Mathematical Induction

Use induction to prove that 3 divides $11^n - 5^n$ for all positive integers n.

Let $P(n)$ denote the proposition that $11^n - 5^n$ is divisible by 3 for all positive integers n.

BASIS STEP: $P(1)$ is true since 3 divides 6.

INDUCTIVE STEP: Let us assume $P(n)$ is true, that is $11^n - 5^n$ is divisible by 3 for an arbitrary positive integer n. This is our inductive hypothesis.

We have to show that $P(n+1)$ is true, $11^{n+1} - 5^{n+1}$ is divisible by 3 assuming the inductive hypothesis $P(n)$.

Proof: $11^{n+1}-5^{n+1} = 11^n \cdot (9+2) - 5^n \cdot (3+2)$

$= (11^n \cdot 9 - 5^n \cdot 3) + (11^n \cdot 2 - 5^n \cdot 2) = 3 \cdot (11^n \cdot 3 - 5^n) + 2(11^n - 5^n)$

$2(11^n - 5^n)$ is divisible by 3 using the inductive hypothesis**.**

$3 \cdot (11^n \cdot 3 - 5^n)$ is divisible by 3 the definition of divisibility since $(11^n \cdot 3 - 5^n)$ is an integer.

Thus, the sum $11^{n+1} - 5^{n+1} = 3 \cdot (11^n \cdot 3 - 5^n) + 2(11^n - 5^n)$ is also divisible by 3.

By the **Principle of Mathematical Induction** (Basis Step and Inductive Step together) $11^n - 5^n$ is divisible by 3 for all positive integers n.