MAT 243 ONLINE WRITTEN HW 5

NAME: ______________________________

(1) **(4 pts)** Fill in the blank in the statements below:

(a) In an inductive proof verifying the condition for $n = 1$ (or the lowest possible value) is called the ______________________________

(b)-(c) In the induction step first we assume $P(n)$, called the __________________________ for __________________, and then we show that $P(n+1)$ is true as well using the inductive hypothesis.

(d) A recurrence relation is an ______________________________

(2) **(10 pts)** Prove that for any positive integer n, $\sum_{k=1}^{n}(4k-3) = 2n^2 - n$ using mathematical induction.

(3) **(10 pts)** Prove that for any integer $n \geq 7$, $3^n < n!$ using mathematical induction.

(4) **(10 pts)** Use induction to prove that 6 divides $9^n - 3^n$ for all integer $n \geq 0$.

(5) **(10 pts)**

(a) Given the recursive definition for the set S below. Describe the elements of S.

$4 \in S$
$x - y \in S$ if $x \in S$ and $y \in S$.

(b) Give a **recursive** definition for the set of positive integers that are powers of 4. (1,4,16, 64,.....)

(c) Give a **recursive** definition for the set of positive integers that are not divisible by 3.

(6) **(10 pts)** Let S be the set of binary strings defined recursively as follows:
Basis step: $1 \in S$
Recursive step: If $x \in S$ then $xx \in S$ and $0x0 \in S$

(If x and y are binary strings then xy is the concatenation of x and y. For instance, if $x = 0111$ and $y = 101$ then $xy = 0111101$.)

(a) **(3 pts)** List the elements of S produced by the first 2 applications of the recursive definition. Find S_0, S_1 and S_2.

(b) **(7 pts)** Use Structural induction to prove that every element of S has even number of 0's in it.

(7) **(10 pts)** Find a closed-form representation of the following recurrence relations:

(a) $a_n = 6a_{n-1} - 9a_{n-2}$ for $n \geq 2$ with initial conditions $a_0 = 4$ and $a_1 = 6$

(b) $a_n = 4a_{n-1} + 5a_{n-2}$ for $n \geq 2$ with initial conditions $a_0 = 2$ and $a_1 = 8$

Show all you steps for full credit.