REVIEW FOR THE FINAL EXAM MAT 243, FALL 2013 The final exam covers Sections 2.4, 3.2, 4.1, 4.2, 4.3, 5.1,5.3, 6.1-6.4, 8.1, 8.2, 8.5, and 9.1; questions on the exam can refer to any of these sections but need to refer to all of them

Think of this review as a starting point for studying for the exam. You will be asked for definitions on the exam. You will be asked to prove statements on the exam.

General outline

1. Series and sequences (a) sequences defined explicitly (closed formula). (b) sequences defined through a recurrence relation (c) ? notation (d) sum various series
2. Growth of functions (a) definition of "f(x) is O(g(x))" (b) popular functions: which is big-O of which? (c) definition of "f(x) is order g(x)"
3. Divisibility and modular arithmetic (a) Definition of a|b, for integers a and b. (b) Definition of a **mod** m and a≡b mod m (c) (a+b) mod m
4. Integer representation (a) Convert between bases.
5. Primes and greatest common divisors (a) Definition of a prime. (b) Prime factorization. (c) greatest common divisor

   (d) Euclidean algorithm
6. Induction
7. Recursive definitions and structural induction.
8. Counting

   (a) Section 6.1: know the basic counting techniques: sum rule (use it when the number of ways to do a task can be decomposed into disjoint subsets), the product rule (use it when the number of ways to do a task can be expressed as the number of ways to do a sequence of tasks) and the inclusion-exclusion principle for the union of two sets.

   (b) Section 6.2: know the Pigeon Hole Principle and also its generalized version. Understand its applications. In your proofs it is important that you identify (or create if necessary) the objects/people and the categories that represent the boxes. It is also important that you are able to determine which two out of the three values N = objects, k = boxes and $\lceil N/k \rceil$ is given, and which one is the question. Be careful with problems where a specific category is in question. PHP only guarantees a category with a certain number of elements, does not specify which one.

   (c) Section 6.3: know the difference between a permutation (order matters) and a combina- tion (order doesn't matter) and the fact that both can only be used if the object or people are distinct. Otherwise you need the product rule. Know their applications for questions about forming committees, test questions, defective items, determining numbers of bit strings with certain properties, books on the shelf, people in a picture, poker hands, fruits in a basket ....etc. Understand how to deal with problems con- taining the phrases at least and at most, and how to use the complement in certain cases.

   (d) Section 6.4: Understand (and hopefully appreciate) the examples for the double counting techniques from the book, class and homework. Be able to STATE and APPLY the binomial theorem. Know how to find the coefficient of any term in the expansion of a binomial or any entry in the Pascal Triangle.

9. Recurrence

(a) Section 8.1: know how to set up a recurrence for the size of a set of objects, where the set depends on an integer, by considering how the the ojects are related to the ones in sets for smaller n. Typical examples include number strings of length n where some condition hold and the Fibonacci numbers. Know how to verify a given solution works.

(b) Section 8.2: know how to solve linear, homogenous recurrence relations with constant co- effients. Know the different techniques for distinct and repeated roots.

(c) Section 8.5: know how to use the inclusion-exclusion formula to find the number of elements in the union of several sets that may overlap. Understand how to set up and use a Venn-diagram for the case n = 3 (three sets).

10. Relations: Section 9.1: know the definitions of a relation, what it means for relation to be reflexive, symmetric, and transitive, and understand how to apply the definitions to prove that a given relation has or does not have these properties.

A few questions:

1. Evaluate (Leave your answer as a fraction, do not simplify and do not evaluate exponents.)

$$\sum_{k=0}^{200}(\frac{1}{3})^k$$

2. Prove that for every positive integer n,

$$1\cdot2\cdot3+2\cdot3\cdot4+...+n(n+1)(n+2)= \frac{n(n+1)(n+2)(n+3)}{4}$$

3. Let x and y be distinct primes, a and b arbitrary integers. Show that if $a \equiv b \bmod x$ and $a \equiv b \bmod y$, then $a \equiv b \bmod xy$.

4. Use the Euclidean algorithm to find gcd(679, 553). (gcd = greatest common divisor)

5. If the product of two integers is $2^7 \cdot 3^8 \cdot 5^2 \cdot 7^{11}$ and their greatest common divisor is $2^3 \cdot 3^4 \cdot 5$, what is their least common multiple?

6. Find the least integer n such that is $\frac{x^4+x^2+1}{x^3+1}$ $O(x^n)$.

7. What is the binary expansion of $(100)_{10}$?

8. Use the definition of "f(x) is O(g(x))" to show that $x^3 + 14x^2 + 7$ is $O(x^3)$.

9. Evaluate these quantities

(a) –45 mod 8 (b) 33 mod 7

(c) ((18 mod 14) + (–35 mod 7)) mod 8

10. Let S be the set of bit strings defined recursively by

Basis step: the empty string is in S, and

Recursive step: if $x \in S$, then $0x \in S$ and $x1 \in S$.

(a) Find all strings of length less than or equal to five. (b) Give an explicit description of the elements of S.

Chapter 6 pg 440: 1, 8, 12, 16, 29, 33

Chapter 8	pg 566: 6, 7
	pg 567: 5, 6,33, 35
Chapter 9	pg 634: 2, 3abd;
	pg 635 1 (ignore antisymmetic), 20.