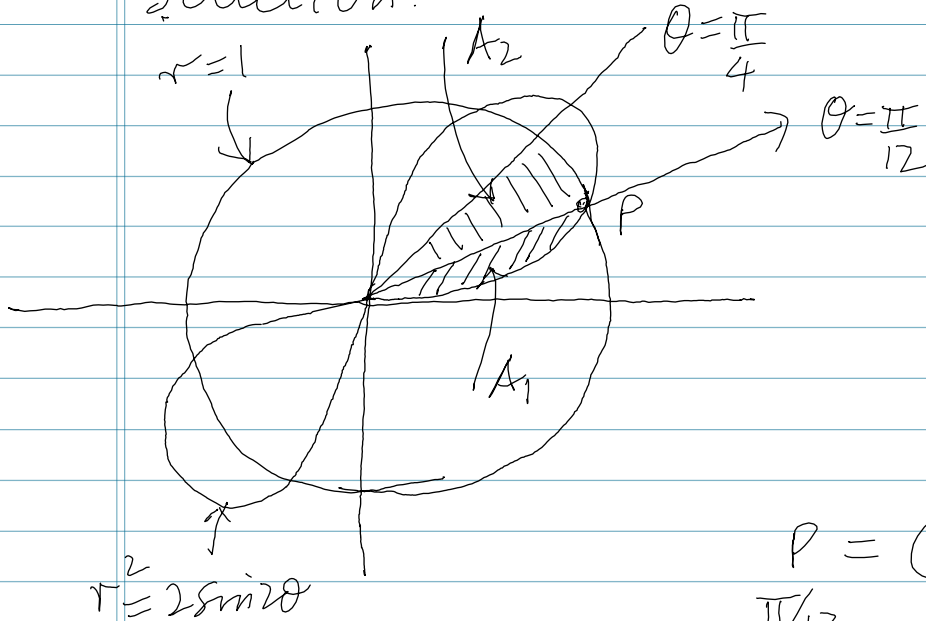


Example: Find the area enclosed by the curve $r=1$ and $r^2=2\sin 2\theta$.
 Solution.



$$\begin{aligned} 2\sin 2\theta &= 1 \\ \sin 2\theta &= \frac{1}{2} \end{aligned}$$

$$2\theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

$$\theta = \frac{\pi}{12} \text{ or } \frac{5\pi}{12}$$

$$P = (1, \pi/12)$$

$$\text{Area}(A_1) = \frac{1}{2} \int_0^{\pi/12} 2\sin 2\theta d\theta$$

$$\begin{aligned} &= \left. -\frac{\cos 2\theta}{2} \right|_0^{\pi/12} = -\frac{\cos(\pi/6)}{2} + \frac{\cos(0)}{2} \\ &= \frac{2 - \sqrt{3}}{4} \end{aligned}$$

$$\begin{aligned} \text{Area}(A_2) &= \frac{1}{2} \int_{\pi/12}^{\pi/4} 2 d\theta = \frac{1}{2} [\theta]_{\pi/12}^{\pi/4} \\ &= \frac{1}{2} \left[\frac{\pi}{4} - \frac{\pi}{12} \right] \end{aligned}$$

$$= \frac{\pi}{12}$$

$$\text{Area} = 4 \left(\text{Area}(A_1) + \text{Area}(A_2) \right)$$

$$= 4 \left[\frac{2 - \sqrt{3}}{4} + \frac{\pi}{12} \right] = 2 - \sqrt{3} + \frac{\pi}{3}$$