MAT 243 Online Written Homework Assignments for Week 2 (units 4-5)

1. Let p = “All the people”, the universe of discourse
   Let k(x) = “x is a kid”, the predicate
   Let i(x) = “x likes ice cream”, the predicate.

   $\forall x \in p, k(x) \rightarrow i(x)$, premise.

   $\neg i(joey)$, premise.

   $\therefore \neg k(joey)$, Modus Tollens

2. Let s and t be odd numbers

   By definition of an odd number, s = (2m + 1) and t =(2n + 1) where
   $m, n\ are\ any\ natural\ number$.

   Then the product $s \times t$ is (2m + 1) × (2n + 1) = (4mn + 2m + 2n + 1)

   It is followed that the product of $s \times t$ = (N + 1), where N = (4mn+2m+2n) is not an odd number.

   Thus, the product $s \times t$ is an odd number.

3. Let p = 2 and q = 10 be two natural numbers.

   By definition, A *natural number* is a positive integer: 1,2, 3, etc.

   If (q – p) > 1, then there exists a natural number between p and q.

   It is followed that (10 – 2) = 8 and 8 > 1, therefore there exists a natural number between 2 and 10.

4. Let $\sqrt{10} = \frac{x}{y}$ where x is not an even number.

   By definition of even number, $x = (2n)\ where\ n\ is\ any\ natural\ number$.

   Then, squaring both side yields

$10 = \frac{x^2}{y^2}$

Followed by $10y^2 = x^2$, or $2(5y^2) = x^2$

x^2 is even, x must also be even. Thus $\sqrt{10}$ cannot be expressed as a rational number, so $\sqrt{10}$ is an irrational number whose square is 10.

It is concluded that there is no natural number whose square is 10.

5. Let p be an even number and q be an odd number.

   By definition of an even number, p = 2n where n is any natural number, and by definition of an odd number, q = (2m + 1) where m is any natural number.

   Then, q + q = (2m + 1) + (2m + 1) = (4m + 2) = 2(2m + 1) = 2n where n = (2m + 1). Thus, P = (q + q), concluding that every even number can be written as the sum of two odd numbers.

6. Let s and t be real numbers and t > s.

   By definition of real numbers, if s and t are real integers, then so are (s + t) and $s \times t$ where $s, t\ are\ any\ integer$.

   If (t – s) > 0 then there exists a real number between s and t.

   Thus, there exists a real number between s and t.