Homework 2-2

Due: 11:59pm on Tuesday, October 28, 2014

To understand how points are awarded, read the Grading Policy for this assignment.

An Object Accelerating on a Ramp

Learning Goal:

Understand that the acceleration vector is in the direction of the change of the velocity vector.

In one dimensional (straight line) motion, acceleration is accompanied by a change in speed, and the acceleration is always parallel (or antiparallel) to the velocity.
When motion can occur in two dimensions (e.g. is confined to a tabletop but can lie anywhere in the x-y plane), the definition of acceleration is

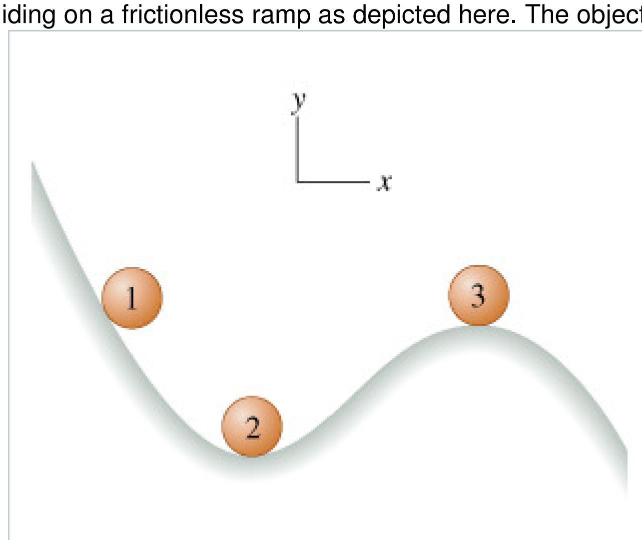
$$\vec{a}(t) = \frac{\vec{v}(t+\Delta t)-\vec{v}(t)}{\Delta t} \text{ in the limit } \Delta t \to 0.$$

In picturing this vector derivative you can think of the derivative of a vector as an instantaneous quantity by thinking of the velocity of the tip of the arrow as the vector changes in time. Alternatively, you can (for small Δt) approximate the acceleration as

$$\vec{a}(t) \approx \frac{\vec{v}(t+\Delta t)-\vec{v}(t)}{\Delta t}.$$

Obviously the difference between $\vec{v}(t+\Delta t)$ and $\vec{v}(t)$ is another vector that can lie in any direction. If it is longer but in the same direction, $\vec{a}(t)$ will be parallel to $\vec{v}(t)$. On the other hand, if $\vec{v}(t+\Delta t)$ has the same magnitude as $\vec{v}(t)$ but is in a slightly different direction, then $\vec{a}(t)$ will be perpendicular to $\vec{v}$. In general, $\vec{v}(t+\Delta t)$ can differ from $\vec{v}(t)$ in both magnitude *and* direction, hence $\vec{a}(t)$ can have any direction relative to $\vec{v}(t)$.

This problem contains several examples of this.Consider an object sliding on a frictionless ramp as depicted here. The object is already moving along the ramp toward position 2 when it is at position 1. The following questions concern the direction of the object's acceleration vector, $\vec{a}$. In this problem, you should find the direction of the acceleration vector by drawing the velocity vector at two points near to the position you are asked about. Note that since the object moves along the track, its velocity vector at a point will be tangent to the track at that point. The acceleration vector will point in the same direction as the vector difference of the two velocities. (This is a result of the equation $\vec{a}(t) \approx (\vec{v}(t+\Delta t)-\vec{v}(t))/\Delta t$ given above.)



Part A

Which direction best approximates the direction of $\vec{a}$ when the object is at position 1?

Hint 1. Consider the change in velocity

At this point, the object's velocity vector is not changing direction; rather, it is increasing in magnitude. Therefore, the object's acceleration is nearly parallel to its velocity.

ANSWER:

- straight up
- downward to the left
- (selected) downward to the right
- straight down

Correct

Part B

Which direction best approximates the direction of $\vec{a}$ when the object is at position 2?

Hint 1. Consider the change in velocity

At this point, the speed has a local maximum; thus the magnitude of $\vec{v}$ is not changing. Therefore, no component of the acceleration vector is parallel to the velocity vector. However, since the direction of $\vec{v}$ is changing there is an acceleration.

ANSWER:

- (selected) straight up
- upward to the right
- straight down
- downward to the left

Correct

Even though the acceleration is directed straight up, this does not mean that the object is moving straight up.

Part C

Which direction best approximates the direction of $\vec{a}$ when the object is at position 3?

Hint 1. Consider the change in velocity

At this point, the speed has a local minimum; thus the magnitude of $\vec{v}$ is not changing. Therefore, no component of the acceleration vector is parallel to the velocity vector. However, since the direction of $\vec{v}$ is changing there is an acceleration.

ANSWER:

- upward to the right
- to the right
- straight down
- downward to the right

Correct

Tactics Box 4.1 Finding the Acceleration Vector

Learning Goal:

To practice Tactics Box 4.1 Finding the Acceleration Vector.

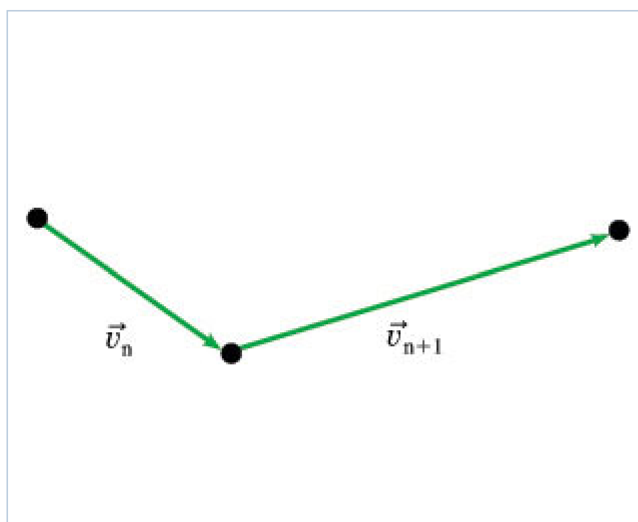
Suppose an object has an initial velocity $\vec{v}_{\rm n}$ at time $t_{\rm n}$ and later, at time $t_{\rm n+1}$, has velocity $\vec{v}_{\rm n+1}$. The fact that the velocity changes tells us that the object undergoes an acceleration during the time interval $\Delta t = t_{\rm n+1} - t_{\rm n}$. From the definition of acceleration,

$$\vec{a} = \frac{\vec{v}_{\rm n+1} - \vec{v}_{\rm n}}{t_{\rm n+1} - t_{\rm n}} = \frac{\Delta \vec{v}}{\Delta t} \,,$$

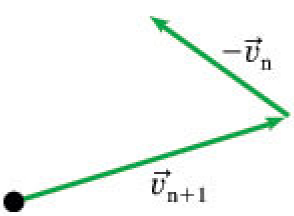
we see that the acceleration vector points in the same direction as the vector $\Delta\vec{v}$. This vector is the change in the velocity $\Delta\vec{v} = \vec{v}_{\rm n+1} - \vec{v}_{\rm n}$, so to know which way the acceleration vector points, we have to perform the vector subtraction $\vec{v}_{\rm n+1} - \vec{v}_{\rm n}$. This Tactics Box shows how to use vector subtraction to find the acceleration vector.

TACTICS BOX 4.1 Finding the acceleration vector.

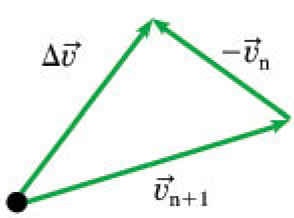
To find the acceleration between velocity $\vec{v}_{\rm n}$ and velocity $\vec{v}_{\rm n+1}$, follow these steps:



1. Draw the velocity vector $\vec{v}_{\text{vec}_{\rm n+1}}$.

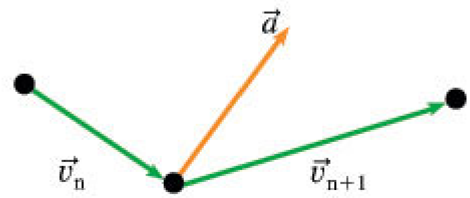


2. Draw $-\vec{v}_{\rm n}$ at the tip of $\vec{v}_{\rm n+1}$.
3. Draw $\Delta\vec{v} = \vec{v}_{\rm n+1} - \vec{v}_{\rm n} = \vec{v}_{\rm n+1} + (-\vec{v}_{\rm n})$. This is the



direction of $\vec{a}$.
4. Return to the original motion diagram. Draw a vector at the middle point in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is

the average acceleration at the midpoint between $\vec{v}_n$ and $\vec{v}_{n+1}$.



Part A

Below is a motion diagram for an object that moves along a curved path. The dots represent the position of the object at three subsequent instants, t_1, t_2, and t_3. The vectors $\vec{v}_1$ and $\vec{v}_2$ show the average velocity of the object for the initial time interval, $\Delta t_i = t_2 - t_1$, and the final time interval, $\Delta t_f = t_3 - t_2$, respectively. Draw the acceleration vector $\vec{a}$ representing the change in average velocity of the object during the total time interval $\Delta t = t_3 - t_1$.

Draw the vectors starting at the appropriate black dots. The location, orientation, and length of the vectors will be graded.

Hint 1. How to draw the acceleration vector

First, draw $\overrightarrow{-v}_1$. Draw $\overrightarrow{-v}_1$ starting at the tip of $\vec{v}_1$ and ending at its tail. Then, move $\overrightarrow{-v}_1$, with the same orientation, so that its tail is at the tip of $\vec{v}_2$. Use the **vector info** button to make sure that the lengths of $\vec{v}_1$ and $\overrightarrow{-v}_1$ are equal. The acceleration vector, $\vec{a}$, starts at the tip of $\vec{v}_1$ (tail of $\vec{v}_2$) and ends at the tip of $\overrightarrow{-v}_1$.

ANSWER:

Correct

Part B

Below is another motion diagram for an object that moves along a curved path. The dots represent the position of the object at five subsequent instants, t_1, t_2, t_3, t_4, and t_5. The vectors $\vec{v}_{21}$, $\vec{v}_{32}$, $\vec{v}_{43}$, and $\vec{v}_{54}$ represent the average velocity of the object during the four corresponding time intervals. Draw the acceleration vectors $\vec{a}_{31}$ and $\vec{a}_{53}$ representing the changes in velocity of the object during the time intervals $\Delta t_{31} = t_3 - t_1$ and $\Delta t_{53} = t_5 - t_3$, respectively.

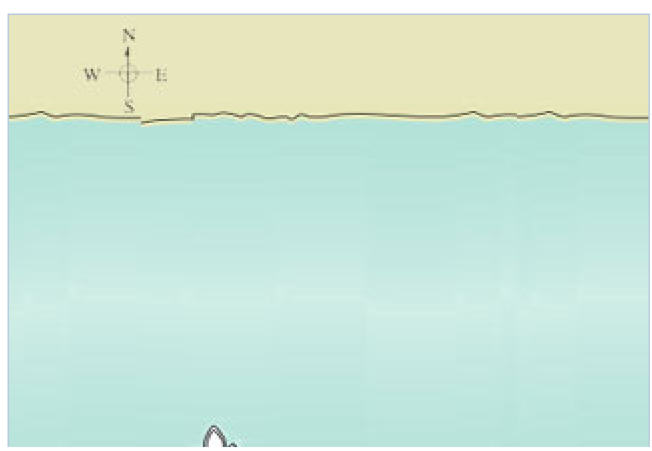
Draw the vectors starting at the appropriate black dots. The location, orientation, and length of the vectors will be graded.

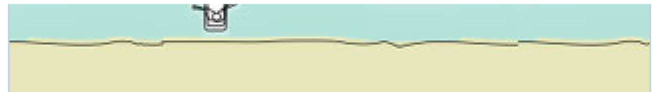
ANSWER:

Correct

± Velocity in a Moving Frame Conceptual Question

You are attempting to row across a stream in your rowboat. Your paddling speed relative to still water is 3.0 $\mathrm{m/s}$ (i.e., if you were to paddle in water without a current, you would move with a speed of 3.0 $\mathrm{m/s}$). You head off by rowing directly north, across the stream.





Part A

Draw the vector $\vec{v}_{\rm still}$, representing your velocity relative to still water.

Draw the vector starting at the rowboat. The location, orientation and length of the vector will be graded. The vector's length is displayed in meters per second.

ANSWER:

Correct

Part B

Now assume that the stream flows east at 4.0 $\rm m/s$. Draw the vectors $\vec{v}_{\rm w}$, representing the velocity of the stream, and $\vec{v}_{\rm tot}$, representing the velocity of your rowboat relative to the stream bank. Be sure to draw both vectors.

Draw $\vec{v}_{\rm tot}$ and $\vec{v}_{\rm w}$ starting at the tail and tip of $\vec{v}_{\rm still}$ respectively. The location, orientation and length of the vectors will be graded. Each vector's length is displayed in meters per second.

Hint 1. How to find the total velocity

The total velocity of your motion is the vector sum of the velocity you would have in still water and the velocity of the stream.
To add two vectors, place the tail of the second vector at the tip of the first. Together, the two vectors now form two legs of a triangle. The sum of these two vectors is the third side of this triangle. The tail of the sum should

coincide with the tail of the first vector, and the tip of the sum should coincide with the tip of the second vector. Place the vectors carefully to be sure that your sum vector is accurate.

ANSWER:

Correct

Part C

Based on the vector diagram in Part B, determine how far downstream of your starting point you will finally reach the opposite shore if the stream is 6.0 meters wide.

Hint 1. Find the time to cross the stream

Notice that your velocity has been given in component form: 3.0 $\mathrm{m/s}$ across the stream and 4.0 $\mathrm{m/s}$ along the stream. Based on these velocity components, how long will it take you to cross the 6.0 meter wide stream?

Express your answer in seconds.

ANSWER:

2.00 s

Correct

Now that you know the time spent crossing the stream, you can use the component of the velocity directed along the river to find out how far down river you end up.

ANSWER:

d = 8 m

Correct

Conceptual Problem about Projectile Motion

Learning Goal:

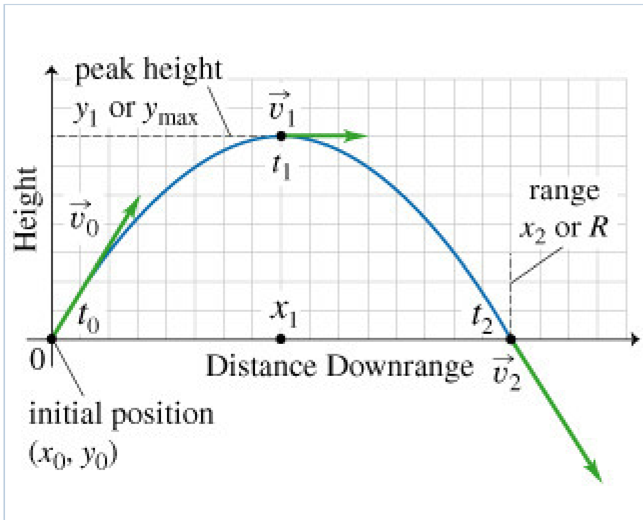
To understand projectile motion by considering horizontal constant velocity motion and vertical constant acceleration motion independently.

Projectile motion refers to the motion of unpowered objects (called projectiles) such as balls or stones moving near the surface of the earth under the influence of the earth's gravity alone. In this analysis we assume that air resistance can be neglected.

An object undergoing projectile motion near the surface of the earth obeys the following rules:

1. An object undergoing projectile motion travels horizontally at a constant rate. That is, the *x* component of its velocity, v_x, is constant.
2. An object undergoing projectile motion moves vertically with a constant downward acceleration whose magnitude, denoted by *g*, is equal to 9.80 $\mathrm{m/s^2}$ near the surface of the earth. Hence, the *y* component of its velocity, v_y, changes continuously.
3. An object undergoing projectile motion will undergo the horizontal and vertical motions described above from the instant it is launched until the instant it strikes the ground again. Even though the horizontal and vertical motions can be treated independently, they are related by the fact that they occur for exactly the same amount of time, namely the time t the projectile is in the air.

The figure shows the trajectory (i.e., the path) of a ball undergoing projectile motion over level ground. The time $t_0 = 0 \mathrm{~s}$ corresponds to the moment just after the ball is launched from position $x_0 = 0 \mathrm{~m}$ and $y_0 = 0 \mathrm{~m}$. Its launch velocity, also called the initial velocity, is $\vec{v}_0$.



Two other points along the trajectory are indicated in the figure.

- One is the moment the ball reaches the peak of its trajectory, at time t_1 with velocity $\vec{v}_1$. Its position at this moment is denoted by (x_1, y_1) or $(x_1, y_{\max})$ since it is at its maximum height.
- The other point, at time t_2 with velocity $\vec{v}_2$, corresponds to the moment just before the ball strikes the ground on the way back down. At this time its position is (x_2, y_2), also known as $(x_{\max}, y_2)$ since it is at its maximum horizontal range.

Projectile motion is symmetric about the peak, provided the object lands at the same vertical height from which is was launched, as is the case here. Hence $y_2 = y_0 = 0 \mathrm{~m}$.

Part A

How do the speeds v_0, v_1, and v_2 (at times t_0, t_1, and t_2) compare?

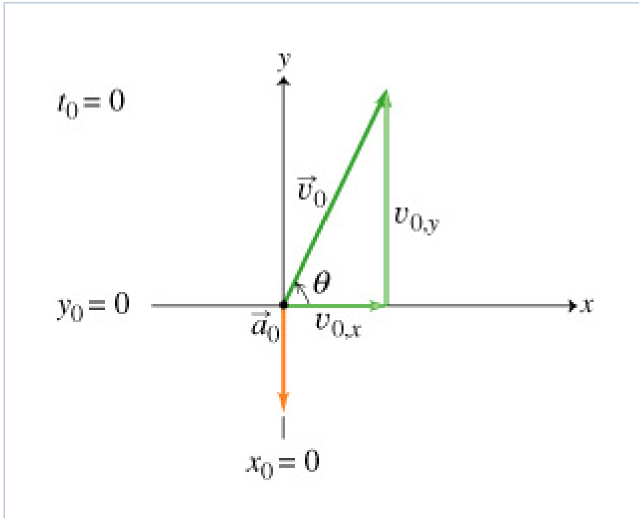
ANSWER:

- $v_0 = v_1 = v_2 > 0$
- $v_0 = v_2 > v_1 = 0$
- (selected) $v_0 = v_2 > v_1 > 0$
- $v_0 > v_1 > v_2 > 0$
- $v_0 > v_2 > v_1 = 0$

Correct

Here v_0 equals v_2 by symmetry and both exceed v_1. This is because v_0 and v_2 include vertical speed as well as the constant horizontal speed.

Consider a diagram of the ball at time t_0. Recall that t_0 refers to the instant just after the ball has been launched, so it is still at ground level ($x_0 = y_0 = 0 \text{ m}$). However, it is already moving with initial velocity $\vec{v}_0$, whose magnitude is $v_0 = 30.0 \text{ m/s}$ and direction is $\theta = 60.0 \text{ degrees}$ counterclockwise from the positive *x* direction.



Part B

What are the values of the intial velocity vector components $v_{0,x}$ and $v_{0,y}$ (both in m/s) as well as the acceleration vector components $a_{0,x}$ and $a_{0,y}$ (both in m/s^2)? Here the subscript 0 means "at time t_0."

Hint 1. Determining components of a vector that is aligned with an axis

If a vector points along a single axis direction, such as in the positive *x* direction, its *x* component will be its full magnitude, whereas its *y* component will be zero since the vector is perpendicular to the *y* direction. If the vector points in the negative *x* direction, its *x* component will be the negative of its full magnitude.

Hint 2. Calculating the components of the initial velocity

Notice that the vector $\vec{v}_0$ points up and to the right. Since "up" is the positive *y* axis direction and "to the right" is the positive *x* axis direction, $v_{0,x}$ and $v_{0,y}$ will both be positive.

As shown in the figure, $v_{0,x}$, $v_{0,y}$, and v_0 are three sides of a right triangle, one angle of which is θ. Thus $v_{0,x}$

and $v_{0,y}$ can be found using the definition of the sine and cosine functions given below. Recall that $v_0 = 30.0\ \mathrm{m/s}$ and $\theta = 60.0$ degrees and note that

$$\sin(\theta) = \frac{\text{length of opposite side}}{\text{length of hypotenuse}} = \frac{v_{0,y}}{v_0},$$

$$\cos(\theta) = \frac{\text{length of adjacent side}}{\text{length of hypotenuse}} = \frac{v_{0,x}}{v_0}.$$

What are the values of $v_{0,x}$ and $v_{0,y}$?

Enter your answers numerically in meters per second separated by a comma.

ANSWER:

15.0,26.0 $\mathrm{m/s}$

Correct

ANSWER:

- 30.0, 0, 0, 0
- 0, 30.0, 0, 0
- 15.0, 26.0, 0, 0
- 30.0, 0, 0, -9.80
- 0, 30.0, 0, -9.80
- 15.0, 26.0, 0, -9.80
- 15.0, 26.0, 0, +9.80

Correct

Also notice that at time t_2, just before the ball lands, its velocity components are $v_{2,x} = 15\ \mathrm{m/s}$ (the same as always) and $v_{2,y} = -26.0\ \mathrm{m/s}$ (the same size but opposite sign from $v_{0,y}$ by symmetry). The acceleration at time t_2 will have components (0, -9.80 $\mathrm{m/s^2}$), exactly the same as at t_0, as required by Rule 2.

The peak of the trajectory occurs at time t_1. This is the point where the ball reaches its maximum height $y_{\max}$. At the peak the ball switches from moving up to moving down, even as it continues to travel horizontally at a constant rate.

Part C

What are the values of the velocity vector components $v_{1,x}$ and $v_{1,y}$ (both in $\mathrm{m/s}$) as well as the acceleration vector components $a_{1,x}$ and $a_{1,y}$ (both in $\mathrm{m/s^2}$)? Here the subscript 1 means that these are all at time t_1.

ANSWER:

- 0, 0, 0, 0
- 0, 0, 0, -9.80
- 15.0, 0, 0, 0
- 15.0, 0, 0, -9.80
- 0, 26.0, 0, 0
- 0, 26.0, 0, -9.80
- 15.0, 26.0, 0, 0
- 15.0, 26.0, 0, -9.80

Correct

At the peak of its trajectory the ball continues traveling horizontally at a constant rate. However, at this moment it stops moving up and is about to move back down. This constitutes a downward-directed change in velocity, so the ball is accelerating downward even at the peak.

The flight time refers to the total amount of time the ball is in the air, from just after it is launched (t_0) until just before it lands (t_2). Hence the flight time can be calculated as $t_2 - t_0$, or just t_2 in this particular situation since $t_0 = 0$. Because the ball lands at the same height from which it was launched, by symmetry it spends half its flight time traveling up to the peak and the other half traveling back down. The flight time is determined by the initial vertical component of the velocity and by the acceleration. The flight time does not depend on whether the object is moving horizontally while it is in the air.

Part D

If a second ball were dropped from rest from height $y_{\rm max}$, how long would it take to reach the ground? Ignore air resistance.

Check all that apply.

Hint 1. Kicking a ball of cliff; a related problem

Consider two balls, one of which is dropped from rest off the edge of a cliff at the same moment that the other is kicked horizontally off the edge of the cliff. Which ball reaches the level ground at the base of the cliff first? Ignore air resistance.

Hint 1. Comparing position, velocity, and acceleration of the two balls

Both balls start at the same height and have the same initial *y* velocity ($v_{0,y} = 0$) as well as the same acceleration ($\vec{a} = g$ downward). They differ only in their *x* velocity (one is zero, the other nonzero). This difference will affect their *x* motion but not their *y* motion.

ANSWER:

- The ball that falls straight down strikes the ground first.
- The ball that was kicked so it moves horizontally as it falls strikes the ground first.
- Both balls strike the ground at the same time.

Correct

The fact that one ball moves horizontally as it falls does not influence its vertical motion. Hence both balls are at the same height at all moments in time and thus they strike the ground at the same instant.
Now return to the original question, in which you are asked to compare the flight time for a ball that rises from the ground to a peak and then falls back down to the ground with the flight time for a second ball that only needs to fall from the peak height to the ground.

ANSWER:

- [ ] t_0
- [x] $t_1 - t_0$
- [ ] t_2
- [x] $t_2 - t_1$
- [x] $\frac{t_2 - t_0}{2}$

All attempts used; correct answer displayed

In projectile motion over level ground, it takes an object just as long to rise from the ground to the peak as it takes for it to fall from the peak back to the ground.

The *range* R of the ball refers to how far it moves horizontally, from just after it is launched until just before it lands. Range is defined as $x_2 - x_0$, or just x_2 in this particular situation since $x_0 = 0$.

Range can be calculated as the product of the flight time t_2 and the *x* component of the velocity v_x (which is the same at all times, so $v_x = v_{0,x}$). The value of v_x can be found from the launch speed v_0 and the launch angle θ using trigonometric functions, as was done in Part B. The flight time is related to the initial *y* component of the velocity, which may also be found from v_0 and θ using trig functions.
The following equations may be useful in solving projectile motion problems, but these equations apply only to a projectile launched over level ground from position $(x_0 = y_0 = 0)$ at time $t_0 = 0$ with initial speed v_0 and launch angle θ measured from the horizontal. As was the case above, t_2 refers to the flight time and R refers to the range of the projectile.

$$\text{flight time: } t_2 = \frac{2v_{0,y}}{g} = \frac{2v_0 \sin(\theta)}{g}$$

$$\text{range: } R = v_x t_2 = \frac{v_0^2 \sin(2\theta)}{g}$$

In general, a high launch angle yields a long flight time but a small horizontal speed and hence little range. A low launch angle gives a larger horizontal speed, but less flight time in which to accumulate range. The launch angle that achieves the maximum range for projectile motion over level ground is 45 degrees.

Part E

Which of the following changes would increase the range of the ball shown in the original figure?

Check all that apply.

ANSWER:

- [x] Increase v_0 above 30 $\mathrm{m/s}$.
- [ ] Reduce v_0 below 30 $\mathrm{m/s}$.
- [x] Reduce θ from 60 $\mathrm{degrees}$ to 45 $\mathrm{degrees}$.
- [ ] Reduce θ from 60 $\mathrm{degrees}$ to less than 30 $\mathrm{degrees}$.
- [ ] Increase θ from 60 $\mathrm{degrees}$ up toward 90 $\mathrm{degrees}$.

Correct

A solid understanding of the concepts of projectile motion will take you far, including giving you additional insight into the solution of projectile motion problems numerically. Even when the object does not land at the same height from which is was launched, the rules given in the introduction will still be useful.
Recall that air resistance is assumed to be negligible here, so this projectile motion analysis may not be the best choice for describing things like frisbees or feathers, whose motion is strongly influenced by air. The value of the gravitational free-fall acceleration g is also assumed to be constant, which may not be appropriate for objects that move vertically through distances of hundreds of kilometers, like rockets or missiles. However, for problems that involve relatively dense projectiles moving close to the surface of the earth, these assumptions are reasonable.

PSS 4.1 Projectile Motion Problems

Learning Goal:

To practice Problem-Solving Strategy 4.1 for projectile motion problems.

A rock thrown with speed $11.5 \mathrm{m/s}$ and launch angle 30.0° (above the horizontal) travels a horizontal distance of d = 20.0m before hitting the ground. From what height was the rock thrown? Use the value g = 9.810$\mathrm{m/s^2}$ for the free-fall acceleration.

PROBLEM-SOLVING STRATEGY 4.1 Projectile motion problems

MODEL: Make simplifying assumptions, such as treating the object as a particle. Is it reasonable to ignore air resistance?

VISUALIZE: Use a pictorial representation. Establish a coordinate system with the *x* axis horizontal and the *y* axis vertical. Show important points in the motion on a sketch. Define symbols, and identify what you are trying to find.

SOLVE: The acceleration is known: $a_x = 0$ and $a_y = -g$. Thus, the problem becomes one of two-dimensional kinematics. The kinematic equations are

$$x_\mathrm{f} = x_\mathrm{i} + v_{\mathrm{i}x}\Delta t, \qquad y_\mathrm{f} = y_\mathrm{i} + v_{\mathrm{i}y}\Delta t - \frac{1}{2}g(\Delta t)^2,$$

$$v_{\mathrm{f}x} = v_{\mathrm{i}x} = \mathrm{constant, and} \qquad \mathrm{v_{fy} = v_{iy} - g\Delta t.}$$

Δt is the same for the horizontal and vertical components of the motion. Find Δt from one component, and then use that value for the other component.

ASSESS: Check that your result has the correct units, is reasonable, and answers the question.

Model

Start by making simplifying assumptions: Model the rock as a particle in free fall. You can ignore air resistance because the rock is a relatively heavy object moving relatively slowly.

Visualize

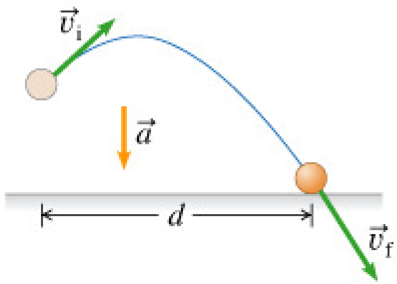
Part A

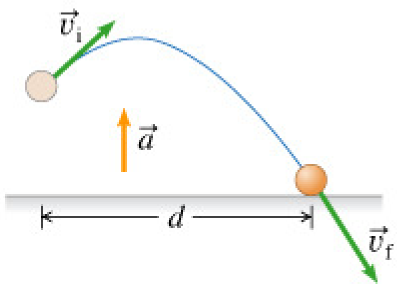
Which diagram represents an accurate sketch of the rock's trajectory?

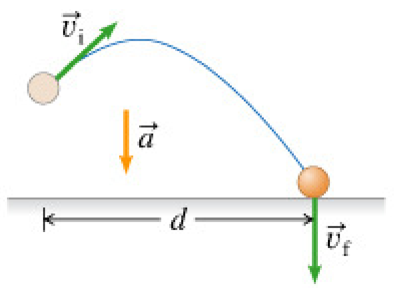
Hint 1. The launch angle

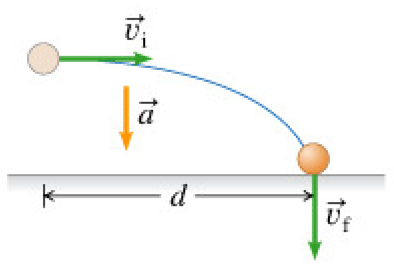
In a projectile's motion, the angle of the initial velocity $\vec{v}_{\rm i}$ above the horizontal is called the *launch angle*.

ANSWER:









Correct

Part B

As stated in the strategy, choose a coordinate system where the *x* axis is horizontal and the *y* axis is vertical. Note that in the strategy, the *y* component of the projectile's acceleration, a_y, is taken to be negative. This implies that the positive *y* axis is upward. Use the same convention for your *y* axis, and take the positive *x* axis to be to the right.

Where you choose your origin doesn't change the answer to the question, but choosing an origin can make a problem easier to solve (even if only a bit). Usually it is nice if the majority of the quantities you are given and the quantity you are trying to solve for take positive values relative to your chosen origin. Given this goal, what location for the origin of the coordinate system would make this problem easiest?

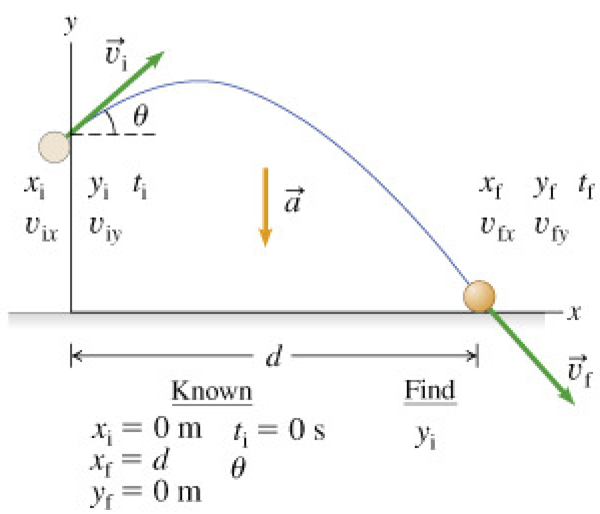
ANSWER:

- At ground level below the point where the rock is launched
- At the point where the rock is released
- At the point where the rock strikes the ground
- At the peak of the trajectory
- At ground level below the peak of the trajectory

Correct

It's best to place the origin of the coordinate system at ground level below the launching point because in this way all the points of interest (the launching point and the landing point) will have positive coordinates. (Based on your experience, you know that it's generally easier to work with positive coordinates.) Keep in mind, however, that this is an arbitrary choice. *The correct solution of the problem will not depend on the location of the origin of your coordinate system.*

Now, define symbols representing initial and final position, velocity, and time. Your target variable is y_i, the initial *y* coordinate of the rock. Your pictorial representation should be complete now, and similar to the picture below:



Solve

Part C

Find the height y_i from which the rock was launched.

Express your answer in meters to three significant figures.

Hint 1. How to approach the problem

The time Δt needed to move horizontally to the final position $x_\mathrm{f} = d$ = $20.0 \mathrm{m}$ is the same time needed for the rock to rise from the initial position y_i to the peak of its trajectory and then fall to the ground. Use the information you have about motion in the horizontal direction to solve for Δt. Knowing this time will allow you to use the equations of motion for the vertical direction to solve for y_i.

Hint 2. Find the time spent in the air

How long (Δt) is the rock in the air?

Express your answer in seconds to three significant figures.

Hint 1. Determine which equation to use

Which of the equations given in the strategy and shown below is the most appropriate to calculate the time Δt the rock spent in the air?

ANSWER:

- $x_{\mathrm{f}} = x_{\mathrm{i}} + v_{\mathrm{i}x}\Delta t$
- $y_{\mathrm{f}} = y_{\mathrm{i}} + v_{\mathrm{i}y}\Delta t - \frac{1}{2}g(\Delta t)^2$
- $v_{\mathrm{f}y} = v_{\mathrm{i}y} - g\Delta t$

Correct

Now, solve for Δt and plug in the values of the known quantities. Note that even though you do not know $v_{\mathrm{i}x}$, you can compute it using the information about the initial speed and the launch angle given in the problem introduction.

Hint 2. Find the *x* component of the initial velocity

What is the *x* component of the rock's initial velocity?

Express your answer in meters per second to three significant figures.

ANSWER:

$v_{\mathrm{i}x}$ = 9.96 $\mathrm{m/s}$

Correct

If you use the answer to this hint in a subsequent part or hint, please use your unrounded answer and only round as a final step before submitting your answer.

ANSWER:

Δt = 2.01 s

Correct

If you use the answer to this hint in a subsequent part or hint, please use your unrounded answer and only round as a final step before submitting your answer.

Now, use the equation for the other component of motion (vertical component) and find the initial height y_{i}. To do that, first you will need to calculate the *y* component of the rock's initial velocity.

Hint 3. Find the *y* component of the initial velocity

What is the *y* component of the rock's initial velocity?

Express your answer in meters per second to three significant figures.

ANSWER:

v_{iy} = 5.75 m/s

Correct

If you use the answer to this hint in a subsequent part or hint, please use your unrounded answer and only round as a final step before submitting your answer.

ANSWER:

y_i = 8.23 m

Correct

Assess

Part D

A second rock is thrown straight upward with a speed 5.750m/s . If this rock takes 2.008s to fall to the ground, from what height H was it released?

Express your answer in meters to three significant figures.

Hint 1. Identify the known variables

What are the values of y_f, v_{iy}, Δt, and a for the second rock? Take the positive *y* axis to be upward and the origin to be located on the ground where the rock lands.

Express your answers to four significant figures in the units shown to the right, separated by commas.

ANSWER:

y_f, v_{iy}, Δt, a = 0,5.750,2.008,-9.810 $m, m/s, s, m/s^2$

Correct

Hint 2. Determine which equation to use to find the height

Which equation should you use to find H? Keep in mind that if the positive *y* axis is upward and the origin is located on the ground, $y_i = H$.

ANSWER:

- $y_f = y_i + v_{iy}\Delta t - \frac{1}{2}g(\Delta t)^2$
- $v_{fy} = v_{iy} - g\Delta t$
- $v_{fy}^2 = v_{iy}^2 - 2g(y_f - y_i)$

Correct

ANSWER:

H = 8.23 m

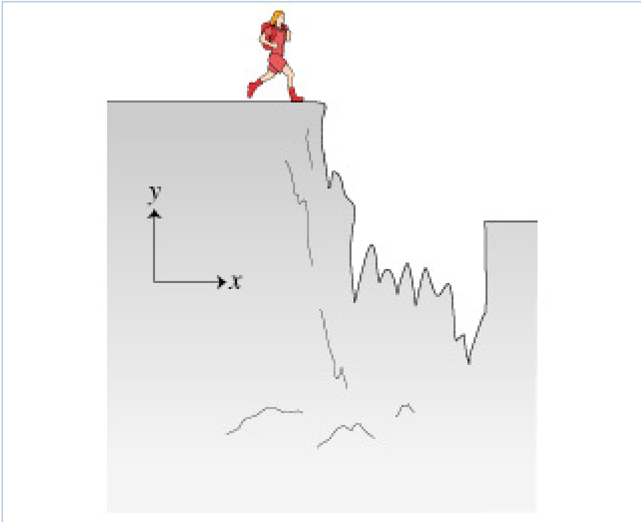
Correct

Projectile motion is made up of two independent motions: uniform motion at constant velocity in the horizontal direction and free-fall motion in the vertical direction. Because both rocks were thrown with the same initial vertical velocity, $v_{iy} = 5.750 \mathrm{m/s}$, and fell the same vertical distance of 8.23m , they were in the air for the same amount of time. This result was expected and helps to confirm that you did the calculation in Part C correctly.

Direction of Velocity at Various Times in Flight for Projectile Motion Conceptual Question

For each of the motions described below, determine the algebraic sign (positive, negative, or zero) of the *x* component and *y* component of velocity of the object at the time specified. For all of the motions, the positive *x* axis points to the right and the positive *y* axis points upward.

Alex, a mountaineer, must leap across a wide crevasse. The other side of the crevasse is below the point from which he leaps, as shown in the figure. Alex leaps horizontally and successfully makes the jump.



Part A

Determine the algebraic sign of Alex's *x* velocity and *y* velocity at the instant he leaves the ground at the beginning of the jump.

Type the algebraic signs of the *x* velocity and the *y* velocity separated by a comma (examples: +,− and 0,+).

Hint 1. Algebraic sign of velocity

The algebraic sign of the velocity is determined solely by comparing the direction in which the object is moving with the direction that is defined to be positive. In this example, to the right is defined to be the positive *x* direction and upward the positive *y* direction. Therefore, any object moving to the right, whether speeding up, slowing down, or even simultaneously moving upward or downward, has a positive *x* velocity. Similarly, if the object is moving downward, regardless of any other aspect of its motion, its *y* velocity is negative.

Hint 2. Sketch Alex's initial velocity

On the diagram below, sketch the vector representing Alex's velocity the instant after he leaves the ground at the beginning of the jump.

ANSWER:

Correct

ANSWER:

+,0

Correct

Part B

Determine the algebraic signs of Alex's *x* velocity and *y* velocity the instant before he lands at the end of the jump.

Type the algebraic signs of the *x* velocity and the *y* velocity separated by a comma (examples: `+,–` and `0,+`).

Hint 1. Sketch Alex's final velocity

On the diagram below, sketch the vector representing Alex's velocity the instant before he safely lands on the other side of the crevasse.

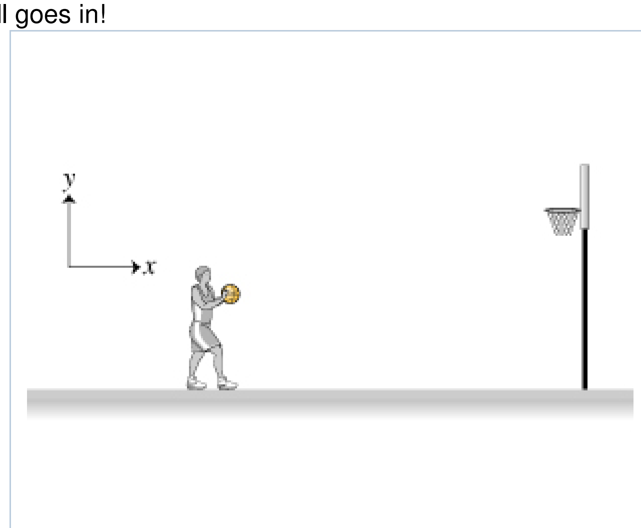
ANSWER:

ANSWER:

+,-

Correct

At the buzzer, a basketball player shoots a desperation shot. The ball goes in!



Part C

Determine the algebraic signs of the ball's *x* velocity and *y* velocity the instant after it leaves the player's hands.

Type the algebraic signs of the *x* velocity and the *y* velocity separated by a comma (examples: +, – and 0, +).

Hint 1. Sketch the basketball's initial velocity

On the diagram below, sketch the vector representing the velocity of the basketball the instant after it leaves the player's hands.

ANSWER:

ANSWER:

+,+

Correct

Part D

Determine the algebraic signs of the ball's *x* velocity and *y* velocity at the ball's maximum height.

Type the algebraic signs of the *x* velocity and the *y* velocity separated by a comma (examples: +, – and 0, +).

Hint 1. Sketch the basketball's velocity at maximum height

On the diagram below, sketch the vector representing the velocity of the basketball the instant it reaches its maximum height.

ANSWER:

ANSWER:

+,0

Correct

Enhanced EOC: Problem 4.14

A supply plane needs to drop a package of food to scientists working on a glacier in Greenland. The plane flies 160m above the glacier at a speed of 200$\mathrm{m/s}$.

You may want to review (pages 91 - 95) .

For help with math skills, you may want to review:

Mathematical Expressions Involving Squares

For general problem-solving tips and strategies for this topic, you may want to view a Video Tutor Solution of Dock jumping.

Part A

How far short of the target should it drop the package?

Express your answer using three significant figures with the appropriate units.

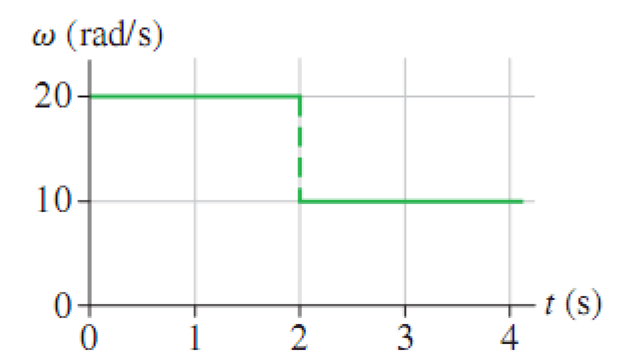
ANSWER:

1140 m

Correct

Enhanced EOC: Problem 4.20

The figure shows the angular-velocity-versus-time graph for a particle moving in a circle.



You may want to review (page) .

For help with math skills, you may want to review:

The Definite Integral

Part A

How many revolutions does the object make during the first 3.0s ?

Express your answer using two significant figures.

Hint 1. How to approach the problem

What is the calculus relationship between the angular velocity and the angle in radians?

How can you obtain the angle in radians if you know the angular velocity as a function of time and the initial angle in radians?

How do you integrate a function graphically?

What is the relation between the angle in radians and the number of revolutions?

ANSWER:

$n =$ 8.0

All attempts used; correct answer displayed

Problem 4.67

Your car tire is rotating at 3.7$\mathrm{rev/s}$ when suddenly you press down hard on the accelerator. After traveling 250m , the tire's rotation has increased to 5.3$\mathrm{rev/s}$. The radius of the tire is 32 cm.

Part A

What was the tire's angular acceleration? Give your answer in $\mathrm{rad/s^2}$.

Express your answer to two significant figures and include the appropriate units.

ANSWER:

α = 0.36 $\frac{\mathrm{rad}}{\mathrm{s}^2}$

Correct

Problem 4.82

A skateboarder starts up a 1.0-m-high, 30° ramp at a speed of 6.8$\mathrm{m/s}$. The skateboard wheels roll without friction. At the top, she leaves the ramp and sails through the air.

Part A

How far from the end of the ramp does the skateboarder touch down?

Express your answer to two significant figures and include the appropriate units.

ANSWER:

l = 3.5 m

All attempts used; correct answer displayed

Score Summary:

Your score on this assignment is 79.6%.
You received 7.96 out of a possible total of 10 points.