

1. (1 pt) Let

$$f(x) = \frac{x^2 + 1}{1 - x^2}.$$

Find each point of discontinuity of f , and for each give the value of the point of discontinuity and evaluate the indicated one-sided limits.

NOTE: When using interval notation in WeBWorK, remember that:

You use 'INF' for ∞ and '-INF' for $-\infty$.

If you have more than one point, give them in numerical order, from smallest to largest.

If you have extra boxes, fill each in with an 'x'.

Point 1: $C =$ _____

$$\lim_{x \rightarrow C^-} f(x) =$$

$$\lim_{x \rightarrow C^+} f(x) =$$

Point 2: $C =$ _____

$$\lim_{x \rightarrow C^-} f(x) =$$

$$\lim_{x \rightarrow C^+} f(x) =$$

Point 3: $C =$ _____

$$\lim_{x \rightarrow C^-} f(x) =$$

$$\lim_{x \rightarrow C^+} f(x) =$$

Answer(s) submitted:

- -1
- -Inf
- Inf
- 1
- Inf
- -Inf
- x
- x
- x

(correct)

Correct Answers:

- -1
- -INF
- INF
- 1
- INF
- -INF
- x
- x
- x

2. (1 pt) Let

$$f(x) = \frac{1}{x + 2}.$$

Find each point of discontinuity of f , and for each give the value of the point of discontinuity and evaluate the indicated one-sided limits.

NOTE: When using interval notation in WeBWorK, remember that:

You use 'INF' for ∞ and '-INF' for $-\infty$.

If you have more than one point, give them in numerical order, from smallest to largest.

If you have extra boxes, fill each in with an 'x'.

Point 1: $C =$ _____

$$\lim_{x \rightarrow C^-} f(x) =$$

$$\lim_{x \rightarrow C^+} f(x) =$$

Point 2: $C =$ _____

$$\lim_{x \rightarrow C^-} f(x) =$$

$$\lim_{x \rightarrow C^+} f(x) =$$

Point 3: $C =$ _____

$$\lim_{x \rightarrow C^-} f(x) =$$

$$\lim_{x \rightarrow C^+} f(x) =$$

Answer(s) submitted:

- -2
- -Inf
- Inf
- x
- x
- x
- x
- x
- x

(correct)

Correct Answers:

- -2
- -INF
- INF
- x
- x
- x
- x
- x
- x

3. (1 pt)

A function $f(x)$ is said to have a **removable** discontinuity at $x = a$ if:

1. f is either not defined or not continuous at $x = a$.
2. $f(a)$ could either be defined or redefined so that the new function IS continuous at $x = a$.

$$\text{Let } f(x) = \begin{cases} \frac{8}{x} + \frac{-7x+40}{x(x-5)}, & \text{if } x \neq 0, 5 \\ 5, & \text{if } x = 0 \end{cases}$$

Show that $f(x)$ has a removable discontinuity at $x = 0$ and determine what value for $f(0)$ would make $f(x)$ continuous at $x = 0$. Must redefine $f(0) =$ _____.

Hint: Try combining the fractions and simplifying.

The discontinuity at $x = 5$ is actually NOT a removable discontinuity, just in case you were wondering.

Answer(s) submitted:

- -1/5

(correct)

Correct Answers:

- -0.2

4. (1 pt)

A function $f(x)$ is said to have a **jump** discontinuity at $x = a$ if:

1. $\lim_{x \rightarrow a^-} f(x)$ exists.
2. $\lim_{x \rightarrow a^+} f(x)$ exists.
3. The left and right limits are not equal.

$$\text{Let } f(x) = \begin{cases} 3x - 7, & \text{if } x < 5 \\ \frac{4}{x+4}, & \text{if } x \geq 5 \end{cases}$$

Show that $f(x)$ has a jump discontinuity at $x = 5$ by calculating the limits from the left and right at $x = 5$.

$$\lim_{x \rightarrow 5^-} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 5^+} f(x) = \underline{\hspace{2cm}}$$

Now for fun, try to graph $f(x)$.

Answer(s) submitted:

- 8
- 4/9

(correct)

Correct Answers:

- 8
- 0.4444444444444444

5. (1 pt) Let

$$f(x) = \begin{cases} 6 + x, & x < -5, \\ 3 - x, & x \geq -5. \end{cases}$$

Find the indicated one-sided limits of f , and determine the continuity of f at the indicated point.

NOTE: Type DNE if a limit does not exist.

You should also sketch a graph of $y = f(x)$, including hollow and solid circles in the appropriate places.

$$\begin{aligned} \lim_{x \rightarrow -5^-} f(x) &= \underline{\hspace{2cm}} \\ \lim_{x \rightarrow -5^+} f(x) &= \underline{\hspace{2cm}} \\ \lim_{x \rightarrow -5} f(x) &= \underline{\hspace{2cm}} \\ f(-5) &= \underline{\hspace{2cm}} \end{aligned}$$

Is f continuous at $x = -5$? (YES/NO) _____

Answer(s) submitted:

- 1
- 8
- DNE
- 8
- NO

(correct)

Correct Answers:

- 1
- 8
- DNE
- 8
- NO

6. (1 pt) Let

$$f(x) = \begin{cases} -6x, & x < 6, \\ 1, & x = 6, \\ 6x, & x > 6. \end{cases}$$

Find the indicated one-sided limits of f , and determine the continuity of f at the indicated point.

NOTE: Type DNE if a limit does not exist.

You should also sketch a graph of $y = f(x)$, including hollow and solid circles in the appropriate places.

$$\begin{aligned} \lim_{x \rightarrow 6^-} f(x) &= \underline{\hspace{2cm}} \\ \lim_{x \rightarrow 6^+} f(x) &= \underline{\hspace{2cm}} \\ \lim_{x \rightarrow 6} f(x) &= \underline{\hspace{2cm}} \\ f(6) &= \underline{\hspace{2cm}} \end{aligned}$$

Is f continuous at $x = 6$? (YES/NO) _____

Answer(s) submitted:

- -36
- 36
- DNE
- 1
- NO

(correct)

Correct Answers:

- -36
- 36
- DNE
- 1
- NO

7. (1 pt) Let

$$f(x) = \frac{x-7}{(x-2)(x+1)}.$$

Use interval notation to indicate where $f(x)$ is continuous.

NOTE: When using interval notation in WeBWorK, remember that:

You use 'INF' for ∞ and '-INF' for $-\infty$.

And use 'U' for the union symbol.

Interval(s) of Continuity:

Answer(s) submitted:

- $(-\text{Inf}, -1) \cup (-1, 2) \cup (2, \text{Inf})$

(correct)

Correct Answers:

- $(-\text{infinity}, -1) \cup (-1, 2) \cup (2, \text{infinity})$

8. (1 pt) Let

$$f(x) = 8x^8 - 5x^4 + 1.$$

Use interval notation to indicate where $f(x)$ is continuous.

Note: Use 'INF' for ∞ , '-INF' for $-\infty$, and use 'U' for the union symbol.

Interval(s) of Continuity:

Answer(s) submitted:

- $(-\text{Inf}, \text{Inf})$

(correct)

Correct Answers:

- $(-\text{infinity}, \text{infinity})$

9. (1 pt) Let

$$f(x) = \sqrt{x-4}.$$

Use interval notation to indicate where $f(x)$ is continuous.

NOTE: Use 'INF' for ∞ , '-INF' for $-\infty$ and 'U' for the union symbol.

Interval(s) of Continuity:

Answer(s) submitted:

- $[4, \text{Inf})$

(correct)

Correct Answers:

- $[4, \text{infinity})$

10. (1 pt) Let

$$f(x) = \sqrt[3]{x-3}.$$

Use interval notation to indicate where $f(x)$ is continuous.

NOTE: Use 'INF' for ∞ , '-INF' for $-\infty$ and 'U' for the union symbol.

Interval(s) of Continuity:

Answer(s) submitted:

- $(-\text{Inf}, 3] \cup [3, \text{Inf})$

(correct)

Correct Answers:

- $(-\text{infinity}, \text{infinity})$

11. (1 pt) For what value of the constant c is the function f continuous on $(-\infty, \infty)$ where

$$f(t) = \begin{cases} t^2 - c & \text{if } t \in (-\infty, 3) \\ ct + 2 & \text{if } t \in [3, \infty) \end{cases}$$

$c =$ _____

Answer(s) submitted:

- $7/4$

(correct)

Correct Answers:

- 1.75

12. (1 pt)

A function $f(x)$ is said to have a **jump** discontinuity at $x = a$ if:

1. $\lim_{x \rightarrow a^-} f(x)$ exists.
2. $\lim_{x \rightarrow a^+} f(x)$ exists.
3. The left and right limits are not equal.

$$\text{Let } f(x) = \begin{cases} x^2 + 3x + 3, & \text{if } x < 5 \\ 6, & \text{if } x = 5 \\ -6x + 4, & \text{if } x > 5 \end{cases}$$

Show that $f(x)$ has a jump discontinuity at $x = 5$ by calculating the limits from the left and right at $x = 5$.

$$\lim_{x \rightarrow 5^-} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 5^+} f(x) = \underline{\hspace{2cm}}$$

Now for fun, try to graph $f(x)$.

Answer(s) submitted:

- 43
- -26

(correct)

Correct Answers:

- 43
- -26

13. (1 pt) Determine if the Intermediate Value Theorem implies that the equation $x^3 - 3x - 9.9 = 0$ has a root in the interval $(0, 1)$.

The equation above ☐ have a root in that interval.

Answer(s) submitted:

- does not

(correct)

Correct Answers:

- DOES NOT

14. (1 pt) Let f be a continuous function such that $f(-8) = -1$ and $f(8) = 1$. Using the Intermediate Value Theorem classify the following statements as

- (*A*) Always true
(*B*) Never True, or
(*C*) True in some cases; False in others.

1. $f(0) = 0$

Answer:(*A*, *B*, or *C*) ____

2. For some c , where $-8 \leq c \leq 8$, $f(c) = 0$.

Answer:(*A*, *B*, or *C*) ____

Answer(s) submitted:

- *C*
- *A*

(correct)

Correct Answers:

- *C*
- *A*