MAT 243 Online Written Homework Assignments for Week 3 (units 6-9)

1. The power set of S = {1} has 2 elements, namely P(S) = {{}, {1}}. The power set of the power set of S, P(P(S)), has 4 elements, namely P(P(S)) = {{}, {{}}, {1}, {{}, 1}}.

2. The image of f(x) = $|x|$ on (-1, 2] is [0, 2). The preimage of f(x) = $|x|$ on (0,1] is [0, 1).

3. In order for f(x) to be bijective, f(x) has to be both injective and surjective.
   Check for injectivity:
   $\forall x \forall y \in \mathbb{R}$
   $f(x_1) = f(x_2)$
   $x_1 + 1 = x_2 + 1$ by definition of f
   Subtracting 1 from both sides yields $x_1 = x_2$
   $\therefore f \text{ } is \text{ } injective$
   Check for surjectivity:
   $\forall y \in \mathbb{R}$
   $\mathrm{y} = x + 1$
   $x = y - 1$
   $\exists x \in \mathbb{R} \mid f(x) = y$
   $\therefore f \text{ } is \text{ } surjective$

   Therefore, f(x) is bijective.

4. Check for injectivity:
   $\forall m \forall n \in \mathbb{N}$
   $f(n) = f(m)$

$\mathrm{n} + 1 = m + 1$

Subtracting 1 from both sides yields $n = m$

$\therefore f \ is \ injective$

Check for surjectivity:

$f(n) = n + 1$

$\neg\forall n \in \mathbb{N} | f(n) = 1$

$0 \neq \mathbb{N}$

$\therefore f \ is \ not \ surjective$

5. $x, y \in (1,\infty)$

$f(x) = f(y)$

$\frac{1}{x} = \frac{1}{y}$

$\frac{y}{x} = 1$

$y = x$

$\therefore f \ is \ injective$

$y \in (1,\infty)$

$x = \frac{1}{y},$

$1 < y < \infty$

$0 < x < 1$

$f(x) = f(\frac{1}{y})$

$\frac{1}{\frac{1}{y}} = y$

$\therefore f \ is \ surjective$

Therefore, f(x) is bijective.

6. $\sum_{n=100}^{200} \frac{5^{2n+3}}{3^{2n+1}} = \frac{5^{203}}{3^{201}} + \frac{5^{205}}{3^{203}} + \dots + \frac{5^{401}}{3^{399}} + \frac{5^{403}}{3^{401}} \approx \frac{4.84074x10^{281}}{2.11652x10^{191}}$