Chapter 6 Circular Motion and Other Applications of Newton's Laws

P6.3 (a) $F = \dfrac{mv^2}{r} = \dfrac{\left(9.11 \times 10^{-31} \text{ kg}\right)\left(2.20 \times 10^6 \text{ m/s}\right)^2}{0.530 \times 10^{-10} \text{ m}} = \boxed{8.32 \times 10^{-8} \text{ N inward}}$

(b) $a = \dfrac{v^2}{r} = \dfrac{\left(2.20 \times 10^6 \text{ m/s}\right)^2}{0.530 \times 10^{-10} \text{ m}} = \boxed{9.13 \times 10^{22} \text{ m/s}^2 \text{ inward}}$

P6.5 (a) $\boxed{\text{static friction}}$

(b) $m a \hat{\mathbf{i}} = f \hat{\mathbf{i}} + n \hat{\mathbf{j}} + mg\left(-\hat{\mathbf{j}}\right)$

$\sum F_y = 0 = n - mg$

thus $n = mg$ and $\sum F_r = m \dfrac{v^2}{r} = f = \mu n = \mu mg$.

Then $\mu = \dfrac{v^2}{rg} = \dfrac{\left(50.0 \text{ cm/s}\right)^2}{\left(30.0 \text{ cm}\right)\left(980 \text{ cm/s}^2\right)} = \boxed{0.085\,0}$.

P6.6 Neglecting relativistic effects. $F = m a_c = \dfrac{mv^2}{r}$

$F = \left(2 \times 1.661 \times 10^{-27} \text{ kg}\right) \dfrac{\left(2.998 \times 10^7 \text{ m/s}\right)^2}{\left(0.480 \text{ m}\right)} = \boxed{6.22 \times 10^{-12} \text{ N}}$

P6.8 $T \cos 5.00° = mg = \left(80.0 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)$

(a) $T = 787 \text{ N} : \quad \vec{\mathbf{T}} = \boxed{\left(68.6 \text{ N}\right)\hat{\mathbf{i}} + \left(784 \text{ N}\right)\hat{\mathbf{j}}}$

(b) $T \sin 5.00° = m a_c : \quad \boxed{a_c = 0.857 \text{ m/s}^2}$ toward the center of the circle.

The length of the wire is unnecessary information. We could, on the other hand, use it to find the radius of the circle, the speed of the bob, and the period of the motion.
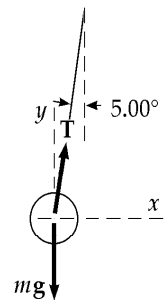

FIG. P6.8

P6.13 $M = 40.0$ kg , $R = 3.00$ m , $T = 350$ N

(a) $\sum F = 2T - Mg = \dfrac{Mv^2}{R}$

   $v^2 = (2T - Mg)\left(\dfrac{R}{M}\right)$

   $v^2 = \left[700 - (40.0)(9.80)\right]\left(\dfrac{3.00}{40.0}\right) = 23.1\ \left(\text{m}^2/\text{s}^2\right)$

   $\boxed{v = 4.81\ \text{m/s}}$

(b) $n - Mg = F = \dfrac{Mv^2}{R}$

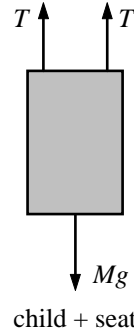 $n = Mg + \dfrac{Mv^2}{R} = 40.0\left(9.80 + \dfrac{23.1}{3.00}\right) = \boxed{700\ \text{N}}$

child + seat   child alone
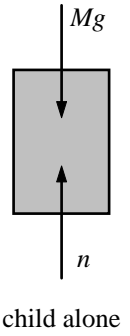
FIG. P6.13(a)   FIG. P6.13(b)

P6.14   (a) $v = 20.0$ m/s,

   $n =$ force of track on roller coaster, and

   $R = 10.0$ m .

FIG. P6.14

   $\sum F = \dfrac{Mv^2}{R} = n - Mg$

   From this we find

   $n = Mg + \dfrac{Mv^2}{R} = (500\ \text{kg})(9.80\ \text{m/s}^2) + \dfrac{(500\ \text{kg})(20.0\ \text{m/s}^2)}{10.0\ \text{m}}$

   $n = 4\,900\ \text{N} + 20\,000\ \text{N} = \boxed{2.49 \times 10^4\ \text{N}}$

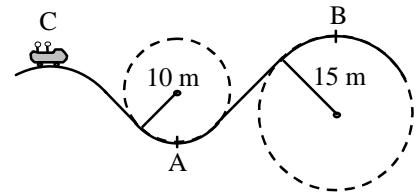(b)   At B, $n - Mg = -\dfrac{Mv^2}{R}$

   The maximum speed at B corresponds to

   $n = 0$

   $-Mg = -\dfrac{Mv_{max}^2}{R} \Rightarrow v_{max} = \sqrt{Rg} = \sqrt{15.0(9.80)} = \boxed{12.1\ \text{m/s}}$

*P6.16   (a) Consider radial forces on the object, taking inward as positive.

   $\Sigma F_r = ma_r$:

   $T - (0.5\ \text{kg})(9.8\ \text{m/s}^2)\cos 20° = mv^2/r = 0.5\ \text{kg}(8\ \text{m/s})^2/2\ \text{m}$

   $T = 4.60\ \text{N} + 16.0\ \text{N} = \boxed{20.6\ \text{N}}$

(b) We already found the radial component of acceleration,

   $(8\ \text{m/s})^2/2\ \text{m} = \boxed{32.0\ \text{m/s}^2\ \text{inward}}$.

   Consider tangential forces on the object.

   $\Sigma F_t = ma_t$:

$(0.5 \text{ kg})(9.8 \text{ m/s}^2)\sin 20 = 0.5 \text{ kg } a_t$

$a_t = \boxed{3.35 \text{ m/s}^2 \text{ downward tangent to the circle}}$

(c) $a = [32^2 + 3.35^2]^{1/2} \text{ m/s}^2$ inward and below the cord at angle $\tan^{-1}(3.35/32)$

$= \boxed{32.2 \text{ m/s}^2 \text{ inward and below the cord at } 5.98°}$

(d) No change. If the object is swinging down it is gaining speed. If the object is swinging up it is losing speed but its acceleration is the same size and its direction can be described in the same terms.

P6.19   (a) $\sum F_x = Ma$,

$a = \dfrac{T}{M} = \dfrac{18.0 \text{ N}}{5.00 \text{ kg}} = \boxed{3.60 \text{ m/s}^2}$ to the right.

(b)   If $v =$ const, $a = 0$, so $\boxed{T = 0}$ (This is also an equilibrium situation.)

(c)   Someone in the car (noninertial observer) claims that the forces on the mass along x are T and a fictitious force $(-Ma)$. Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x-direction.
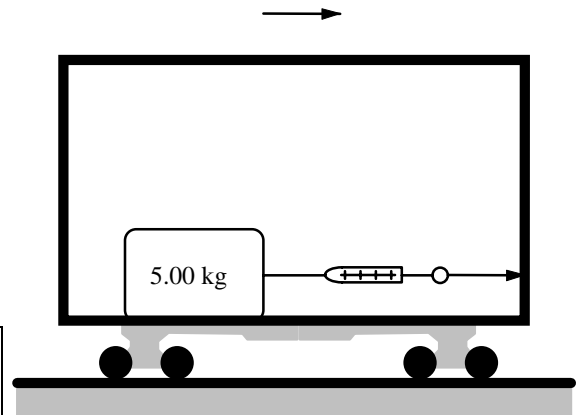

FIG. P6.19

P6.23 $F_{max} = F_g + ma = 591 \text{ N}$
$F_{min} = F_g - ma = 391 \text{ N}$

(a)   Adding, $2F_g = 982 \text{ N}$, $F_g = \boxed{491 \text{ N}}$

(b)   Since $F_g = mg$, $m = \dfrac{491 \text{ N}}{9.80 \text{ m/s}^2} = \boxed{50.1 \text{ kg}}$

(c)   Subtracting the above equations,

$2ma = 200 \text{ N}$ $\therefore a = \boxed{2.00 \text{ m/s}^2}$

P6.25 $a_r = \left(\dfrac{4\pi^2 R_e}{T^2}\right)\cos 35.0° = 0.027\ 6\ \text{m/s}^2$

We take the y axis along the local vertical.

$\left(a_{net}\right)_y = 9.80 - \left(a_r\right)_y = 9.77\ \text{m/s}^2$

$\left(a_{net}\right)_x = 0.015\ 8\ \text{m/s}^2$

$\theta = \arctan\dfrac{a_x}{a_y} = \boxed{0.092\ 8°}$

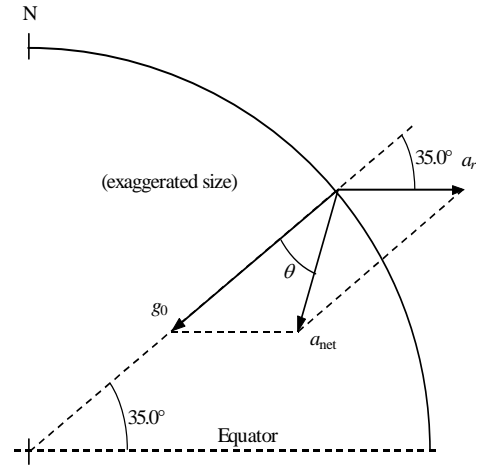**FIG. P6.25**

P6.31 (a)   At terminal velocity, $R = v_T b = mg$

$$\therefore b = \frac{mg}{v_T} = \frac{\left(3.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{2.00\times10^{-2}\ \text{m/s}} = \boxed{1.47\ \text{N}\cdot\text{s/m}}$$

(b)   In the equation describing the time variation of the velocity, we have

$v = v_T\left(1 - e^{-bt/m}\right)$ $v = 0.632 v_T$ when $e^{-bt/m} = 0.368$

or at time $t = -\left(\dfrac{m}{b}\right)\ln\left(0.368\right) = \boxed{2.04\times10^{-3}\ \text{s}}$

(c)   At terminal velocity, $R = v_T b = mg = \boxed{2.94\times10^{-2}\ \text{N}}$

P6.33 $v = \left(\dfrac{mg}{b}\right)\left[1 - \exp\left(\dfrac{-bt}{m}\right)\right]$ where $\exp(x) = e^x$ is the exponential function.

At $t \to \infty$ $v \to v_T = \dfrac{mg}{b}$

At $t = 5.54\ \text{s}$ $0.500 v_T = v_T\left[1 - \exp\left(\dfrac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right)\right]$

$\exp\left(\dfrac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right) = 0.500$

$\dfrac{-b(5.54\ \text{s})}{9.00\ \text{kg}} = \ln 0.500 = -0.693$

$b = \dfrac{\left(9.00\ \text{kg}\right)\left(0.693\right)}{5.54\ \text{s}} = 1.13\ \text{kg/s}$

(a) $v_T = \dfrac{mg}{b}$ $v_T = \dfrac{\left(9.00\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{1.13\ \text{kg/s}} = \boxed{78.3\ \text{m/s}}$

(b) $\qquad 0.750v_T = v_T\left[1 - \exp\left(\dfrac{-1.13t}{9.00\text{ s}}\right)\right]$ $\qquad \exp\left(\dfrac{-1.13t}{9.00\text{ s}}\right) = 0.250$

$$t = \dfrac{9.00(\ln 0.250)}{-1.13}\text{ s} = \boxed{11.1\text{ s}}$$

(c) $\qquad \dfrac{dx}{dt} = \left(\dfrac{mg}{b}\right)\left[1 - \exp\left(-\dfrac{bt}{m}\right)\right];\quad \displaystyle\int_{x_0}^{x} dx = \int_{0}^{t}\left(\dfrac{mg}{b}\right)\left[1 - \exp\left(\dfrac{-bt}{m}\right)\right]dt$

$$x - x_0 = \dfrac{mgt}{b} + \left(\dfrac{m^2 g}{b^2}\right)\exp\left(\dfrac{-bt}{m}\right)\Big|_0^t = \dfrac{mgt}{b} + \left(\dfrac{m^2 g}{b^2}\right)\left[\exp\left(\dfrac{-bt}{m}\right) - 1\right]$$

At $t = 5.54$ s,

$$x = 9.00\text{ kg}(9.80\text{ m/s}^2)\dfrac{5.54\text{ s}}{1.13\text{ kg/s}} + \left(\dfrac{(9.00\text{ kg})^2(9.80\text{ m/s}^2)}{(1.13\text{ kg/s})^2}\right)\left[\exp(-0.693) - 1\right]$$

$$x = 434\text{ m} + 626\text{ m}(-0.500) = \boxed{121\text{ m}}$$

P6.33 $\qquad v = \left(\dfrac{mg}{b}\right)\left[1 - \exp\left(\dfrac{-bt}{m}\right)\right]$ where $\exp(x) = e^x$ is the exponential function.

At $t \to \infty$ $\qquad\qquad\qquad\qquad\qquad v \to v_T = \dfrac{mg}{b}$

At $t = 5.54$ s $\qquad\qquad\qquad\qquad 0.500v_T = v_T\left[1 - \exp\left(\dfrac{-b(5.54\text{ s})}{9.00\text{ kg}}\right)\right]$

$$\exp\left(\dfrac{-b(5.54\text{ s})}{9.00\text{ kg}}\right) = 0.500$$

$$\dfrac{-b(5.54\text{ s})}{9.00\text{ kg}} = \ln 0.500 = -0.693$$

$$b = \dfrac{(9.00\text{ kg})(0.693)}{5.54\text{ s}} = 1.13\text{ kg/s}$$

(a) $\qquad v_T = \dfrac{mg}{b}$ $\qquad\qquad\qquad\qquad v_T = \dfrac{(9.00\text{ kg})(9.80\text{ m/s}^2)}{1.13\text{ kg/s}} = \boxed{78.3\text{ m/s}}$

(b) $\qquad 0.750v_T = v_T\left[1 - \exp\left(\dfrac{-1.13t}{9.00\text{ s}}\right)\right]$ $\qquad \exp\left(\dfrac{-1.13t}{9.00\text{ s}}\right) = 0.250$

$$t = \dfrac{9.00(\ln 0.250)}{-1.13}\text{ s} = \boxed{11.1\text{ s}}$$

(c) $\qquad \dfrac{dx}{dt} = \left(\dfrac{mg}{b}\right)\left[1 - \exp\left(-\dfrac{bt}{m}\right)\right];\quad \displaystyle\int_{x_0}^{x} dx = \int_{0}^{t}\left(\dfrac{mg}{b}\right)\left[1 - \exp\left(\dfrac{-bt}{m}\right)\right]dt$

$$x - x_0 = \dfrac{mgt}{b} + \left(\dfrac{m^2 g}{b^2}\right)\exp\left(\dfrac{-bt}{m}\right)\Big|_0^t = \dfrac{mgt}{b} + \left(\dfrac{m^2 g}{b^2}\right)\left[\exp\left(\dfrac{-bt}{m}\right) - 1\right]$$

At $t = 5.54$ s,

$$x = 9.00 \text{ kg}\left(9.80 \text{ m/s}^2\right)\frac{5.54 \text{ s}}{1.13 \text{ kg/s}} + \left(\frac{\left(9.00 \text{ kg}\right)^2 \left(9.80 \text{ m/s}^2\right)}{\left(1.13 \text{ kg/s}\right)^2}\right)\left[\exp\left(-0.693\right) - 1\right]$$

$$x = 434 \text{ m} + 626 \text{ m}\left(-0.500\right) = \boxed{121 \text{ m}}$$

P6.42   When the cloth is at a lower angle θ, the radial component of $\sum F = ma$ reads

$$n + mg\sin\theta = \frac{mv^2}{r}$$
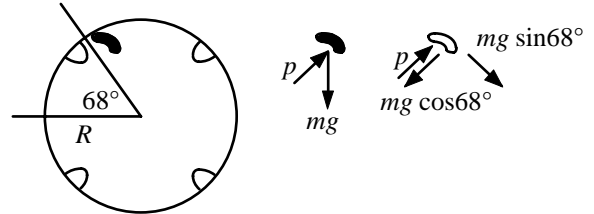
FIG. P6.42

At $\theta = 68.0°$, the normal force drops to zero and $g\sin 68° = \dfrac{v^2}{r}$.

$$v = \sqrt{rg\sin 68°} = \sqrt{\left(0.33 \text{ m}\right)\left(9.8 \text{ m/s}^2\right)\sin 68°} = 1.73 \text{ m/s}$$

The rate of revolution is

$$\text{angular speed} = \left(1.73 \text{ m/s}\right)\left(\frac{1 \text{ rev}}{2\pi r}\right)\left(\frac{2\pi r}{2\pi\left(0.33 \text{ m}\right)}\right) = \boxed{0.835 \text{ rev/s}} = 50.1 \text{ rev/min}$$

*P6.50 (a)   Since the object of mass m_2 is in equilibrium, $\sum F_y = T - m_2 g = 0$

or $T = \boxed{m_2 g}$

(b)   The tension in the string provides the required centripetal acceleration of the puck.

Thus, $F_c = T = \boxed{m_2 g}$

(c)   From $F_c = \dfrac{m_1 v^2}{R}$

we have $v = \sqrt{\dfrac{RF_c}{m_1}} = \boxed{\sqrt{\left(\dfrac{m_2}{m_1}\right)gR}}$

(d) $\boxed{\text{The puck will spiral inward, gaining speed as it does so.}}$ It gains speed because the extra-large string tension produces forward tangential acceleration as well as inward radial acceleration of the puck, pulling at an angle of less than 90° to the direction of the inward-spiraling velocity.

(e) $\boxed{\text{The puck will spiral outward, slowing down as it does so.}}$

*P6.55　(a)　　$n = \dfrac{mv^2}{R}$　　　　　　　　$f - mg = 0$

　　　　　　　$f = \mu_s n$　　　　　　　　$v = \dfrac{2\pi R}{T}$

　　　　　　　$T = \sqrt{\dfrac{4\pi^2 R \mu_s}{g}}$

　　(b)　　$T = \boxed{2.54 \text{ s}}$

　　　　　　$\#\ \dfrac{\text{rev}}{\text{min}} = \dfrac{1 \text{ rev}}{2.54 \text{ s}}\left(\dfrac{60 \text{ s}}{\text{min}}\right) = \boxed{23.6\ \dfrac{\text{rev}}{\text{min}}}$
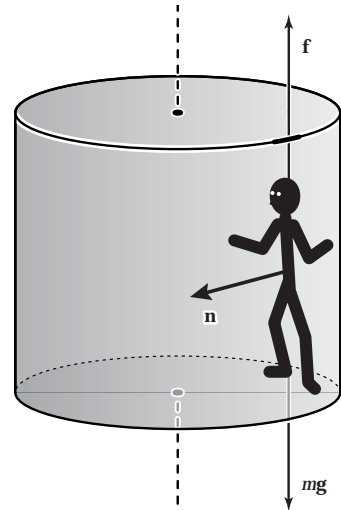

FIG. P6.55

(c) The gravitational and frictional forces remain constant. The normal force increases. The person
　　remains in motion with the wall.

(d) The gravitational force remains constant. The normal and frictional forces decrease. The person
　　slides relative to the wall and downward into the pit.

P6.57   (a)   If the car is about to slip *down* the incline, f is directed up the incline.

$$\sum F_y = n\cos\theta + f\sin\theta - mg = 0 \text{ where } f = \mu_s n \text{ gives}$$

$$n = \frac{mg}{\cos\theta(1+\mu_s\tan\theta)} \text{ and } f = \frac{\mu_s mg}{\cos\theta(1+\mu_s\tan\theta)}$$

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\dfrac{v_{min}^2}{R}$ yields

$$\boxed{v_{min} = \sqrt{\frac{Rg(\tan\theta - \mu_s)}{1+\mu_s\tan\theta}}}$$

When the car is about to slip *up* the incline, f is directed down the incline. Then, $\sum F_y = n\cos\theta - f\sin\theta - mg = 0$ with $f = \mu_s n$ yields

$$n = \frac{mg}{\cos\theta(1-\mu_s\tan\theta)} \text{ and } f = \frac{\mu_s mg}{\cos\theta(1-\mu_s\tan\theta)}$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\dfrac{v_{max}^2}{R}$, which gives

$$\boxed{v_{max} = \sqrt{\frac{Rg(\tan\theta + \mu_s)}{1-\mu_s\tan\theta}}}$$

(b)   If $v_{min} = \sqrt{\dfrac{Rg(\tan\theta - \mu_s)}{1+\mu_s\tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.

(c) $v_{min} = \sqrt{\dfrac{(100 \text{ m})(9.80 \text{ m/s}^2)(\tan 10.0° - 0.100)}{1+(0.100)\tan 10.0°}} = \boxed{8.57 \text{ m/s}}$

$v_{max} = \sqrt{\dfrac{(100 \text{ m})(9.80 \text{ m/s}^2)(\tan 10.0° + 0.100)}{1-(0.100)\tan 10.0°}} = \boxed{16.6 \text{ m/s}}$
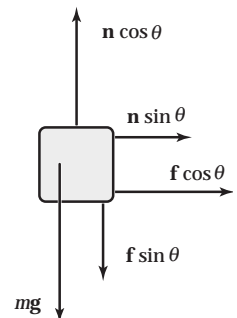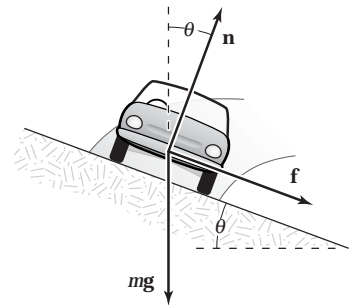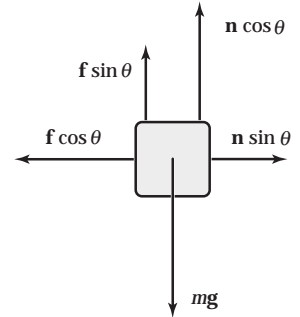

FIG. P6.57