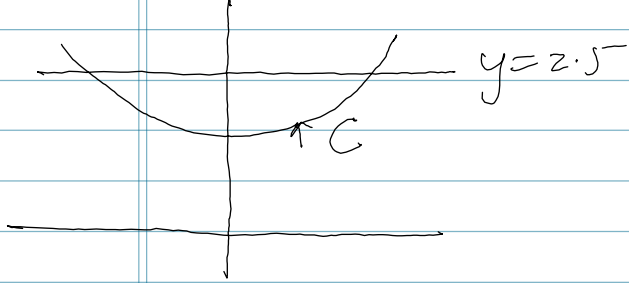


Example. Find the area bounded by the curve $C: x = t - \frac{1}{t}, y = t + \frac{1}{t}$, and the line $y = 2.5$.

Solution.



$$t + \frac{1}{t} = \frac{5}{2}$$

$$\Rightarrow 2t^2 - 5t + 2 = 0$$

$$\Rightarrow (2t - 1)(t - 2) = 0$$

$$\Rightarrow t = \frac{1}{2} \text{ or } t = 2.$$

$$t = \frac{1}{2}, x = \frac{1}{2} - 2 = -\frac{3}{2}.$$

$$t = 2, x = 2 - \frac{1}{2} = \frac{3}{2}.$$

for

$$\frac{1}{2} < t < 2, y = t + \frac{1}{t} < 2.5.$$

$$A = \int_{-3/2}^{3/2} (2.5 - y) dy.$$

$$= \int_{1/2}^2 (2.5 - (t + \frac{1}{t})) (1 + \frac{1}{t^2}) dt$$

$$y = t + \frac{1}{t}$$

$$= \int_{1/2}^2 (\frac{5}{2} - t - 2t^{-1} + \frac{5}{2}t^{-2} - t^{-3}) dt \quad dy = (1 + \frac{1}{t^2}) dt.$$

$$= \left(\frac{5}{2}t - \frac{t^2}{2} - 2\ln|t| - \frac{5}{2t} + \frac{1}{2t^2} \right) \Big|_{1/2}^2$$

$$= \frac{15}{4} - 4\ln 2.$$