Mathematical Induction

Use induction to prove that $3^n > n$ for all positive integers n.

Let $P(n)$ denote the proposition that $3^n > n$, where n is a positive integer.

BASIS STEP: $P(1)$ is true since $3 > 1$.

INDUCTIVE STEP: Let us assume $P(n)$ is true, that is $3^n > n$ is true for an arbitrary positive integer n. This is our inductive hypothesis.

We have to show that $P(n+1)$, $3^{n+1} > n+1$ is also true assuming the inductive hypothesis $P(n)$.

Proof:

$\mathbf{3^{n+1}} = 3 \cdot 3^n > 3 \cdot n$ using the inductive hypothesis.

$3 \cdot n = n + 2n \geq n + 1,$ when $n \geq 1$.

By the **Principle of Mathematical Induction** (Basis Step and Inductive Step together) $3^n > n$ for all positive integers n.