MAT 243 ONLINE WRITTEN HW 4

NAME: ______________________________

(1) **(4 pts)** Fill in the blank in the statements below:

(a) A function $f(x)$ is big-O of $g(x)$ if and only if ______________________________

(b) An integer a divides integer b if and only if ______________________________

(c) Integers a and b are congruent mod m if and only if ________________________

(d) If gcd(a,b)=1 then we say that a and b are ______________________________

(2) **(10 pts)** For each of the given the functions find the best big-O estimate. Explain all your steps by referring to the theorems you are using.

(a) $f(x) = (x^3 + log(x^6))(log(x) + 17)$

(b) $g(n) = (2^n + log(n))(log(n!) + n^2)$

(3) **(10 pts)** Compute 4^{1033} mod 9 using fast modular exponentiation. Show and explain all your steps.

(4) **(10 pts)** Prove or disprove: If $a \equiv b \pmod{2m}$ then $a \equiv b \pmod{m}$.

(5) **(10 pts)** Follow the idea of the example below and generalize the argument to prove that for any non-negative integer n if the sum of the digits of n is divisible by 9 then n is divisible by 9.

$$\begin{aligned} 4257 &= 4(1000) + 2(100) + 5(10) + 7 \\ &= 4(999 + 1) + 2(99 + 1) + 5(9 + 1) + 7 \\ &= 4(999) + 2(99) + 5(9) + (4 + 2 + 5 + 7) \end{aligned}$$

Hint: Let $n = a_n 10^n + a_{n-1}10^{n-1} + \ldots\ a_1 10 + a_0$

(6) **(4 pts)** Find gcd(847,161) using the Euclidean algorithm. Show all your steps.