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1

(a) The power set of S, denoted $\mathcal{P}$(S), is the set of all subsets of the set S.

(b) $A - B = \{x|x \in A \wedge x \notin B\}$

(c) A function $f: A \rightarrow B$ is one-on-one if for each b ∈ B there is at most one a ∈ A with $f\ (a)\ =\ b.$

(d) A sequence is a function from a subset of the set of integers to a set S.

2

$\mathcal{P}$(A)= $\{\emptyset, \{1\}, \{2\}, \{1,2\}\}$, $\mathcal{P}$(B)=$\{\emptyset, \{2\}, \{3\}, \{2,3\}\}$,

$\mathcal{P}(A) \cup \mathcal{P}(B) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\} \cup \{\emptyset, \{2\}, \{3\}, \{2,3\}\}$

$= \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}\}$

$\mathcal{P}(A \cup B) = \mathcal{P}(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

My conjecture: $\mathcal{P}(A) \cup \mathcal{P}(B) \neq \mathcal{P}(A \cup B)$

3

$$Suppose\ f(n) = f(x) \rightarrow (2n + 3)$$

$$= (2x + 3) \rightarrow (2n = 2x) \rightarrow (n = x)$$

$$\therefore f\ is\ one - to - one\quad function$$

$$\exists y \in \mathbb{Z} | y = -2$$

$$Then, (2n + 3) = \ y = (-2)$$

$$2n = -5$$

$$n = \frac{-5}{2}$$

$$n \notin \mathbb{Z}$$

$$\therefore f\ is\ not\ onto$$

4

$$g(n) = \left\lceil \frac{n-1}{2} \right\rceil$$

By definition of the ceiling function, for any $y \in \mathbb{Z}$ we can also find

any $n \in \mathbb{Z} | g(n) = y$

Thus, $g(1) = \frac{1-1}{2} = 0$

$g(2) = \frac{2-1}{2} = \frac{1}{2}$

$g(1) \neq g(2) \rightarrow g \text{ is not one} - \text{to} - \text{one}$

$\exists y \in \mathbb{Z} | y = -2$

$$Then, \left\lceil \frac{n-1}{2} \right\rceil = y = (-2)$$

$n - 1 = -4$

$n = -3$

$n \in \mathbb{Z}$

$\therefore g \text{ is onto}$

5

$\sum_{k=2}^{50} (k+3)^2 = (2+3)^2 + (3+3)^2 + (4+3)^2 + \cdots + (50+3)^2$

$= 5^2 + 6^2 + 7^2 + \cdots + 53^2$

$= (1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + \cdots + 53^2) - (1^2 + 2^2 + 3^2 + 4^2)$

$$= \frac{(53)(53+1)((2 \times 53)+1)}{6} - (1 + 4 + 9 + 16)$$

$$= \frac{(53)(54)(107)}{6} - (30)$$

$$= \left((53) \times (9) \times (107)\right) - (30)$$

$$= 51039 - 30 = 51009$$

$$\therefore \sum_{k=2}^{50} (k+3)^2 = 51009$$