Using TI-Nspire CAS Technology in Teaching Engineering Mathematics: ODE's

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ABSTRACT

In September 2011, our engineering school has adopted the TI-Nspire CAS CX calculator (and software) for all new students entering bachelor level. With both platforms and integrated facilities, Nspire CAS is becoming a more complete solution than Voyage 200. Although we already started to use this new device in the classroom, we will have a transition period where both the TI Voyage 200 and the new CX handheld calculators can be found on the students' desks. This talk will demonstrate examples of how this technology is used in classrooms when teaching differential equations.

Nspire CX handheld with its faster CPU and extended graphing capabilities gives us even more options for doing math in our classes. Starting with first order equations, the use of an Nspire-CAS calculator makes it easy for students to explore slope fields, numerical solutions with Euler's method or even Picard's approximations for well behaved equations. With Nspire, we can now see graphically how a numerical solution fits when compared to an exact solution. Consider applications of first order ODE's, Newton's law of cooling for example. With Nspire, students can hook up a temperature probe to their calculator and gather data which will then be compared to the theoretical solution.

Looking at second order equations, a CAS calculator can help in computing part of the algebraic solution. In the method of undetermined coefficients, we still teach the classical approach but let students use their handheld to substitute the candidate in the equation and solve the resulting system of equations. When teaching the method of variation of parameters, we show them step by step solution (with the help of their calculator) instead of giving them a unique formula for the solution.

For teachers, having this technology can be very useful for grading exams. One can construct a small program that will yield step by step solution for the variation of parameters solution. If a student makes an error at the beginning (wrong solutions for the homogeneous equation), the teacher could easily obtain what should be the remaining solution for this student. We will demonstrate how simple it is to create such a program.

Our students must learn skills in regards to technology: this is now part of the curriculum and will be presented. In exams or in take-home work, some problems will require the use of Nspire CAS (or Voyage 200) technology. Students should also learn to validate manual results using these tools and understand unexpected results obtained by CAS systems. We will present examples where the benefit of using this approach is obvious. Moreover, the majority of students will spend a lot of energy exploring these math concepts if these tools are used. Everything is more visual and they love to play with technology, as long as they are told which direction to follow.

I. First order equations

Consider the differential equation $\dfrac{dy}{dx} = y \cdot (4 - y)$, with the initial condition $y(0) = 0.2$.

We want to find the value of y when $x = 1$.

We will graph the slope field of this equation, the numerical solution (Euler method) and the exact solution, all in the same window. Also we will approximate the solution with a polynomial, using Picard's iterations; we will graph Picard's 3rd iteration.

We will also show the required value $y(1)$ estimated by Euler method (in a table of values) and compare it with the exact value.

The left screen[1] gives the slope field, and the right screen gives the slope field with the numerical solution, by Euler, using a plot step of 0.1 :
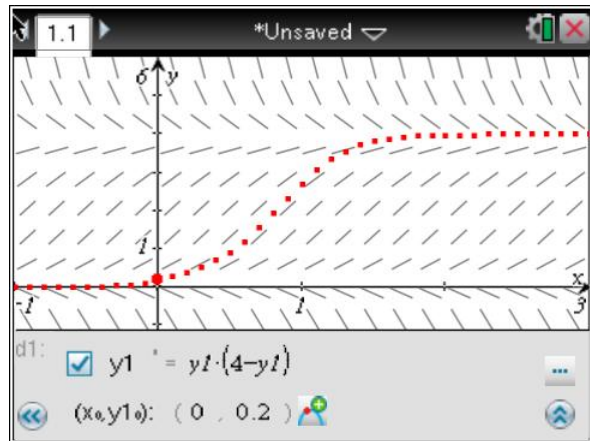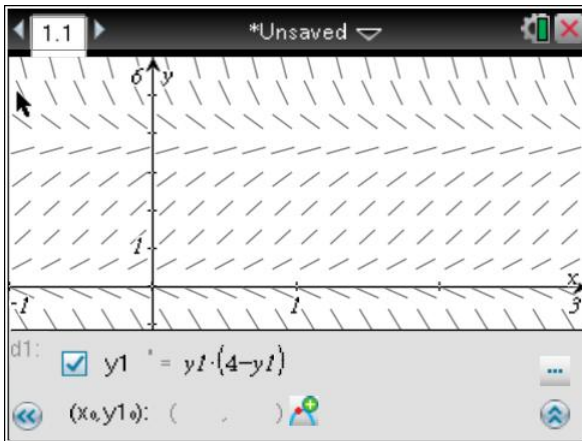

Again, the slope field and the numerical solution, but with a plot step of 0.01 :

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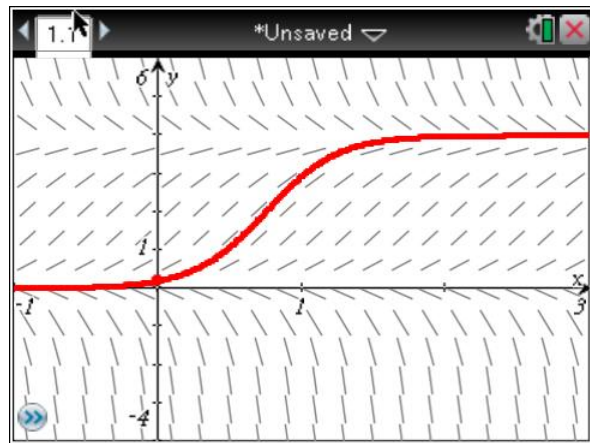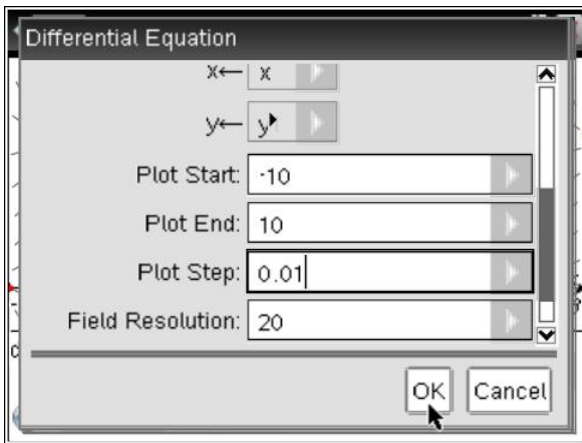
[1] All solutions and screens were done using TI-Nspire CAS CX, with O.S. 3.1

Let's now compute the exact solution; this is possible because the differential equation is a simple (separable) equation and the calculator has a command for finding exact solution of well-behaved 1st and 2nd order equations.

We graph this exact solution, comparing it with the Euler solution. We come back with a plot step of 0.1 for the numerical solution since it will help us see clearly the difference between the 2 solutions:
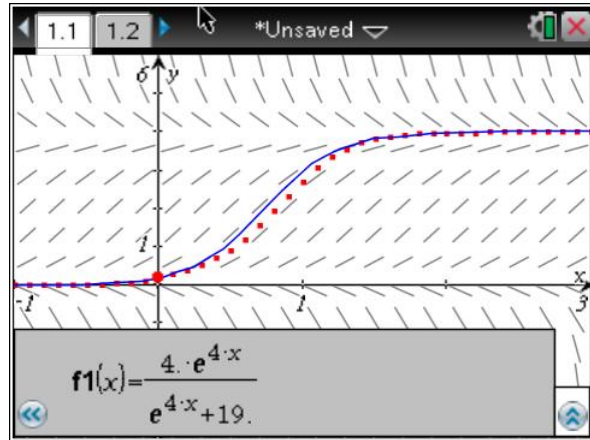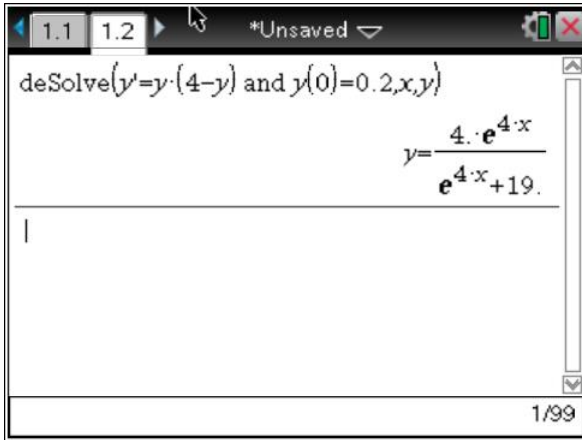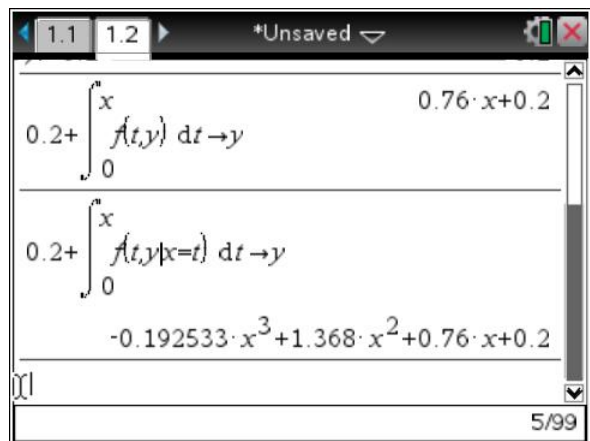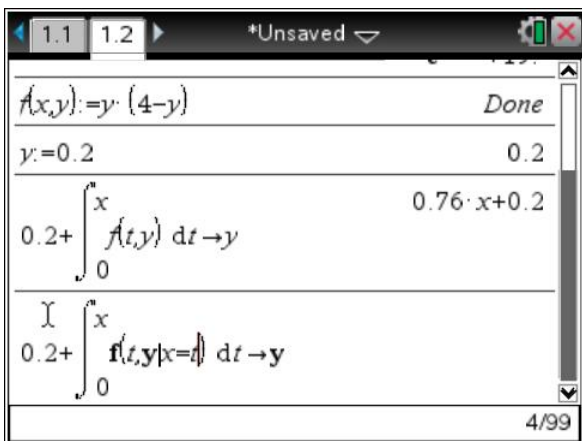

Picard's iterations

Let's recall that given the differential equation $\dfrac{dy}{dx} = f(x, y)$ subject to the initial condition

$y(x_0) = y_0$, the formula for Picard's iterations is $y_{n+1}(x) = y_0 + \displaystyle\int_{x_0}^{x} f(t, y_n(t))\, dt$.

In our example we have $f(x, y) = y \cdot (4 - y)$, $x_0 = 0$ and $y_0 = 0.2$.

On the left screen, we first store the function $f(x, y)$, and then we prepare to get the iterations. On the right screen we press ENTER several times to iterate this formula.

Our goal is to graph an approximate polynomial solution (see the right screen below). We chose Picard's 3rd iteration which is a 7th degree polynomial.
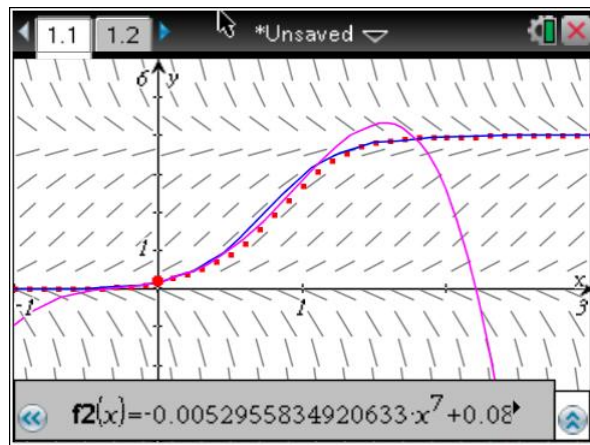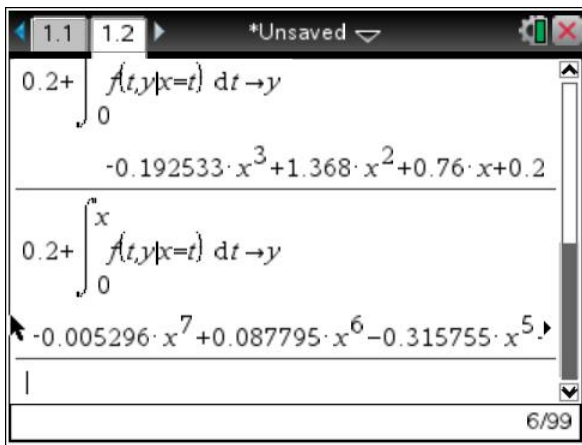
Looking at this last graph, we see that, as expected, the polynomial solution does a good job near $x = 0$ but is much in error when the x-value grows greater than 1.

Table of values

We now want to compare the numerical values obtained by these different methods. That's why we use a table of values :
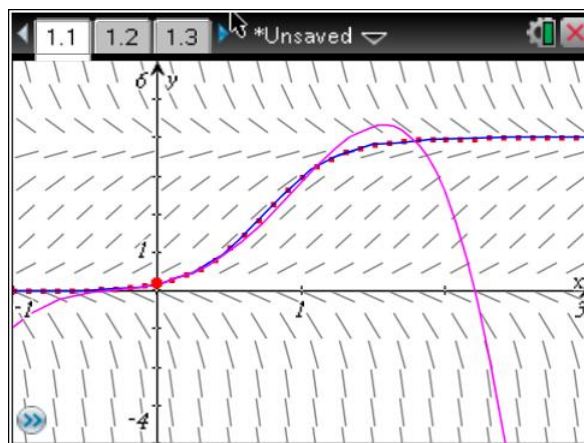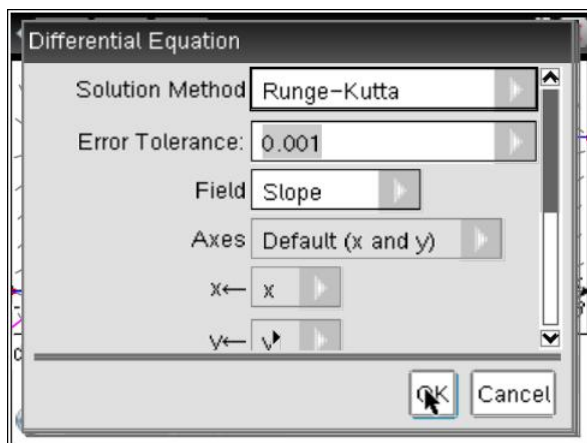

The first column gives the exact (6 digits) values of the solution, so that $y(1) = 2.96737$; the second column is the Picard's 3rd iteration : 2.85069, and the third column gives the value obtained by Euler, using a step size of 0.1 : 2.71611.

There is another numerical method we can use, which is far better than Euler, and this is the special version of the Runge-Kutta method implemented in TI-Nspire (a 3rd order adaptive step version). As we can see below, if we change the solution method to "Runge-Kutta" in the "Differential Equation Settings" window, the values in the 3rd column of our table will change. We see below that the estimated value for $y(1)$, by Runge-Kutta, is the same as the exact solution when values are rounded up two decimals. When rounding up to 3 decimals, the difference between the 2 values is only 0.001. Thus we observe a good improvement in the numerical solution and we notice that the graph obtained with this method is very close to the one of the exact solution!

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II. Newton's law of cooling, using a temperature probe

Here is a classic example of an application of 1st order differential equations in real life; we want to analyze the temperature of a liquid that is cooling down.

Newton's law states that this situation will be modeled by the equation $\dfrac{dT}{dt} = k \cdot (T - T_E)$, where T is the temperature of the liquid, T_E is the ambient temperature (supposed constant) and k is a parameter depending on the environment of the experience. The solution is $T(t) = T_E + C \cdot e^{kt}$ where C is the difference between initial temperature and ambient temperature.

With the new TI-Nspire CX CAS, we can get real experimental data in the calculator using a temperature-probe. Knowing the ambient temperature, we will use two of the observations to get the theoretical solution and see how it compares to the data collected.

Here is the experiment. We took temperatures of water cooling, from 100ºC. You can see on the picture the test tube filled with hot water, the probe connected to a calculator and a fan to keep ambient temperature constant. We set the calculator to take a temperature reading every 30 seconds, for 15 minutes.

The results are shown below. As we can see, we started a little late; water had already cooled below 95°C. The ambient temperature was 24.0 °C.

	Time	Temp
1	0	92.4
2	30	88.1
3	60	84.4
4	90	81.1
5	120	77.9
6	150	74.7
7	180	71.9
8	210	69.1
9	240	66.6
10	270	64.2
11	300	61.9
12	330	59.7
13	360	57.7
14	390	55.8
15	420	54.1
16	450	52.4
17	480	50.7
18	510	49.2
19	540	47.9
20	570	46.5
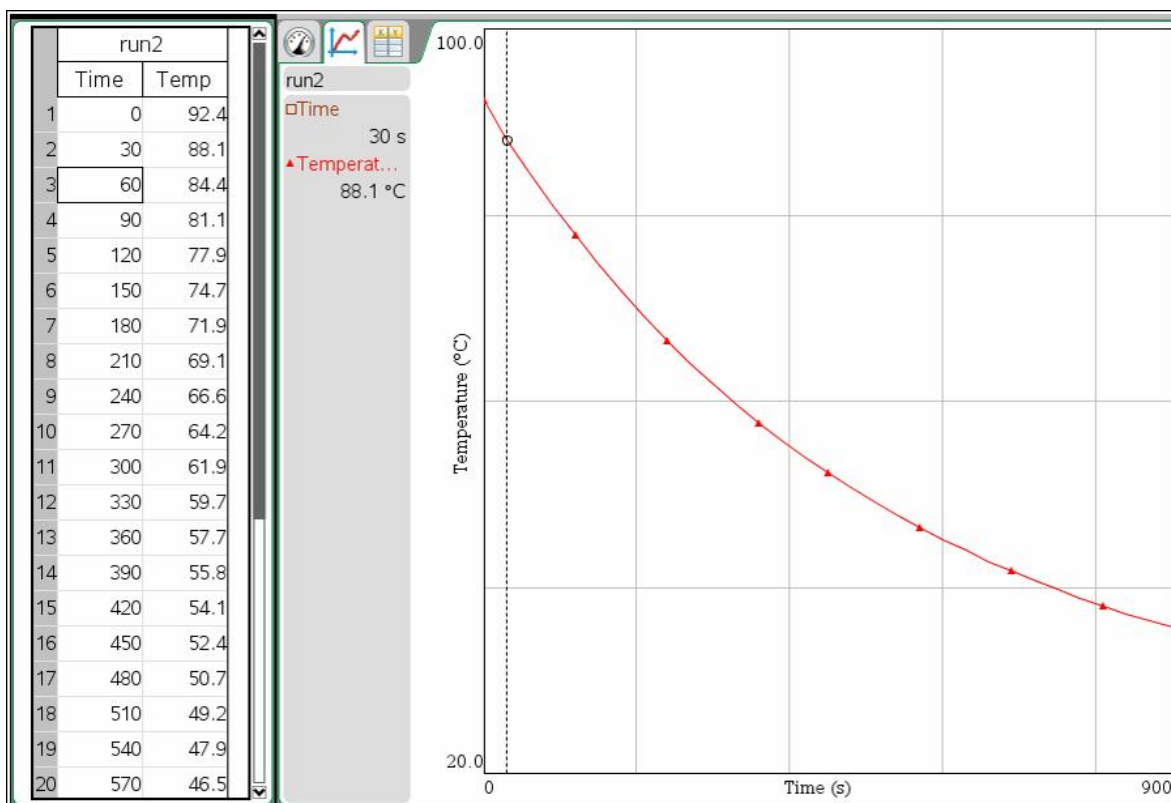
On the left we have the temperature every 30 seconds. On the right, the points are some of these values (about 1 over 4), and the software automatically connects them.

We need 2 observations to determine the parameters C and k in the expression of temperature $T(t) = 24 + C \cdot e^{kt}$.

To find these values, we solve a system of 2 equations. Let's take data at $t = 0$ and 900 seconds. This is an arbitrary choice, we could choose other points which would give us similar, but not necessarily the exact same values for C and k.

We work with time in minutes instead of seconds, so that k is not too small. We have, taking tr for temperature: $tr(0) = 92.4$ and $tr(15) = 35.8$.

$tr(t) := c \cdot e^{k \cdot t} + 24$	*Done*
$\text{solve}\left(\begin{cases} tr(0) = 92.4 \\ tr(15) = 35.8 \end{cases}, \{c, k\}\right)$	$c = 68.4$ and $k = -0.117152$
$tr(t) := c \cdot e^{k \cdot t} + 24 \mid c = 68.4$ and $k = -0.11715155287713$	*Done*
$tr(t)$	$68.4 \cdot (0.88945)^t + 24$

Thus we find that $tr(t) = 24 + 68.4 \cdot e^{-0.117152t}$. This is the solution according to the theoretical model.

We use this function to check whether it gives the results we had obtained by experiment at 300, 600 and 30 seconds.

1.1 1.2 ▶	*temp_june19* ▽	
		Done
$tr(t)$		$68.4 \cdot (0.88945)^t + 24$
$tr(5)$		62.0772
$tr(10)$		45.1969
$tr\left(\dfrac{1}{2}\right)$		88.5085
		7/99

Let's recall the temperatures obtained by experiment :

$T(300) = 61.9$
$T(600) = 45.2$
and $T(30) = 88.1$.

We see that the results are quite satisfying, the theoretical solution giving values near what was observed.

It is important, while we do the experiment, not to move or touch the recipient and we need to have a fan to insure a constant temperature for the surrounding environment. Look at the curve below. It

represents the results of another experiment we had done earlier where we had moved the probe, and touched the sides of the recipient with it while having no fan.
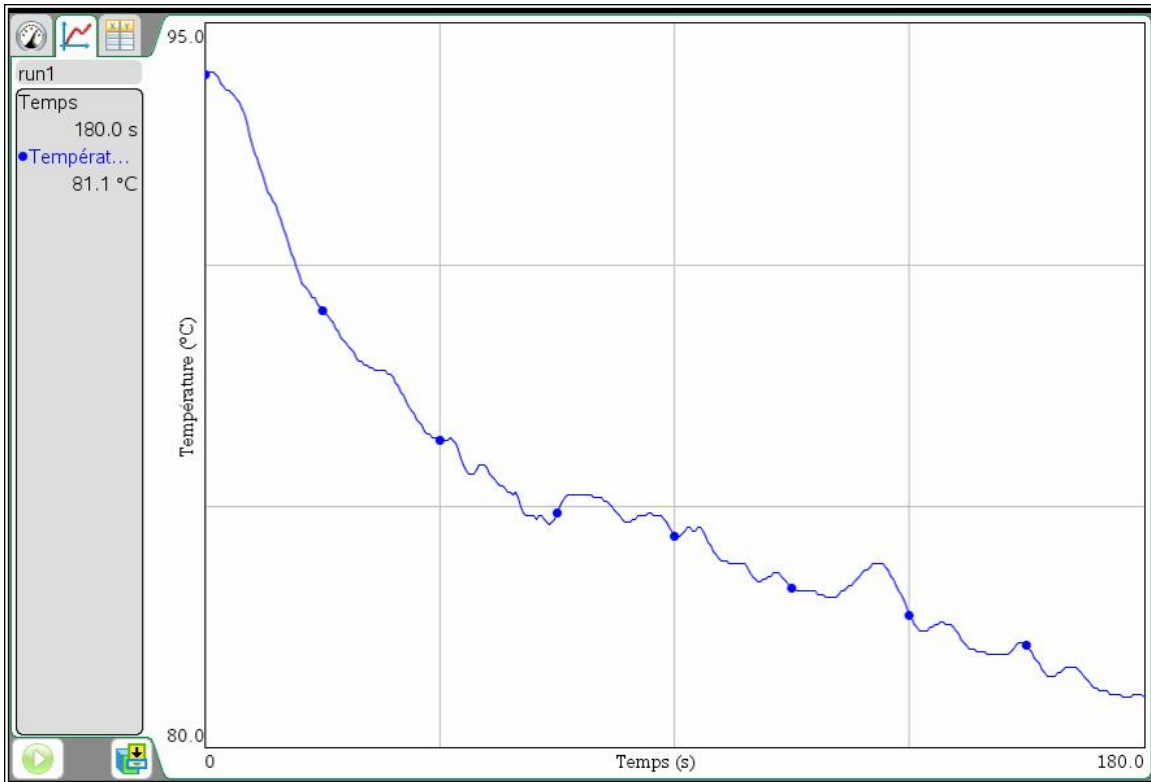
III. Second order equations
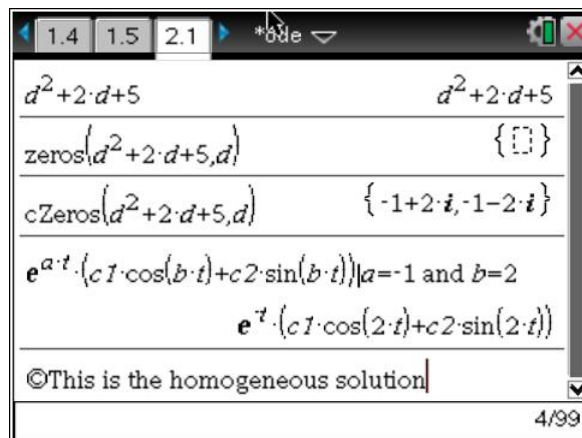# A- Undetermined coefficients

Let's consider the equation $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + 5y = 61\sin(9t)$.

We want our students to solve this equation by the method of undetermined coefficients. We show them how their CAS calculators can help in doing some parts of the solution.

First they have find the roots of the characteristic polynomial and write the homogeneous solution.

They then write the candidate for the particular solution, this candidate is $A\sin(9t)+B\cos(9t)$. Now, the problem is to find the values of A and B so that this candidate gives a solution when substituted in the differential equation.

They can substitute it in the equation using the operator op( ) we have shown them to automate this substitution, and then solve the resulting system of equations.
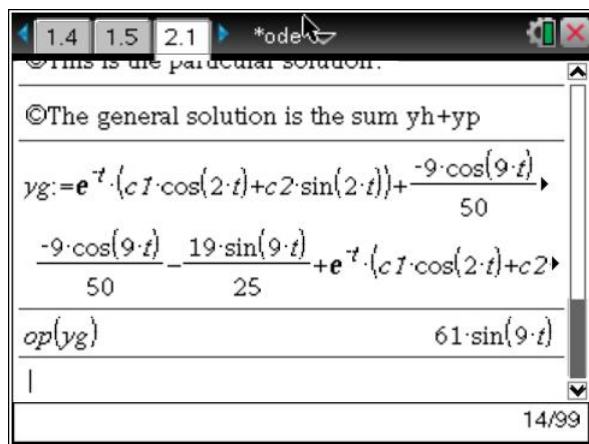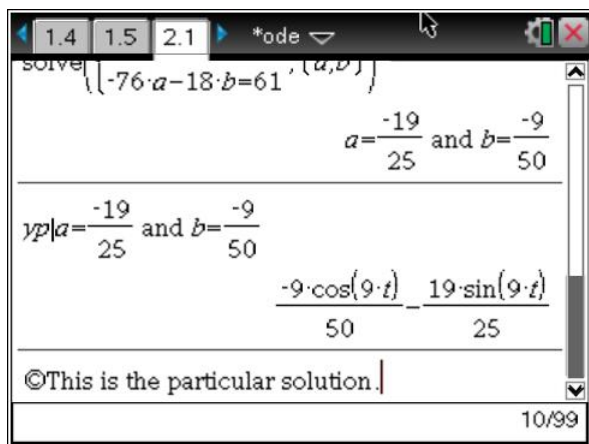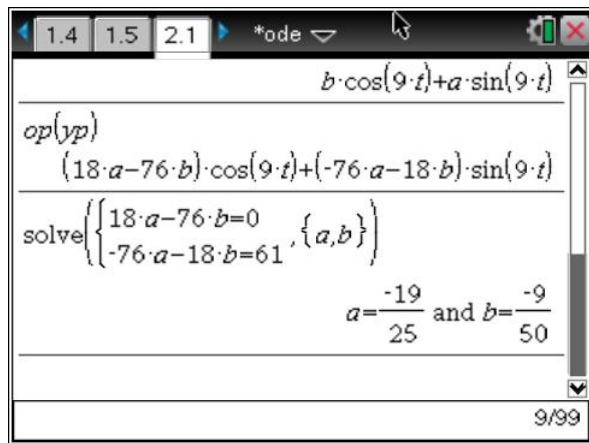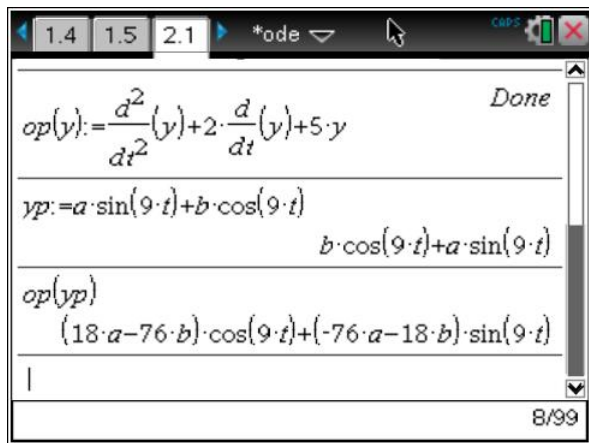
Thus they obtain the general solution $y_g=\dfrac{-9}{50}\cos(3t)-\dfrac{19}{25}\sin(3t)+e^{-t}\cdot\left(c_1\cos(2t)+c_2\sin(2t)\right)$, and they even can verify that the solution is good by substituting it in the differential equation, using op( ).

As we saw earlier, we could use a command of the calculator to get the solution in one step. The next screen shows this but, notice the huge difference between our previous solution and the general solution obtained with the DeSolve command.

Using the op( ) operator, we can verify that this long solution is indeed a solution of the differential equation, with a *little* use of trigonometric identities :

©This is the particular solution.

©The general solution is the sum yh+yp

$yg:=e^{-t}\cdot(c1\cdot\cos(2\cdot t)+c2\cdot\sin(2\cdot t))+\dfrac{-9\cdot\cos(9\cdot t)}{50}-\dfrac{19\cdot\sin(9\cdot t)}{25}$ $\dfrac{-9\cdot\cos(9\cdot t)}{50}-\dfrac{19\cdot\sin(9\cdot t)}{25}+e^{-t}\cdot(c1\cdot\cos(2\cdot t)+c2\cdot\sin(2\cdot t))$

op(yg) $61\cdot\sin(9\cdot t)$

deSolve(y''+2·y'+5·y=61·sin(9·t),t,y)

$y=c1\cdot e^{-t}\cdot\cos(2\cdot t)+c2\cdot e^{-t}\cdot\sin(2\cdot t)-\dfrac{176\cdot(\cos(t))^9}{25}-\dfrac{4864\cdot\sin(t)\cdot(\cos(t))^8}{25}+\dfrac{128\cdot(\cos(t))^7}{5}+\dfrac{8512\cdot\sin(t)\cdot(\cos(t))^6}{25}-\dfrac{(976\cdot(\sin$

©We see the homogeneous solution we already had found.

$op\left(\dfrac{-176\cdot(\cos(t))^9}{25}-\dfrac{4864\cdot\sin(t)\cdot(\cos(t))^8}{25}+\dfrac{128\cdot(\cos(t))^7}{5}+\dfrac{8512\cdot\sin(t)\cdot(\cos(t))^6}{25}-\dfrac{(976\cdot(\sin(t))^4+227)\cdot(\cos(t))^5}{25}-\dfrac{912\cdot\sin(}{}\right.$

$\dfrac{-2928\cdot(\cos(t))^9}{5}+2656\cdot\sin(t)\cdot(\cos(t))^8+\dfrac{4336\cdot(\cos(t))^7}{25}-\dfrac{64\cdot\sin(t)\cdot(982\cdot(\sin(t))^2+161)\cdot(\cos(t))^6}{5}+\left(\dfrac{-976\cdot(\sin(t))^4}{5}-\dfrac{44464}{}\right.$

$\dfrac{\sin(t))^6}{}+\dfrac{27722\cdot(\sin(t))^2}{25}-\dfrac{8202}{25}\right)\cdot(\cos(t))^3-\dfrac{171\cdot\sin(t)\cdot(1704\cdot(\sin(t))^2-331)\cdot(\cos(t))^2}{100}+\dfrac{\sin(t)\cdot(9217\cdot(\sin(t))^2-3117)}{100}\blacktriangleright\sin$

$\dfrac{61\cdot\sin(t)\cdot(256\cdot(\sin(t))^8-576\cdot(\sin(t))^6+432\cdot(\sin(t))^4-120\cdot(\sin(t))^2+9)}{}$

tCollect$(61\cdot\sin(t)\cdot(256\cdot(\sin(t))^8-576\cdot(\sin(t))^6+432\cdot(\sin(t))^4-120\cdot(\sin(t))^2+9))$ $61\cdot\sin(9\cdot t)$

B- Variation of parameters (Lagrange method)

We ask our students to solve the differential equation $\dfrac{d^2 y}{dt^2}+6\dfrac{dy}{dt}+9y=45t^4 e^{-3t}$, using the method of variation of parameters.

First they have to find the roots of the characteristic polynomial and write the homogeneous solution : $y_h = c_1 u_1 + c_2 u_2$. According to this method, a particular solution will be of the form $y_p = L_1 u_1 + L_2 u_2$, where L_1 and L_2 can be variable (functional) coefficients. They have to find L_1 and L_2.

The system of equations to solve is $\begin{cases} L_1' u_1 + L_2' u_2 = 0 \\ L_1' u_1' + L_2' u_2' = 45t^4 e^{-3t} \end{cases}$

They obtain L_1' and L_2', integrate to find L_1 and L_2, and give the desired solution.

We could ask them to do all this by manual calculation (classic textbook approach) but let's get a little help from the CAS calculator.

Screen 1/5:

$d^2+6\cdot d+9$ $\qquad d^2+6\cdot d+9$

$\text{zeros}(d^2+6\cdot d+9,d)$ $\qquad \{-3\}$

$\text{factor}(d^2+6\cdot d+9)$ $\qquad (d+3)^2$

$e^{-3\cdot t}\cdot(c1+c2\cdot t)$ $\qquad (c2\cdot t+c1)\cdot e^{-3\cdot t}$

©This is the homogeneous solution yh;
$u1=e^{-3t}$ and $u2=t*e^{-3t}$

Screen 9/99:

©First equation of the system:

$a\cdot e^{-3\cdot t}+b\cdot t\cdot e^{-3\cdot t}=0$ $\qquad (b\cdot t+a)\cdot e^{-3\cdot t}=0$

©Second equation of the system:

$a\cdot\dfrac{d}{dt}\left(e^{-3\cdot t}\right)+b\cdot\dfrac{d}{dt}\left(t\cdot e^{-3\cdot t}\right)=45\cdot t^4\cdot e^{-3\cdot t}$

$\qquad (-3\cdot b\cdot t-3\cdot a+b)\cdot e^{-3\cdot t}=45\cdot t^4\cdot e^{-3\cdot t}$

Screen 11/99:

$(-3\cdot b\cdot t-3\cdot a+b)\cdot e^{-3\cdot t}=45\cdot t^4\cdot e^{-3\cdot t}$

$\text{solve}\left(\left\{\begin{array}{l}(b\cdot t+a)\cdot e^{-3\cdot t}=0\\(-3\cdot b\cdot t-3\cdot a+b)\cdot e^{-3\cdot t}=45\cdot t^4\cdot e^{-3\cdot t}\end{array}\right\},\{a,b\}\right)$

$\qquad a=-45\cdot t^5$ and $b=45\cdot t^4$

©So we have L1'=-45*t⁵ and L2'=45*t⁴
We only have to integrate

Screen 14/99:

We only have to integrate

$l1:=\int a\ dt\,|\,a=-45\cdot t^5$ and $b=45\cdot t^4$ $\qquad \dfrac{-15\cdot t^6}{2}$

$l2:=\int b\ dt\,|\,a=-45\cdot t^5$ and $b=45\cdot t^4$ $\qquad 9\cdot t^5$

$yp:=l1\cdot e^{-3\cdot t}+l2\cdot t\cdot e^{-3\cdot t}$ $\qquad \dfrac{3\cdot t^6\cdot e^{-3\cdot t}}{2}$

Screen 16/99:

$yp:=l1\cdot e^{-3\cdot t}+l2\cdot t\cdot e^{-3\cdot t}$ $\qquad \dfrac{3\cdot t^6\cdot e^{-3\cdot t}}{2}$

©The general solution is the sum of yp + yh

$yg:=yp+e^{-3\cdot t}\cdot(c1+c2\cdot t)$

$\qquad \left(\dfrac{3\cdot t^6}{2}+c2\cdot t+c1\right)\cdot e^{-3\cdot t}$

Screen 18/99:

$\left(\dfrac{3\cdot t^6}{2}+c2\cdot t+c1\right)\cdot e^{-3\cdot t}$

©And we can compare with desolve:

$\text{deSolve}\left(y''+6\cdot y'+9\cdot y=45\cdot t^4\cdot e^{-3\cdot t},t,y\right)$

$\qquad y=\left(\dfrac{3\cdot t^6}{2}+c3\cdot t+c4\right)\cdot e^{-3\cdot t}$

We could give them a direct formula to get this result but we prefer a step by step approach.

We made a program with Nspire to test these results, even if students make a mistake, for example if they take the wrong homogeneous solution. This program outputs the intermediate steps we want to see in the students' solution.

Define LibPub **vap**$(u,v,f,x)=$
Prgm
©order of parameters: u1, u2, f(x), x
Local a,s,w

$$w:=\begin{bmatrix} u & v \\ \dfrac{d}{dx}(u) & \dfrac{d}{dx}(v) \end{bmatrix}$$

Disp "wronskian = ",w

$$s:=\text{propFrac}\left(w^{-1}\cdot\begin{bmatrix} 0 \\ f \end{bmatrix}\right)$$

Disp "L1' and L2' = ",s
propFrac$\left(\int s\ dx\right)\to a$
Disp "L1 and L2 = ",a
$yp:=\text{propFrac}(a[1,1]\cdot u+a[2,1]\cdot v)$
Disp "yp = ",yp
$yg:=\text{propFrac}(yp+c1\cdot u+c2\cdot v)$
Disp "yg = ",yg
EndPrgm

$$y=\left(\cdots\frac{}{2}+c3\cdot t+c4\right)\cdot e$$

fonct265\vap $\left(e^{-3\cdot t}, t\cdot e^{-3\cdot t}, 45\cdot t^4\cdot e^{-3\cdot t}, t\right)$

$$\text{wronskian} = \begin{bmatrix} e^{-3\cdot t} & t\cdot e^{-3\cdot t} \\ -3\cdot e^{-3\cdot t} & (1-3\cdot t)\cdot e^{-3\cdot t} \end{bmatrix}$$

$$\text{L1' and L2'} = \begin{bmatrix} -45\cdot t^5 \\ 45\cdot t^4 \end{bmatrix}$$

$$\text{L1 and L2} = \begin{bmatrix} \dfrac{-15\cdot t^6}{2} \\ 9\cdot t^5 \end{bmatrix}$$

$$yp = \frac{3\cdot t^6\cdot e^{-3\cdot t}}{2}$$

$$yg = \left(\frac{3\cdot t^6}{2}+c2\cdot t+c1\right)\cdot e^{-3\cdot t}$$

Done

If they make a mistake in finding the homogeneous solution, for example if they forget the middle term $6\dfrac{dy}{dt}$, they will have $y_h = c_1 \cos(3t) + c_2 \sin(3t)$ and their particular solution will not be good either, but we still can easily grade their solutions.

$$fonct265\backslash vap\left(\cos(3{\cdot}t),\sin(3{\cdot}t),45{\cdot}t^4{\cdot}e^{-3{\cdot}t},t\right)$$

$$\text{wronskian} = \begin{bmatrix} \cos(3{\cdot}t) & \sin(3{\cdot}t) \\ -3{\cdot}\sin(3{\cdot}t) & 3{\cdot}\cos(3{\cdot}t) \end{bmatrix}$$

$$\text{L1' and L2'} = \begin{bmatrix} -15{\cdot}t^4{\cdot}e^{-3{\cdot}t}{\cdot}\sin(3{\cdot}t) \\ 15{\cdot}t^4{\cdot}e^{-3{\cdot}t}{\cdot}\cos(3{\cdot}t) \end{bmatrix}$$

$$\text{L1 and L2} = \begin{bmatrix} \left(\dfrac{5{\cdot}t^4}{2}+\dfrac{10{\cdot}t^3}{3}+\dfrac{5{\cdot}t^2}{3}-\dfrac{5}{27}\right){\cdot}e^{-3{\cdot}t}{\cdot}\cos(3{\cdot}t)+\left(\dfrac{5{\cdot}t^4}{2}-\dfrac{5{\cdot}t^2}{3}-\dfrac{10{\cdot}t}{9}-\dfrac{5}{27}\right){\cdot}e^{-3{\cdot}t}{\cdot}\sin(3{\cdot}t) \\ \left(\dfrac{-5{\cdot}t^4}{2}+\dfrac{5{\cdot}t^2}{3}+\dfrac{10{\cdot}t}{9}+\dfrac{5}{27}\right){\cdot}e^{-3{\cdot}t}{\cdot}\cos(3{\cdot}t)+\left(\dfrac{5{\cdot}t^4}{2}+\dfrac{10{\cdot}t^3}{3}+\dfrac{5{\cdot}t^2}{3}-\dfrac{5}{27}\right){\cdot}e^{-3{\cdot}t}{\cdot}\sin(3{\cdot}t) \end{bmatrix}$$

$$yp = \left(\dfrac{5{\cdot}t^4}{2}+\dfrac{10{\cdot}t^3}{3}+\dfrac{5{\cdot}t^2}{3}-\dfrac{5}{27}\right){\cdot}e^{-3{\cdot}t}$$

$$yg = c1{\cdot}\cos(3{\cdot}t)+c2{\cdot}\sin(3{\cdot}t)+\left(\dfrac{5{\cdot}t^4}{2}+\dfrac{10{\cdot}t^3}{3}+\dfrac{5{\cdot}t^2}{3}-\dfrac{5}{27}\right){\cdot}e^{-3{\cdot}t}$$

Done

20/99

IV. Mass-spring system

The harmonic motion is often modeled with a mass-spring system, as we see in this picture :
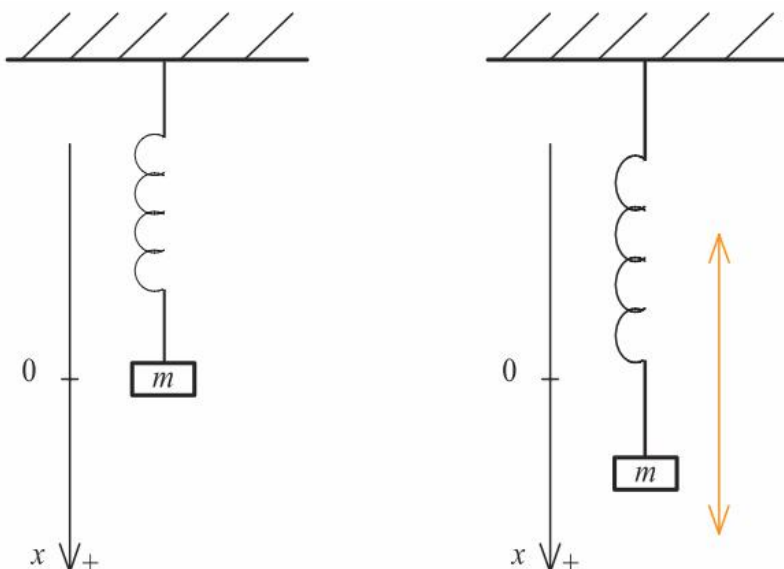
An object of mass m is hanging at the end of a spring.

We set an x-axis such that $x = 0$ at rest, and x is positive when the spring is stretched.

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Let's take a mass-spring system that leads to the equation

$$x'' + 2x' + 26x = 2\delta(t-3) + 2\delta(t-5), \ x(0) = 0, \ x'(0) = 1.$$

As we saw, x is the position of the mass so that x' is its velocity and x'' is its acceleration. And the Dirac Delta function $\delta(t-a)$ could be visualized as hammer blow given on the object at $t = a$, so that we have 2 hammer blows, the first-one after 3 seconds, and the second-one after 5 seconds.

We ask our students to solve this differential equation, with Laplace transforms. They are allowed to use a program they installed on their calculators. This program from Lars Fredericksen was originally done for the TI-89 and it was named « Laplace ». It has been modified for TI-Nspire by Philippe Fortin (Lycée Louis Barthou – Pau)[2], and re-named « specfunc ». Finally the comments have been translated in French by Chantal Trottier, and the program received the name of « ETS-specfunc ».

Once they get the solution, we ask them to graph it. This presents some difficulties.

They have to get the solution, then define the function $u(t)$, which is the unit-step function (Heaviside function); its value is 0 if $t < 0$, and 1 if $t > 0$. They don't want this function to be understood by the calculator when they use this program, so they will have to delete it after use. Or they could create a permanent Heaviside function in *step*(*t*) and replace in the output solution all occurrences of the function $u(t)$ by *step*(*t*).

$\text{ets\_specfunc\\solved}\left(\dfrac{d^2}{dt^2}(x(t)) + 2\cdot\dfrac{d}{dt}(x(t)) + 26\cdot x(t) = 2\cdot\delta(t-3) + 2\cdot\delta(t-5), \{x(t), 0, 1\}\right)$

$$x(t) = \frac{2\cdot e^3 \cdot \sin(5\cdot t - 15)\cdot u(t-3)}{5\cdot e^t} + \frac{2\cdot e^5 \cdot \sin(5\cdot t - 25)\cdot u(t-5)}{5\cdot e^t} + \frac{\sin(5\cdot t)\cdot u(t)}{5\cdot e^t}$$

$\text{right}\left(x(t) = \dfrac{2\cdot e^3 \cdot \sin(5\cdot t - 15)\cdot u(t-3)}{5\cdot e^t} + \dfrac{2\cdot e^5 \cdot \sin(5\cdot t - 25)\cdot u(t-5)}{5\cdot e^t} + \dfrac{\sin(5\cdot t)\cdot u(t)}{5\cdot e^t}\right)$

$$\frac{2\cdot e^{3-t}\cdot \sin(5\cdot t - 15)\cdot u(t-3)}{5} + \frac{2\cdot e^{5-t}\cdot \sin(5\cdot t - 25)\cdot u(t-5)}{5} + \frac{e^{-t}\cdot \sin(5\cdot t)\cdot u(t)}{5}$$

$u(t) := \begin{cases} 0, t < 0 \\ 1, t > 0 \end{cases}$ $\qquad\qquad$ Done

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It remains only to graph it, and to answer some questions to demonstrate their comprehension, such as « Is the object going down or up when the first hammer blow occurs? » « And the second? », « What is the greatest distance from equilibrium, when does the object reach it, and is the spring stretched or compressed at that point? »
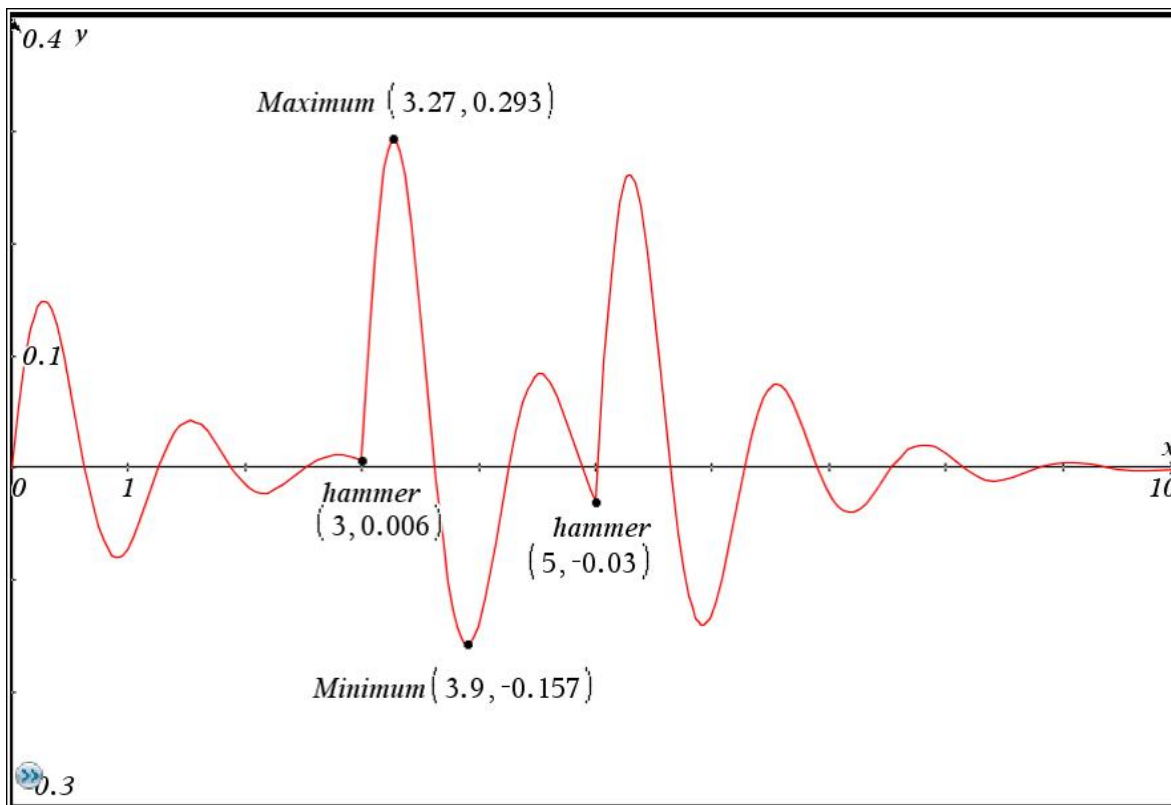
These were some examples of how we can benefit from the use of CAS technology when teaching a basic course in differential equations. We still teach the classic topics using a paper and pencil approach for simple problem. We then show how technology can be used to aid in long and tedious computations. This gives us time to explore even more mathematics with students.