Homework Assignment #5

Energy of Harmonic Oscillators

Description: Several questions, both qualitative and computational, related to kinetic and potential energy of a mass-spring system vibrating horizontally.
Learning Goal: To learn to apply the law of conservation of energy to the analysis of harmonic oscillators.

Systems in simple harmonic motion, or *harmonic oscillators*, obey the law of conservation of energy just like all other systems do. Using energy considerations, one can analyze many aspects of motion of the oscillator. Such an analysis can be simplified if one assumes that mechanical energy is not dissipated. In other words,

$$E = K + U = \text{constant},$$

where E is the total mechanical energy of the system, K is the kinetic energy, and U is the potential energy.
As you know, a common example of a harmonic oscillator is a mass attached to a spring. In this problem, we will consider a *horizontally* moving block attached to a spring. Note that, since the gravitational potential energy is not changing in this case, it can be excluded from the calculations.

For such a system, the potential energy is stored in the spring and is given by

$$U = \frac{1}{2}kx^2,$$

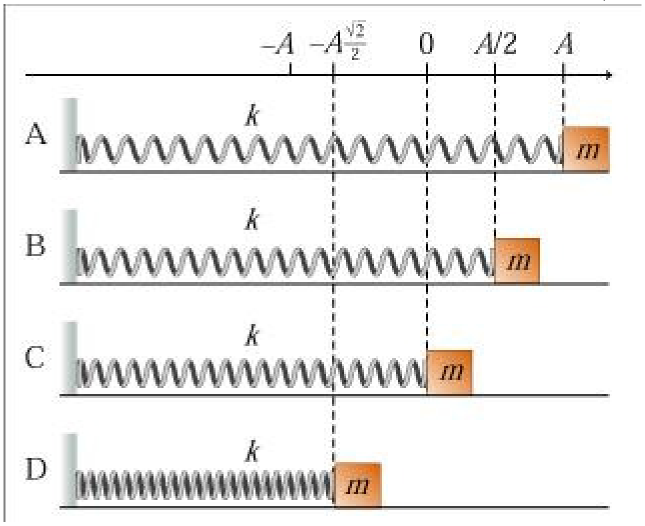
where k is the force constant of the spring and x is the distance from the equilibrium position.

The kinetic energy of the system is, as always,

$$K = \frac{1}{2}mv^2,$$

where m is the mass of the block and v is the speed of the block.

We will also assume that there are no resistive forces; that is, $E = \text{constant}$.

Consider a harmonic oscillator at four different moments, labeled A, B, C, and D, as shown in the figure 

. Assume that the force constant k, the mass of the block, m, and the amplitude of vibrations, A, are given. Answer the following questions.

Part A

Which moment corresponds to the maximum potential energy of the system?

Hint A.1 Consider the position of the block

Recall that $U = \frac{1}{2}kx^2$, where x is the distance from equilibrium. Thus, the farther the block is from equilibrium, the greater the potential energy. When is the block farthest from equilibrium?

ANSWER:

- (•) A
- ( ) B
- ( ) C
- ( ) D

Part B

Which moment corresponds to the minimum kinetic energy of the system?

Hint B.1 How does the velocity change?

Recall that $K = \frac{1}{2}mv^2$, where v is the speed of the block. When is the speed at a minimum? Keep in mind that speed is the *magnitude* of the velocity, so the lowest value that it can take is zero.

ANSWER:

- (•) A
- ( ) B
- ( ) C
- ( ) D

When the block is displaced a distance A from equilibrium, the spring is stretched (or compressed) the most, and the block is momentarily at rest. Therefore, the maximum potential energy is $U_{\rm max} = \frac{1}{2}kA^2$. At that moment, of course, $K = K_{\rm min} = 0$. Recall that $E = K + U$. Therefore,

$$E = \frac{1}{2}kA^2$$
.

In general, the mechanical energy of a harmonic oscillator equals its potential energy at the maximum or minimum displacement.

Part C

Consider the block in the process of oscillating.

ANSWER:

If the kinetic energy of the block is increasing, the block *must* be

- ○ at the equilibrium position.
- ○ at the amplitude displacement.
- ○ moving to the right.
- ○ moving to the left.
- ○ moving away from equilibrium.
- ◉ moving toward equilibrium.

Part D

Which moment corresponds to the maximum kinetic energy of the system?

Hint D.1 Consider the velocity of the block

As the block begins to move away from the amplitude position, it gains speed. As the block approaches equilibrium, the force applied by the spring—and, therefore, the acceleration of the block—decrease. The speed of the block is at a maximum when the acceleration becomes zero. At what position does the object begin to slow down?

ANSWER:

- ○ A
- ○ B
- ◉ C
- ○ D

Part E

Which moment corresponds to the minimum potential energy of the system?

Hint E.1 Consider the distance from equilibrium

The smallest potential energy corresponds to the smallest distance from equilibrium.

ANSWER:
- ○ A
- ○ B
- ◉ C
- ○ D

When the block is at the equilibrium position, the spring is not stretched (or compressed) at all. At that moment, of course, $U = U_{\rm min} = 0$. Meanwhile, the block is at its maximum speed ($v_{\rm max}$). The maximum kinetic energy can then be written as $K_{\rm max} = \frac{1}{2}mv_{\rm max}^2$. Recall that $E = K + U$ and that $U = 0$ at the equilibrium position. Therefore,

$$E = \frac{1}{2}mv_{\rm max}^2.$$

Recalling what we found out before,

$$E = \frac{1}{2}kA^2,$$

we can now conclude that

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{\rm max}^2,$$

or

$$v_{\rm max} = \sqrt{\frac{k}{m}}A = \omega A.$$

Part F

At which moment is $K = U$?

Hint F.1 Consider the potential energy

At this moment, $U = \frac{1}{2}U_{\rm max}$. Use the formula for $U_{\rm max}$ to obtain the corresponding distance from equilibrium.

ANSWER:
- ○ A
- ○ B

- ○ C
- ◉ D

Part G

Find the kinetic energy K of the block at the moment labeled B.

Hint G.1 How to approach the problem

Find the potential energy first; then use conservation of energy.

Hint G.2 Find the potential energy

Find the potential energy U of the block at the moment labeled B.

Express your answer in terms of k and A.

ANSWER: $U = \frac{1}{8}kA^2$

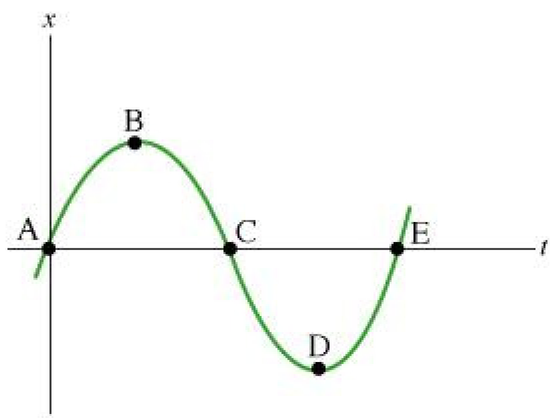
Using the facts that the total energy $E = \frac{1}{2}kA^2$ and that $E = K + U$, you can now solve for the kinetic energy K at moment B.

Express your answer in terms of k and A.

ANSWER: $K = \frac{3}{8}kA^2$

Position, Velocity, and Acceleration of an Oscillator

Description: Where are position, velocity and acceleration are positive, negative and equal to zero on a sine wave?
Learning Goal: To learn to find kinematic variables from a graph of position vs. time.
The graph of the position of an oscillating object as a function of time is shown.



Some of the questions ask you to determine ranges on the graph over which a statement is true. When answering these questions, choose the *most complete* answer. For example, if the answer "B to D" were correct, then "B to C" would technically also be correct--but you will only recieve credit for choosing the most complete answer.

Part A

Where on the graph is $x > 0$?

ANSWER:

- A to B
- A to C
- C to D
- C to E
- B to D
- A to B and D to E

Part B

Where on the graph is $x < 0$?

ANSWER:

- A to B

- A to C
- C to D
- (selected) C to E
- B to D
- A to B and D to E

Part C

Where on the graph is $x = 0$?

ANSWER:

- A only
- C only
- E only
- A and C
- (selected) A and C and E
- B and D

Part D

Where on the graph is the velocity $v > 0$?

Hint D.1 Finding instantaneous velocity

Instantaneous velocity is the derivative of the position function with respect to time,

$$v(t) = \frac{dx(t)}{dt}.$$

Thus, you can find the velocity at any time by calculating the slope of the x vs. t graph. When is the slope greater than 0 on this graph?

ANSWER:

- A to B
- A to C
- C to D

- [ ] C to E
- [ ] B to D
- [x] A to B and D to E

Part E

Where on the graph is the velocity $v < 0$?

ANSWER:

- [ ] A to B
- [ ] A to C
- [ ] C to D
- [ ] C to E
- [x] B to D
- [ ] A to B and D to E

Part F

Where on the graph is the velocity $v = 0$?

Hint F.1 **How to tell if $v = 0$**

The velocity is zero when the slope of the *x* vs. *t* curve is zero: $\frac{dx(t)}{dt} = 0$.

ANSWER:

- [ ] A only
- [ ] B only
- [ ] C only
- [ ] D only
- [ ] E only
- [ ] A and C
- [ ] A and C and E
- [x] B and D

Part G

Where on the graph is the acceleration $a > 0$?

Hint G.1 Finding acceleration

Acceleration is the second derivative of the position function with respect to time:

$$a = \frac{d^2x(t)}{dt^2}.$$

This means that the sign of the acceleration is the same as the sign of the curvature of the *x* vs. *t* graph. The acceleration of a curve is negative for downward curvature and positive for upward curvature. Where is the curvature greater than 0?

ANSWER:

- ○ A to B
- ○ A to C
- ○ C to D
- ● C to E
- ○ B to D
- ○ A to B and D to E

Part H

Where on the graph is the acceleration $a < 0$?

ANSWER:

- ○ A to B
- ● A to C
- ○ C to D
- ○ C to E
- ○ B to D
- ○ A to B and D to E

Part I

Where on the graph is the acceleration $a = 0$?

Hint I.1 How to tell if $a = 0$

The acceleration is zero at the inflection points of the *x* vs. *t* graph. Inflection points are where the curvature of the graph changes sign.

ANSWER:

- ○ A only
- ○ B only
- ○ C only
- ○ D only
- ○ E only
- ○ A and C
- ◉ A and C and E
- ○ B and D

Changing the Period of a Pendulum

Description: Short quantitative problem requiring students to determine the impact different masses and accelerations of gravity have on a simple pendulum's period. Requires students to use proportional reasoning. This problem is based on Young/Geller Quantitative Analysis 11.4

A simple pendulum consisting of a bob of mass m attached to a string of length L swings with a period T.

Part A

If the bob's mass is doubled, approximately what will the pendulum's new period be?

Hint A.1 Period of a simple pendulum

The period T of a simple pendulum of length L is given by

$$T = 2\pi\sqrt{\frac{L}{g}},$$

where g is the acceleration due to gravity.

ANSWER:

- ○ $T/2$
- ◉ T
- ○ $\sqrt{2}T$
- ○ $2T$

Part B

If the pendulum is brought on the moon where the gravitational acceleration is about $g/6$, approximately what will its period now be?

Hint B.1 How to approach the problem

Recall the formula of the period of a simple pendulum. Since the gravitational acceleration appears in the denominator, the period must increase when the gravitational acceleration decreases.

ANSWER:

- ○ $T/6$
- ○ $T/\sqrt{6}$
- ◉ $\sqrt{6}T$
- ○ $6T$

Part C

If the pendulum is taken into the orbiting space station what will happen to the bob?

Hint C.1 How to approach the problem

Recall that the oscillations of a simple pendulum occur when a pendulum bob is raised above its equilibrium position and let go, causing the pendulum bob to fall. The gravitational force acts to bring the bob back to its equilibrium position. In the space station, the earth's gravity acts on both the station and everything inside it, giving them the same acceleration. These objects are said to be in free fall.

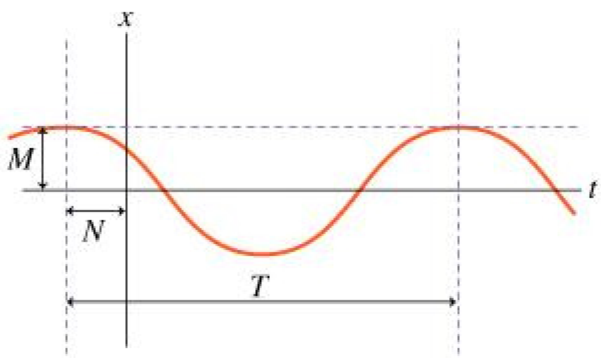
ANSWER:

- ○ It will continue to oscillate in a vertical plane with the same period.
- ○ It will no longer oscillate because there is no gravity in space.
- ◉ It will no longer oscillate because both the pendulum and the point to which it is attached are in free fall.
- ○ It will oscillate much faster with a period that approaches zero.

In the space station, where all objects undergo the same acceleration due to the earth's gravity, the tension in the string is zero and the bob does not fall relative to the point to which the string is attached.

Cosine Wave

Description: Find amplitude, period, phi for displayed graph. Mult. choice
The graph shows the position x of an oscillating object as a function of time t.



The equation of the graph is

$$x(t) = A\cos(\omega t + \phi),$$

where A is the amplitude, ω is the angular frequency, and ϕ is a phase constant. The quantities M, N, and T are measurements to be used in your answers.

Part A

What is A in the equation?

Hint A.1 **Maximum of $x(t)$**

What is the maximum value of x on the graph and what is the maximum of $x(t)$ as described by the equation? The equation is just a constant multiplied by a cosine function. Cosine can only range from -1 to 1.

ANSWER:
- ○ T
- ◉ M
- ○ $2M$
- ○ M/T
- ○ $T/2$

Part B

What is ω in the equation?

Hint B.1 **Period**

Think of the simpler equation $x = \cos(\omega t)$. The period T is the same as before. What does x equal when $t = T$? Use the result to solve for ω.

ANSWER:
- ○ T
- ○ M
- ○ $2\pi T$
- ◉ $2\pi/T$
- ○ $2/T$
- ○ $1/T$

Part C

What is ϕ in the equation?

Hint C.1 Using the graph and trigonometry

What is x equal to when $t = -N$? Use your result for ω to solve for ϕ in terms of T, M, and N.

Hint C.2 Using the graph and Part B

You might be able to find ϕ in terms of ω and then use your result from Part B.

ANSWER:
- ○ N
- ○ $T - N$
- ◉ $2\pi N/T$
- ○ $-2\pi N/T$
- ○ $\arccos(2\pi N/T)$

Energy of a Spring

Description: Short quantitative problem relating the total, potential, and kinetic energies of a mass that is attached to a spring and undergoing simple harmonic motion. This problem is based on Young/Geller Quantitative Analysis 11.1

An object of mass m attached to a spring of force constant k oscillates with simple harmonic motion. The maximum displacement from equilibrium is A and the total mechanical energy of the system is E.

Part A

What is the system's potential energy when its kinetic energy is equal to $\frac{3}{4}E$?

Hint A.1 How to approach the problem

Since the sum of kinetic and potential energies of the system is equal to the system's total energy, if you know the fraction of total energy corresponding to kinetic energy you can calculate how much energy is potential energy. Moreover, using conservation of energy you can calculate the system's total energy in terms of the given quantities k and A. At this point you simply need to combine those results to find the potential energy of the system in terms of k and A.

Hint A.2 Find the fraction of total energy that is potential energy

When the kinetic energy of the system is equal to $\frac{3}{4}E$, what fraction of the total energy E is potential energy?

Hint A.2.1 Conservation of mechanical energy

In a system where no forces other than gravitational and elastic forces do work, the sum of kinetic energy K and potential energy U is conserved. That is, the total energy E of the system, given by $E = K + U$, is constant.

Express your answer numerically.

ANSWER: $\frac{1}{4}$

0.250

Hint A.3 Find the total energy of the system

What is the total mechanical energy of the system, E?

Hint A.3.1 How to approach the problem

If you apply conservation of energy to the system when the object reaches its maximum displacement, you can calculate the system's total energy E in terms of the given quantities A and k. In fact, when the object is at its maximum displacement from equilibrium, its speed is momentarily zero and so is its kinetic energy. It follows that the system's energy at this point is entirely potential, that is, $E = U$, where U is the spring's elastic potential energy.

Hint A.3.2 Elastic potential energy

The elastic potential energy U of a spring that has been compressed or stretched by a distance x is given by

$$U = \frac{1}{2}kx^2,$$

where k is the force constant of the spring.

Express your answer in terms of some or all of the variables m, k, and A.

ANSWER: $E = \frac{kA^2}{2}$

ANSWER:

- ○ kA^2
- ○ $\frac{kA^2}{2}$
- ○ $\frac{kA^2}{4}$
- ◉ $\frac{kA^2}{8}$

Part B

What is the object's velocity when its potential energy is $\frac{2}{3}E$?

Hint B.1 How to approach the problem

You can calculate the object's velocity using energy considerations. Calculate the fraction of the system's total energy that is kinetic energy and then find the object's velocity from the definition of kinetic energy. To simplify your expression write the total energy in terms of k and A. Alternatively, you could directly use the formula for the object's velocity in terms of the variables k, m, A, and displacement x derived from energy considerations. The only unknown quantity in such a formula would be the object's displacement x, which can be calculated from the system's potential energy.

Hint B.2 Find the kinetic energy

If the system's potential energy is $\frac{2}{3}E$, what is the system's kinetic energy?

Hint B.2.1 Conservation of mechanical energy

In a system where no forces other than gravitational and elastic forces do work, the sum of kinetic energy K and potential energy U is conserved. That is, the total energy E of the system, given by $E = K + U$, is constant.

Hint B.2.2 Total energy of the system

The total energy of a system consisting of an object attached to a horizontal spring of force constant k is given by

$$E = \frac{1}{2}kA^2,$$

where A is the maximum displacement of the object from its equilibrium position.

ANSWER:

- ○ $\frac{1}{2}kA^2$
- ○ $\frac{1}{3}kA^2$
- ◉ $\frac{1}{6}kA^2$
- ○ $\frac{2}{3}kA^2$

Now use your result and the definition of kinetic energy as $\frac{1}{2}mv^2$ to find the object's speed v.

Hint B.3 Formula for the velocity in terms of position

The velocity of an object of mass m undergoing simple harmonic motion is given by

$$v = \pm\sqrt{\frac{k}{m}}\sqrt{A^2 - x^2},$$

where k is the force constant of the system, x is the object's position, and A is maximum displacement.

Hint B.4 Find the object's position

When the system's potential energy is $\frac{2}{3}E$, what is the displacement x of the object from its equilibrium position?

Hint B.4.1 Elastic potential energy

The elastic potential energy U of a spring that has been compressed or stretched by a distance x is given by

$$U = \frac{1}{2}kx^2$$
,

where k is the force constant of the spring.

Hint B.4.2 Total energy of the system

The total energy of a system consisting of an object attached to a horizontal spring of force constant k is given by

$$E = \frac{1}{2}kA^2$$
,

where A is the maximum displacement of the object from its equilibrium position.

ANSWER:

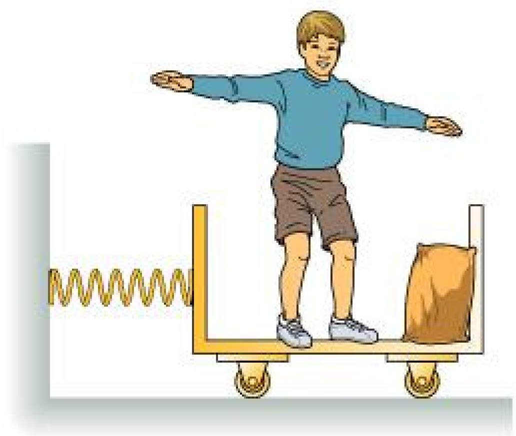
- ○ $\frac{2}{3}A$
- ◉ $\pm\sqrt{\frac{2}{3}}A$
- ○ $\pm\sqrt{\frac{2}{3k}}A$
- ○ $\pm\sqrt{\frac{1}{3}}A$

ANSWER:

- ○ $\pm\sqrt{\frac{k}{m}}A$
- ○ $\pm\sqrt{\frac{k}{m}}\sqrt{\frac{2}{3}}A$
- ◉ $\pm\sqrt{\frac{k}{m}}\frac{A}{\sqrt{3}}$
- ○ $\pm\sqrt{\frac{k}{m}}\frac{A}{\sqrt{6}}$

Mass and Simple Harmonic Motion Conceptual Question

Description: Conceptual question on the effect of mass on simple harmonic motion.



The *shaker cart,* shown in the figure, is the latest extreme sport craze. You stand inside of a small cart attached to a heavy-duty spring, the spring is compressed and released, and you shake back and forth, attempting to maintain your balance. Note that there is also a sandbag in the cart with you.
At the instant you pass through the equilibrium position of the spring, you drop the sandbag out of the cart onto the ground.

Part A

What effect does dropping the sandbag out of the cart at the equilibrium position have on the amplitude of your oscillation?

Hint A.1 How to approach the problem

Dropping the sandbag at the equilibrium point may or may not change the total energy in the spring-cart system. First, determine if the sandbag itself has energy at the equilibrium point. If so, it will carry that energy out of the system. In that case, determine how (if at all) the loss of energy and the loss of the sandbag's mass affects the amplitude of the motion. If not, the system's total energy will not change. In that case, determine how (if at all) the loss of the sandbag's mass affects the amplitude of the motion.

Hint A.2 Energy and the equilibrium point

At equilibrium, both the cart and the bag are moving at their maximum speed and the displacement is zero. Recall that the energy of the system is the sum of the kinetic energy and the potential energy stored in the spring. Note that the total energy of the system is equal to the kinetic energy at equilibrium, $(1/2)mv_{\rm max}^2$, which is equal to the potential energy at maximum displacement, $(1/2)kA^2$.

ANSWER:
- ○ It increases the amplitude.
- ◉ It decreases the amplitude.
- ○ It has no effect on the amplitude.

At equilibrium the sandbag has kinetic energy, which leaves the system with the sandbag. Since the amplitude is related to the energy of the system by the equation $E = \frac{1}{2} k A^2$, a reduction in the energy of the system must lead to a reduction in the amplitude of the motion.

Part B

What effect does dropping the sandbag out of the cart at the equilibrium position have on the maximum speed of the cart?

Hint B.1 How to approach the problem

At the equilibrium point, the sandbag is moving, so it has kinetic energy. It will carry that energy out of the system. It also carries its own mass out of the system. Does the speed of the cart change when the sandbag is dropped out?

Note that the sandbag leaves the cart while the cart is moving at its maximum speed. Whatever the speed of the cart is the instant after the sandbag leaves, the maximum speed will keep this value into the future.

ANSWER:

- ○ It increases the maximum speed.
- ○ It decreases the maximum speed.
- ● It has no effect on the maximum speed.

Dropping the sandbag doesn't change the speed of the cart. Since energy doesn't leave the system at any time after the bag is dropped, the cart will always return to the equilibrium point with the same kinetic energy, and therefore the same maximum speed.

Instead of dropping the sandbag as you pass through equilibrium, you decide to drop the sandbag when the cart is at its maximum distance from equilibrium.

Part C

What effect does dropping the sandbag at the cart’s maximum distance from equilibrium have on the amplitude of your oscillation?

Hint C.1 How to approach the problem

Dropping the sandbag at the point of maximum displacement may or may not change the total energy in the spring-cart system. First, determine if the sandbag itself has energy at the point of maximum displacement. If so, it will carry that energy out of the system. In that case, determine how (if at all) the loss of energy and the loss of the sandbag's mass affects the amplitude of the motion. If not, the system's total energy will not change. In that case, determine how (if at all) the loss of the sandbag's mass affects the amplitude of the motion.

Hint C.2 Energy at maximum displacement

At its maximum distance from equilibrium, both the cart and the bag are at rest. By dropping the bag at this point, no energy is lost from the spring-cart system, since all of the energy is stored as potential energy in the spring. Therefore, both the elastic potential energy at maximum displacement and the kinetic energy at equilibrium must remain constant.

ANSWER:

- ○ It increases the amplitude.

- ○ It decreases the amplitude.
- ◉ It has no effect on the amplitude.

At the point of maximum displacement, all of the system's energy is stored in the spring. therefore, dropping the sandbag has no effect on the system's total energy E. Since $E = \frac{1}{2} k A^2$, the amplitude cannot change.

Part D

What effect does dropping the sandbag at the cart's maximum distance from equilibrium have on the maximum speed of the cart?

Hint D.1 How to approach the problem

By dropping the sandbag at maximum displacement, you reduce the mass of the system without reducing the total energy. How will the maximum speed, attained when all of this energy is converted to kinetic energy, compare to the maximum speed before the sandbag was dropped?

ANSWER:

- ◉ It increases the maximum speed.
- ○ It decreases the maximum speed.
- ○ It has no effect on the maximum speed.

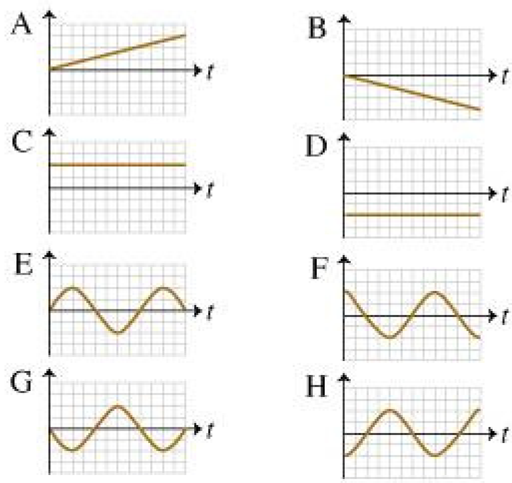
The system has not changed energy, but its mass has decreased. Since $E = \frac{1}{2} m v_{\rm max}^2$, the maximum speed must increase to balance out the decreased mass.

Simple Harmonic Motion Conceptual Question

Description: Identify graphs of position, velocity and acceleration vs. time describing an object undergoing simple harmonic motion.

An object of mass m is attached to a vertically oriented spring. The object is pulled a short distance below its equilibrium position and released from rest. Set the origin of the coordinate system at the equilibrium position of the object and choose upward as the positive direction. Assume air resistance is so small that it can be ignored.

Refer to these graphs



when answering the following questions.

Part A

Beginning the instant the object is released, select the graph that best matches the position vs. time graph for the object.

Hint A.1 How to approach the problem

To find the graph that best matches the object's position vs. time, first determine the initial value of the position. This will narrow down your choices of possible graphs. Then, interpret what the remaining graphs say about the subsequent motion of the object. You should find that only one graph describes the position of the object correctly.

Hint A.2 Find the initial position

The origin of the coordinate system is set at the equilibrium position of the object, with the positive direction upward. The object is pulled below equilibrium and released. Therefore, is the initial position positive, negative, or zero?

ANSWER:

- ○ positive
- ◉ negative
- ○ zero

ANSWER:

Part B

Beginning the instant the object is released, select the graph that best matches the velocity vs.

time graph for the object.

Hint B.1 Find the initial velocity

The object is released from rest. Is the initial velocity positive, negative, or zero?

ANSWER:

- ○ positive
- ○ negative
- ◉ zero

Hint B.2 Find the velocity a short time later

After the object is released from rest, in which direction will it initially move?

ANSWER:

- ◉ upward (positive)
- ○ downward (negative)

ANSWER:

Part C

Beginning the instant the object is released, select the graph that best matches the acceleration vs. time graph for the object.

Hint C.1 Find the initial acceleration

The object is released from rest, and a short time later it is moving upward. Based on this observation, what is the direction of the initial acceleration?

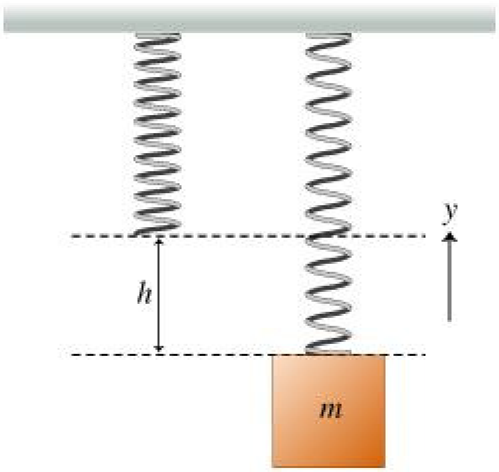
ANSWER:

- ◉ positive
- ○ negative
- ○ neither positive nor negative (i.e., there is no acceleration)

ANSWER:

Vertical Mass-and-Spring Oscillator

Description: A mass hangs from a spring. Find the spring constant and frequency of oscillation. A block of mass m is attached to the end of an ideal spring. Due to the weight of the block, the block remains at rest when the spring is stretched a distance h from its equilibrium length.



The spring has an unknown spring constant k.

Part A

What is the spring constant k?

Hint A.1 Sum of forces acting on the block

Since the block is not accelerating, the net force acting on the block must be zero. Taking the positive *y* direction to be upward, write an expression for the net vertical force $\sum F_y$ acting on the block.

Hint A.1.1 Force due to spring

What is F_s, the force that the spring exerts on the block?

ANSWER: $F_s =$ hk

Hint A.1.2 Force due to gravity

What is F_g, the gravitational force on the block?

ANSWER: $F_g =$ $-mg$

Express the sum of the vertical forces in terms of m, h, k, and g, the magnitude of the acceleration due to gravity.

ANSWER: $\sum F_y = 0$ $hk - mg$

Express the spring constant in terms of given quantities and g, the magnitude of the acceleration due to gravity.

ANSWER:

$$k = \frac{mg}{h}$$

Part B

Suppose that the block gets bumped and undergoes a small vertical displacement. Find the resulting angular frequency ω of the block's oscillation about its equilibrium position.

Hint B.1 Formula for angular frequency

The angular frequency of simple harmonic motion for a body of mass m acted on by a restoring force with force constant k is given by $\omega = \sqrt{k/m}$.

Express the frequency in terms of given quantities and g, the magnitude of the acceleration due to gravity.

ANSWER:

$$\omega = \sqrt{\frac{g}{h}}$$

It may seem that this result for the frequency does not depend on either the mass of the block or the spring constant, which might make little sense. However, these parameters are what would determine the extension h of the spring when the block is hanging: $h = mg/k$.

One way of thinking about this problem is to consider both k and g as unknowns. By measuring h and ω (both fairly simple measurements), and knowing the mass, you can determine the value of the spring constant and the acceleration due to gravity experimentally.

14.1. IDENTIFY: We want to relate the characteristics of various waves, such as the period, frequency and angular frequency.

SET UP: The frequency f in Hz is the number of cycles per second. The angular frequency ω is $\omega = 2\pi f$ and has units of radians per second. The period T is the time for one cycle of the wave and has units of seconds. The period and frequency are related by $T = \frac{1}{f}$.

EXECUTE: **(a)** $T = \frac{1}{f} = \frac{1}{466\text{ Hz}} = 2.15 \times 10^{-3}\text{ s}.$

$\omega = 2\pi f = 2\pi(466\text{ Hz}) = 2.93 \times 10^{3}\text{ rad/s}.$

(b) $f = \frac{1}{T} = \frac{1}{50.0 \times 10^{-6}\text{ s}} = 2.00 \times 10^{4}\text{ Hz}.$ $\omega = 2\pi f = 1.26 \times 10^{5}\text{ rad/s}.$

(c) $f = \frac{\omega}{2\pi}$ so f ranges from $\frac{2.7 \times 10^{15}\text{ rad/s}}{2\pi\text{ rad}} = 4.3 \times 10^{14}\text{ Hz}$ to

$\frac{4.7\times10^{15}\text{ rad/s}}{2\pi\text{ rad}} = 7.5\times10^{14}\text{ Hz}$. $T = \frac{1}{f}$ so T ranges from

$\frac{1}{7.5\times10^{14}\text{ Hz}} = 1.3\times10^{-15}\text{ s}$ to $\frac{1}{4.3\times10^{14}\text{ Hz}} = 2.3\times10^{-15}\text{ s}$.

(d) $T = \frac{1}{f} = \frac{1}{5.0\times10^{6}\text{ Hz}} = 2.0\times10^{-7}\text{ s}$ and $\omega = 2\pi f = 2\pi(5.0\times10^{6}\text{ Hz}) = 3.1\times10^{7}\text{ rad/s}$.

EVALUATE: Visible light has much higher frequency than either sounds we can hear or ultrasound. Ultrasound is sound with frequencies higher than what the ear can hear. Large f corresponds to small T.

14.4. IDENTIFY: The period is the time for one cycle and the amplitude is the maximum displacement from equilibrium. Both these values can be read from the graph.

SET UP: The maximum x is 10.0 cm. The time for one cycle is 16.0 s.

EXECUTE: **(a)** $T = 16.0\text{ s}$ so $f = \frac{1}{T} = 0.0625\text{ Hz}$.

(b) $A = 10.0\text{ cm}$.

(c) $T = 16.0\text{ s}$

(d) $\omega = 2\pi f = 0.393\text{ rad/s}$

EVALUATE: After one cycle the motion repeats.

14.18. IDENTIFY: The general expression for $v_x(t)$ is $v_x(t) = -\omega A\sin(\omega t + \phi)$. We can determine ω and A by comparing the equation in the problem to the general form.

SET UP: $\omega = 4.71\text{ rad/s}$. $\omega A = 3.60\text{ cm/s} = 0.0360\text{ m/s}$.

EXECUTE: **(a)** $T = \frac{2\pi}{\omega} = \frac{2\pi\text{ rad}}{4.71\text{ rad/s}} = 1.33\text{ s}$

(b) $A = \frac{0.0360\text{ m/s}}{\omega} = \frac{0.0360\text{ m/s}}{4.71\text{ rad/s}} = 7.64\times10^{-3}\text{ m} = 7.64\text{ mm}$

(c) $a_{max} = \omega^2 A = (4.71\text{ rad/s})^2(7.64\times10^{-3}\text{ m}) = 0.169\text{ m/s}^2$

(d) $\omega = \sqrt{\frac{k}{m}}$ so $k = m\omega^2 = (0.500\text{ kg})(4.71\text{ rad/s})^2 = 11.1\text{ N/m}$.

EVALUATE: The overall positive sign in the expression for $v_x(t)$ and the factor of $-\pi/2$ both are related to the phase factor ϕ in the general expression.

14.47. IDENTIFY: Since the cord is much longer than the height of the object, the system can be modeled as a simple pendulum. We will assume the amplitude of swing is small, so that $T = 2\pi\sqrt{\frac{L}{g}}$.

SET UP: The number of swings per second is the frequency $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$.

EXECUTE: $f = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{1.50\text{ m}}} = 0.407$ swings per second.

EVALUATE: The period and frequency are both independent of the mass of the object.