MAT 243 HW 2

Exercise #1. (4 pts) Fill in the blank in the statements below:

(1) An argument is valid if and only if _____

(2) The argument with premises p and $p \to q$ and conclusion q is called _____

(3) If premises q and $p \to q$ are given and someone concludes p, it is called _____

(4) A corollary is _____

Exercise #2. (a) (8 pts) Use the rules of inference to prove $p \wedge s$, given the following premises. Write your solution as a numbered sequence of statements. Identify each statement as either a premise, or a conclusion that follows according to a rule of inference from previous statements. In that case, state the rule of inference and refer by number to the previous statements that the rule of inference used.

(1) $\neg r$

(2) s

(3) $q \vee r$

(4) $q \to p$

(b) (6 pts) Write the following argument in symbolic form. Identify the propositional functions and the universe of discourse you are using. Determine if the argument is valid or invalid. Identify if Universal Modus Ponens, Universal Modus Tollens or one of the fallacies were used.

No swimmers are overweight. Piroska is overweight. Therefore Piroska is not a swimmer.

Exercise #3. (12 pts) Prove or disprove the following statements.

(a) There exists a real number x such that $x + 9 > x^2$.

(b) For all integer x there exists an integer y such that $x = y^2 - 1$.

Exercise #4. (10 pts) Let m be an even integer and n be an even integer. Prove that their sum is even.

Exercise #5. (10 pts) Use proof by contraposition to show that if mn is even then m is even or n is even.