Mathematical Induction

Use induction to prove that 2 divides $\boldsymbol{n^2 + n}$ for all positive integers $\boldsymbol{n}$.

Let $P(n)$ denote the proposition that $n^2 + n$ is divisible by 2 for all positive integers n.

BASIS STEP: $P(1)$ is true since 2 divides 2.

INDUCTIVE STEP: Let us assume $P(n)$, that is $n^2 + n$ is divisible by 2 for an arbitrary positive integer n. This is our inductive hypothesis.

We have to show that $P(n + 1)$,
that is $(n + 1)^2 + (n + 1)$ is also divisible by 2 assuming the inductive hypothesis $P(n)$.

Proof: $(n+1)^2+(n+1)= n^2+n+2(n+1)$

n^2+n is divisible by 2 using the inductive hypothesis.

$2(n+1)$ is divisible by 2 the definition of divisibility since $n+1$ is an integer.

Thus, the sum $(n+1)^2+(n+1)= n^2+n+2(n+1)$ is also divisible by 2.

By the **Principle of Mathematical Induction** (Basis Step and Inductive Step together) n^2+n is divisible by 2 for all positive integers n.