Chapter 4 Motion in Two Dimensions

P4.2 (a) $\vec{\mathbf{r}} = \boxed{18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}}$

(b) $\vec{\mathbf{v}} = \boxed{(18.0\ \text{m/s})\hat{\mathbf{i}} + \left[4.00\ \text{m/s} - \left(9.80\ \text{m/s}^2\right)t\right]\hat{\mathbf{j}}}$

(c) $\vec{\mathbf{a}} = \boxed{\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}}$

(d) by substitution, $\vec{\mathbf{r}}(3.00\ \text{s}) = \boxed{(54.0\ \text{m})\hat{\mathbf{i}} - (32.1\ \text{m})\hat{\mathbf{j}}}$

(e) $\vec{\mathbf{v}}(3.00\ \text{s}) = \boxed{(18.0\ \text{m/s})\hat{\mathbf{i}} - (25.4\ \text{m/s})\hat{\mathbf{j}}}$

(f) $\vec{\mathbf{a}}(3.00\ \text{s}) = \boxed{\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}}$

P4.9 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If time t elapses before it hits the ground, then since there is no horizontal acceleration, $x_f = v_{xi}t$, i.e.,

$$t = \frac{x_f}{v_{xi}} = \frac{(1.40\ \text{m})}{v_{xi}}$$

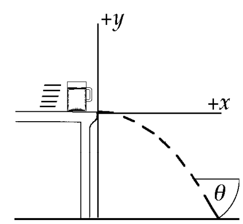

FIG. P4.9

In the same time it falls a distance of 0.860 m with acceleration downward of $9.80\ \text{m/s}^2$. Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2: \ \ 0 = 0.860\ \text{m} + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)\left(\frac{1.40\ \text{m}}{v_{xi}}\right)^2$$

Thus,

$$v_{xi} = \sqrt{\frac{\left(4.90\ \text{m/s}^2\right)\left(1.96\ \text{m}^2\right)}{0.860\ \text{m}}} = \boxed{3.34\ \text{m/s}}$$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t = 0 + \left(-9.80\ \text{m/s}^2\right)\left(\frac{1.40\ \text{m}}{3.34\ \text{m/s}}\right) = -4.11\ \text{m/s}$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9°}$$

P4.11 $x = v_{xi}t = v_i \cos\theta_i t$

$$x = (300\ \text{m/s})(\cos 55.0°)(42.0\ \text{s})$$

$$x = \boxed{7.23\times10^3\ \text{m}}$$

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

$$y = (300\ \text{m/s})(\sin 55.0°)(42.0\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(42.0\ \text{s})^2 = \boxed{1.68\times10^3\ \text{m}}$$

P4.17 (a) We use the trajectory equation:

$$y_f = x_f \tan\theta_i - \frac{gx_f^2}{2v_i^2\cos^2\theta_i}$$

With

$$x_f = 36.0\ \text{m},\ v_i = 20.0\ \text{m/s},\ \text{and}\ \theta = 53.0°$$

we find

$$y_f = (36.0\ \text{m})\tan 53.0° - \frac{(9.80\ \text{m/s}^2)(36.0\ \text{m})^2}{2(20.0\ \text{m/s})^2\cos^2(53.0°)} = 3.94\ \text{m}$$

The ball clears the bar by

$$(3.94 - 3.05)\ \text{m} = \boxed{0.889\ \text{m}}$$

(b) The time the ball takes to reach the maximum height is

$$t_1 = \frac{v_i \sin\theta_i}{g} = \frac{(20.0\ \text{m/s})(\sin 53.0°)}{9.80\ \text{m/s}^2} = 1.63\ \text{s}$$

The time to travel 36.0 m horizontally is $t_2 = \frac{x_f}{v_{ix}}$

$$t_2 = \frac{36.0\ \text{m}}{(20.0\ \text{m/s})(\cos 53.0°)} = 2.99\ \text{s}$$

Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}$.

P4.18 When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$x_f = \sqrt{(3.25\ \text{km})^2 - (2.15\ \text{km})^2} = 2.437\ \text{km}$$

$$y_f = x_f \tan\theta - \frac{gx_f^2}{2v_i^2\cos^2\theta_i}$$

$$-2\,150\ \text{m} = (2\,437\ \text{m})\tan\theta_i - \frac{(9.8\ \text{m/s}^2)(2\,437\ \text{m})^2}{2(280\ \text{m/s})^2\cos^2\theta_i}$$

$$\therefore -2\,150\ \text{m} = (2\,437\ \text{m})\tan\theta_i - (371.19\ \text{m})(1+\tan^2\theta_i)$$

$$\therefore \tan^2\theta - 6.565\tan\theta_i - 4.792 = 0$$

$$\therefore \tan\theta_i = \frac{1}{2}\left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)}\right) = 3.283 \pm 3.945$$

Select the negative solution, since θ_i is below the horizontal.

$$\therefore \tan\theta_i = -0.662,\ \boxed{\theta_i = -33.5°}$$

P4.20 From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80\ \text{m/s}^2)(1.85 - 1.02)\ \text{m}$. From this, $v_{yi} = 4.03\ \text{m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(0.900 - 1.85)\ \text{m}$. Thus the vertical velocity just before he lands is

$$v_{yf} = -4.32\ \text{m/s}$$

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32\ \text{m/s} = 4.03\ \text{m/s} + (-9.80\ \text{m/s}^2)t$$

or $t = \boxed{0.852\ \text{s}}$.

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80\ \text{m} = v_{xi}(0.852\ \text{s})$$

which yields $v_{xi} = \boxed{3.29\ \text{m/s}}$.

(c) $v_{yi} = \boxed{4.03\ \text{m/s}}$. See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\dfrac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\dfrac{4.03\ \text{m/s}}{3.29\ \text{m/s}}\right) = \boxed{50.8°}$.

(e) Similarly for the deer, the upward part of the flight gives $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$:

$$0 = v_{yi}^2 + 2(-9.80\ \text{m/s}^2)(2.50 - 1.20)\ \text{m}$$

so $v_{yi} = 5.04\ \text{m/s}$.

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields

$v_{yf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(0.700 - 2.50)\ \text{m}$ and $v_{yf} = -5.94\ \text{m/s}$.

The hang time is then found as $v_{yf} = v_{yi} + a_y t$:

$-5.94\ \text{m/s} = 5.04\ \text{m/s} + (-9.80\ \text{m/s}^2)t$ and

$$\boxed{t = 1.12\ \text{s}}$$

P4.21 The horizontal kick gives zero vertical velocity to the rock. Then its time of flight follows from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-40.0\ \text{m} = 0 + 0 + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$$

$$t = 2.86\ \text{s}$$

The extra time $3.00\ \text{s} - 2.86\ \text{s} = 0.143\ \text{s}$ is the time required for the sound she hears to travel straight back to the player. It covers distance

$$(343\ \text{m/s})0.143\ \text{s} = 49.0\ \text{m} = \sqrt{x^2 + (40.0\ \text{m})^2}$$

where x represents the horizontal distance the rock travels.

$$x = 28.3\text{ m} = v_{xi}t + 0t^2$$

$$\therefore v_{xi} = \frac{28.3\text{ m}}{2.86\text{ s}} = \boxed{9.91\text{ m/s}}$$

P4.23 For the smallest impact angle

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right)$$

we want to minimize v_{yf} and maximize $v_{xf} = v_{xi}$. The final y component of velocity is related to v_{yi} by $v_{yf}^2 = v_{yi}^2 + 2gh$, so we want to minimize v_{yi} and maximize v_{xi}. Both are accomplished by making the initial velocity horizontal. Then $v_{xi} = v$, $v_{yi} = 0$, and $v_{yf} = \sqrt{2gh}$. At last, the impact angle is

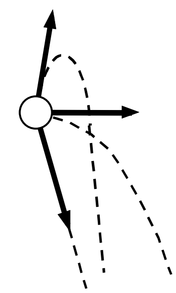

FIG. P4.23

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \boxed{\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)}$$

P4.24 $a = \dfrac{v^2}{R}$, $T = 24\text{ h}(3\,600\text{ s/h}) = 86\,400\text{ s}$

$$v = \frac{2\pi R}{T} = \frac{2\pi(6.37\times10^6\text{ m})}{86\,400\text{ s}} = 463\text{ m/s}$$

$$a = \frac{(463\text{ m/s})^2}{6.37\times10^6\text{ m}} = \boxed{0.033\,7\text{ m/s}^2\text{ directed toward the center of Earth}}$$

P4.25 $a_c = \dfrac{v^2}{r} = \dfrac{(20.0\text{ m/s})^2}{1.06\text{ m}} = \boxed{377\text{ m/s}^2}$ The mass is unnecessary information.

P4.27 (a) $v = r\omega$

At 8.00 rev/s, $v = (0.600\text{ m})(8.00\text{ rev/s})(2\pi\text{ rad/rev}) = 30.2\text{ m/s} = 9.60\pi\text{ m/s}$.

At 6.00 rev/s, $v = (0.900\text{ m})(6.00\text{ rev/s})(2\pi\text{ rad/rev}) = 33.9\text{ m/s} = 10.8\pi\text{ m/s}$.

$\boxed{6.00\text{ rev/s}}$ gives the larger linear speed.

(b) Acceleration $= \dfrac{v^2}{r} = \dfrac{(9.60\pi\text{ m/s})^2}{0.600\text{ m}} = \boxed{1.52\times10^3\text{ m/s}^2}$.

(c) At 6.00 rev/s, acceleration $= \dfrac{(10.8\pi\text{ m/s})^2}{0.900\text{ m}} = \boxed{1.28\times10^3\text{ m/s}^2}$. So 8 rev/s gives the higher acceleration.

P4.31 $r = 2.50\text{ m}$, $a = 15.0\text{ m/s}^2$

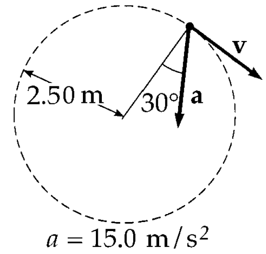

FIG. P4.31

(a) $a_c = a\cos 30.0° = (15.0\text{ m/s}^2)(\cos 30°) = \boxed{13.0\text{ m/s}^2}$

(b) $a_c = \dfrac{v^2}{r}$

so $v^2 = ra_c = 2.50\text{ m}(13.0\text{ m/s}^2) = 32.5\text{ m}^2/\text{s}^2$

$v = \sqrt{32.5}\text{ m/s} = \boxed{5.70\text{ m/s}}$

(c) $a^2 = a_t^2 + a_r^2$

so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{(15.0\text{ m/s}^2)^2 - (13.0\text{ m/s}^2)} = \boxed{7.50\text{ m/s}^2}$

P4.32 Let i be the starting point and f be one revolution later. The curvilinear motion with constant tangential acceleration is described by

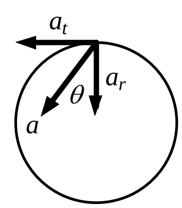

FIG. P4.32

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$

$$2\pi r = 0 + \frac{1}{2}a_t t^2$$

$$a_t = \frac{4\pi r}{t^2}$$

and $v_{xf} = v_{xi} + a_x t$, $v_f = 0 + a_t t = \dfrac{4\pi r}{t}$. The magnitude of the radial acceleration is

$a_r = \dfrac{v_f^2}{r} = \dfrac{16\pi^2 r^2}{t^2 r}$. Then $\tan\theta = \dfrac{a_t}{a_r} = \dfrac{4\pi r\, t^2}{t^2 16\pi^2 r} = \dfrac{1}{4\pi}$ $\qquad \theta = \boxed{4.55°}$

P4.36 The bumpers are initially $100\text{ m} = 0.100\text{ km}$ apart. After time t the bumper of the leading car travels 40.0 t, while the bumper of the chasing car travels 60.0t. Since the cars are side by side at time t, we have

$$0.100 + 40.0t = 60.0t$$

yielding

$$t = 5.00 \times 10^{-3}\text{ h} = \boxed{18.0\text{ s}}$$

P4.37 To guess the answer, think of v just a little less than the speed c of the river. Then poor Alan will spend most of his time paddling upstream making very little progress. His time-averaged speed will be low and Beth will win the race.

Now we calculate: For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$.

Therefore, the total time for Alan is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \boxed{\frac{2L/c}{1 - v^2/c^2}}$$

For Beth, her cross-stream speed (both ways) is

$$\sqrt{c^2 - v^2}$$

Thus, the total time for Beth is $t_2 = \frac{2L}{\sqrt{c^2 - v^2}} = \boxed{\frac{2L/c}{\sqrt{1 - v^2/c^2}}}$.

Since $1 - \frac{v^2}{c^2} < 1$, $t_1 > t_2$, or Beth, who swims cross-stream, returns first.

***P4.38** We can find the time of flight of the can by considering its horizontal motion:
16 m = (9.5 m/s) t + 0 $\quad\quad t = 1.68$ s

(a) For the boy to catch the can at the same location on the truck bed, he must throw it $\boxed{\text{straight up, at } 0° \text{ to the vertical}}$.

(b) For the free fall of the can, $y_f = y_i + v_{yi}t + (1/2)a_y t^2$:
$0 = 0 + v_{yi}(1.68\text{ s}) - (1/2)(9.8\text{ m/s}^2)(1.68\text{ s})^2 \quad v_{yi} = \boxed{8.25\text{ m/s}}$

(c) The boy sees the can always over his head, traversing $\boxed{\text{a straight line segment upward and then downward}}$.

(d) The ground observer sees the can move as a projectile on $\boxed{\text{a symmetric section of a parabola opening downward}}$. Its initial velocity is $(9.5^2 + 8.25^2)^{1/2}$ m/s = $\boxed{12.6\text{ m/s north at } \tan^{-1}(8.25/9.5) = 41.0° \text{ above the horizontal}}$

P4.41 Choose the x axis along the 20-km distance. The y components of the displacements of the ship and the speedboat must agree:

$$(26\text{ km/h})t\sin(40° - 15°) = (50\text{ km/h})t\sin\alpha$$
$$\alpha = \sin^{-1}\frac{11.0}{50} = 12.7°$$

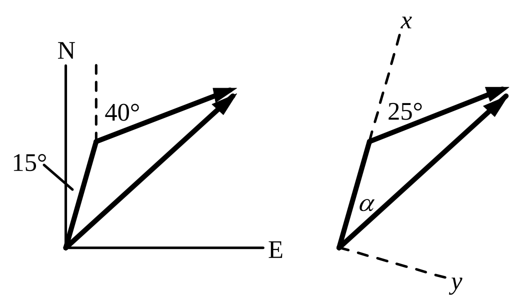

FIG. P4.41

The speedboat should head

$$15° + 12.7° = \boxed{27.7° \text{ east of north}}$$

P4.51 The special conditions allowing use of the horizontal range equation applies. For the ball thrown at 45°,

$$D = R_{45} = \frac{v_i^2 \sin 90}{g}$$

For the bouncing ball,

$$D = R_1 + R_2 = \frac{v_i^2 \sin 2\theta}{g} + \frac{(v_i/2)^2 \sin 2\theta}{g}$$

where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g} = \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g}$$

$$\sin 2\theta = \frac{4}{5}$$

$$\boxed{\theta = 26.6°}$$

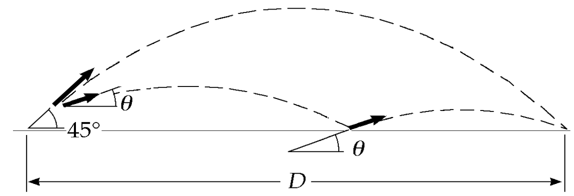

FIG. P4.51

(b) The time for any symmetric parabolic flight is given by

$$y_f = v_{yi} t - \frac{1}{2} g t^2$$

$$0 = v_i \sin\theta_i t - \frac{1}{2} g t^2$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin\theta_i}{g}$ is the time at landing.

So for the ball thrown at 45.0°

$$t_{45} = \frac{2v_i \sin 45.0°}{g}$$

For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2(v_i / 2)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}$$

The ratio of this time to that for no bounce is

$$\frac{3v_i \sin 26.6°/ g}{2v_i \sin 45.0°/ g} = \frac{1.34}{1.41} = \boxed{0.949}$$

P4.62 We follow the steps outlined in Example 4.7, eliminating $t = \frac{d\cos\phi}{v_i \cos\theta}$ to find

$$\frac{v_i \sin\theta\, d\cos\phi}{v_i \cos\theta} - \frac{g d^2 \cos^2\phi}{2v_i^2 \cos^2\theta} = -d\sin\phi$$

Clearing of fractions,

$$2v_i^2 \cos\theta \sin\theta \cos\phi - g d \cos^2\phi = -2v_i^2 \cos^2\theta \sin\phi$$

To maximize d as a function of θ, we differentiate through with respect to θ and set $\frac{dd}{d\theta} = 0$:

$$2v_i^2 \cos\theta \cos\theta \cos\phi + 2v_i^2 \sin\theta(-\sin\theta)\cos\phi - g\frac{dd}{d\theta}\cos^2\phi = -2v_i^2 2\cos\theta(-\sin\theta)\sin\phi$$

We use the trigonometric identities from Appendix B4 $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to find $\cos\phi\cos 2\theta = \sin 2\theta \sin\phi$. Next, $\frac{\sin\phi}{\cos\phi} = \tan\phi$ and $\cot 2\theta = \frac{1}{\tan 2\theta}$ give $\cot 2\phi = \tan\phi = \tan(90° - 2\theta)$ so $\phi = 90° - 2\theta$ and $\theta = 45° - \frac{\phi}{2}$.