Mathematical Induction

Use induction to prove that

$$\sum_{i=0}^{n-1} \frac{1}{2^i} = 1 + \frac{\mathbf{1}}{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{4}} + \cdots + \frac{\mathbf{1}}{\mathbf{2}^{\boldsymbol{n}-\mathbf{1}}} = 2 - \frac{1}{2^{n-1}}$$

for all positive integers $\boldsymbol{n}$.

Let P(n) denote the proposition $\sum_{i=0}^{n-1} \frac{1}{2^i}$ $=1+\frac{\mathbf{1}}{\mathbf{2}}+\frac{\mathbf{1}}{\mathbf{4}}+\cdots+\frac{\mathbf{1}}{\mathbf{2^{n-1}}}=2-\frac{1}{2^{n-1}}$,
where n is a positive integer.

BASIS STEP: P(1) is true since $\sum_{i=0}^{0} \frac{1}{2^i}$ $=1$ and $2-\frac{\mathbf{1}}{\mathbf{1}}$ $=1$.

INDUCTIVE STEP: Let us assume P(n), that is
$\sum_{i=0}^{\boldsymbol{n}-1} \frac{1}{2^i}=1+\frac{\mathbf{1}}{\mathbf{2}}+\frac{\mathbf{1}}{\mathbf{4}}+\cdots+\frac{\mathbf{1}}{\mathbf{2^{n-1}}}=2-\frac{1}{2^{n-1}}$ is true for an arbitrary positive integer n. This is our inductive hypothesis.

We have to show the statement P(n+1),
$\sum_{i=0}^{n} \frac{1}{2^i}=1+\frac{\mathbf{1}}{\mathbf{2}}+\frac{\mathbf{1}}{\mathbf{4}}+\cdots+\frac{\mathbf{1}}{\mathbf{2^{n-1}}}+\frac{\mathbf{1}}{\mathbf{2^{n}}}=2-\frac{1}{2^{n}}$ is true assuming the inductive hypothesis P(n).

Proof:

$$\sum_{i=0}^{n} \frac{1}{2^i} = \sum_{i=0}^{n-1} \frac{1}{2^i} + \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}} + \frac{1}{2^n} =$$

$$2 - \frac{1}{2^{n-1}} + \frac{1}{2^n} = 2 - \frac{1}{2^n}(2-1) = 2 - \frac{1}{2^n}$$

using the inductive hypothesis.

By the Principle of Mathematical Induction (Basis Step and Inductive Step together) $\sum_{i=0}^{n-1} \frac{1}{2^i} = 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{n-1}} = 2 - \frac{1}{2^{n-1}}$ for all positive interges n.