Examples with conditional statements.

Conditional statement as a disjunction

$\boldsymbol{P \rightarrow Q \equiv (\neg P \vee Q)}$**, where $\boldsymbol{P}$ is the hypothesis and $\boldsymbol{Q}$ is the conclusion**.

Express the following conditional statements as a disjunction.

Definition: A real number x is said to be rational if there are integers a and $b \neq 0$ such that $x = \frac{a}{b}$.
A real number x is said to be irrational if it is not rational.

- **Conditional: If x and y are both rational numbers then x+y is also a rational number .**
- **Disjunction form: x is an irrational or y is an irrational number or x+y is a rational number.**

- **Conditional: If it is not raining then I go to the beach or I go hiking. (I could do both.)**
- **Disjunction form: It is raining or I go to the beach or I go hiking.**

- Conditional: ∀ **x** ∀ **y(xy=0** →**(x=0** ∨ **y=0)), where the domain of discourse is the set of real numbers.**
- **Disjunction Form**: ∀ **x** ∀ **y(xy≠0**∨**(x=0** ∨ **y=0))**

Negation of a conditional statement

$\neg(P \rightarrow Q) \equiv \neg(\neg P \vee Q) \equiv (P \wedge \neg Q)$, **where P is the hypothesis and Q is the conclusion.**

Express the negation of the following conditional statements.

- **Conditional: If x and y are both rational numbers then x+y is also a rational number .**
- **Negation: x and y are both rational numbers and x+y is an irrational number.**

- **Conditional: If it is not raining then I go to the beach or I go hiking. (I could do both.)**
- **Negation: It is not raining and I do not go to the beach and I do not go hiking.**

- **Conditional: $\forall x \forall y(xy=0 \rightarrow (x=0 \vee y=0))$, where the domain of discourse is the set of real numbers.**
- Negation: $\exists x \exists y(xy=0 \wedge (x \neq 0 \wedge y \neq 0))$

Conditional statement and its contrapositive

$\neg Q \rightarrow \neg P$ is the contrapositive of the conditional $P \rightarrow Q$, where P is the hypothesis and Q is the conclusion of the original conditional. Note that, $P \rightarrow Q \equiv \neg Q \rightarrow \neg P$, that is the conditional is equivalent to its contrapositive.

Express the **contrapositive** of the following conditional statements.

- **Conditional: If x and y are both rational numbers then x+y is also a rational number .**
- **Contrapositive: x+y is an irrational number then x or y is an irrational number.**

- **Conditional: If it is not raining then I go to the beach or I go hiking. (I could do both.)**
- **Contrapositive: If I do not go to the beach and I do not go hiking then it is raining.**

- **Conditional:** $\forall x \forall y(xy=0 \rightarrow (x=0 \vee y=0))$**, where the domain of discourse is the set of real numbers.**
- **Contrapositive:** $\forall x \forall y((x \neq 0 \wedge y \neq 0) \rightarrow xy \neq 0)$

Conditional statement and its converse

$Q \to P$ is the converse of the conditional $P \to Q$, where P is the hypothesis and Q is the conclusion of the original conditional. Note that, $P \to Q$ is NOT equivalent to $Q \to P$, that is the conditional is NOT equivalent to its converse.

Express the **converse** of the following conditional statements.

- **Conditional: If x and y are both rational numbers then x+y is also a rational number .**
- **Converse: If x+y is a rational number then x and y are both rational numbers.**

- **Conditional: If it is not raining then I go to the beach or I go hiking. (I could do both.)**
- **Converse: If I go to the beach or I go hiking then it is not raining.**

- **Conditional: ∀ x ∀ y(xy=0 →(x=0 ∨ y=0)), where the domain of discourse is the set of real numbers.**
- **Converse: ∀ x ∀ y((x=0 ∨ y=0) → xy = 0)**

Conditional statement and its inverse

$\neg P \to \neg Q$ is the inverse of the conditional $P \to Q$, where P is the hypothesis and Q is the conclusion of the original conditional. Note that, $P \to Q$ is NOT equivalent to $\neg P \to \neg Q$, that is the conditional is NOT equivalent to its inverse. However, $Q \to P \equiv \neg P \to \neg Q$, that is the converse of $P \to Q$ is equivalent to the inverse of $P \to Q$. In fact, the inverse is the contrapositive of the converse.

Express the **inverse** of the following conditional statements.

- **Conditional: If x and y are both rational numbers then x+y is also a rational number .**
- **Inverse: If x or y is an irrational number then x+y is an irrational number.**

- **Conditional: If it is not raining then I go to the beach or I go hiking. (I could do both.)**
- **Inverse: If it is raining then I do not go to the beach and I do not go hiking.**

- **Conditional: ∀ x ∀ y(xy=0 →(x=0 ∨ y=0)), where the domain of discourse is the set of real numbers.**
- **Inverse**: **∀ x ∀ y(xy ≠ 0 →(x≠0 ∧ y ≠ 0))**