# Homework 2-1

**Due: 11:59pm on Tuesday, October 28, 2014**

To understand how points are awarded, read the Grading Policy for this assignment.

## Going for a Drive

**Learning Goal:**

To gain a qualitative understanding of kinematics and how the qualitative nature of position and velocity versus time graphs relates to the equations of kinematics.

In this problem, you will explore kinematics using an applet that simulates a car moving under constant acceleration. When you open the applet, you will see three sliders that allow you to adjust the initial position $x_0$, the initial velocity $v_0$, and the acceleration $a$. Set the initial position to 0 $\mathrm{m}$, the initial velocity to $-10\ \mathrm{m/s}$ and the acceleration to 5 $\mathrm{m/s^2}$.

Run the simulation. Notice that, as the movie proceeds, pictures of the car remain at certain points. Once the simulation is over, these pictures form a motion diagram--a representation of motion consisting of pictures taken at equal time intervals during the motion. In this case, the interval between pictures is one second.

Below the movie, the position of the car as a function of time is graphed in green. Run the simulation several times, paying attention to how the graph and the motion diagram/movie of the car's motion relate to each other.

### Part A

Which of the following describe the relationship between the motion diagram/movie and the graph?

**Check all that apply.**

ANSWER:

- [x] When the slope of the graph is close to zero, the pictures in the motion diagram are close together.
- [x] When the slope of the graph is steep, the car is moving quickly.
- [ ] When the slope of the graph is positive, the car is to the right of its starting position.
- [ ] When the *x* position on the graph is negative, the car moves backward. When the *x* position on the graph is positive, the car moves forward.
- [ ] When the slope of the graph is negative, the car *always* moves slower than when the slope of the graph is positive.
- [x] When the *x* position on the graph is negative, the car is to the left of its starting position. When the *x* position on the graph is positive, the car is to the right of its starting position.

**All attempts used; correct answer displayed**

Notice that the first and second options are always true, regardless of the values of $x_0$, $v_0$, and $a$. The last option, however, is only true when $x_0 = 0$. Frequently, you will be able to pick your coordinate system. In such cases, making $x_0 = 0$ is often a good choice.

### Part B

Run the simulation, paying close attention to the graph of position. Press reset and change the value of $x_0$. Run the simulation again, noting any changes in the graph. How does varying $x_0$ affect the graph of position?

ANSWER:

- Increasing $x_0$ increases the width of the graph, whereas decreasing $x_0$ decreases the width.
- Increasing $x_0$ shifts the graph to the right, whereas decreasing it shifts the graph to the left.
- Increasing $x_0$ shifts the graph to the left, whereas decreasing it shifts the graph to the right.
- Increasing $x_0$ shifts the graph upward, whereas decreasing it shifts the graph downward.
- Increasing $x_0$ shifts the graph downward, whereas decreasing it shifts the graph upward.
- Changing $x_0$ does not affect the graph.

**Correct**

## Part C

Now, run the simulation with different values of $v_0$, but don't use any positive values. Note any changes in the graph. How does varying $v_0$ affect the graph of position?

**Choose the best answer.**

ANSWER:

- Increasing $v_0$ increases the width of the graph, whereas decreasing $v_0$ decreases the width.
- Increasing $v_0$ shifts the graph to the right and upward, whereas decreasing it shifts the graph to the left and downward.
- Increasing $v_0$ shifts the graph to the left and upward, whereas decreasing it shifts the graph to the right and downward.
- Increasing $v_0$ shifts the graph to the right and downward, whereas decreasing it shifts the graph to the left and upward.
- Increasing $v_0$ shifts the graph to the left and downward, whereas decreasing it shifts the graph to the right and upward.
- Changing $v_0$ does not affect the graph.

**Correct**

This behavior may be a bit difficult to understand by just looking at the equation for position vs. time that you know from kinematics. If you complete the square to get the equation into standard form for a parabola, this should become more apparent.

Now that you've seen how $v_0$ affects the graph, run the simulation with a few different values of acceleration. You should see that increasing the acceleration decreases the width of the graph and decreasing the acceleration increases the width. (Decreasing the acceleration below 0 makes the parabola open downward instead of upward.)

## Part D

Enter the equation for position $x(t)$ as a function of time. Before submitting your answer, check that it is consistent with the qualities of the graph that you have identified. For instance, if you increase $x_0$ in the equation, would it move the graph upward?

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**Express your answer in terms of time $t$, initial position $x_0$, initial velocity $v_0$, and acceleration $a$.**

ANSWER:

$x(t)$ = $x_0 + v_0 t + \frac{1}{2} a t^2$

**Correct**

## Part E

Now, open this applet. This applet looks like the previous applet, but when you run the simulation, you will now get graphs of both position and velocity. Run the simulation several times with different values of $x_0$. How does changing $x_0$ affect the graph of velocity?

ANSWER:

- Increasing $x_0$ increases the slope of the graph, whereas decreasing $x_0$ decreases the slope.
- Increasing $x_0$ shifts the graph to the right, whereas decreasing it shifts the graph to the left.
- Increasing $x_0$ shifts the graph to the left, whereas decreasing it shifts the graph to the right.
- Increasing $x_0$ shifts the graph upward, whereas decreasing it shifts the graph downward.
- Increasing $x_0$ shifts the graph downward, whereas decreasing it shifts the graph upward.
- Changing $x_0$ does not affect the graph.

**Correct**

## Part F

Run the simulation again, with the following settings: $x_0 = 10 \text{ m}$, $v_0 = -15 \text{ m/s}$, and $a = 5 \text{ m/s}^2$. The units of time in the graph are seconds. At what time $t$ is the velocity equal to zero?

**Express your answer in seconds to the nearest integer.**

ANSWER:

$t(v=0)$ = 3 $\text{s}$

**Correct**

Notice that the position graph has a minimum when velocity equals zero. This should make sense to you. Since velocity is the derivative of position, position has a local minimum or maximum when velocity is zero.

## Part G

Suppose that a car starts from rest at position $-3.31 \text{ m}$ and accelerates with a constant acceleration of 4.15 $\text{m/s}^2$. At what time $t$ is the velocity of the car 19.2 $\text{m/s}$? Use the applet to be certain that your answer is reasonable.

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**Express your answer in seconds to three significant figures.**

**Hint 1.** Choose the kinematic equation

You have seen a number of kinematic equations. Choose the one from the following list that will be the most useful in this problem. Again, $a$ is the acceleration, $v$ and $x$ are, respectively, velocity and position at time $t$, while $v_0$ and $x_0$ are, respectively, the initial velocity and initial position.

ANSWER:

- $v = v_0 + at$
- $x = x_0 + v_0 t + \frac{1}{2} at^2$
- $x - x_0 = \left(\frac{v_0+v}{2}\right)t$
- $v^2 = v_0^2 + 2a(x - x_0)$

**Correct**

**Hint 2.** Using the applet to check your answer

The applet does not allow you to enter numbers to the precision asked for in the problem. However, you can check that your answer is reasonable by selecting values close to the ones asked for in the problem. For instance, you cannot use the acceleration 4.15 $\mathrm{m/s^2}$ with the applet, but you can use 4 $\mathrm{m/s^2}$. If you set all of the values on the applet as close as you can to the values in the problem, then the answer seen on the applet should be close to the correct answer. If the applet shows an answer of 10 $\mathrm{s}$ and your calculations give you 5 $\mathrm{s}$, then you should check your calculations for errors and be sure that you're using the right equations and principles. However, if your calculations give 9.5 $\mathrm{s}$, there is a good chance that you are right.

ANSWER:

$t$ = 4.63 $\mathrm{s}$

**Correct**

## Part H

For the same initial conditions as in the last part, what is the car's position $x$ at time 4.05 $\mathrm{s}$? Again, be sure to use the applet to check that your answer is reasonable.

**Express your answer in meters to three significant figures.**

**Hint 1.** Choose the kinematic equation

You have seen a number of kinematic equations. Choose the one from the following list that will be the most useful in this problem. Again, $a$ is the acceleration, $v$ and $x$ are, respectively, velocity and position at time $t$, while $v_0$ and $x_0$ are, respectively, the initial velocity and initial position.

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ANSWER:

- $v = v_0 + at$
- $x = x_0 + v_0 t + \frac{1}{2}at^2$
- $x - x_0 = \left(\frac{v_0+v}{2}\right)t$
- $v^2 = v_0^2 + 2a(x - x_0)$

**Correct**

ANSWER:

$x$ = 30.7 $\mathrm{m}$

**Correct**

Any time that you are working a physics problem, you should check that your answer is reasonable. Even when you don't have an applet with which to check, you have a wealth of personal experience. For example, if you obtain an answer such as "the distance from New York to Los Angeles is 3.96 $\mathrm{m}$," you know it must be wrong. You should always try to relate situations from physics class to real-life situations.

## Catching the Dot

This applet shows a dot moving with constant acceleration.

### Part A

Change the values of the initial position, initial velocity, and the acceleration of the car so that its center follows the same path as the dot. What is the equation $x(t)$ that describes the motion of the dot?

**Express your answer in terms of $t$. Enter all of the numbers in the equation without their units, as it is understood that you are using meters, meters per second, and meters per second squared for distances, speeds, and accelerations, respectively.**

**Hint 1.** How to approach the problem

Think about which quantity ( $x_0$, $v_0$, or $a$) can be determined most easily. Once you find that value as accurately as possible, decide which of the remaining two will be easiest to determine. Once you get to the third quantity, you may have to refine the other two slightly based on how close you are to matching the dot's trajectory.

Once you have determined the correct values, think about which kinematic formula gives the value of $x$ in terms of $x_0$, $v_0$, $a$, and $t$. Substitute the appropriate values into the formula and you will have the equation describing the trajectory of the dot.

**Hint 2.** Find $x_0$

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Find the initial position $x_0$ of the dot. This should be relatively easy: Look at the scale and estimate where on the *x*-axis the dot is located. Move the slider so that the car is at the same point and then look at the picture to be sure that the center of the car is lined up with the dot.

**Express your answer in meters to three significant figures.**

ANSWER:

$x_0$ = 12.0 $\mathrm{m}$

**Correct**

### **Hint 3.** Find $v_0$

Find the initial velocity $v_0$ of the dot. This will be much easier if you find $x_0$ first and set the car and dot to the same $x_0$. Then, you may want to adjust the $v_0$ and $a$ settings for the car at the same time to help make the trajectory of the car more similar to that of the dot.

**Express your answer in meters per second to three significant figures.**

#### **Hint 1.** Recognizing differences in speed and acceleration

Matching $x_0$ is relatively easy, because the dot is not moving. You can simply adjust the slider until the car and dot line up. However, values for $v_0$ and $a$ can only be discerned based on the motion of the car, and each contributes differently. To sort out differences in $v_0$ and $a$ between the dot and the car, you must think about what velocity and acceleration mean.

Velocity is the rate at which the position of an object is changing. Acceleration is the rate at which the velocity is changing. Thus, when you look at the position of the dot at times shortly after you press the run button, the position will be determined almost completely by the initial velocity. If the dot has a large acceleration, then the speed will change quickly and you will notice that the distance covered by the dot in one second will change significantly from second to second.

It is best to change the initial velocity until the dot and the car seem to move roughly together immediately after you press the "run" button. Next, adjust the acceleration so that their speeds seem to change in the same way. Then, go back to get the initial velocities the same, and finally adjust the accelerations so that the car and the dot move together at all times. Realistically, you may have to go back and forth refining $v_0$ and $a$ several times before the car and dot move together perfectly. This technique, known as *iteration*, can often be helpful in solving problems that defy straightforward analytic solutions.

ANSWER:

$v_0$ = -12.0 $\mathrm{m/s}$

**Correct**

### **Hint 4.** Find $a$

Find the acceleration $a$ of the dot. This will be much easier if you find $x_0$ first and set the car and dot to the same $x_0$. You may want to adjust the $v_0$ and $a$ settings for the car at the same time to help make the trajectory of the

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car closer to that of the dot. It will be helpful to get the value of $v_0$ close to that of the dot before trying to match the accelerations.

**Express your answer in meters per second squared to two significant figures.**

**Hint 1.** Recognizing differences in speed and acceleration

Matching $x_0$ is relatively easy, because the dot is not moving. You can simply adjust the slider until the car and dot line up. However, values for $v_0$ and $a$ can only be discerned based on the motion of the car, and each contributes differently. To sort out differences in $v_0$ and $a$ between the dot and the car, you must think about what velocity and acceleration mean.

Velocity is the rate at which the position of an object is changing. Acceleration is the rate at which the velocity is changing. Thus, when you look at the position of the dot at times shortly after you press the run button, the position will be determined almost completely by the initial velocity. If the dot has a large acceleration, then the speed will change quickly and you will notice that the distance covered by the dot in one second will change significantly from second to second.

It is best to change the initial velocity until the dot and the car seem to move roughly together immediately after you press the "run" button. Next, adjust the acceleration so that their speeds seem to change in the same way. Then, go back to get the initial velocities the same, and finally adjust the accelerations so that the car and the dot move together at all times. Realistically, you may have to go back and forth refining $v_0$ and $a$ several times before the car and dot move together perfectly. This technique, known as *iteration*, can often be helpful in solving problems that defy straightforward analytic solutions.

ANSWER:

$a$ = 2.0 $\mathrm{m/s^2}$

Correct

**Hint 5.** The appropriate equation

Recall that the kinematic equation describing the motion of an object with constant acceleration is the following:

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2.$$

ANSWER:

$x(t)$ = $12 - 12t + t^2$

Correct

## Part B

The simulation runs until $t = 20\ \mathrm{s}$, even though the dot is far off of the screen by then. What is the position $x(20\ \mathrm{s})$ of the dot at time $t = 20\ \mathrm{s}$?

Typesetting math: 100% **[...]swer in meters to at least two significant figures.**

ANSWER:

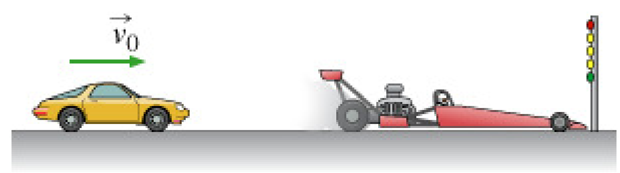
$x(20\ \mathrm{s})$ = 172 $\mathrm{m}$

**Correct**

## Rearending Drag Racer

To demonstrate the tremendous acceleration of a top fuel drag racer, you attempt to run your car into the back of a dragster that is "burning out" at the red light before the start of a race. (Burning out means spinning the tires at high speed to heat the tread and make the rubber sticky.)

You drive at a constant speed of $v_0$ toward the stopped dragster, not slowing down in the face of the imminent collision. The dragster driver sees you coming but waits until the last instant to put down the hammer, accelerating from the starting line at constant acceleration, $a$. Let the time at which the dragster starts to accelerate be $t = 0$.




### Part A

What is $t_{\mathrm{max}}$, the longest time after the dragster begins to accelerate that you can possibly run into the back of the dragster if you continue at your initial velocity?

**Hint 1.** Calculate the velocity

At $t_{\mathrm{max}}$, what will the velocity of the drag car be?

**Your answer should not contain $t_{\mathrm{max}}$, as that time is not yet known.**

**Hint 1.** Consider the speed of both cars

No collision can occur if the dragster has greater speed than the speed of the car behind it.

ANSWER:

$v_{\mathrm{d}}(t_{\mathrm{max}})$ = $v_0$

ANSWER:

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$t_{max}$ = $\frac{v_0}{a}$

**Correct**

## Part B

Assuming that the dragster has started at the last instant possible (so your front bumper *almost* hits the rear of the dragster at $t = t_{max}$), find your distance from the dragster when he started. If you calculate positions on the way to this solution, choose coordinates so that the position of the drag car is 0 at $t = 0$. Remember that you are solving for a distance (which is a magnitude, and can never be negative), not a position (which *can* be negative).

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**Hint 1.** Drag car position at time $t_{max}$

Taking $x = 0$ at the position of the dragster at $t = 0$, find $x_d(t_{max})$, the position of the dragster at $t_{max}$.

**Express your answer in terms of $t_{max}$ and given quantities.**

ANSWER:

$x_d(t_{max})$ = $\frac{1}{2} a{t_{max}}^2$ .

**Hint 2.** Distance car travels until $t_{max}$

Find $D_{car}$, the distance you travel from $t = 0$ to $t_{max}$.

ANSWER:

$D_{car}$ = $v_0 t_{max}$

**Hint 3.** Starting position of car

Express $D_{car}$, the distance the car travels in terms of the starting distance of the car from the starting line at time $t = 0$, $D_{start}$, and the position of the drag car at time $t_{max}$, $x_d(t_{max})$. Note that $D_{start}$ is a distance and can't be negative. This should affect your use of signs.

**Express your answer in terms of $t_{max}$, $x_d(t_{max})$, and $D_{start}$.**

ANSWER:

$D_{car}$ = $x_d(t_{max}) + D_{start}$

**Hint 4.** Obtaining the Solution

Equate your two expressions for the distance traveled by the car up to $t_{max}$, substitute for $t_{max}$ in terms of $v_0$ and $a$, and solve for $D_{start}$, the initial distance of the car from the starting line.
Your answer should be in terms of $v_0$ and $a$.

ANSWER:

$D_{start}$ = $\frac{{v_0}^2}{2a}$

**Correct**

## Part C

Find numerical values for $t_{max}$ and $D_{start}$ in seconds and meters for the (reasonable) values

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$v_0 = 60\ \mathrm{mph}$ (26.8 m/s) and $a = 50\ \mathrm{m/s^2}$.

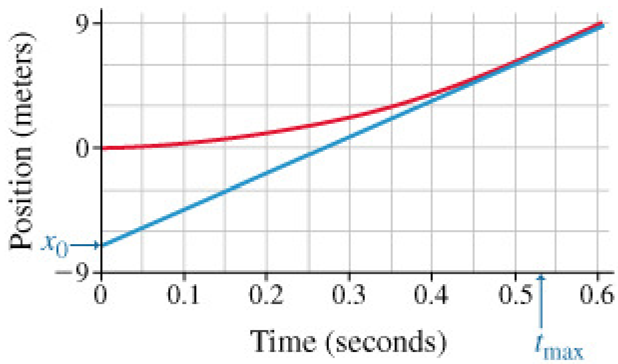
**Separate your two numerical answers by commas, and give your answer to two significant figures.**

ANSWER:

$t_{\mathrm{max}}, D_{\mathrm{start}}$ = 0.54,7.2 s, m

### Correct

The blue curve shows how the car, initially at $x_0$, continues at constant velocity (blue) and just barely touches the accelerating drag car (red) at $t_{\mathrm{max}}$.




## A Flower Pot Falling Past a Window

As you look out of your dorm window, a flower pot suddenly falls past. The pot is visible for a time $t$, and the vertical length of your window is $L_{\mathrm{w}}$. Take down to be the positive direction, so that downward velocities are positive and the acceleration due to gravity is the positive quantity $g$.

Assume that the flower pot was dropped by someone on the floor above you (rather than thrown downward).

### Part A

From what height $h$ above the bottom of your window was the flower pot dropped?

**Express your answer in terms of $L_{\mathrm{w}}$, $t$, and $g$.**

**Hint 1.** How to approach the problem

The initial velocity of the pot is zero. Find the velocity $v_{\mathrm{b}}$ of the pot at the bottom of the window. Then using the kinematic equation that relates initial and final velocities, acceleration, and distance traveled, you can solve for the distance $h$.

**Hint 2.** Find the velocity at the bottom of the window

What is the velocity $v_{\mathrm{b}}$ of the flower pot at the instant it passes the bottom of your window?

**Express your answer in terms of $L_{\rm w}$, $t$, and $g$.**

### **Hint 1.** Find the average velocity

What is the average velocity $v_{\rm avg}$ of the flower pot as it passes by your window?

**Express your answer in terms of $L_{\rm w}$ and $t$.**

ANSWER:

$v_{\rm avg}$ = $\frac{L_w}{t}$

### **Hint 2.** Find the time when $v = v_{\rm avg}$

As the pot falls past your window, there will be some instant when the pot's velocity equals the average velocity $v_{\rm avg}$. How much time $\tau$ does it take, after the pot's instantaneous velocity equals its average velocity, for the pot to reach the bottom of the window? Recall that, under constant acceleration, velocity changes linearly with time. This means that the average velocity during a time interval will occur at the middle of that time interval.

**Express your answer in terms of $t$.**

ANSWER:

$\tau$ = $\frac{t}{2}$

ANSWER:

$v_{\rm b}$ = $\frac{L_w}{t} + \frac{gt}{2}$

### **Hint 3.** The needed kinematic equation

To solve this problem most easily, you should use the kinematic equation $v_{\rm f}^2 - v_{\rm i}^2 = 2a(x_{\rm f} - x_{\rm i})$. Note that you are looking for $x_{\rm f} - x_{\rm i}$, the distance traveled by the flower pot from the moment it was dropped until it reaches the height of the bottom of your window.

ANSWER:

$h$ = $\frac{\left(\frac{L_w}{t}\right)^2 + \left(\frac{L_w}{t} gt\right) + \left(\frac{1}{2} gt\right)^2}{2g}$

**Correct**

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If the bottom of your window is a height $h_{\rm b}$ above the ground, what is the velocity $v_{\rm ground}$ of the pot as it hits the ground? You may introduce the new variable $v_{\rm b}$, the speed at the bottom of the window, defined by

$$v_{\rm b} = \frac{L_{\rm w}}{t} + \frac{gt}{2}.$$

**Express your answer in terms of some or all of the variables $h_{\rm b}$, $L_{\rm w}$, $t$, $v_{\rm b}$, and $g$.**

**Hint 1.** Needed kinematic equation

The initial velocity of the pot is zero. The total distance it falls is $h_{\rm total}$. Using the kinematic equation that relates initial and final velocities, the total distance traveled, and the acceleration, you can solve for the pot's final velocity.

Alternatively, you could use the same kinematic equation, but set $x_{\rm i} = h_{\rm b}$ and $v_{\rm i} = v_{\rm b}$.

**Hint 2.** Find the initial height of the pot

From what height $h_{\rm total}$ above the ground was the pot dropped?

**Express your answer in terms of some or all of the variables $h_{\rm b}$, $L_{\rm w}$, $t$, $v_{\rm b}$, and $g$.**

ANSWER:

$h_{\rm total}$ = $h_b + \frac{\left(\frac{Lw}{t}+\frac{gt}{2}\right)^2}{2g}$

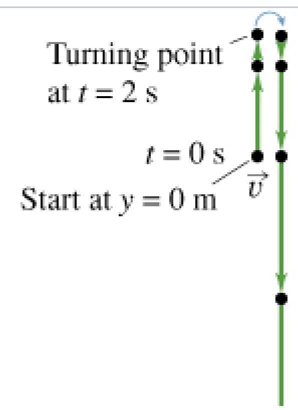
ANSWER:

$v_{\rm ground}$ = $\sqrt{(2gh_b)+\left(\frac{Lw}{t}+\frac{gt}{2}\right)^2}$

**Correct**

## Motion Diagram and Gravity Graphing Question

A stone is thrown upward from the edge of a cliff, reaches its maximum height, and then falls down into the valley below. A motion diagram for this situation is given, beginning the instant the stone leaves the thrower's hand. Construct the corresponding motion graphs taking the acceleration due to gravity as exactly **10** $\rm m/s^2$ . Ignore air resistance. In all three motion graphs, the unit of time is in seconds and the unit of displacement is in meters. In plotting the points, round-off the coordinate values to the nearest integer.

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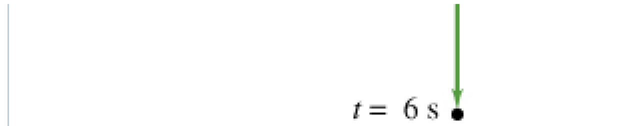


## Part A

Construct a graph corresponding to the stone's vertical displacement, $y(t)$.

### Hint 1. How to approach the problem

The motion diagram indicates the position of the stone at successive values of time, as well as the coordinate system being used to analyze the motion. By examining the first "dot" on the motion diagram, you can determine the initial value of the position of the stone, which is where your position graph should begin. By examining the last "dot" on the motion diagram, you can determine the final position of the stone, which is where your position graph should end.

### Hint 2. Find the initial value of the stone's position

Is the initial value of position positive, negative, or zero?

ANSWER:

- positive
- negative
- zero (selected)

**Correct**

### Hint 3. Find the final value of the stone's position

Is the final value of position positive, negative, or zero?

ANSWER:

- positive
- negative (selected)
- zero

**Correct**

ANSWER:

**All attempts used; correct answer displayed**

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## Part B

Construct a graph corresponding to the stone's vertical velocity, $v_y(t)$.

**Hint 1.** Find the initial value of the stone's velocity

Is the initial value of velocity positive, negative, or zero?

ANSWER:

- (•) positive
- ( ) negative
- ( ) zero

**Correct**

**Hint 2.** Find the final value of the stone's velocity

Is the final value of the stone's velocity positive, negative, or zero?

ANSWER:

- ( ) positive
- (•) negative
- ( ) zero

**Correct**

ANSWER:

**All attempts used; correct answer displayed**

## Part C

Construct a graph corresponding to the stone's vertical acceleration, $a_y(t)$.

**Hint 1.** How to approach the problem

Once the stone leaves the thrower's hand, it is acted on by only the force of gravity. The force of gravity causes acceleration that is constant in both magnitude and direction.

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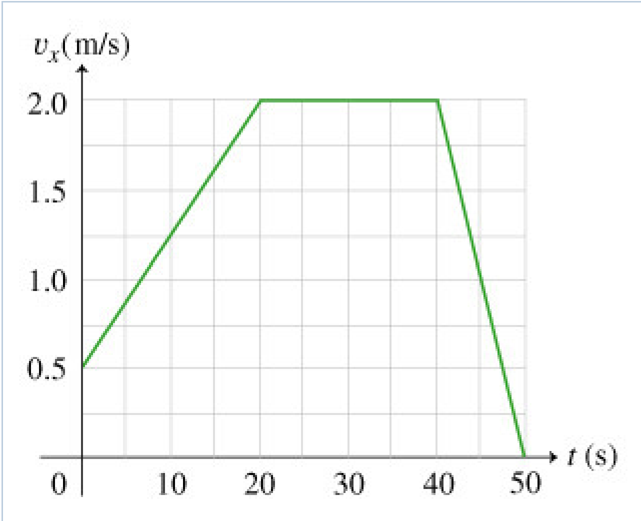
ANSWER:

**Correct**

## What Velocity vs. Time Graphs Can Tell You

A common graphical representation of motion along a straight line is the *v vs. t graph*, that is, the graph of (instantaneous) velocity as a function of time. In this graph, time $t$ is plotted on the horizontal axis and velocity $v$ on the vertical axis. Note that by definition, velocity and acceleration are vector quantities. In straight-line motion, however, these vectors have only a single nonzero component in the direction of motion. Thus, in this problem, we will call $v$ the velocity and $a$ the acceleration, even though they are really the components of the velocity and acceleration vectors in the direction of motion, respectively.

Here is a plot of velocity versus time for a particle that travels along a straight line with a varying velocity. Refer to this plot to answer the following questions.




### Part A

What is the initial velocity of the particle, $v_0$?

**Express your answer in meters per second.**

**Hint 1.** Initial velocity

The initial velocity is the velocity at $t = 0$ s.

**Hint 2.** How to read a v vs. t graph

Recall that in a graph of velocity versus time, time is plotted on the horizontal axis and velocity on the vertical axis. For example, in the plot shown in the figure, $v = 2.00 \text{ m/s}$ at $t = 30.0$ s.

ANSWER:

$v_0$ = 0.5 $\text{m/s}$

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**Correct**

## Part B

What is the total distance $\Delta x$ traveled by the particle?

**Express your answer in meters.**

**Hint 1.** How to approach the problem

Recall that the area of the region that extends over a time interval $\Delta t$ under the v vs. t curve is always equal to the distance traveled in $\Delta t$. Thus, to calculate the total distance, you need to find the area of the entire region under the v vs. t curve. In the case at hand, the entire region under the v vs. t curve is not an elementary geometrical figure, but rather a combination of triangles and rectangles.

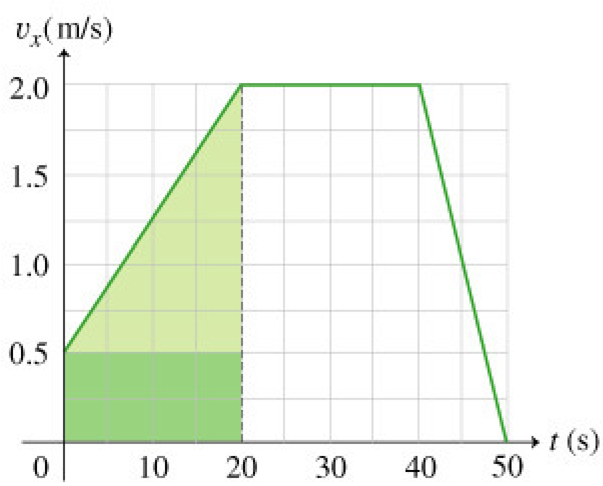
**Hint 2.** Find the distance traveled in the first 20.0 seconds

What is the distance $\Delta x_1$ traveled in the first 20 seconds of motion, between $t = 0.0$ s and $t = 20.0$ s?

**Express your answer in meters.**

**Hint 1.** Area of the region under the v vs. t curve

The region under the v vs. t curve between $t = 0.0$ s and $t = 20.0$ s can be divided into a rectangle of dimensions $20.0$ s by $0.50$ m/s, and a triangle of base $20.0$ s and height $1.50$ m/s, as shown in the figure.




ANSWER:

$\Delta x_1$ = 25 m

**Correct**

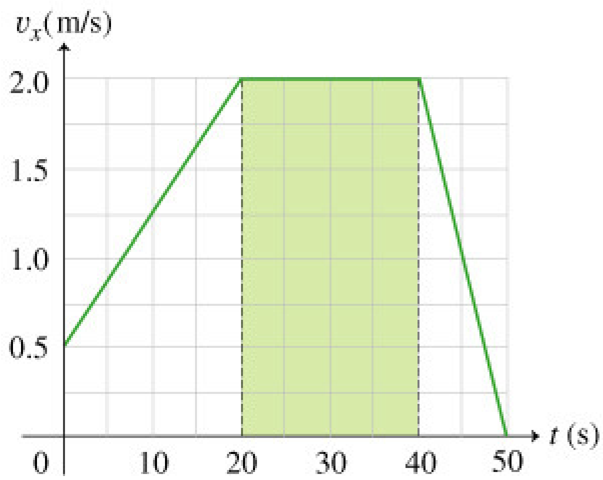
Typesetting math: 100% the distance traveled in the second 20.0 seconds

What is the distance $\Delta x_2$ traveled in the second 20 seconds of motion, from $t = 20.0 \text{ s}$ to $t = 40.0 \text{ s}$?

**Express your answer in meters.**

**Hint 1.** Area of the region under the v vs. t curve

The region under the v vs. t curve between $t = 20.0 \text{ s}$ and $t = 40.0 \text{ s}$ is a rectangle of dimensions $20.0 \text{ s}$ by $2.00 \text{ m/s}$, as shown in the figure.




ANSWER:

$\Delta x_2$ = 40 $\text{m}$

**Hint 4.** Find the distance traveled in the last 10.0 seconds

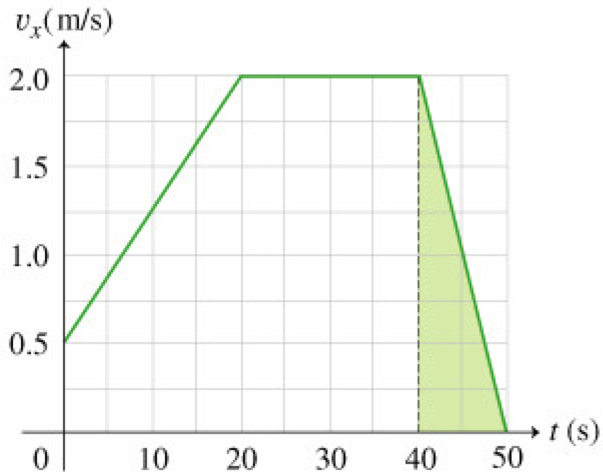
What is the distance $\Delta x_3$ traveled in the last 10 seconds of motion, from $t = 40.0 \text{ s}$ to $t = 50.0 \text{ s}$?

**Express your answer in meters.**

**Hint 1.** Area of the region under the v vs. t curve

The region under the v vs. t curve between $t = 40.0 \text{ s}$ and $t = 50.0 \text{ s}$ is a triangle of base $10.0 \text{ s}$ and height $2.00 \text{ m/s}$, as shown in the figure.

Typesetting math: 100%




ANSWER:

$\Delta x_3$ = 10 m

ANSWER:

$\Delta x$ = 75 m

**Correct**

## Part C

What is the average acceleration $a_{\rm av}$ of the particle over the first 20.0 seconds?

**Express your answer in meters per second per second.**

### Hint 1. Definition and graphical interpretation of average acceleration

The average acceleration $a_{\rm av}$ of a particle that travels along a straight line in a time interval $\Delta t$ is the ratio of the change in velocity $\Delta v$ experienced by the particle to the time interval $\Delta t$, or

$$a_{\rm av} = \frac{\Delta v}{\Delta t}.$$

In a v vs. t graph, then, the average acceleration equals the slope of the line connecting the two points representing the initial and final velocities.

### Hint 2. Slope of a line

The slope $m$ of a line from point A, of coordinates $(x_{\rm A}, y_{\rm A})$, to point B, of coordinates $(x_{\rm B}, y_{\rm B})$, is equal to the "rise" over the "run," or

$$m = \frac{y_{\rm B} - y_{\rm A}}{x_{\rm B} - x_{\rm A}}.$$

Typesetting math: 100%

ANSWER:

$a_{av}$ = 0.075 $\mathrm{m/s^2}$

**Correct**

The average acceleration of a particle between two instants of time is the slope of the line connecting the two corresponding points in a v vs. t graph.

---

## Part D

What is the instantaneous acceleration $a$ of the particle at $t = 45.0 \mathrm{\ s}$?

**Hint 1.** Graphical interpretation of instantaneous acceleration

The acceleration of a particle at any given instant of time or at any point in its path is called the instantaneous acceleration. If the v vs. t graph of the particle's motion is known, you can directly determine the instantaneous acceleration at any point on the curve. The instantaneous acceleration at any point is equal to the slope of the line tangent to the curve at that point.

**Hint 2.** Slope of a line

The slope $m$ of a line from point A, of coordinates $(x_\mathrm{A}, y_\mathrm{A})$, to point B, of coordinates $(x_\mathrm{B}, y_\mathrm{B})$, is equal to the "rise" over the "run," or

$$m = \frac{y_\mathrm{B} - y_\mathrm{A}}{x_\mathrm{B} - x_\mathrm{A}}.$$

ANSWER:

$a$ =

- ○ 1 $\mathrm{m/s^2}$
- ○ 0.20 $\mathrm{m/s^2}$
- ◉ -0.20 $\mathrm{m/s^2}$
- ○ 0.022 $\mathrm{m/s^2}$
- ○ -0.022 $\mathrm{m/s^2}$

**Correct**

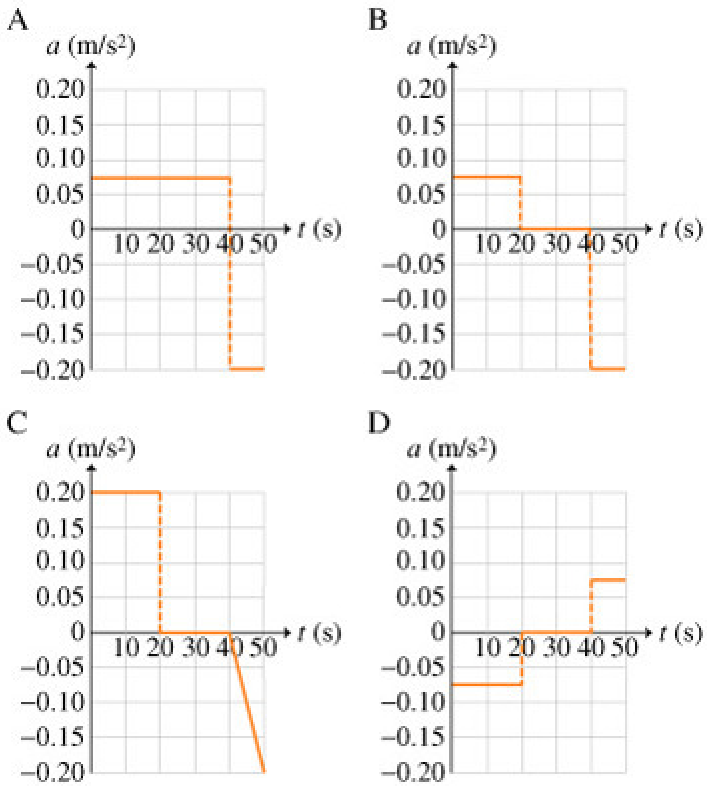
The instantaneous acceleration of a particle at any point on a v vs. t graph is the slope of the line tangent to the curve at that point. Since in the last 10 seconds of motion, between $t = 40.0 \mathrm{\ s}$ and $t = 50.0 \mathrm{\ s}$, the curve is a straight line, the tangent line is the curve itself. Physically, this means that the instantaneous acceleration of the particle is *constant* over that time interval. This is true for any motion where velocity increases linearly with time. In the case at hand, can you think of another time interval in which the acceleration of the particle is constant?

Now that you have reviewed how to plot variables as a function of time, you can use the same technique and draw an acceleration vs. time graph, that is, the graph of (instantaneous) acceleration as a function of time. As usual in these types of graphs, time $t$ is plotted on the horizontal axis, while the vertical axis is used to indicate acceleration $a$.

Typesetting math: 100%

## Part E

Which of the graphs shown below is the correct acceleration vs. time plot for the motion described in the previous parts?




### Hint 1. How to approach the problem

Recall that whenever velocity increases linearly with time, acceleration is constant. In the example here, the particle's velocity increases linearly with time in the first 20.0 s of motion. In the second 20.0 s , the particle's velocity is constant, and then it decreases linearly with time in the last 10 s. This means that the particle's acceleration is constant over each time interval, but its value is different in each interval.

### Hint 2. Find the acceleration in the first 20 s

What is $a_1$, the particle's acceleration in the first 20 s of motion, between $t = 0.0$ s and $t = 20.0$ s?

**Express your answer in meters per second per second.**

#### Hint 1. Constant acceleration

Since we have already determined that in the first 20 s of motion the particle's acceleration is constant, its constant value will be equal to the average acceleration that you calculated in Part C.

ANSWER:

$a_1$ = 0.075 $\mathrm{m/s^2}$

### Hint 3. Find the acceleration in the second 20 s

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What is $a_2$, the particle's acceleration in the second 20 s of motion, between $t = 20.0$ s and $t = 40.0$ s?

**Express your answer in meters per second per second.**

**Hint 1.** Constant velocity

In the second 20 s of motion, the particle's velocity remains unchanged. This means that in this time interval, the particle does not accelerate.

ANSWER:

$a_2$ = 0 $\mathrm{m/s^2}$

**Hint 4.** Find the acceleration in the last 10 s

What is $a_3$, the particle's acceleration in the last 10 s of motion, between $t = 40.0$ s and $t = 50.0$ s?

**Express your answer in meters per second per second.**

**Hint 1.** Constant acceleration

Since we have already determined that in the last 10 s of motion the particle's acceleration is constant, its constant value will be equal to the instantaneous acceleration that you calculated in Part D.

ANSWER:

$a_3$ = -0.20 $\mathrm{m/s^2}$

ANSWER:

- Graph A
- Graph B
- Graph C
- Graph D

**Correct**

In conclusion, graphs of velocity as a function of time are a useful representation of straight-line motion. If read correctly, they can provide you with all the information you need to study the motion.

## One-Dimensional Kinematics with Constant Acceleration

**Learning Goal:**

To understand the meaning of the variables that appear in the equations for one-dimensional kinematics with constant

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Motion with a constant, nonzero acceleration is not uncommon in the world around us. Falling (or thrown) objects and cars starting and stopping approximate this type of motion. It is also the type of motion most frequently involved in introductory kinematics problems.

The kinematic equations for such motion can be written as

$$x(t) = x_{\rm i} + v_{\rm i} t + \frac{1}{2} a t^2,$$

$$v(t) = v_{\rm i} + a t,$$

where the symbols are defined as follows:

- $x(t)$ is the position of the particle;
- $x_{\rm i}$ is the *initial* position of the particle;
- $v(t)$ is the velocity of the particle;
- $v_{\rm i}$ is the *initial* velocity of the particle;
- $a$ is the acceleration of the particle.

In anwering the following questions, assume that the acceleration is constant and nonzero: $a \neq 0$.

## Part A

The quantity represented by $x$ is a function of time (i.e., is not constant).

ANSWER:

- (selected) true
- false

**Correct**

## Part B

The quantity represented by $x_{\rm i}$ is a function of time (i.e., is not constant).

ANSWER:

- true
- (selected) false

**Correct**

Recall that $x_{\rm i}$ represents an initial value, not a variable. It refers to the position of an object at some initial moment.

## Part C

The quantity represented by $v_{\rm i}$ is a function of time (i.e., is not constant).

Typesetting math: 100%

ANSWER:

- true
- false (selected)

**Correct**

## Part D

The quantity represented by $v$ is a function of time (i.e., is not constant).

ANSWER:

- true (selected)
- false

**Correct**

The velocity $v$ always varies with time when the linear acceleration is nonzero.

## Part E

Which of the given equations is not an explicit function of $t$ and is therefore useful when you don't know or don't need the time?

ANSWER:

- $x = x_{\mathrm{i}} + v_{\mathrm{i}}t + \frac{1}{2}at^2$
- $v = v_{\mathrm{i}} + at$
- $v^2 = v_{\mathrm{i}}^2 + 2a(x - x_{\mathrm{i}})$ (selected)

**Correct**

## Part F

A particle moves with constant acceleration $a$. The expression $v_{\mathrm{i}} + at$ represents the particle's velocity at what instant in time?

ANSWER:

Typesetting math: 100%

- only at time $t = 0$
- only at the "initial" time
- when a time $t$ has passed since the particle's velocity was $v_{\rm i}$

**Correct**

More generally, the equations of motion can be written as

$$x(t) = x_{\rm i} + v_{\rm i}\ \Delta t + \tfrac{1}{2}\, a\, (\Delta t)^2$$

and

$$v(t) = v_{\rm i} + a\ \Delta t.$$

Here $\Delta t$ is the time that has elapsed since the beginning of the particle's motion, that is, $\Delta t = t - t_{\rm i}$, where $t$ is the current time and $t_{\rm i}$ is the time at which we start measuring the particle's motion. The terms $x_{\rm i}$ and $v_{\rm i}$ are, respectively, the position and velocity at $t = t_{\rm i}$. As you can now see, the equations given at the beginning of this problem correspond to the case $t_{\rm i} = 0$, which is a convenient choice if there is only one particle of interest.

To illustrate the use of these more general equations, consider the motion of two particles, A and B. The position of particle A depends on time as $x_{\rm A}(t) = x_{\rm i} + v_{\rm i}t + (1/2)at^2$. That is, particle A starts moving at time $t = t_{\rm iA} = 0$ with velocity $v_{\rm iA} = v_{\rm i}$, from $x_{\rm iA} = x_{\rm i}$. At time $t = t_1$, particle B has twice the acceleration, half the velocity, and the same position that particle A had at time $t = 0$.

---

## Part G

What is the equation describing the position of particle B?

**Hint 1.** How to approach the problem

The general equation for the distance traveled by particle B is

$$x_{\rm B}(t) = x_{\rm iB} + v_{\rm iB}\Delta t + \tfrac{1}{2}\, a_{\rm B}(\Delta t)^2,$$

or

$$x_{\rm B}(t) = x_{\rm B}(t = t_1) + v_{\rm B}(t = t_1)(t - t_1) + \tfrac{1}{2}\, a_{\rm B}(t - t_1)^2,$$

since $\Delta t = t - t_1$ is a good choice for B. From the information given, deduce the correct values of the constants that go into the equation for $x_B(t)$ given here, in terms of A's constants of motion.

ANSWER:

Typesetting math: 100%

- $x_B(t) = x_i + 2v_i t + \frac{1}{4}at^2$
- $x_B(t) = x_i + 0.5v_i t + at^2$
- $x_B(t) = x_i + 2v_i(t + t_1) + \frac{1}{4}a(t + t_1)^2$
- $x_B(t) = x_i + 0.5v_i(t + t_1) + a(t + t_1)^2$
- $x_B(t) = x_i + 2v_i(t - t_1) + \frac{1}{4}a(t - t_1)^2$
- $x_B(t) = x_i + 0.5v_i(t - t_1) + a(t - t_1)^2$

**Correct**

## Part H

At what time does the velocity of particle B equal that of particle A?

### Hint 1. Velocity of particle A

Type an expression for particle A's velocity as a function of time.

**Express your answer in terms of $t$ and some or all of the variables $x_i$, $v_i$, and $a$.**

#### Hint 1. How to approach this part

Look at the general expression for $v(t)$ given in the problem introduction.

ANSWER:

$v_A(t)$ = $v_i + at$

### Hint 2. Velocity of particle B

Type an expression for particle B's velocity as a function of time.

**Express your answer in terms of $t$ and some or all of the variables $t_1$, $x_i$, $v_i$, and $a$.**

#### Hint 1. How to approach this part

The general expression for $v_B(t)$ is

$$v_B(t) = v_B(t = t_1) + a_B(t - t_1).$$

From the information given, deduce the correct values of the constants that go into this equation in terms of particle A's constants of motion.

ANSWER:

Typesetting math: 100%

$v_B(t)$ = $0.5v_i + 2a(t - t_1)$

ANSWER:

- $t = t_1 + \frac{v_i}{4a}$
- $t = 2t_1 + \frac{v_i}{2a}$
- $t = 3t_1 + \frac{v_i}{2a}$
- The two particles never have the same velocity.

**Correct**

## Problem 2.58

A toy train is pushed forward and released at $x_0 = 2.0$ m with a speed of 2.0 $\mathrm{m/s}$. It rolls at a steady speed for 2.0 s, then one wheel begins to stick. The train comes to a stop 6.0 m from the point at which it was released.

### Part A

What is the magnitude of the train's acceleration after its wheel begins to stick? Assume acceleration is constant after wheel begins to stick.

**Express your answer to two significant figures and include the appropriate units.**

ANSWER:

$a$ = 1.0 $\frac{\mathrm{m}}{\mathrm{s}^2}$

**Correct**

## Problem 2.33

A particle's velocity is described by the function $v_x = kt^2$ m/s, where $v_x$ is in m/s, $t$ is in s, and $k$ is a constant. The particle's position at $t_0 = 0$ s is $x_0$ = -8.00m . At $t_1$ = 2.00s , the particle is at $x_1$ = 6.60m .

### Part A

Determine the units of $k$ in terms of m and s.

ANSWER:

$\frac{m}{s^3}$

Typesetting math: 100%

**Correct**

## Part B

Determine the value of the constant $k$.

**Express your answer to three significant figures.**

ANSWER:

$k$ = 5.48 $\mathrm{m/s^3}$

**Correct**

# Problem 2.83

The Starship Enterprise returns from warp drive to ordinary space with a forward speed of 60$\mathrm{km/s}$ . To the crew's great surprise, a Klingon ship is 150$\mathrm{km}$ directly ahead, traveling in the same direction at a mere 20$\mathrm{km/s}$ . Without evasive action, the Enterprise will overtake and collide with the Klingons in just about 3.8$\mathrm{s}$ . The Enterprise's computers react instantly to brake the ship.

## Part A

What magnitude acceleration does the Enterprise need to just barely avoid a collision with the Klingon ship? Assume the acceleration is constant.
***Hint***: Draw a position-versus-time graph showing the motions of both the Enterprise and the Klingon ship. Let $x_0$ = 0$\mathrm{km}$ be the location of the Enterprise as it returns from warp drive. How do you show graphically the situation in which the collision is "barely avoided"? Once you decide what it looks like graphically, express that situation mathematically.

**Express your answer to two significant figures and include the appropriate units.**

ANSWER:

$a$ = 5.3 $\mathrm{\frac{km}{s^2}}$

**Correct**

**Score Summary:**

Your score on this assignment is 92.9%.
You received 9.29 out of a possible total of 10 points.

Typesetting math: 100%