Mathematical Induction

Use induction to prove that $3^n > n^2$ for all positive integers $n \geq 3$.

Let $P(n)$ denote the proposition that $3^n > n^2$, where n is a positive integer $n \geq 3$.

BASIS STEP: $P(3)$ is true since $27 > 9$.

INDUCTIVE STEP: Let us assume $P(n)$, that is $3^n > n^2$ is true for an arbitrary positive integer $n \geq 3$. This is our inductive hypothesis.

We have to show that $P(n+1)$, $3^{n+1} > (n+1)^2$ is true assuming the inductive hypothesis $P(n)$.

Proof:

$3^{n+1} = 3 \cdot 3^n > 3 \cdot n^2$ using the inductive hypothesis**.**

$3 \cdot n^2 = n^2 + 2n^2 > n^2 + 2n + 1 = (n+1)^2,$ when $n \geq 3$.

$2n^2 > 2n + 1, \text{since } 2n^2 - 2n = 2n(\text{n} - 1) > 1, \text{when } n \geq 3.$

By the **Principle of Mathematical Induction** (Basis Step and Inductive Step together) $3^n > n^2$ for all positive integers $n \geq 3$.