Mathematical Induction

Use induction to prove that $n! > 2^n$ for all positive integers $n \geq 4$.

Let $P(n)$ denote the proposition that $n! > 2^n$, where n is a positive integer $n \geq 4$.

BASIS STEP: $P(4)$ is true since $4! = 24 > 2^4$ =16.

INDUCTIVE STEP: Let us assume $P(n)$, that is $n! > 2^n$ is true for an arbitrary positive integer $n \geq 4$. This is our inductive hypothesis.

We have to show that $P(n+1)$, $(n+1)! > 2^{n+1}$ is also true assuming the inductive hypothesis $P(n)$.

Proof:

$(n+1)! = (n+1) \cdot n! > (n+1) \cdot 2^n$ using the inductive hypothesis.

$(n+1) \cdot 2^n > 2 \cdot 2^n = 2^{n+1}$, when n ≥ 4.

By the **Principle of Mathematical Induction** (Basis Step and Inductive Step together) $n! > 2^n$ for all positive integers $n \geq 4$.