Chapter 11 Angular Momentum

P11.1 $\vec{\mathbf{M}}\times\vec{\mathbf{N}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 6 & 2 & -1\\ 2 & -1 & -3\end{vmatrix}=\hat{\mathbf{i}}(-6-1)+\hat{\mathbf{j}}(-2+18)+\hat{\mathbf{k}}(-6-4)=\boxed{-7.00\hat{\mathbf{i}}+16.0\hat{\mathbf{j}}-10.0\hat{\mathbf{k}}}$

P11.3 (a) $\vec{\mathbf{A}}\times\vec{\mathbf{B}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -3 & 4 & 0\\ 2 & 3 & 0\end{vmatrix}=\boxed{-17.0\hat{\mathbf{k}}}$

(b) $\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|=\left|\vec{\mathbf{A}}\right|\left|\vec{\mathbf{B}}\right|\sin\theta$

$17=5\sqrt{13}\sin\theta$

$\theta=\sin^{-1}\left(\dfrac{17}{5\sqrt{13}}\right)=\boxed{70.6°}$

P11.4 $\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}=-3.00(6.00)+7.00(-10.0)+(-4.00)(9.00)=-124$

$AB=\sqrt{(-3.00)^2+(7.00)^2+(-4.00)^2}\cdot\sqrt{(6.00)^2+(-10.0)^2+(9.00)^2}=127$

(a) $\cos^{-1}\left(\dfrac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right)=\cos^{-1}(-0.979)=\boxed{168°}$

(b) $\vec{\mathbf{A}}\times\vec{\mathbf{B}}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -3.00 & 7.00 & -4.00\\ 6.00 & -10.0 & 9.00\end{vmatrix}=23.0\hat{\mathbf{i}}+3.00\hat{\mathbf{j}}-12.0\hat{\mathbf{k}}$

$\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|=\sqrt{(23.0)^2+(3.00)^2+(-12.0)^2}=26.1$

$\sin^{-1}\left(\dfrac{\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|}{AB}\right)=\sin^{-1}(0.206)=\boxed{11.9°}$ or $168°$

(c) Only $\boxed{\text{the first method}}$ gives the angle between the vectors unambiguously.

P11.6 The cross-product vector must be perpendicular to both of the factors, so its dot product with either factor must be zero:

Does $\left(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}\right)\cdot\left(4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)=0$?

We have $8-9-4=-5\neq 0$ so the answer is

$\boxed{\text{No. The cross product could not work out that way.}}$

P11.7 $\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|=\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\quad\Rightarrow\quad AB\sin\theta=AB\cos\theta\quad\Rightarrow\quad\tan\theta=1$ or

$\theta = \boxed{45.0°}$

P11.9 $\boxed{\left|\vec{\mathbf{F}}_3\right| = \left|\vec{\mathbf{F}}_1\right| + \left|\vec{\mathbf{F}}_2\right|}$

The torque produced by $\vec{\mathbf{F}}_3$ depends on the perpendicular distance **OD**, therefore translating the point of application of $\vec{\mathbf{F}}_3$ to any other point along **BC** $\boxed{\text{will not change the net torque}}$.

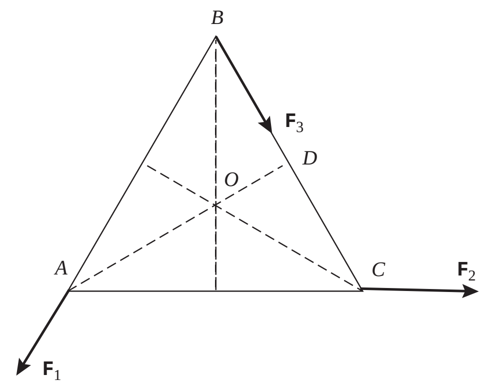

FIG. P11.9

P11.11 $L = \sum m_i v_i r_i$

$= (4.00 \text{ kg})(5.00 \text{ m/s})(0.500 \text{ m}) + (3.00 \text{ kg})(5.00 \text{ m/s})(0.500 \text{ m})$

$L = 17.5 \text{ kg} \cdot \text{m}^2/\text{s}$, and

$\boxed{\vec{\mathbf{L}} = \left(17.5 \text{ kg} \cdot \text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$

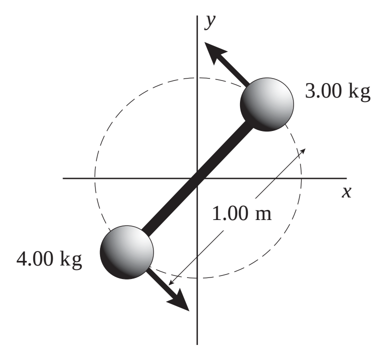

FIG. P11.11

P11.13 $\vec{\mathbf{r}} = \left(6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}} \text{ m}\right)$ $\qquad$ $\vec{\mathbf{v}} = \dfrac{d\vec{\mathbf{r}}}{dt} = 5.00\hat{\mathbf{j}} \text{ m/s}$

so $\quad \vec{\mathbf{p}} = m\vec{\mathbf{v}} = 2.00 \text{ kg}\left(5.00\hat{\mathbf{j}} \text{ m/s}\right) = 10.0\hat{\mathbf{j}} \text{ kg} \cdot \text{m/s}$

and $\quad \vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6.00 & 5.00t & 0 \\ 0 & 10.0 & 0 \end{vmatrix} = \boxed{\left(60.0 \text{ kg} \cdot \text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$

P11.14 $\sum F_x = ma_x \qquad T\sin\theta = \dfrac{mv^2}{r}$

$\sum F_y = ma_y \qquad T\cos\theta = mg$

So $\dfrac{\sin\theta}{\cos\theta} = \dfrac{v^2}{rg} \qquad v = \sqrt{rg\dfrac{\sin\theta}{\cos\theta}}$

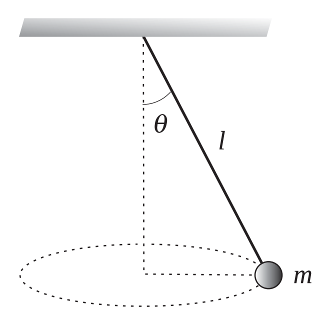

FIG. P11.14

$$L = rmv\sin 90.0°$$

$$L = rm\sqrt{rg\frac{\sin\theta}{\cos\theta}}$$

$$L = \sqrt{m^2 gr^3\frac{\sin\theta}{\cos\theta}}$$

$r = \ell\sin\theta$, so

$$L = \boxed{\sqrt{m^2 g\ell^3\frac{\sin^4\theta}{\cos\theta}}}$$

P11.16 (a) The net torque on the counterweight-cord-spool system is:

$$|\vec{\tau}| = \left|\vec{\mathbf{r}}\times\vec{\mathbf{F}}\right| = 8.00\times 10^{-2}\ \text{m}\,(4.00\ \text{kg})\left(9.80\ \text{m/s}^2\right) = \boxed{3.14\ \text{N}\cdot\text{m}}$$

(b) $\left|\vec{\mathbf{L}}\right| = \left|\vec{\mathbf{r}}\times m\vec{\mathbf{v}}\right| + I\omega$

$$\left|\vec{\mathbf{L}}\right| = Rmv + \frac{1}{2}MR^2\left(\frac{v}{R}\right) = R\left(m + \frac{M}{2}\right)v = \boxed{(0.400\ \text{kg}\cdot\text{m})v}$$

(c) $\tau = \dfrac{dL}{dt} = (0.400\ \text{kg}\cdot\text{m})a \qquad a = \dfrac{3.14\ \text{N}\cdot\text{m}}{0.400\ \text{kg}\cdot\text{m}} = \boxed{7.85\ \text{m/s}^2}$

***P11.19** (a) The vector from P to the falling ball is

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$\vec{\mathbf{r}} = \left(\ell\cos\theta\,\hat{\mathbf{i}} + \ell\sin\theta\,\hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}$$

The velocity of the ball is

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = 0 - gt\hat{\mathbf{j}}$$

So

$$\vec{\mathbf{L}} = \vec{\mathbf{r}}\times m\vec{\mathbf{v}}$$

$$\vec{\mathbf{L}} = m\left[\left(\ell\cos\theta\,\hat{\mathbf{i}} + \ell\sin\theta\,\hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}\right]\times\left(-gt\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{L}} = \boxed{-m\ell gt\cos\theta\,\hat{\mathbf{k}}}$$

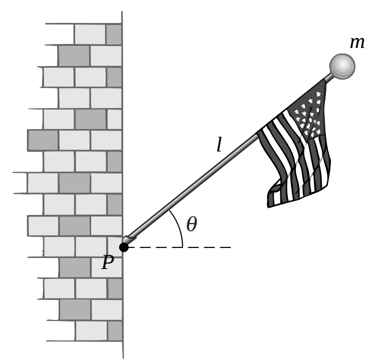

FIG. P11.19

(b) $\boxed{\text{The Earth exerts a gravitational torque on the ball.}}$

(c) Differentiating with respect to time, we have $\boxed{-mg\ell\cos\theta\,\hat{\mathbf{k}}}$ for the rate of change of angular momentum, which is also the torque due to the gravitational force on the ball.

P11.26 The total angular momentum about the center point is given by $L = I_h\omega_h + I_m\omega_m$

with $\qquad I_h = \dfrac{m_h L_h^2}{3} = \dfrac{60.0\ \text{kg}(2.70\ \text{m})^2}{3} = 146\ \text{kg}\cdot\text{m}^2$

and $\qquad I_m = \dfrac{m_m L_m^2}{3} = \dfrac{100\ \text{kg}(4.50\ \text{m})^2}{3} = 675\ \text{kg}\cdot\text{m}^2$

In addition, $\qquad \omega_h = \dfrac{2\pi\ \text{rad}}{12\ \text{h}}\left(\dfrac{1\ \text{h}}{3\ 600\ \text{s}}\right) = 1.45\times 10^{-4}\ \text{rad/s}$

while $\omega_m = \dfrac{2\pi \text{ rad}}{1 \text{ h}}\left(\dfrac{1 \text{ h}}{3\,600 \text{ s}}\right) = 1.75\times 10^{-3} \text{ rad/s}$

Thus, $L = 146 \text{ kg}\cdot\text{m}^2\left(1.45\times 10^{-4} \text{ rad/s}\right) + 675 \text{ kg}\cdot\text{m}^2\left(1.75\times 10^{-3} \text{ rad/s}\right)$

or $\boxed{L = 1.20 \text{ kg}\cdot\text{m}^2/\text{s}}$ The hands turn clockwise, so their vector angular momentum is $\boxed{\text{perpendicularly into the clock face.}}$

P11.29 (a) From conservation of angular momentum for the system of two cylinders:

$$\left(I_1 + I_2\right)\omega_f = I_1\omega_i \qquad \text{or} \qquad \omega_f = \boxed{\frac{I_1}{I_1 + I_2}\omega_i}$$

(b) $$K_f = \frac{1}{2}\left(I_1 + I_2\right)\omega_f^2 \qquad \text{and} \qquad K_i = \frac{1}{2}I_1\omega_i^2$$

so $$\frac{K_f}{K_i} = \frac{\frac{1}{2}\left(I_1 + I_2\right)}{\frac{1}{2}I_1\omega_i^2}\left(\frac{I_1}{I_1 + I_2}\omega_i\right)^2 = \boxed{\frac{I_1}{I_1 + I_2} \text{ which is less than 1}}$$

***P11.34** (a) Let $M =$ mass of rod and $m =$ mass of each bead. From $I_i\omega_i = I_f\omega_f$ between the moment of release and the moment the beads slide off, we have

$$\left[\frac{1}{12}M\ell^2 + 2mr_1^2\right]\omega_i = \left[\frac{1}{12}M\ell^2 + 2mr_2^2\right]\omega_f$$

When $M = 0.3$ kg, $\ell = 0.500$ m, $r_1 = 0.100$ m, $r_2 = 0.250$ m, $\omega_i = 36$/s, we find

$[0.00625 + 0.02\, m]36 = [0.00625 + 0.125\, m]\ \omega_f$

$\boxed{\omega_f = (36/\text{s})(1 + 3.2\, m)/(1 + 20\, m)}$

(b) The denominator of this fraction always exceeds the numerator, so

ω_f decreases smoothly from a maximum value of 36.0 rad/s for $m = 0$ toward a minimum value of $(36 \times 3.2/20) = 5.76$ rad/s as $m \to \infty$.

As a bonus, we find the work that the bar does on the beads as a function of m. Consider the beads alone. Their kinetic energy increases because of work done on them by the bar.

initial kinetic energy + work = final kinetic energy

$(1/2)(2\, m\, r_1^2)(\omega_i)^2 + W_b = (1/2)(2\, m\, r_2^2)(\omega_f)^2$

$m\,(0.1)^2(36)^2 + W_b = m\,(0.25)^2[(36/\text{s})(1 + 3.2\, m)/(1 + 20\, m)]^2$

$W_b = m[81(1 + 3.2m)^2 - 12.96(1 + 20m)^2] / (1 + 20\,m)^2 = (68.04\,m)(1 - 64\,m^2)/(1 + 20\,m)^2$ joules

W_b increases from 0 for $m = 0$ toward a maximum value of about 0.8 J at about m = 0.035 kg, and then decreases and goes negative, diverging to $-\infty$ as $m \to \infty$.

P11.36 When they touch, the center of mass is distant from the center of the larger puck by

$$y_{\text{CM}} = \frac{0 + 80.0\text{ g}(4.00\text{ cm} + 6.00\text{ cm})}{120\text{ g} + 80.0\text{ g}} = 4.00\text{ cm}$$

(a)

$$L = r_1 m_1 v_1 + r_2 m_2 v_2 = 0 + (6.00 \times 10^{-2}\text{ m})(80.0 \times 10^{-3}\text{ kg})(1.50\text{ m/s}) = \boxed{7.20 \times 10^{-3}\text{ kg} \cdot \text{m}^2/\text{s}}$$

(b) The moment of inertia about the CM is

$$I = \left(\frac{1}{2}m_1 r_1^2 + m_1 d_1^2\right) + \left(\frac{1}{2}m_2 r_2^2 + m_2 d_2^2\right)$$

$$I = \frac{1}{2}(0.120\text{ kg})(6.00 \times 10^{-2}\text{ m})^2 + (0.120\text{ kg})(4.00 \times 10^{-2})^2$$

$$+ \frac{1}{2}(80.0 \times 10^{-3}\text{ kg})(4.00 \times 10^{-2}\text{ m})^2 + (80.0 \times 10^{-3}\text{ kg})(6.00 \times 10^{-2}\text{ m})^2$$

$$I = 7.60 \times 10^{-4}\text{ kg} \cdot \text{m}^2$$

Angular momentum of the two-puck system is conserved: $L = I\omega$

$$\omega = \frac{L}{I} = \frac{7.20 \times 10^{-3}\text{ kg} \cdot \text{m}^2/\text{s}}{7.60 \times 10^{-4}\text{ kg} \cdot \text{m}^2} = \boxed{9.47\text{ rad/s}}$$

***P11.39** (a) Consider the system to consist of the wad of clay and the cylinder. No external forces acting on this system have a torque about the center of the cylinder. Thus, angular momentum of the system is conserved about the axis of the cylinder.

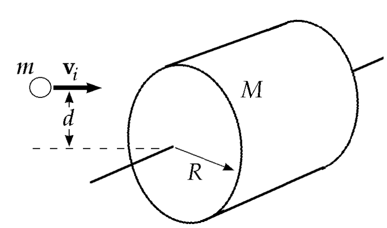

FIG. P11.39

$L_f = L_i$: $\quad I\omega = m v_i d$

or $\quad \left[\frac{1}{2}MR^2 + mR^2\right]\omega = m v_i d$

Thus, $\quad \omega = \boxed{\frac{2 m v_i d}{(M + 2m)R^2}}$

(b) $\boxed{\text{No; some mechanical energy changes into internal energy.}}$

(c) $\boxed{\text{Momentum is not conserved. The axle exerts a backward force on the cylinder.}}$

P11.51 (a) $\tau = \left|\vec{\mathbf{r}} \times \vec{\mathbf{F}}\right| = \left|\vec{\mathbf{r}}\right|\left|\vec{\mathbf{F}}\right| \sin 180° = 0$

Angular momentum is conserved.

$$L_f = L_i$$
$$mrv = mr_i v_i$$
$$v = \boxed{\frac{r_i v_i}{r}}$$

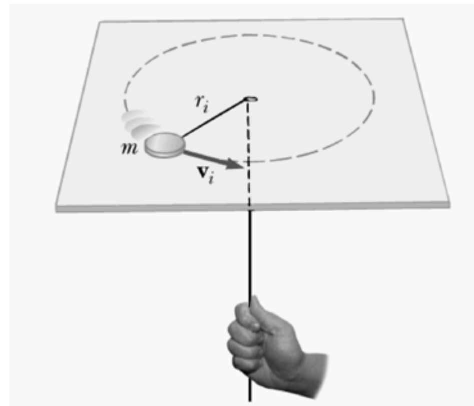

FIG. P11.51

(b) $T = \dfrac{mv^2}{r} = \boxed{\dfrac{m(r_i v_i)^2}{r^3}}$

(c) The work is done by the centripetal force in the *negative-r*, inward direction.

METHOD 1:

$$W = \int F \cdot d\ell = -\int T dr' = -\int_{r_i}^{r} \frac{m(r_i v_i)^2}{(r')^3} dr' = \left.\frac{m(r_i v_i)^2}{2(r')^2}\right|_{r_i}^{r}$$
$$= \frac{m(r_i v_i)^2}{2}\left(\frac{1}{r^2} - \frac{1}{r_i^2}\right) = \boxed{\frac{1}{2} m v_i^2 \left(\frac{r_i^2}{r^2} - 1\right)}$$

METHOD 2:

$$W = \Delta K = \frac{1}{2} m v^2 - \frac{1}{2} m v_i^2 = \boxed{\frac{1}{2} m v_i^2 \left(\frac{r_i^2}{r^2} - 1\right)}$$

(d) Using the data given, we find

$v = \boxed{4.50 \text{ m/s}}$ $\quad T = \boxed{10.1 \text{ N}}$ $\quad W = \boxed{0.450 \text{ J}}$

P11.53 (a) $L_i = m_1 v_{1i} r_{1i} + m_2 v_{2i} r_{2i} = 2mv\left(\dfrac{d}{2}\right)$

$L_i = 2(75.0 \text{ kg})(5.00 \text{ m/s})(5.00 \text{ m})$

$L_i = \boxed{3\,750 \text{ kg} \cdot \text{m}^2/\text{s}}$

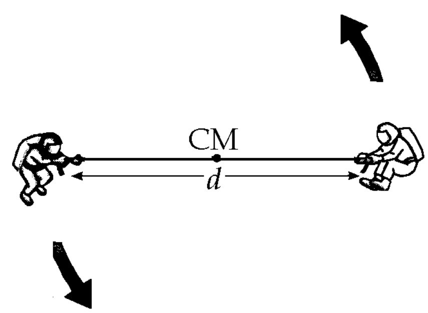

FIG. P11.53

(b) $K_i = \dfrac{1}{2} m_1 v_{1i}^2 + \dfrac{1}{2} m_2 v_{2i}^2$

$K_i = 2\left(\dfrac{1}{2}\right)(75.0 \text{ kg})(5.00 \text{ m/s})^2 = \boxed{1.88 \text{ kJ}}$

(c) Angular momentum is conserved: $L_f = L_i = \boxed{3\,750 \text{ kg} \cdot \text{m}^2/\text{s}}$

(d) $$v_f = \frac{L_f}{2(mr_f)} = \frac{3\,750\ \text{kg}\cdot\text{m}^2/\text{s}}{2(75.0\ \text{kg})(2.50\ \text{m})} = \boxed{10.0\ \text{m/s}}$$

(e) $$K_f = 2\left(\frac{1}{2}\right)(75.0\ \text{kg})(10.0\ \text{m/s})^2 = \boxed{7.50\ \text{kJ}}$$

(f) $$W = K_f - K_i = \boxed{5.62\ \text{kJ}}$$

P11.60 Angular momentum is conserved during the inelastic collision.

$$Mva = I\omega$$

$$\omega = \frac{Mva}{I} = \frac{3v}{8a}$$

The condition, that the box falls off the table, is that the center of mass must reach its maximum height as the box rotates, $h_{\max} = a\sqrt{2}$. Using conservation of energy:

$$\frac{1}{2}I\omega^2 = Mg\left(a\sqrt{2} - a\right)$$

$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\left(\frac{3v}{8a}\right)^2 = Mg\left(a\sqrt{2} - a\right)$$

$$v^2 = \frac{16}{3}ga\left(\sqrt{2} - 1\right)$$

$$v = \boxed{4\left[\frac{ga}{3}\left(\sqrt{2} - 1\right)\right]^{1/2}}$$

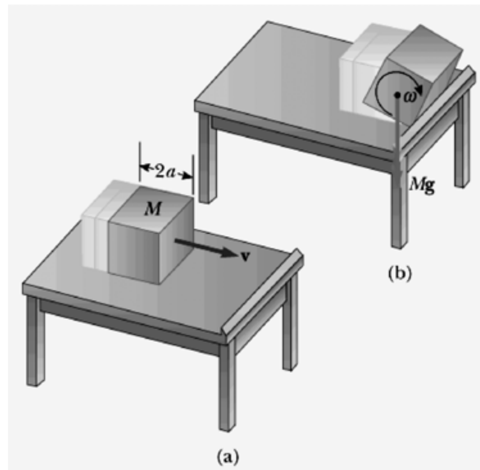

FIG. P11.60