Section 6.5: Approximate Integration

Use (a) the Trapezoid Rule, (b) the Midpoint Rule, and (c) Simpson's Rule to approximate the given integral with the specified value of *n*. (Round your answers to six decimal places)

7. $\int_0^2 \sqrt[4]{1+x^2}\, dx \qquad n=8$

$0 \quad \frac{1}{4} \quad \frac{1}{2} \quad \frac{3}{4} \quad 1 \quad \frac{5}{4} \quad \frac{3}{2} \quad \frac{7}{4} \quad 2$

$\Delta x = \frac{2-0}{8} = \frac{1}{4}$

a) $$T_8 = \frac{\frac{1}{4}}{2}\Big(\sqrt[4]{1+0^2} + 2\sqrt[4]{1+(1/4)^2} + 2\sqrt[4]{1+(1/2)^2} + 2\sqrt[4]{1+(3/4)^2} + 2\sqrt[4]{1+(1)^2}$$
$$+ 2\sqrt[4]{1+(5/4)^2} + 2\sqrt[4]{1+(3/2)^2} + 2\sqrt[4]{1+(7/4)^2} + \sqrt[4]{1+(2)^2}\Big)$$

$\approx 2.41378967045\ldots$

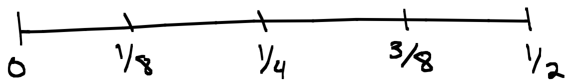
b) $$M_8 = \left(\frac{1}{4}\right)\sqrt[4]{1+(1/8)^2} + \left(\frac{1}{4}\right)\sqrt[4]{1+(3/8)^2} + \left(\frac{1}{4}\right)\sqrt[4]{1+(5/8)^2} + \left(\frac{1}{4}\right)\sqrt[4]{1+(7/8)^2}$$
$$+ \left(\frac{1}{4}\right)\sqrt[4]{1+(9/8)^2} + \left(\frac{1}{4}\right)\sqrt[4]{1+(11/8)^2} + \left(\frac{1}{4}\right)\sqrt[4]{1+(\tfrac{13}{8})^2} + \left(\frac{1}{4}\right)\sqrt[4]{1+(\tfrac{15}{8})^2}$$

$\approx 2.41145300802\ldots$

c) $$S_8 = \frac{\left(\frac{1}{4}\right)}{3}\Big(\sqrt[4]{1+0^2} + 4\sqrt[4]{1+(1/4)^2} + 2\sqrt[4]{1+(1/2)^2} + 4\sqrt[4]{1+(3/4)^2} + 2\sqrt[4]{1+(1)^2}$$
$$+ 4\sqrt[4]{1+(5/4)^2} + 2\sqrt[4]{1+(3/2)^2} + 4\sqrt[4]{1+(7/4)^2} + \sqrt[4]{1+(2)^2}\Big)$$

≈ 2.41223163124

8. $\int_0^{1/2} \sin(x^2)\, dx \quad n=4$



$\Delta x = \frac{1/2 - 0}{4} = 1/8$

$$T_4 = \frac{(1/8)}{2}\left(\sin(0^2) + 2\sin((1/8)^2) + 2\sin((1/4)^2) + 2\sin((3/8)^2) + \sin((1/2)^2) \right)$$

$$\approx .042743454255....$$

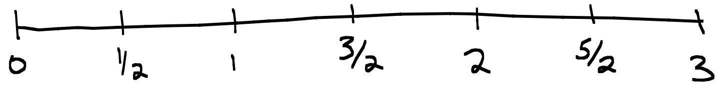
$$M_4 = \frac{1}{8}\sin((1/16)^2) + \frac{1}{8}\sin((3/16)^2) + \frac{1}{8}\sin((5/16)^2) + \frac{1}{8}\sin((7/16)^2)$$

$$\approx .04084950042....$$

$$S_4 = \frac{\left(\frac{1}{8}\right)}{3}\left(\sin(0^2) + 4\sin((1/8)^2) + 2\sin((1/4)^2) + 4\sin((3/8)^2) + \sin((1/2)^2) \right)$$

$$\approx .041477830884$$

10. $\int_0^3 \frac{dt}{1+t^2+t^4} \qquad n=6$



$$\Delta t = \frac{3-0}{6} = \frac{1}{2}$$

$$T_6 = \frac{\left(\frac{1}{2}\right)}{2}\left(\frac{1}{1+0^2+0^4} + (2)\frac{1}{1+(1/2)^2+(1/2)^4} + (2)\frac{1}{1+1^2+1^4} + (2)\frac{1}{1+(3/2)^2+(3/2)^4} \right.$$

$$\left. + (2)\frac{1}{1+(2)^2+(2)^4} + (2)\frac{1}{1+(5/2)^2+(5/2)^4} + \frac{1}{1+3^2+3^4} \right)$$

$$\approx .89512242 1438 \ldots$$

$$M_8 = \frac{1}{2}\left(\frac{1}{1+(1/4)^2+(1/4)^4}\right) + \frac{1}{2}\left(\frac{1}{1+(3/4)^2+(3/4)^4}\right) + \frac{1}{2}\left(\frac{1}{1+(5/4)^2+(5/4)^4}\right)$$

$$+ \frac{1}{2}\left(\frac{1}{1+(7/4)^2+(7/4)^4}\right) + \frac{1}{2}\left(\frac{1}{1+(9/4)^2+(9/4)^4}\right) + \frac{1}{2}\left(\frac{1}{1+(11/4)^2+(11/4)^4}\right)$$

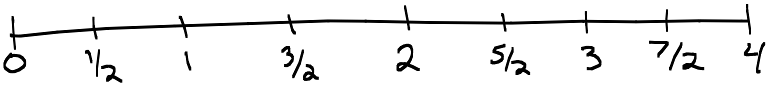
$$\approx .89547841853 6 \ldots$$

$$S_8 = \frac{(1/2)}{3}\left(\frac{1}{1+0^2+0^4} + (4)\frac{1}{1+(1/2)^2+(1/2)^4} + (2)\frac{1}{1+1^2+1^4} + (4)\frac{1}{1+(3/2)^2+(3/2)^4} \right.$$

$$\left. + (2)\frac{1}{1+(2)^2+(2)^4} + (4)\frac{1}{1+(5/2)^2+(5/2)^4} + \frac{1}{1+3^2+3^4} \right)$$

$$\approx .898014266435$$

11. $\int_0^4 e^{\sqrt{t}} \sin t \, dt \quad n=8$



$$\Delta t = \frac{4-0}{8} = \frac{1}{2}$$

$$T_8 = \frac{\left(\frac{1}{2}\right)}{2}\Big[e^{\sqrt{0}}\sin 0 + 2\left(e^{\sqrt{1/2}}\sin(1/2)\right) + 2\left(e^{\sqrt{1}}\sin(1)\right) + 2\left(e^{\sqrt{3/2}}\sin(3/2)\right)$$

$$+2\left(e^{\sqrt{2}}\sin(2)\right) + 2\left(e^{\sqrt{5/2}}\sin(5/2)\right) + 2\left(e^{\sqrt{3}}\sin(3)\right)$$

$$+2\left(e^{\sqrt{7/2}}\sin(7/2)\right) + e^{\sqrt{4}}\sin(4)\Big]$$

$$\approx 4.51361759796\ldots$$

$$M_8 = \frac{1}{2}\left(e^{\sqrt{1/4}}\sin\left(\tfrac{1}{4}\right)\right) + \frac{1}{2}\left(e^{\sqrt{3/4}}\sin(3/4)\right) + \frac{1}{2}\left(e^{\sqrt{5/4}}\sin(5/4)\right) + \frac{1}{2}\left(e^{\sqrt{7/4}}\sin(7/4)\right)$$

$$+\frac{1}{2}\left(e^{\sqrt{9/4}}\sin(9/4)\right) + \frac{1}{2}\left(e^{\sqrt{11/4}}\sin(11/4)\right) + \frac{1}{2}\left(e^{\sqrt{13/4}}\sin(13/4)\right) + \frac{1}{2}\left(e^{\sqrt{15/4}}\sin 15/4\right)$$

$$\approx 4.74825600585$$

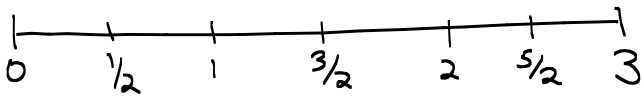
$$S_8 = \frac{\left(\frac{1}{2}\right)}{3}\Big[e^{\sqrt{0}}\sin 0 + 4\left(e^{\sqrt{1/2}}\sin(1/2)\right) + 2\left(e^{\sqrt{1}}\sin(1)\right) + 4\left(e^{\sqrt{3/2}}\sin(3/2)\right)$$

$$+2\left(e^{\sqrt{2}}\sin(2)\right) + 4\left(e^{\sqrt{5/2}}\sin(5/2)\right) + 2\left(e^{\sqrt{3}}\sin(3)\right)$$

$$+4\left(e^{\sqrt{7/2}}\sin(7/2)\right) + e^{\sqrt{4}}\sin(4)\Big]$$

$$\approx 4.67511063485\ldots$$

15. $\int_0^3 \frac{1}{1+y^5}\,dy \qquad n=6$



$$\Delta y = \frac{3-0}{6} = \frac{1}{2}$$

$$T_6 = \frac{\left(\frac{1}{2}\right)}{2}\left(\frac{1}{1+0^5} + 2\left(\frac{1}{1+(1/2)^5}\right) + 2\left(\frac{1}{1+(1)^5}\right) + 2\left(\frac{1}{1+(3/2)^5}\right)\right.$$
$$\left. + 2\left(\frac{1}{1+(2)^5}\right) + 2\left(\frac{1}{1+(5/2)^5}\right) + \frac{1}{1+(3)^5}\right)$$

$$\approx 1.06427451097$$

$$M_6 = \frac{1}{2}\left(\frac{1}{1+(1/4)^5}\right) + \frac{1}{2}\left(\frac{1}{1+(3/4)^5}\right) + \frac{1}{2}\left(\frac{1}{1+(5/4)^5}\right) + \frac{1}{2}\left(\frac{1}{1+(7/4)^5}\right)$$
$$+ \frac{1}{2}\left(\frac{1}{1+(9/4)^5}\right) + \frac{1}{2}\left(\frac{1}{1+(11/4)^5}\right)$$

$$\approx 1.06741564032$$

$$S_6 = \frac{\left(\frac{1}{2}\right)}{3}\left[\frac{1}{1+0^5} + 4\left(\frac{1}{1+(1/2)^5}\right) + 2\left(\frac{1}{1+(1)^5}\right) + 4\left(\frac{1}{1+(3/2)^5}\right)\right.$$
$$\left. + 2\left(\frac{1}{1+(2)^5}\right) + 4\left(\frac{1}{1+(5/2)^5}\right) + \frac{1}{1+(3)^5}\right]$$

$$\approx 1.0749152776\ldots$$