Chapter 12 Static Equilibrium and Elasticity

P12.1 Take torques about $\boldsymbol{P}$.

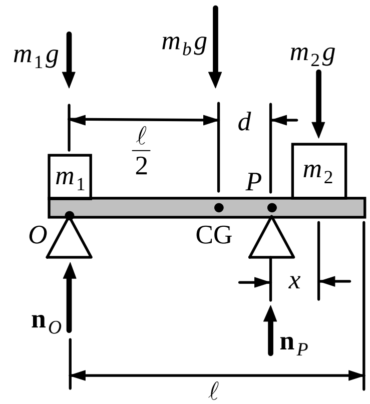

FIG. P12.1

$$\sum \tau_p = -n_0\left[\frac{\ell}{2}+d\right]+m_1g\left[\frac{\ell}{2}+d\right]+m_bgd-m_2gx=0$$

We want to find x for which $n_0 = 0$.

$$x=\frac{(m_1g+m_bg)d+m_1g\frac{\ell}{2}}{m_2g}=\boxed{\frac{(m_1+m_b)d+m_1\frac{\ell}{2}}{m_2}}$$

P12.2 Use distances, angles, and forces as shown. The conditions of equilibrium are:

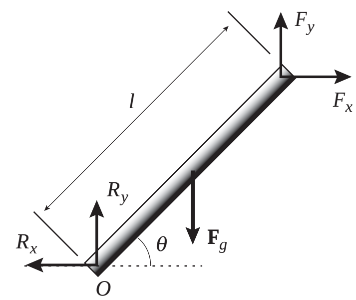

FIG. P12.2

$$\sum F_y = 0 \Rightarrow \boxed{F_y+R_y-F_g=0}$$

$$\sum F_x = 0 \Rightarrow \boxed{F_x-R_x=0}$$

$$\sum \tau = 0 \Rightarrow \boxed{F_y\ell\cos\theta-F_g\left(\frac{\ell}{2}\right)\cos\theta-F_x\ell\sin\theta=0}$$

P12.4 The hole we can count as negative mass

$$x_{CG}=\frac{m_1x_1-m_2x_2}{m_1-m_2}$$

Call σ the mass of each unit of pizza area.

$$x_{CG}=\frac{\sigma\pi R^2 0-\sigma\pi\left(\frac{R}{2}\right)^2\left(-\frac{R}{2}\right)}{\sigma\pi R^2-\sigma\pi\left(\frac{R}{2}\right)^2}$$

$$x_{CG}=\frac{R/8}{3/4}=\boxed{\frac{R}{6}}$$

P12.6 Let σ represent the mass-per-face area. A vertical strip at position x, with width dx and height $\dfrac{(x-3.00)^2}{9}$ has mass

$$dm = \frac{\sigma(x-3.00)^2\,dx}{9}$$

The total mass is

$$M = \int dm = \int_{x=0}^{3.00} \frac{\sigma(x-3)^2\,dx}{9}$$

$$M = \left(\frac{\sigma}{9}\right)\int_0^{3.00}\left(x^2-6x+9\right)dx$$

$$M = \left(\frac{\sigma}{9}\right)\left[\frac{x^3}{3}-\frac{6x^2}{2}+9x\right]_0^{3.00} = \sigma$$

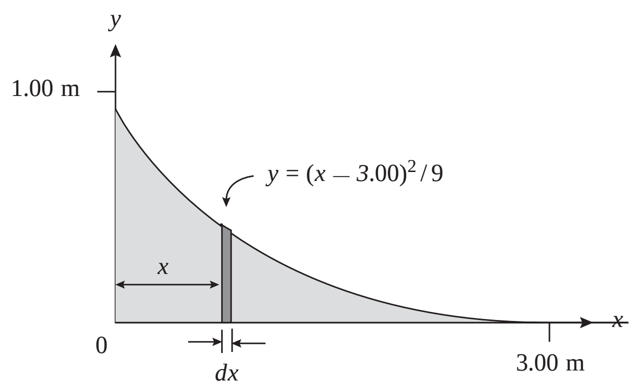

FIG. P12.6

The x-coordinate of the center of gravity is

$$x_{\mathrm{CG}} = \frac{\int x\,dm}{M} = \frac{1}{9\sigma}\int_0^{3.00}\sigma x(x-3)^2\,dx = \frac{\sigma}{9\sigma}\int_0^{3.00}\left(x^3-6x^2+9x\right)dx$$

$$= \frac{1}{9}\left[\frac{x^4}{4}-\frac{6x^3}{3}+\frac{9x^2}{2}\right]_0^{3.00} = \frac{6.75\text{ m}}{9.00} = \boxed{0.750\text{ m}}$$

P12.16 Relative to the hinge end of the bridge, the cable is attached horizontally out a distance $x=(5.00\text{ m})\cos 20.0° = 4.70\text{ m}$ and vertically down a distance $y=(5.00\text{ m})\sin 20.0° = 1.71\text{ m}$. The cable then makes the following angle with the horizontal:

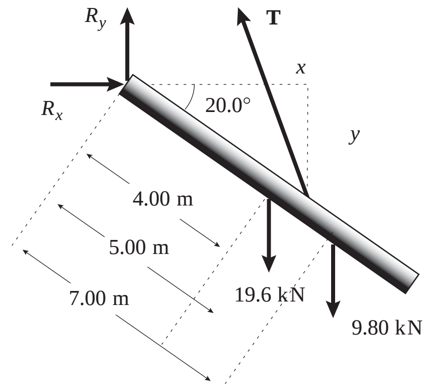

FIG. P12.16

$$\theta = \tan^{-1}\left[\frac{(12.0+1.71)\text{ m}}{4.70\text{ m}}\right] = 71.1°$$

(a) Take torques about the hinge end of the bridge:

$$R_x(0)+R_y(0)-19.6\text{ kN}(4.00\text{ m})\cos 20.0°$$
$$-T\cos 71.1°(1.71\text{ m})+T\sin 71.1°(4.70\text{ m})$$
$$-9.80\text{ kN}(7.00\text{ m})\cos 20.0° = 0$$

which yields $T=\boxed{35.5\text{ kN}}$

(b) $\sum F_x = 0 \Rightarrow R_x - T\cos 71.1° = 0$

or $R_x=(35.5\text{ kN})\cos 71.1° = \boxed{11.5\text{ kN (right)}}$

(c) $\sum F_y = 0 \Rightarrow R_y - 19.6\text{ kN} + T\sin 71.1° - 9.80\text{ kN} = 0$

Thus,

$$R_y = 29.4\text{ kN} - (35.5\text{ kN})\sin 71.1° = -4.19\text{ kN}$$
$$= \boxed{4.19\text{ kN down}}$$

P12.18 Call the required force F, with components $F_x = F\cos 15.0°$ and $F_y = -F\sin 15.0°$, transmitted to the center of the wheel by the handles.

Just as the wheel leaves the ground, the ground exerts no force on it.

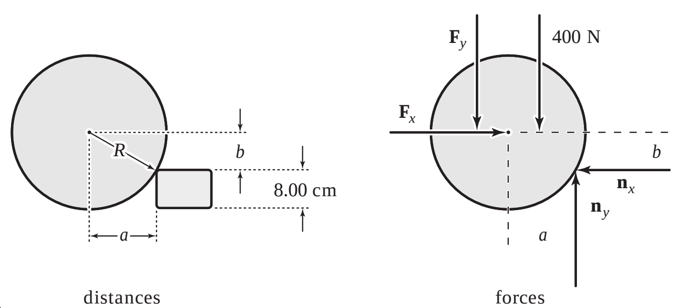

FIG. P12.18

$$\sum F_x = 0:\ F\cos 15.0° - n_x \quad (1)$$
$$\sum F_y = 0:\ -F\sin 15.0° - 400\text{ N} + n_y = 0 \quad (2)$$

Take torques about its contact point with the brick. The needed distances are seen to be:

$$b = R - 8.00\text{ cm} = (20.0-8.00)\text{ cm} = 12.0\text{ cm}$$
$$a = \sqrt{R^2-b^2} = 16.0\text{ cm}$$

(a) $\sum \tau = 0: \; -F_x b + F_y a + (400 \text{ N})a = 0$, or

$$F\left[-(12.0 \text{ cm})\cos 15.0° + (16.0 \text{ cm})\sin 15.0°\right] + (400 \text{ N})(16.0 \text{ cm}) = 0$$

so

$$F = \frac{6\,400 \text{ N} \cdot \text{cm}}{7.45 \text{ cm}} = \boxed{859 \text{ N}}$$

(b) Then, using Equations (1) and (2),

$n_x = (859 \text{ N})\cos 15.0° = 830 \text{ N}$ and

$n_y = 400 \text{ N} + (859 \text{ N})\sin 15.0° = 622 \text{ N}$

$$n = \sqrt{n_x^2 + n_y^2} = \boxed{1.04 \text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{n_y}{n_x}\right) = \tan^{-1}(0.749) = \boxed{36.9° \text{ to the left and upward}}$$

P12.19 When $x = x_{\text{min}}$, the rod is on the verge of slipping, so

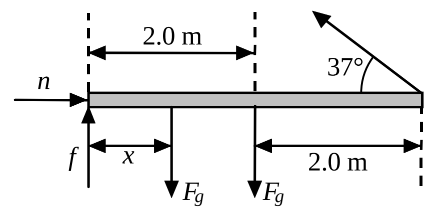

FIG. P12.19

$$f = (f_s)_{\text{max}} = \mu_s n = 0.50n$$

From $\sum F_x = 0$, $n - T\cos 37° = 0$, or $n = 0.799T$.

Thus,

$$f = 0.50(0.799T) = 0.399T$$

From $\sum F_y = 0$, $f + T\sin 37° - 2F_g = 0$,

or

$0.399T - 0.602T - 2F_g = 0$, giving $T = 2.00F_g$

Using $\sum \tau = 0$ for an axis perpendicular to the page and through the left end of the beam gives $-F_g \cdot x_{\text{min}} - F_g(2.0 \text{ m}) + \left[(2F_g)\sin 37°\right](4.0 \text{ m}) = 0$, which reduces to $x_{\text{min}} = \boxed{2.82 \text{ m}}$

P12.21 To find *U*, measure distances and forces from point A. Then, balancing torques,

$$(0.750)U = 29.4(2.25) \qquad \boxed{U = 88.2 \text{ N}}$$

To find *D*, measure distances and forces from point B. Then, balancing torques,

$$(0.750)D = (1.50)(29.4) \qquad \boxed{D = 58.8 \text{ N}}$$

Also, notice that $U = D + F_g$, so $\sum F_y = 0$.

P12.39 Using $\sum F_x = \sum F_y = \sum \tau = 0$, choosing the origin at the left end of the beam, we have (neglecting the weight of the beam)

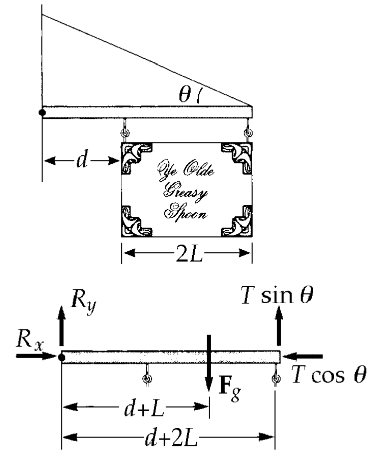

FIG. P12.39

$$\sum F_x = R_x - T\cos\theta = 0$$
$$\sum F_y = R_y + T\sin\theta - F_g = 0$$

and $\sum \tau = -F_g(L+d) + T\sin\theta(2L+d) = 0$.

Solving these equations, we find:

(a) $$T = \boxed{\frac{F_g(L+d)}{\sin\theta(2L+d)}}$$

(b) $$R_x = \boxed{\frac{F_g(L+d)\cot\theta}{2L+d}} \qquad R_y = \boxed{\frac{F_g L}{2L+d}}$$

***P12.50** Considering the torques about the point at the bottom of the bracket yields:

$$W(0.050\,0\text{ m}) - F(0.060\,0\text{ m}) = 0 \text{ so } F = 0.833W$$

(a) With $W = 80.0$ N, $F = 0.833(80\text{ N}) = \boxed{66.7\text{ N}}$.

(b) Differentiate with respect to time: $dF/dt = 0.833\, dW/dt$

The force exerted by the screw is increasing at the rate $dF/dt = 0.833(0.15\text{ N/s}) =$ $\boxed{0.125\text{ N/s}}$

P12.51 From geometry, observe that

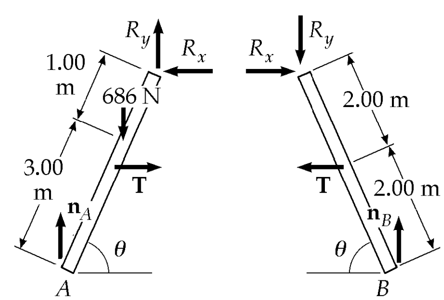

FIG. P12.51

$$\cos\theta = \frac{1}{4} \qquad \text{and} \qquad \theta = 75.5°$$

For the left half of the ladder, we have

$$\sum F_x = T - R_x = 0 \qquad (1)$$
$$\sum F_y = R_y + n_A - 686\text{ N} = 0 \qquad (2)$$
$$\sum \tau_{\text{top}} = 686\text{ N}(1.00\cos 75.5°) + T(2.00\sin 75.5°)$$
$$-n_A(4.00\cos 75.5°) = 0 \qquad (3)$$

For the right half of the ladder we have

$$\sum F_x = R_x - T = 0$$
$$\sum F_y = n_B - R_y = 0 \qquad (4)$$
$$\sum \tau_{\text{top}} = n_B(4.00\cos 75.5°) - T(2.00\sin 75.5°) = 0 \qquad (5)$$

Solving equations 1 through 5 simultaneously yields:

(a) $\boxed{T = 133 \text{ N}}$

(b) $\boxed{n_A = 429 \text{ N}}$ and $\boxed{n_B = 257 \text{ N}}$

(c) $\boxed{R_x = 133 \text{ N}}$ and $\boxed{R_y = 257 \text{ N}}$

The force exerted by the left half of the ladder on the right half is to the right and downward.

P12.60 When the car is on the point of rolling over, the normal force on its inside wheels is zero.

$$\sum F_y = ma_y: \quad n - mg = 0$$

$$\sum F_x = ma_x: \quad f = \frac{mv^2}{R}$$

Take torque about the center of mass: $fh - n\frac{d}{2} = 0$.

Then by substitution $\frac{mv_{\max}^2}{R}h - \frac{mgd}{2} = 0 \qquad v_{\max} = \sqrt{\frac{gdR}{2h}}$

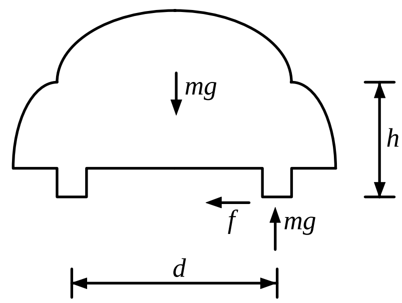

FIG. P12.60

A wider wheelbase (larger d) and a lower center of mass (smaller h) will reduce the risk of rollover.