MAT 243 Online Written Homework Assignments for Week 4 (units 10-13)

Multiple Choice Question

1. Using fast modular exponentiation, how many operations (squaring and mod count as one operation) does it take to compute $11^{2^{500}} \bmod 19$?

    a. 18
    b. 19
    c. 500
    d. 2^{500}
    e. $11^{2^{500}}$

Free Response Questions

2. True or false? If the statement is true, give an explanation. If it is false, give a counterexample.

    "If $f_1(x)$ is of order $g(x)$, and $f_2(x)$ is of order $g(x)$, then $f_1(x) + f_2(x)$ is of order $g(x)$ too."

3. Compute $3^{1048576} \bmod 7$ using fast modular exponentiation. Hint: $1048576 = 2^{20}$. Show all your steps.

4. Find the prime factorization of $2^{16} - 1$. (A mere calculator solution based on evaluation of $2^{16} - 1$ is unacceptable. Hint: factor the expression.)

5. Prove: if n is a natural number, then $2^{2n} - 1$ must be divisible by 3.

6. Given the base-k expansion of a natural number n, how can we tell whether n is divisible by k? You must prove your answer, based on the definition of base-k expansion.

7. In the lectures, we discussed an algorithm for primality testing. For testing whether a natural number n is prime, the number of trial divisions needed for this algorithm is of order n^p for some real number p. Determine p and explain your answer. You do not need to give an exact inequality- based proof based on the definition of order. Using reasonable approximations is enough.