Mathematical Induction

Use induction to prove that

$$\sum_{i=1}^{n}(2i-1) = 1 + 3 + 5 + \cdots + (2n-1) = n^2$$

for all positive integers n.

Let P(n) denote the proposition $\sum_{i=1}^{n}(2i-1) = 1+3+5+\cdots+(2\mathbf{n}-1) = \mathbf{n}^2$, where n is a positive integer.

BASIS STEP: P(1) is true since $\sum_{i=1}^{1}(2i-1) = 1\cdot 2-1 = 1$ and $1^2 = 1$

INDUCTIVE STEP: Let us assume P(n), that is
$\sum_{i=1}^{n}(2i-1) = 1+3+5+\cdots+(2\mathbf{n}-1) = n^2$ is true for an arbitrary positive integer n . This is our inductive hypothesis.

We have to show the statement P(n+1),

$$\sum_{i=1}^{n+1}(2i-1) = 1+3+5+\cdots+(2\mathbf{n}-1)+(2(\mathbf{n}+1)-1)$$
$$= 1+3+5+\cdots+(2\mathbf{n}-1)+(2\mathbf{n}+1) = (\mathbf{n}+1)^2$$

is true assuming the inductive hypothesis P(n).

Proof:

$$\sum_{i=1}^{n+1}(2i-1) = \sum_{i=1}^{n}(2i-1) + (2n+1) = 1+3+5+\cdots+(2n-1) + (2n+1) =$$

$$n^2 + (2n+1) = (n+1)^2$$

using the inductive hypothesis.

By the Principle of Mathematical Induction (Basis Step and Inductive Step together) $\sum_{i=1}^{n}(2i-1) = 1+3+5+\cdots+(2n-1) = n^2$ for all positive integers n.