Mathematical Induction

Use induction to prove that $\mathbf{133}$ divides $\mathbf{11^{n+1} + 12^{2n-1}}$

for all positive integers $\boldsymbol{n}$.

Let $P(n)$ denote the proposition that $11^{n+1} + 12^{2n-1}$ is divisible by 133 for all positive integers n.

BASIS STEP: $P(1)$ is true since 133 divides 133.

INDUCTIVE STEP: Let us assume $P(n)$, that is $11^{n+1} + 12^{2n-1}$is divisible by 133 for an arbitrary positive integer n. This is our inductive hypothesis.

We have to show that $P(n+1)$, $11^{(n+1)+1}$+$12^{2(n+1)-1}$ is divisible by 133 assuming the inductive hypothesis $P(n)$.

Note that: $11^{(n+1)+1}$+$12^{2(n+1)-1}$=$11^{n+2} + 12^{2n+1}$

Proof: $11^{n+2} + 12^{2n+1} = 11^{n+1} \cdot 11 + 12^{2n-1} \cdot 144 =$
$11^{n+1} \cdot 11 + 12^{2n-1} \cdot (133 + 11) = (11^{n+1}+12^{2n-1}) \cdot 11 + 12^{2n-1} \cdot 133$

$11^{n+1} + 12^{2n-1}$ is divisible by 133 using the inductive hypothesis.

$12^{2n-1} \cdot 133$ is divisible by 133 the definition of divisibility since 12^{2n-1} is an integer.

Thus, the sum $11^{n+2} + 12^{2n+1}$=$(11^{n+1}+12^{2n-1}) \cdot 11 + 12^{2n-1} \cdot 133$ is also divisible by 133.

By the **Principle of Mathematical Induction** (Basis Step and Inductive Step together) $11^{n+1} - 12^{2n-1}$ is divisible by 133 for all positive integers n.