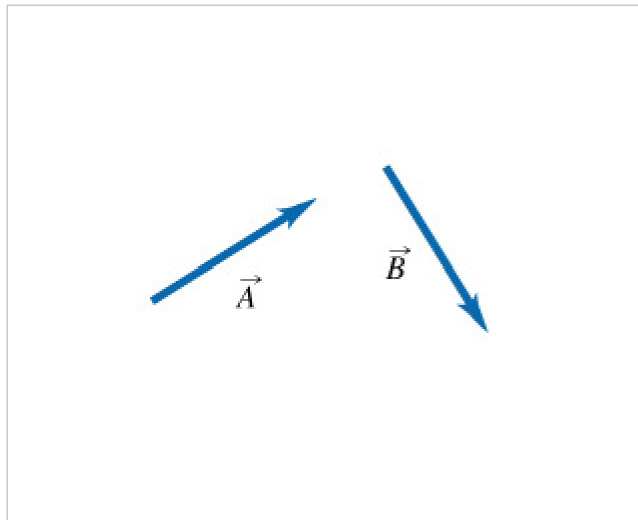
1. Draw $\vec{A}$.





2. Place the tail of $\vec{B}$ at the tip of $\vec{A}$.
3. Draw an arrow from the tail of $\vec{A}$ to the tip of $\vec{B}$. This is vector $\vec{A}+\vec{B}$.





Part A

Create the vector $\vec{R}=\vec{A}+\vec{B}$ by following the steps in the Tactics Box above. When moving vector $\vec{B}$, keep in mind that its direction should remain unchanged.

The location, orientation, and length of your vectors will be graded.

ANSWER:

Correct

Part B

Create the vector $\vec{R} = \vec{C} + \vec{D}$ by following the steps in the Tactics Box above. When moving vector $\vec{D}$, keep in mind that its direction should remain unchanged.

The location, orientation, and length of your vectors will be graded.

ANSWER:

Correct

Tactics Box 1.2 Vector Subtraction

Learning Goal:

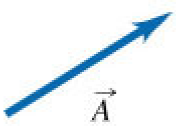
To practice Tactics Box 1.2 Vector Subtraction.

Vector subtraction has some similarities to the subtraction of two scalar quantities. With numbers, subtraction is the same as the addition of a negative number. For example, $5 - 3$ is the same as $5 + (-3)$. Similarly, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$. We can use the rules for vector addition and the fact that $-\vec{B}$ is a vector opposite in direction to $\vec{B}$ to form rules for vector subtraction, as explained in this tactics box.

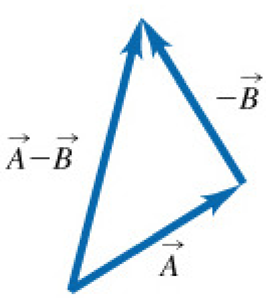
TACTICS BOX 1.2 Vector subtraction

To subtract $\vec{B}$ from $\vec{A}$, perform these steps:

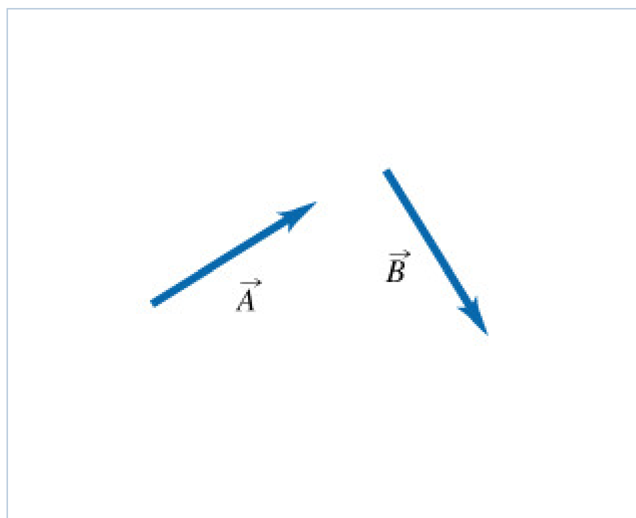
1. Draw $\vec{A}$.



2. Place the tail of $-\vec{B}$ at the tip of $\vec{A}$.
3. Draw an arrow from the tail of $\vec{A}$ to the tip of $-\vec{B}$. This is



vector $\vec{A} - \vec{B}$.



Part A

Find vector $\vec{C} = \vec{A} - \vec{B}$ by following the steps in the tactics box above. When drawing $-\vec{B}$, keep in mind that it has the same magnitude as $\vec{B}$ but opposite direction.

The location, orientation, and length of your vectors will be graded.

ANSWER:

Correct

Part B

Find vector $\vec{F} = \vec{D} - \vec{E}$ by following the steps in the tactics box above. When drawing $-\vec{E}$, keep in mind that it has the same magnitude as $\vec{E}$ but opposite direction.

The location, orientation, and length of your vectors will be graded.

ANSWER:

Correct

PSS 1.1 Motion Diagrams

Learning Goal:

To practice Problem-Solving Strategy 1.1 for motion diagram problems.

A car is traveling with constant velocity along a highway. The driver notices he is late for work, so he stomps down on the gas pedal and the car begins to speed up. The car has just achieved double its initial velocity when the driver spots a police officer behind him and applies the brakes. The car then slows down, coming to rest at a stoplight ahead.

Draw a complete motion diagram for this situation.

PROBLEM-SOLVING STRATEGY 1.1 Motion diagrams

MODEL: Represent the moving object as a particle. Make simplifying assumptions when interpreting the problem statement.

VISUALIZE: A complete motion diagram consists of:

- The position of the object in each frame of the film, shown as a dot. Use five or six dots to make the motion clear but without overcrowding the picture. More complex motions may need more dots.
- The average velocity vectors, found by connecting each dot in the motion diagram to the next with a vector arrow. There is *one* velocity vector linking each set of *two* position dots. Label the row of velocity vectors $\vec{v}$.
- The average acceleration vectors, found using Tactics Box 1.3. There is *one* acceleration vector linking each set of *two* velocity vectors. Each acceleration vector is drawn at the dot between the two velocity vectors it links. Use $\vec{0}$ to indicate a point at which the acceleration is zero. Label the row of acceleration vectors $\vec{a}$.

Model

It is appropriate to use the particle model for the car. You should also make some simplifying assumptions.

Part A

The car's motion can be divided into three different stages: its motion before the driver realizes he's late, its motion after the driver hits the gas (but before he sees the police car), and its motion after the driver sees the police car. Which of the following simplifying assumptions is it reasonable to make in this problem?

A. During *each* of the three different stages of its motion, the car is moving with constant acceleration.
B. During *each* of the three different stages of its motion, the car is moving with constant velocity.
C. The highway is straight (i.e., there are no curves).
D. The highway is level (i.e., there are no hills or valleys).

Enter all the correct answers in alphabetical order without commas. For example, if statements C and D are correct, enter CD.

ANSWER:

ACD

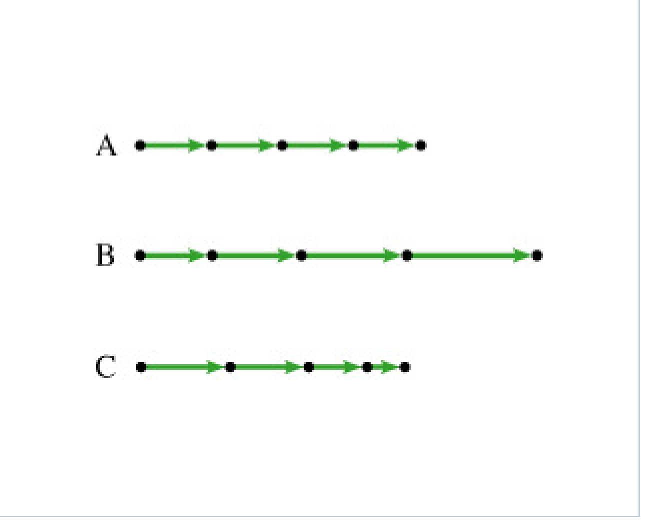
All attempts used; correct answer displayed

In addition to the assumptions listed above, in the rest of this problem assume that the car is moving in a straight line to the right.

Visualize

Part B

In the three diagrams shown to the left, the position of the car at five subsequent instants of time is represented by black dots, and the car's average velocity is represented by green arrows. Which of these diagrams best describes the position and the velocity of the car before the driver notices he is late?



ANSWER:

- ◉ A
- ○ B
- ○ C

Correct

Part C

Which of the diagrams shown to the left best describes the position and the velocity of the car after the driver hits the gas, but before he notices the police officer?

ANSWER:

- ○ A
- ◉ B
- ○ C

Correct

Part D

Which of the diagrams shown to the left best describes the position and the velocity of the car after the driver notices the police officer?

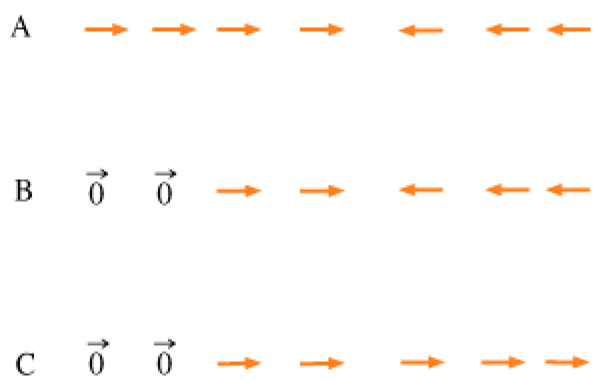
ANSWER:

- ○ A
- ○ B
- ◉ C

Correct

Part E

Which of the diagrams shown below most accurately depicts the average *acceleration* vectors of the car during the events described in the problem introduction?

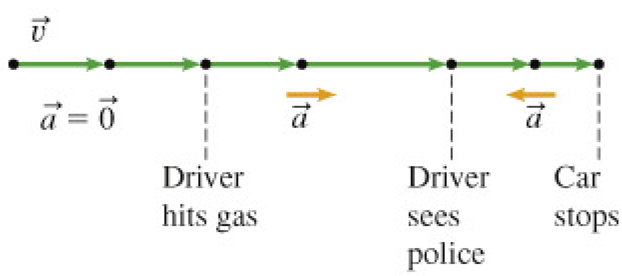


ANSWER:

- A
- B
- C

Correct

You can now draw a complete motion diagram for the situation described in this problem. Your diagram should look like this:



Tactics Box 1.3 Finding the Acceleration Vector

Learning Goal:

To practice Tactics Box 1.3 Finding the Acceleration Vector.

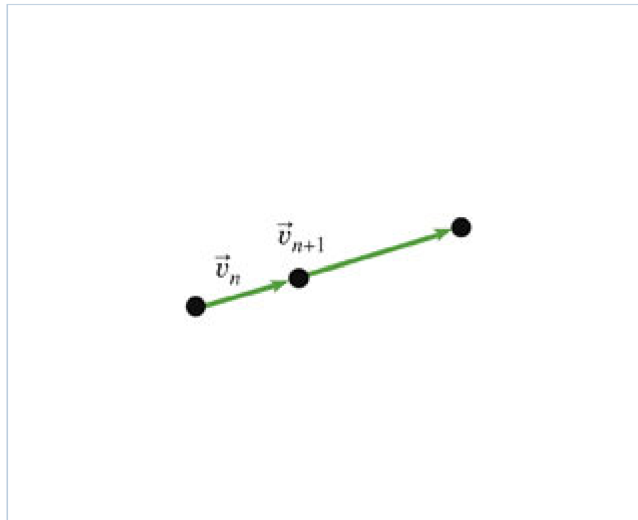
Suppose an object has an initial velocity $\vec{v}_n$ at time t_n and later, at time t_{n+1}, has velocity $\vec{v}_{n+1}$. The fact that the velocity changes tells us the object undergoes an acceleration during the time interval $\Delta t = t_{n+1} - t_n$. From the definition of average acceleration,

$$\vec{a} = \frac{\vec{v}_{n+1} - \vec{v}_n}{t_{n+1} - t_n} = \frac{\Delta \vec{v}}{\Delta t},$$

we see that the acceleration vector points in the same direction as the vector $\Delta\vec{v}$. This vector is the change in the velocity $\Delta\vec{v} = \vec{v}_{n+1} - \vec{v}_n$, so to know which way the acceleration vector points, we have to perform the vector subtraction $\vec{v}_{n+1} - \vec{v}_n$. This Tactics Box shows how to use vector subtraction to find the acceleration vector.

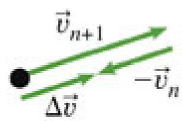
TACTICS BOX 1.3 Finding the acceleration vector

To find the acceleration as the velocity changes from $\vec{v}_n$ to $\vec{v}_{n+1}$:

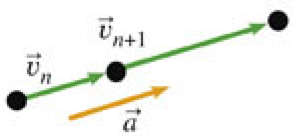


1. Draw the velocity vector $\vec{v}_{n+1}$.

2. Draw $-\vec{v}_n$ at the tip of $\vec{v}_{n+1}$.
3. Draw $\Delta\vec{v} = \vec{v}_{n+1} - \vec{v}_n = \vec{v}_{n+1} + (-\vec{v}_n)$. This is the direction of $\vec{a}$.



4. Return to the original motion diagram. Draw a vector at the middle point in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration at the midpoint between $\vec{v}_n$ and $\vec{v}_{n+1}$.



Part A

Below is a motion diagram for an object that moves along a linear path. The dots represent the position of the object at three subsequent instants, t_1, t_2, and t_3. The vectors $\vec{v}_{21}$ and $\vec{v}_{32}$ show the average velocity of the object for the initial time interval, $\Delta t_{21} = t_2 - t_1$, and the final time interval, $\Delta t_{32} = t_3 - t_2$, respectively. Draw the vector $\overrightarrow{-v}_{21}$ and the acceleration vector $\vec{a}$ representing the change in average velocity of the object during the total time interval $\Delta t = t_3 - t_1$. Assume for this problem that $\Delta t = 1$ s.

The orientation and length of the vectors will be graded. The location of the vectors will not be graded.

Hint 1. How to draw the acceleration vector

First, draw $\overrightarrow{-v}_{21}$. Draw $\overrightarrow{-v}_{21}$ starting at the tip of $\vec{v}_{21}$ and ending at its tail. Then, move $\overrightarrow{-v}_{21}$, with the same orientation, so that its tail is at the tip of $\vec{v}_{32}$. Use the **vector info** button to make sure that the lengths of $\vec{v}_{21}$ and $\overrightarrow{-v}_{21}$ are equal. The acceleration vector, $\vec{a}$, starts at the tail of $\vec{v}_{32}$ and ends at the tip of $\overrightarrow{-v}_{21}$.

ANSWER:

Correct

Part B

Below is another motion diagram for an object that moves along a linear path. The dots represent the position of the object at five subsequent instants, t_1, t_2, t_3, t_4, and t_5. The vectors $\vec{v}_{21}$, $\vec{v}_{32}$, $\vec{v}_{43}$, and $\vec{v}_{54}$ represent the average velocity of the object during the four corresponding time intervals. Draw the velocity vectors $\overrightarrow{-v}_{21}$ and $\overrightarrow{-v}_{43}$ and the acceleration vectors $\vec{a}_{31}$ and $\vec{a}_{53}$ representing the changes in average velocity of the object during the time intervals $\Delta t_{31} = t_3 - t_1$ and $\Delta t_{53} = t_5 - t_3$, respectively. Assume for this problem that $\Delta t_{31} = \Delta t_{53} = 1$ s.

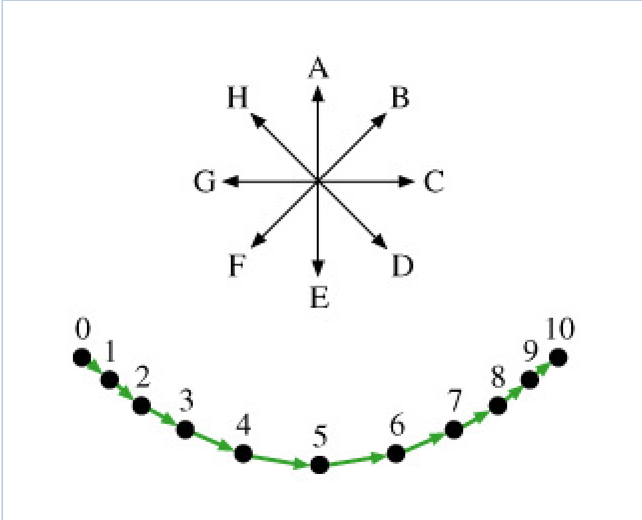
The orientation and length of the vectors will be graded. The location of the vectors will not be graded.

ANSWER:

Correct

Curved Motion Diagram

The motion diagram shown in the figure represents a pendulum released from rest at an angle of 45° from the vertical. The dots in the motion diagram represent the positions of the pendulum bob at eleven moments separated by equal time intervals. The green arrows represent the average velocity between adjacent dots. Also given is a "compass rose" in which directions are labeled with the letters of the alphabet.



Part A

What is the direction of the acceleration of the object at moment 5?

Enter the letter of the arrow with this direction from the compass rose in the figure. Type z if the acceleration vector has zero length.

Hint 1. How to approach the problem

The acceleration of the object at moment 5 is the acceleration found from the change in velocity between moments 4 and 5 and moments 5 and 6.

Hint 2. Definition of acceleration

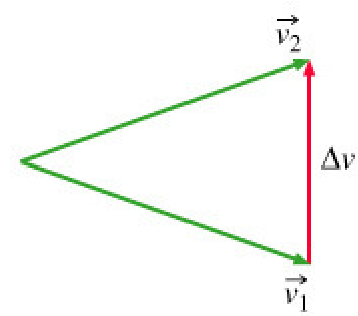
Acceleration is defined as the change in *velocity* per unit time. Mathematically,

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}.$$

Since velocity is a vector, acceleration is a vector that points in the direction of the *change* in the velocity.

Hint 3. Change of velocity: a graphical interpretation

Let us assume that in a second the velocity of an object changes from an initial value $\vec{v}_1$ to a final value $\vec{v}_2$. Then, the change in velocity that the object undergoes in that interval of time is $\Delta\vec{v} = \overrightarrow{v_2} - \overrightarrow{v_1}$. If one represents the velocity of the object graphically with vectors, then the change of velocity can be evaluated simply by applying the rule of subtraction of vectors, as shown in the picture.



ANSWER:

A

All attempts used; correct answer displayed

Part B

What is the direction of the acceleration of the object at moments 0 and 10?

Enter the letters corresponding to the arrows with these directions from the compass rose in the figure, separated by commas. Type z if the acceleration vector has zero length.

Hint 1. Find the direction of the velocity

What is the direction of the velocity of this object at moments 1 and 9?

Enter the letters of the corresponding directions from the compass rose, separated by commas. Type z if the velocity vector has zero length.

ANSWER:

directions at time step 1, time step 9 = D,B

Hint 2. Definition of acceleration

Acceleration is defined as the change in *velocity* per unit time. Mathematically,

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}.$$

Since velocity is a vector, acceleration is a vector that points in the direction of the *change* in the velocity.

Hint 3. Applying the definition of acceleration

To find the acceleration at moment 0, subtract the (vector) velocity at moment 0 from the velocity at moment 1. Similarly, to find the acceleration at moment 10, subtract the (vector) velocity at moment 9 from the velocity at moment 10.

ANSWER:

directions at time step 0, time step 10 = D,F

All attempts used; correct answer displayed

Enhanced EOC: Problem 1.10

You may want to review (pages 12 - 16) .

For general problem-solving tips and strategies for this topic, you may want to view a Video Tutor Solution of Tossed ball.

Part A

Figure shows two dots of a motion diagram and vector $\vec{v}_1$. Draw the vector $\vec{v}_2$ if the acceleration vector $\vec{a}$ at dot 2 points up.

Draw the vector with its tail at the dot 2. The orientation of your vectors will be graded. The exact length of your vectors will not be graded but the relative length of one to the other will be graded.

ANSWER:

Correct

Part B

Figure shows two dots of a motion diagram and vector $\vec{v}_1$. Draw the vector $\vec{v}_2$ if the acceleration vector $\vec{a}$ at dot 2 points down.

Draw the vector with its tail at the dot 2. The orientation of your vectors will be graded. The exact length of your vectors will not be graded but the relative length of one to the other will be graded.

ANSWER:

Correct

Problem 1.11

Part A

Figure shows two dots of a motion diagram and vector $\vec{v}_2$. Draw the vector $\vec{v}_3$ if the acceleration vector $\vec{a}$ at dot 3 points to the right.

Draw the vector with its tail at the dot 3. The orientation of your vector will be graded. The exact length of your vector will not be graded but the relative length of one to the other will be graded.

ANSWER:

Correct

Part B

Figure shows two dots of a motion diagram and vector $\vec{v}_2$. Draw the vector $\vec{v}_3$ if the acceleration vector $\vec{a}$ at dot 3 points to the left.

Draw the vector with its tail at the dot 3. The orientation of your vector will be graded. The exact length of your vector will not be graded but the relative length of one to the other will be graded.

ANSWER:

Correct

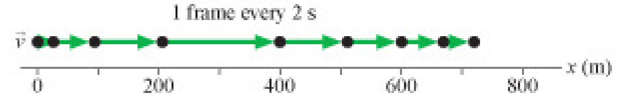
Enhanced EOC: Problem 1.18

The figure shows the motion diagram of a drag racer. The camera took one frame every 2 s.

You may want to review (pages 16 - 19) .

For help with math skills, you may want to review:

Plotting Points on a Graph



Part A

Make a position-versus-time graph for the drag racer.

Hint 1. How to approach the problem

Based on Table 1.1 in the book/e-text, what two observables are associated with each point? Which position or point of the drag racer occurs first? Which position occurs last?

If you label the first point as happening at $t = 0 \text{ s}$, at what time does the next point occur? At what time does the last position point occur?

What is the position of a point halfway in between $x = 0 \text{ m}$ and $x = 200 \text{ m}$? Can you think of a way to estimate the positions of the points using a ruler?

ANSWER:

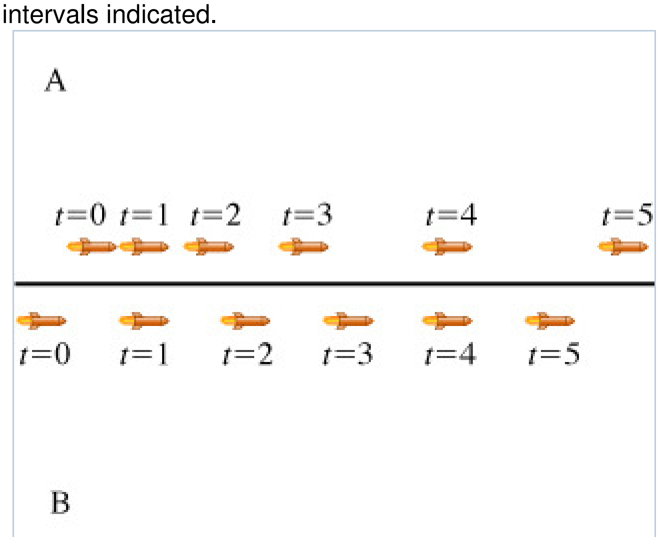
Correct

Motion of Two Rockets

Learning Goal:

To learn to use images of an object in motion to determine velocity and acceleration.

Two toy rockets are traveling in the same direction (taken to be the *x* axis). A diagram is shown of a time-exposure image where a stroboscope has illuminated the rockets at the uniform time intervals indicated.



Part A

At what time(s) do the rockets have the same velocity?

Hint 1. How to determine the velocity

The diagram shows position, not velocity. You can't find instantaneous velocity from this diagram, but you can determine the average velocity between two times t_1 and t_2:

$$v_{\text{avg}}[t_1, t_2] = \frac{x(t_2)-x(t_1)}{t_2-t_1}.$$

Note that no position values are given in the diagram; you will need to estimate these based on the distance between successive positions of the rockets.

ANSWER:

- at time $t = 1$ only
- at time $t = 4$ only
- at times $t = 1$ and $t = 4$
- at some instant in time between $t = 1$ and $t = 4$
- at no time shown in the figure

Correct

Part B

At what time(s) do the rockets have the same *x* position?

ANSWER:

- at time $t = 1$ only
- at time $t = 4$ only
- at times $t = 1$ and $t = 4$
- at some instant in time between $t = 1$ and $t = 4$
- at no time shown in the figure

Correct

Part C

At what time(s) do the two rockets have the same acceleration?

Hint 1. How to determine the acceleration

The velocity is related to the spacing between images in a stroboscopic diagram. Since acceleration is the rate at which velocity changes, the acceleration is related to the how much this spacing changes from one interval to the next.

ANSWER:

- at time $t = 1$ only
- at time $t = 4$ only
- at times $t = 1$ and $t = 4$
- at some instant in time between $t = 1$ and $t = 4$
- at no time shown in the figure

Correct

Part D

The motion of the rocket labeled A is an example of motion with uniform (i.e., constant) ___________.

ANSWER:

- and nonzero acceleration
- velocity
- displacement
- time

Correct

Part E

The motion of the rocket labeled B is an example of motion with uniform (i.e., constant) ___________.

ANSWER:

- and nonzero acceleration
- velocity
- displacement
- time

Correct

Part F

At what time(s) is rocket A ahead of rocket B?

Hint 1. Use the diagram

You can answer this question by looking at the diagram and identifying the time(s) when rocket A is to the right of rocket B.

ANSWER:

- before $t = 1$ only
- after $t = 4$ only
- before $t = 1$ and after $t = 4$
- between $t = 1$ and $t = 4$
- at no time(s) shown in the figure

Correct

Tactics Box 1.5 Drawing a Pictorial Representation

Learning Goal:

To practice Tactics Box 1.5 Drawing a Pictorial Representation.

You will find that motion problems and other physics problems often have several variables and other pieces of information to keep track of. The best way to tackle such problems is to draw a pictorial representation of the problem that shows all of the important details. This Tactics Box explains how to draw a pictorial representation of a motion problem.

TACTICS BOX 1.5 Drawing a pictorial representation

1. **Draw a motion diagram.** The motion diagram develops your intuition for the motion and, especially important, determines whether the signs of v and a are positive or negative.
2. **Establish a coordinate system.** Select your axes and origin to match the motion. For one-dimensional motion, you want either the *x* axis or the *y* axis parallel to the motion.
3. **Sketch the situation.** Not just any sketch. Show the object at the *beginning* of the motion, at the *end*, and at any point where the character of the motion changes. Show the object, not just a dot, but very simple drawings are adequate.
4. **Define symbols.** Use the sketch to define symbols representing quantities such as position, velocity, acceleration, and time. Every variable used later in the mathematical solution should be defined on the sketch. Some will have known values, and others are initially unknown, but all should be given symbolic names.
5. **List known information.** Make a table of the quantities whose values you can determine from the problem statement or that can be found quickly with simple geometry or unit conversions. Some quantities are implied by the problem, rather than explicitly given. Others are determined by your choice of coordinate system.
6. **Identify the desired unknowns.** What quantity or quantities will allow you to answer the question? These should have been defined as symbols in step 4. Don't list every unknown, only the one or two needed to answer the question.

Follow the steps above to draw a pictorial representation of the following problem: A light train is traveling on a straight section of track at a constant speed of 15 $\mathrm{m/s}$. As it approaches the next station, it starts to slow down at a rate of 5 $\mathrm{m/s^2}$

until it stops at the station. From the moment the train starts to slow down, how long does it take for the train to reach the station? Note that you are not expected to solve this problem, only to draw the pictorial representation.

Part A

Draw a motion diagram for the train. Assume that the train is moving toward the right and starts to slow down at $t = 0$. The separation of each dot represents an elapsed time of one second. Include velocity vectors and the acceleration vector in your drawing. Keep in mind that the acceleration of the train has a magnitude of 5 $\mathrm{m/s^2}$.

The orientation and length of the velocity vectors will be graded. Only the direction of the acceleration vector will be graded. The location of the vectors will not be graded.

Hint 1. How to draw the vectors with correct lengths

Draw each velocity vector between the corresponding two black dots. To draw the acceleration with the correct length, click on the **vector info** button; adjust the vector until the length displayed in the properties window has the desired value.

ANSWER:

Correct

Part B

Draw the coordinate system that is most appropriate for the train's motion. Since this problem deals with one-dimensional motion, you need only the *x*-axis. Note that $x_{\rm i}$, $x_{\rm f}$, $t_{\rm i}$, $t_{\rm f}$, $(v_x)_{\rm i}$ and $(v_x)_{\rm f}$ are the initial position, final position, initial time, final time, initial velocity, and final velocity of the train, respectively.

The orientation of the vector will be graded. The location and length of the vector will not be graded.

ANSWER:

Correct

Keep in mind that the choice of coordinate system is arbitrary. However, in problems involving one-dimensional motion it is often convenient to select the *x*-axis to match the motion. Thus, in this case the positive end of the *x*-axis is chosen to be to the right. It is also convenient in this problem to place the origin at the location where the train starts to slow down.

Part C

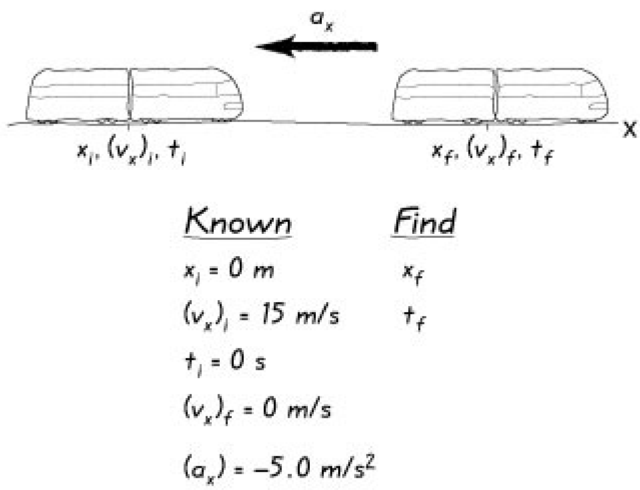
To complete your pictorial representation of the problem, you should compile two lists: one of known quantities and one of the unknown quantities that will allow you to answer the question in the problem. Below are all of the relevant quantities in this problem. Sort them accordingly.

Drag the appropriate items to their respective bins.

ANSWER:

All attempts used; correct answer displayed

Now that you have identified the list of known and unknown quantities, you can add this information to your pictorial representation in the form of a table. The actual sketch that you draw might look like this:



PSS 1.2 General Problem-Solving Strategy

Learning Goal:

To practice Problem-Solving Strategy 1.2 for general problems.

Two hockey pucks, labeled A and B, are initially at rest on a smooth ice surface and are separated by a distance of $18.0 \mathrm{m}$. Simultaneously, each puck is given a quick push, and they begin to slide directly toward each other. Puck A moves with a speed of $2.70 \mathrm{m/s}$, and puck B moves with a speed of $4.30 \mathrm{m/s}$.

What is the distance covered by puck A by the time the two pucks collide?

PROBLEM-SOLVING STRATEGY 1.2 General problem-solving strategy

MODEL: It is impossible to treat every detail of the situation. Simplify the problem with a model that captures the essential features. For example, the object in a mechanics problem is usually represented as a particle.

VISUALIZE: This is where expert problem solvers put most of their effort.

- Draw a *pictorial representation.* This helps you visualize important aspects of the physics and assess the information you are given. It starts the process of translating the problem into symbols.
- Use a *graphical representation* if it is appropriate for the problem.
- Go back and forth between these representations; they need not be done in any particular order.

SOLVE: Only after modeling and visualizing are complete is it time to develop a *mathematical representation* with specific equations that must be solved. All symbols used here should have been defined in the pictorial representation.

ASSESS: Is your result believable? Does it have proper units? Does it make sense?

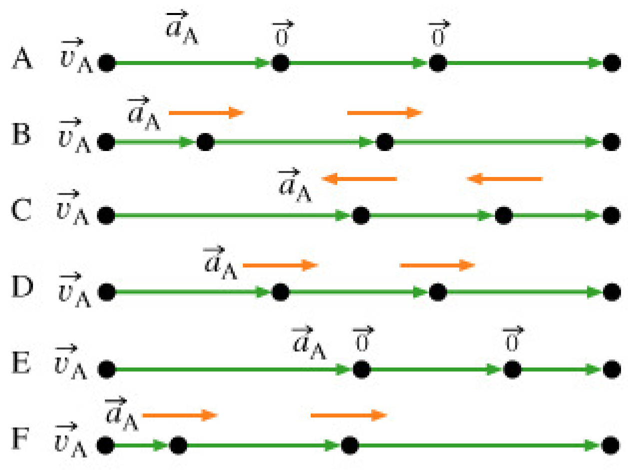
Model

In this problem, the objects of interest are the pucks. Since their dimensions are small compared to the distance traveled, it is reasonable to model them as small particles, as suggested in the strategy. Another reasonable assumption is that the effects of air resistance on small, slowly moving objects such as the pucks, and of friction between the puck and the smooth ice, are both very small. Therefore, in the absence of friction and air resistance, the pucks can be modeled as particles moving with constant velocity. In addition, because the problem states that the pucks are given a "quick" push, it is appropriate to ignore the motion of the pucks as they are being pushed and simply think of them as already moving with constant velocity.

Visualize

Part A

Which of the motion diagrams shown here best represents the motion of puck A *prior to* the collision?



ANSWER:

- A
- B
- C
- D
- E
- F

Correct

Now, draw a motion diagram for puck B. Recall that a complete motion diagram for this problem should include both objects of interest, puck A as well as puck B.

Part B

After completing your motion diagram, it's time to choose an appropriate coordinate system for this problem. Note that there is no single correct way to visualize this problem, but for the questions that follow, assume that the pucks are moving along the *x* axis with *+x* directed to the right. Take the initial position of puck A, when it is at rest, to be at the origin, and take the initial position of puck B to be to the right of puck A. Start your clock at the instant when the pucks begin to move.

Based on the assumptions listed above, sort the following quantities as known or unknown.

Drag the appropriate items to their respective bins.

ANSWER:

Correct

Part C

Which of the following relationships follows from the problem statement?

Note that $(x_1)_\mathrm{A}$ and $(x_1)_\mathrm{B}$ are the positions of the pucks at the moment of their collision.

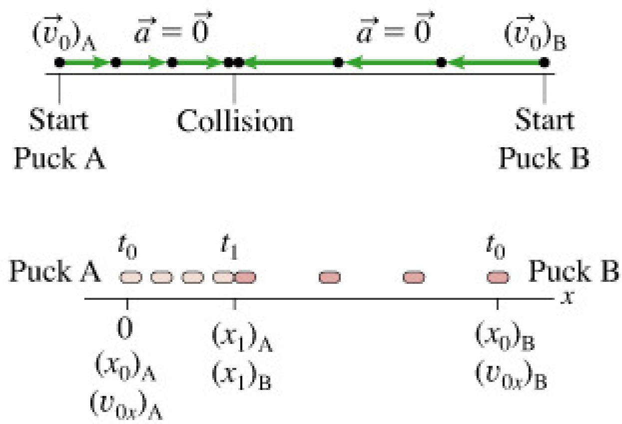
ANSWER:

- $(x_1)_\mathrm{A} + (x_1)_\mathrm{B} = 18.0\mathrm{m}$
- $(x_1)_\mathrm{A} = (x_1)_\mathrm{B}$
- $(x_1)_\mathrm{A} + (x_1)_\mathrm{B} = 0\ \mathrm{m}$
- $(x_1)_\mathrm{B} - (x_1)_\mathrm{A} = 18.0\mathrm{m}$

Correct

For the pucks to collide, they must be at the same position at the same time! So, now the number of unknown quantities in this problem is reduced to two: the final position of the pucks, which you may simply call x_1, and the time of collision, t_1. Make certain to identify what the problem is trying to find.

Now, put all this information together and create your pictorial representation for this problem. Your effort should produce a sketch like this:



A complete pictorial representation would also include a list of knowns and unknowns similar to the following:

Known: $(x_0)_A = 0$ m, $(x_0)_B =$ 18.0m , $(v_{0x})_A =$ 2.70m/s , $(v_{0x})_B =$- 4.30m/s , $t_0 = 0$, $a_A = a_B = 0$

Unknown: t_1, $(x_1)_A = (x_1)_B = x_1$

Note that your target variable is the distance covered by puck A by the time the two pucks collide, that is, $(x_1)_A - (x_0)_A$. Since we selected a coordinate system whose origin is at the initial position of puck A, $(x_0)_A = 0$ m and your target variable becomes simply x_1.

In addition to the pictorial representation, you could also draw a graphical representation of the problem, such as plotting (on the same graph) the position of each puck as a function of time.

Solve

Now you can use the information and the insights that you have accumulated to construct the necessary mathematical representation of this problem and to derive the solution.

Here are two relevant equations that you may not yet know how to derive (the notation corresponds to the notation used in the pictorial representation above):

$$(x_1)_A = (x_0)_A + (v_{0x})_A t_1$$

and

$$(x_1)_B = (x_0)_B - (v_{0x})_B t_1.$$

(The two equations have different signs because the pucks are moving in opposite directions.)

Part D

What is x_1, the distance that puck A covers prior to the collision?

Express your answer in meters to three significant figures. Do not include units in your answer.

Hint 1. How to approach this question

In the Visualize step, you determined that $x_1 = (x_1)_A = (x_1)_B$. Combine this information with the equations you were given. You will then obtain the following system of equations:

$$x_1 = (x_0)_A + (v_{0x})_A t_1$$

$$x_1 = (x_0)_B - (v_{0x})_B t_1$$

Solve this system to find an expression for x_1 in terms of known quantities. Then, substitute the values of the known quantities into your expression to find a numerical result. If necessary, refer back to the pictorial representation you drew in the Visualize step for the values of the known quantities.

Hint 2. Find a general expression for the distance traveled prior to the collision

Solve the system of equations given in the previous hint, and find an expression for x_1 in terms of the following variables: x_{0A}, the initial position of puck A; x_{0B}, the initial position of puck B; v_{0A}, the initial speed of puck A; and v_{0B}, the initial speed of puck B.

Express your answer in terms of some or all of the variables x_{0A}, x_{0B}, v_{0A}, and v_{0B}.

ANSWER:

$$x_1 = \frac{x_{0B} + \frac{v_{0B} x_{0A}}{v_{0A}}}{1 + \frac{v_{0B}}{v_{0A}}}$$

ANSWER:

x_1 = 6.94 m

Correct

Assess

Part E

Your answer should satisfy common sense. For instance, can you decide which of the following values for the distance covered by puck A would definitely be wrong, regardless of the speed of the two pucks and considering that the two pucks are sliding toward each other?

A. $19\ \mathrm{m}$
B. $5\ \mathrm{m}$
C. $16\ \mathrm{m}$
D. $1\ \mathrm{m}$
E. $25\ \mathrm{m}$

Type the letters corresponding to the *definitely wrong* answers. Do not use commas. For instance, if options C and D are definitely wrong, type CD.

ANSWER:

AE

Correct

Even without solving the problem, we can conclude that the distance covered by puck A must be less than or equal to $18.0 \mathrm{m}$. You can verify that your answer from Part D does satisfy this common-sense evaluation.

In the future, whenever possible, use two simple methods, *units check* and *common-sense evaluation*, to assess your results. They will be useful in catching some incorrect answers without following the entire solution through. However, beware that these methods do not catch all incorrect answers.

Tactics Box 1.6 Using Significant Figures

Learning Goal:

To practice Tactics Box 1.6 Using Significant Figures.

You can think of a significant figure as being a digit that is reliably known. For example, a length measurement of $6.2 \mathrm{cm}$ has *two* significant figures, the 6 and the 2. The next decimal place—the one-hundredths—is not reliably known and is thus not a significant figure.

Determining the proper number of significant figures is straightforward, but there a few definite rules to follow. These are summarized in this tactics box.

TACTICS BOX 1.6 Using significant figures.

It is acceptable to keep one or two extra digits during *intermediate* steps of a calculation. The goal here is to minimize round-off errors in the calculation. But the *final* answer must be reported with the proper number of significant figures.

1. When multiplying or dividing several numbers, or when taking roots, the number of significant figures in the answer should match the number of significant figures of the least precisely known number used in the calculation.
2. When adding or subtracting several numbers, the number of decimal places in the answer should match the smallest number of decimal places of any number used in the calculation.
3. It is acceptable to keep one or two extra digits during *intermediate* steps of a calculation, as long as the final answer is reported with the proper number of significant figures. The goal here is to minimize round-off errors in the calculation. But keep only one or two extra digits, not the seven or eight shown in your calculator display.

Part A

Your bedroom has a rectangular shape, and you want to measure its area. You use a tape that is precise to $0.001 \mathrm{m}$ and find that the shortest wall in the room is $3.547 \mathrm{m}$ long. The tape, however, is too short to measure the length of the second wall, so you use a second tape, which is longer but only precise to $0.01 \mathrm{m}$. You measure the second wall to be $4.79 \mathrm{m}$ long. Which of the following numbers is the most precise estimate that you can obtain from your measurements for the area of your bedroom?

Hint 1. Find how many significant figures should be in your answer

When you calculate the area of your bedroom, you have to multiply the measurements of the lengths of the walls. Considering that the measurements have different precision, how many significant figures should be in your answer in this case?

Express your answer as an integer.

ANSWER:

3

ANSWER:

- ◉ 17.0 m^2
- ○ 16.990 m^2
- ○ 16.99 m^2
- ○ 16.9 m^2
- ○ 16.8 m^2

Correct

Part B

Using the measurements described in Part A, which of the following numbers is the most precise estimate for the perimeter of your bedroom?

Hint 1. Find how many decimal places should be in your answer

When you calculate the perimeter of your bedroom, you have to add the measurements of the lengths of all the walls. Considering that the measurements have different precision, how many *decimal places* should be in your answer in this case?

Express your answer as an integer.

ANSWER:

2

ANSWER:

- ○ 16.674 m
- ◉ 16.67 m
- ○ 16.68 m
- ○ 16.7 m
- ○ 17 m

Correct

Part C

If your bedroom has a circular shape, and its diameter measured 6.32 m, which of the following numbers would be the most precise value for its area?

Hint 1. The area of a circle

The area A of a circle of radius r is

$$A = \pi r^2.$$

Hint 2. Find how many significant figures should be in your answer

In this case, when you calculate the area of the bedroom, you need to carry out a series of multiplications and divisions that involve exact numbers and numbers that are correct to a certain precision. How many significant figures should be in your answer in this case?

Express your answer as an integer.

ANSWER:

3

ANSWER:

- 30 m^2
- 31.0 m^2
- 31.4 m^2
- 31.37 m^2
- 31.371 m^2

Correct

To calculate the area A of a circle given its diameter d, you need to first divide the diameter by 2, then multiply your result by itself (or take the square of it), and finally multiply everything by π:

$$A = \pi \left(\frac{d}{2}\right)^2.$$

Since here the number 2 and π are exact numbers, they do not change the accuracy of the measured numbers involved in the calculation. Therefore, your answer should be expressed to the same number of significant figures as that used in the given diameter.

Score Summary:

Your score on this assignment is 88.4%.
You received 10.61 out of a possible total of 12 points.