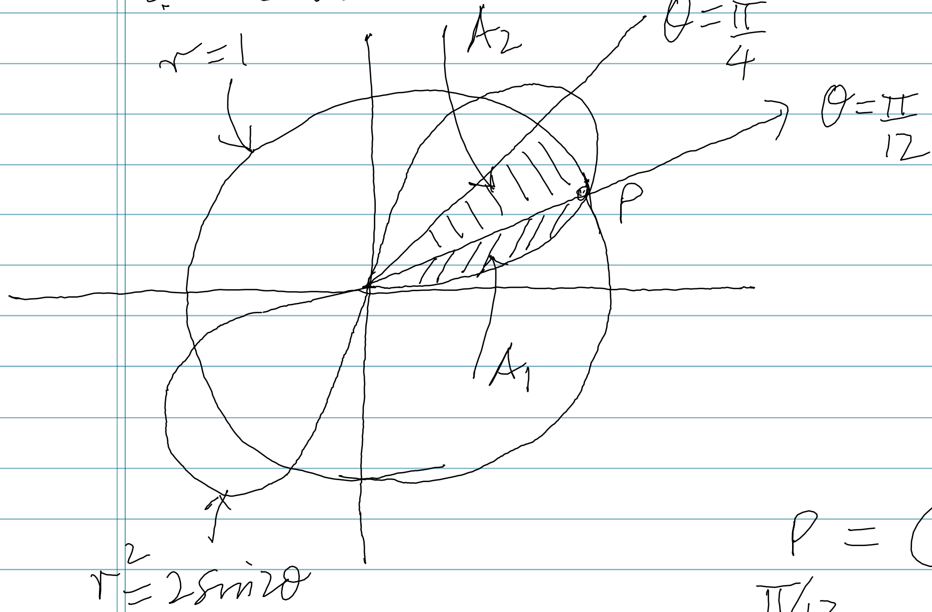
Example: Find the area enclosed by the curve $r=1$ and $r^2 = 2\sin 2\theta$.

Solution.



$$2\sin 2\theta = 1$$
$$\sin 2\theta = \frac{1}{2}$$
$$2\theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$
$$\theta = \frac{\pi}{12} \text{ or } \frac{5\pi}{12}$$

$$P = (1, \pi/12)$$

$$\text{Area}(A_1) = \frac{1}{2}\int_0^{\pi/12} 2\sin 2\theta \, d\theta$$
$$= -\frac{\cos 2\theta}{2}\Big|_0^{\pi/12} = -\frac{\cos(\pi/6)}{2} + \frac{\cos(0)}{2}$$
$$= \frac{2-\sqrt{3}}{4}$$

$$\text{Area}(A_2) = \frac{1}{2}\int_{\pi/12}^{\pi/4} 1^2 \, d\theta = \frac{1}{2}\left[\theta\right]_{\pi/12}^{\pi/4}$$
$$= \frac{1}{2}\left[\frac{\pi}{4} - \frac{\pi}{12}\right]$$
$$= \frac{\pi}{12}.$$

$$\text{Area} = 4\left(\text{Area}(A_1) + \text{Area}(A_2)\right)$$
$$= 4\left[\frac{2-\sqrt{3}}{4} + \frac{\pi}{12}\right] = 2-\sqrt{3} + \frac{\pi}{3}$$