%Exercise 1

A = floor(10*rand(4,3))

A =

- 9 8 8
- 2 7 4
- 6 4 6
- 4 0 7

E1 = eye(4);

E1([1,3],:)=E1([3,1],:)

E1 =

- 0 0 1 0
- 0 1 0 0
- 1 0 0 0
- 0 0 0 1

E2 = eye(4);

E2(2,2)=5

E2 =

- 1 0 0 0
- 0 5 0 0
- 0 0 1 0
- 0 0 0 1

E3 = eye(4);

E3(4,1)=7

E3 =

- 1 0 0 0
- 0 1 0 0
- 0 0 1 0
- 7 0 0 1

E1*A

ans =

- 6 4 6
- 2 7 4
- 9 8 8
- 4 0 7

E2*A

ans =

- 9 8 8
- 10 35 20
- 6 4 6
- 4 0 7

E3*A

ans =

- 9 8 8
- 2 7 4
- 6 4 6
- 67 56 63

%For E1*A, the rows were swapped/exchanged.

%For E2*A, the second row was multiplied by 5.

%For E3*A, Row 4 was the original row 4 plus 7*row1

%Exercise 2

%Part A

A = [1,-2,3;2,-6,5;-1,-4,0];

E1 = eye(3);

E1(2,1) = -2

E1 =

- 1 0 0
- -2 1 0
- 0 0 1

E1*A

ans =

- 1 -2 3
- 0 -2 -1
- -1 -4 0

E2 = eye(3);

$$E2(3,1) = 1$$

E2 =

- 1 0 0
- 0 1 0
- 1 0 1

E2*E1*A

ans =

- 1 -2 3
- 0 -2 -1
- 0 -6 3

E3 = eye(3);

$$E3(3,2) = -3$$

E3 =

- 1 0 0
- 0 1 0
- 0 -3 1

U = E3*E2*E1*A

U =

- 1 -2 3
- 0 -2 -1
- 0 0 6

%U is the upper triangular matrix. %Part B

L = inv(E1)*inv(E2)*inv(E3)

L=

- 1 0 0
- 2 1 0
- -1 3 1

%L is the lower triangular matrix.

A=L*U

A =

- 1 -2 3
- 2 -6 5
- -1 -4 0

%L*U produced matrix A.

%Exercise 3

%Part A

p = [3,1,2,5,4];

E = eye(length(p));

E = E(p,:)

E =

- 0 0 1 0 0
- 1 0 0 0 0
- 0 1 0 0 0
- 0 0 0 0 1
- 0 0 0 1 0

A = floor(10*rand(5))

A =

- 9 9 8 6 4
- 7 4 0 2 9
- 1 8 1 1 4
- 4 0 2 0 4

9 3 1 7 8

E*A

ans =

1 8 1 1 4 9 9 8 6 4 7 4 0 2 9 9 3 1 7 8 4 0 2 0 4

A*E

ans =

9 8 9 4 6 4 0 7 9 2 8 1 1 4 1 0 2 4 4 0 3 1 9 8 7

%For E*A operation, the rows were swapped in the same order as E matrix. %For A*E operation, the columns were swapped in matrix A. %Part B inv(E)

ans =

E'

ans =

 %E and E' produced the same results.

%Exercise 4

%Part A

A = [2,2,4,6;1,2,1,4;-4,-3,-7,-8;2,1,3,3];

b = [6;8;-11;3];

[L,U,P] = Iu(A)

L=

1.0000 0 0 0

-0.2500 1.0000 0 0

-0.5000 0.4000 1.0000 0

-0.5000 -0.4000 -1.0000 1.0000

U =

-4.0000 -3.0000 -7.0000 -8.0000

0 1.2500 -0.7500 2.0000

0 0.8000 1.2000

0 0 0 1.0000

P =

0 0 1 0

0 1 0 0

1 0 0 0

0 0 0 1

L*U

ans =

-4 -3 -7 -8

1 2 1 4

2 2 4 6

2 1 3 3

P*A-L*U (If PA = LU, their differences evaluate to 0)

ans =

- 0 0 0 0
- 0 0 0 0
- 0 0 0 0
- 0 0 0 0

%Part B

 $y = L \setminus (P*b);$

x = U y

x =

- -1.0000
- 8.0000
- 1.0000
- -2.0000

P*b

ans =

- -11
- 8
- 6
- 3

P*A*x

ans =

- -11
- 8
- 6
- 3

%Part C

z = [-1;8;1;-2];

norm (x-z)

```
ans =
 2.2204e-016
%Exercise 5
%Part A
A = rand(500);
x = ones(500,1);
b = A*x;
tic; R = rref([A,b]); x_rref = R(:,end); toc
elapsed_time =
  20.7520
%Part B
tic; [L,U,P] = Iu(A); y = L\setminus(P*b); x_Iu = U\setminus y; toc
elapsed_time =
  0.0370
%The LU decomposition produced faster result than Part A.
%Part C
norm(x_rref - x)
ans =
 1.5236e-011
norm(x_lu - x)
ans =
 1.9308e-011
%The solutions are different. The LU decomposition likely the most accurate.
diary off
```