Let A and B be sets.

Show that $P(A \cap B) \subseteq P(A) \cap P(B)$

Definition of subsets: $A \subseteq B \leftrightarrow \forall x(x \in A \rightarrow x \in B)$

We have to show that, if X is an element of P(A ∩ B) then X is an element of P(A) ∩ P(B).

Show that: If $X \in P(A \cap B)$ then $X \in P(A) \cap P(B)$.

Let X be an arbitrary element of P(A ∩ B).

Let $X \in P(A \cap B)$.

Then X is a subset of A ∩ B.

Then $X \subseteq A \cap B$.

Then X is a subset of A and a subset of B (why?).

Then $X \subseteq A$ and $X \subseteq B$.

If X is a subset of A then X is an element of P(A).
If $X \subseteq A$ *then* $X \in P(A)$ *.*

If X is a subset of B then X is an element of P(B).
If $X \subseteq B$ *then* $X \in P(B)$*.*

If X is an element of P(A) and P(B) then
X is and element of $P(A) \cap P(B)$*.*
If $X \in P(A \cap B)$ *then* $X \in P(A) \cap P(B)$*.*