Answer to 3.2 problem 19

(a) implies (b)
Recall that, by definition, N(A) is the subspace of all solutions of the homogeneous system **Ax** = **0**.
If N(A) = {**0**}, then the only solution to the homogeneous system **Ax** = **0** is the trivial solution (zero vector), thus the matrix is singular by the Theorem on Equivalent Conditions on Nonsingularity (page 8 of Lecture Notes on Elementary Matrices – Section 1.5)

(b) implies (c)
If A is nonsingular, then it is invertible and **Ax** = **b** if and only if $\mathbf{x} = A^{-1}\mathbf{b}$. Thus $A^{-1}\mathbf{b}$ is the only solution to **Ax** = **b**.

(c) implies (a)
If the equation **Ax** = **b** has a unique solution for each **b**, then in particular for **b** = **0** the solution **x** =**0** must be unique. Therefore N(A) = {**0**}.