# MAT 243 Online Written Homework Assignments for Week 6 (units 16-18)

Free Response Questions

1. Compute the value of the following alternating sum and show all your work.

$$\binom{30}{3} 4 - \binom{30}{4} 16 + \binom{30}{5} 64 \mp \cdots - \binom{30}{30} 4^{28}$$

   Leave expressions of the form $m^n$ in your answer without attempting to evaluate them. Solutions that rely solely on calculator computation of the terms of the sum are unacceptable.

2. A medical research survey categorizes adults as follows:

   - by gender (male or female)
   - by age group (age groups are 18-25, 26-35, 36-50, 51+)
   - by income (less than 30k/year, 30k-50k/year, 51k-80k per year, more than 80k/year)
   - for women only: by whether they have been pregnant (yes/no)
   - for men only: by frequency of undergoing prostate exams (frequently, rarely, never).

   What minimum size of a set of adults will guarantee that there are two people in it with matching characteristics in all categories? Just the number, no explanation.

3. In how many ways can you select a subset of 5 elements from a set of 7 elements? Give the answer in terms of combinations or permutations and explain.

4. 100 people have come to the annual pet owner's meeting. A quick show of hands demonstrates that 60 people own cats and 55 own dogs. Give the numerical answer and a <u>brief</u> explanation for each of the following questions:

   a. If a committee is randomly selected from these 100 people, how many people must be selected to guarantee that there will be at least 3 dog owners on the committee?

   b. How many people must be randomly selected for a committee to guarantee at least 2 dog owners, and at least 2 cat owners?

   c. How many people must be randomly selected for a committee to guarantee at least 2 people who don't own a dog, and at least 2 people who don't own a cat?

5. How many positive divisors does the number 248832 have?

6. The selection of how many distinct (positive) factors of 50 will guarantee that at least two of them have a product of 50? Explain.

7. Suppose a committee of the UN contains 193 diplomats, exactly one representing each of the 193 nation states. In how many ways can a subcommittee of 20 be selected

    a. If the representative from Switzerland must be on the committee?

    b. If exactly 5 members must be from the 54 African states?

    c. If a head of state wants to receive all 193 diplomats and greet them in person, in how many different ways can they be lined up to be received by her?

       You do not need to explain your answers. Give all answers in terms of combinations, permutations or factorials. There is no need to evaluate them.