1. Using Mersenne prime for my attempted guess, 2^{500} operations will be taken for the task. So the answer is:

   d. 2^{500}

2. True. Let $f_1(x) = (2x^2 + 3x + 1)$ and $f_2(x) = (x^2 - 7x – 4)$ where $g(x) = O(x^2)$. Then $f_1(x) + f_2(x) = [(2x^2 + 3x + 1) + (x^2 - 7x – 4)] = (3x^2 – 4x -3)$, which is still $O(x^2)$.

3. n is the exponent = 1048576, y = 1 and x ≡ 3 mod 7 = 3

   n = 1048576 is even, keep y = 1 as is
   next sub step, determine x^2 mod p
   x^2 mod p = 3^2 mod 7
   x^2 mod p = 9 mod 7
   9 mod 7 = 2, reset x to 2

   Then, cut n in half and take the integer value:

   $$1048576 \div 2 = 524288$$

   n = 524288 is even
   524288 is even, keep y = 1 as is
   next sub step, determine x^2 mod p
   x^2 mod p = 2^2 mod 7
   x^2 mod p = 4 mod 7
   4 mod 7 = 4, reset x to this value

   Then, cut n in half and take the integer value:

   $$524288 \div 2 = 262144$$

   n = 262144 is even
   262144 is even, keep y = 1 as is
   next sub step, determine x^2 mod p
   x^2 mod p = 4^2 mod 7
   x^2 mod p = 16 mod 7
   16 mod 7 = 2, reset x to this value

Then, cut n in half and take the integer value:

$$262144 \div 2 = 131072$$

n = 131072 is even
131072 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 2^2 mod 7
x^2 mod p = 4 mod 7
4 mod 7 = 4, reset x to this value

Then, cut n in half and take the integer value:

$$131072 \div 2 = 65536$$

n = 65536 is even
65536 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 4^2 mod 7
x^2 mod p = 16 mod 7
16 mod 7 = 2, reset x to this value

Then, cut n in half and take the integer value : 65536 ÷ 2 = 32768

n = 32768 is even
32768 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 2^2 mod 7
x^2 mod p = 4 mod 7
4 mod 7 = 4, reset x to this value

Then, cut n in half and take the integer value: 32768 ÷ 2 = 16384

n = 16384 is even
16384 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 4^2 mod 7

x^2 mod p = 16 mod 7
16 mod 7 = 2, reset x to this value

Then, cut n in half and take the integer value: 16384 ÷ 2 = 8192

n = 8192 is even
8192 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 2^2 mod 7
x^2 mod p = 4 mod 7
4 mod 7 = 4, reset x to this value

Then, cut n in half and take the integer value: 8192 ÷ 2 = 4096

n = 4096 is even
4096 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 4^2 mod 7
x^2 mod p = 16 mod 7
16 mod 7 = 2, reset x to this value

Then, cut n in half and take the integer value: 4096 ÷ 2 = 2048

n = 2048 is even
2048 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 2^2 mod 7
x^2 mod p = 4 mod 7
4 mod 7 = 4, reset x to this value

Then, cut n in half and take the integer value: 2048 ÷ 2 = 1024

n = 1024 is even
1024 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 4^2 mod 7
x^2 mod p = 16 mod 7
16 mod 7 = 2, reset x to this value

Then, cut n in half and take the integer value: 1024 ÷ 2 = 512

n = 512 is even
512 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 2^2 mod 7
x^2 mod p = 4 mod 7
4 mod 7 = 4, reset x to this value

Then, cut n in half and take the integer value: 512 ÷ 2 = 256

n = 256 is even
256 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 4^2 mod 7
x^2 mod p = 16 mod 7
16 mod 7 = 2, reset x to this value

Then, cut n in half and take the integer value: 256 ÷ 2 = 128

n = 128 is even
128 is even, keep y = 1 as is
next sub stepc determine x^2 mod p
x^2 mod p = 2^2 mod 7
x^2 mod p = 4 mod 7
4 mod 7 = 4, so we reset x to this value

Then, cut n in half and take the integer value: 128 ÷ 2 = 64

n = 64 is even
64 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 4^2 mod 7
x^2 mod p = 16 mod 7
16 mod 7 = 2, reset x to this value

Then, cut n in half and take the integer value: 64 ÷ 2 = 32

$n = 32$ is even
32 is even, keep $y = 1$ as is
next sub step, determine $x^2 \bmod p$
$x^2 \bmod p = 2^2 \bmod 7$
$x^2 \bmod p = 4 \bmod 7$
$4 \bmod 7 = 4$, reset x to this value

Then, cut n in half and take the integer value: $32 \div 2 = 16$

$n = 16$ is even
16 is even, keep $y = 1$ as is
next sub step, determine $x^2 \bmod p$
$x^2 \bmod p = 4^2 \bmod 7$
$x^2 \bmod p = 16 \bmod 7$
$16 \bmod 7 = 2$, reset x to this value

Then, cut n in half and take the integer value: $16 \div 2 = 8$

$n = 8$ is even
8 is even, keep $y = 1$ as is
next sub step, determine $x^2 \bmod p$
$x^2 \bmod p = 2^2 \bmod 7$
$x^2 \bmod p = 4 \bmod 7$
$4 \bmod 7 = 4$, reset x to this value

Then, cut n in half and take the integer value: $8 \div 2 = 4$

$n = 4$ is even
4 is even, keep $y = 1$ as is
next sub step, determine $x^2 \bmod p$
$x^2 \bmod p = 4^2 \bmod 7$
$x^2 \bmod p = 16 \bmod 7$
$16 \bmod 7 = 2$, reset x to this value

Then, cut n in half and take the integer value: $4 \div 2 = 2$

n = 2 is even
2 is even, keep y = 1 as is
next sub step, determine x^2 mod p
x^2 mod p = 2^2 mod 7
x^2 mod p = 4 mod 7
4 mod 7 = 4, reset x to this value

Then, cut n in half and take the integer value: 2 ÷ 2 = 1

n = 1 is odd
1 is odd, calculate (yx) mod p
yx mod p = (1)(4) mod 7
yx mod p = 4 mod 7
4 mod 7 = 4, now reset y to this value
next sub step, determine x^2 mod p
x^2 mod p = 4^2 mod 7
x^2 mod p = 16 mod 7
16 mod 7 = 2, reset x to this value

Then, cut n in half and take the integer value: 1 ÷ 2 = 0

n = 0, y = **4, so $3^{1048576}$ mod 7 ≡ 4** (Dreadful).

4. Prime factorization of $(2^{16} - 1)$ = 3 x 5 x 17 x 257

5. If **n** is a natural number, then $(2^{2n} - 1)$ must be divisible by 3.
   Definition: **a** is divisible by **b** if (a / b) is a whole number. So, divisible by 3 means divisible by {0,3,6,9,12,15,...,3(n-1), 3n}.
   Let n be an arbitrary natural number, say 6, then $(2^{2n} - 1)$ / 3 = $(2^{12} - 1)$ / 3 = 1365. Hence, the statement is true.

6. Ran out of time.
7. Ran out of time. Must turn HW in now.