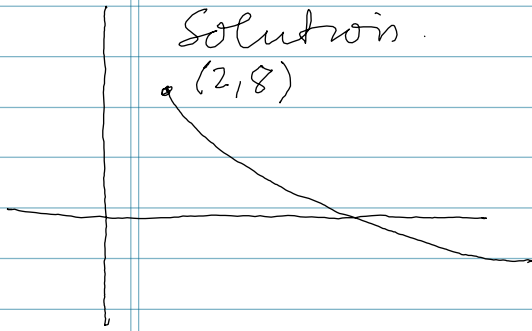


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Example: Find the exact length of
the curve

$$x = e^t + e^{-t}, y = 10 - 2t, 0 \leq t \leq 6.$$

Solution.

(2, 8)



$$x = f(t), y = g(t), a \leq t \leq b.$$

Arc length

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$

$$\frac{dx}{dt} = e^t - e^{-t}, \quad \frac{dy}{dt} = -2.$$

$$\begin{aligned} \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 &= (e^t - e^{-t})^2 + 4 \\ &= (e^t + e^{-t})^2 \end{aligned}$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = e^t + e^{-t} \quad (e^t + e^{-t} > 0)$$

$$\begin{aligned} L &= \int_0^6 (e^t + e^{-t}) dt = e^t - e^{-t} \Big|_0^6 \\ &= (e^6 - e^{-6}) - (1 - 1) \\ &= e^6 - \frac{1}{e^6} \end{aligned}$$