Mathematical Induction

Use induction to prove that $n^n > n!$ for all positive integers $n \geq 2$.

Let $P(n)$ denote the proposition that $n^n > n!$, where n is a positive integer, $n \geq 2$.

BASIS STEP: $P(2)$ is true since $2^2 = 4 > 2! = 2$.

INDUCTIVE STEP: Let us assume $P(n)$, that is $n^n > n!$ is true for an arbitrary positive integer $n \geq 2$. This is our inductive hypothesis.

We have to show that $P(n+1)$, $(n+1)^{(n+1)} > (n+1)!$ is also true assuming the inductive hypothesis $P(n)$.

Proof:

$(n+1)^{n+1} = (n+1) \cdot (n+1)^{n} > (n+1) \cdot n^{n}$ since $n+1 > n$.

$(n+1) \cdot n^{n} \geq (n+1) \cdot n! = (n+1)!$ by the inductive hypothesis.

By the **Principle of Mathematical Induction** (Basis Step and Inductive Step together) $n^{n} > n!$ for all positive integers $n \geq 2$.