Chapter 5 The Laws of Motion

P5.2 For the same force F, acting on different masses

$$F = m_1 a_1$$

and

$$F = m_2 a_2$$

(a) $$\frac{m_1}{m_2} = \frac{a_2}{a_1} = \boxed{\frac{1}{3}}$$

(b) $$F = (m_1 + m_2)a = 4m_1 a = m_1\left(3.00\ \text{m/s}^2\right)$$

$$a = \boxed{0.750\ \text{m/s}^2}$$

P5.3 $m = 4.00\ \text{kg}$, $\vec{\mathbf{v}}_i = 3.00\hat{\mathbf{i}}\ \text{m/s}$, $\vec{\mathbf{v}}_8 = \left(8.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}\right)\ \text{m/s}$, $t = 8.00\ \text{s}$

$$\vec{\mathbf{a}} = \frac{\Delta\vec{\mathbf{v}}}{t} = \frac{5.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}}{8.00}\ \text{m/s}^2$$

$$\vec{\mathbf{F}} = m\vec{\mathbf{a}} = \boxed{\left(2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)\ \text{N}}$$

$$F = \sqrt{(2.50)^2 + (5.00)^2} = \boxed{5.59\ \text{N}}$$

P5.4 (a) Let the x axis be in the original direction of the molecule's motion.

$$v_f = v_i + at:\quad -670\ \text{m/s} = 670\ \text{m/s} + a\left(3.00\times 10^{-13}\ \text{s}\right)$$

$$a = \boxed{-4.47\times 10^{15}\ \text{m/s}^2}$$

(b) For the molecule, $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$. Its weight is negligible.

$$\vec{\mathbf{F}}_{\text{wall on molecule}} = 4.68\times 10^{-26}\ \text{kg}\left(-4.47\times 10^{15}\ \text{m/s}^2\right) = -2.09\times 10^{-10}\ \text{N}$$

$$\vec{\mathbf{F}}_{\text{molecule on wall}} = \boxed{+2.09\times 10^{-10}\ \text{N}}$$

P5.8 We find acceleration:

$$\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$4.20\ \text{m}\hat{\mathbf{i}} - 3.30\ \text{m}\hat{\mathbf{j}} = 0 + \frac{1}{2}\vec{\mathbf{a}}\left(1.20\ \text{s}\right)^2 = 0.720\ \text{s}^2\vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} = \left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right)\ \text{m/s}^2$$

Now $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ becomes

$$\vec{\mathbf{F}}_g + \vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{F}}_2 = 2.80\text{ kg}\left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right)\text{ m/s}^2 + (2.80\text{ kg})\left(9.80\text{ m/s}^2\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}}_2 = \boxed{\left(16.3\hat{\mathbf{i}} + 14.6\hat{\mathbf{j}}\right)\text{ N}}$$

P5.9 (a) $\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \left(20.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)\text{ N}$

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}: \quad 20.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}} = 5.00\vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} = \left(4.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\text{ m/s}^2$$

or

$$\boxed{a = 5.00\text{ m/s}^2 \text{ at } \theta = 36.9°}$$

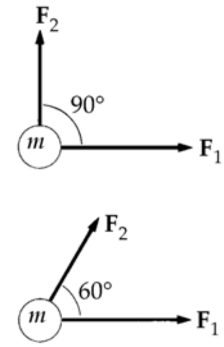

FIG. P5.9

$$F_{2x} = 15.0\cos 60.0° = 7.50\text{ N}$$
$$F_{2y} = 15.0\sin 60.0° = 13.0\text{ N}$$
$$\vec{\mathbf{F}}_2 = \left(7.50\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}}\right)\text{ N}$$

(b) $\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \left(27.5\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}}\right)\text{ N} = m\vec{\mathbf{a}} = 5.00\vec{\mathbf{a}}$

$$\vec{\mathbf{a}} = \boxed{\left(5.50\hat{\mathbf{i}} + 2.60\hat{\mathbf{j}}\right)\text{ m/s}^2 = 6.08\text{ m/s}^2 \text{ at } 25.3°}$$

P5.13 (a) $\boxed{15.0\text{ lb up}}$ to counterbalance the Earth's force on the block

(b) $\boxed{5.00\text{ lb up}}$ The forces on the block are now the Earth pulling down with 15 lb and the rope pulling up with 10 lb.

(c) $\boxed{0}$ The block now accelerates up away from the floor.

P5.14 $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ reads

$$\left(-2.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} + 5.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}}\right)\text{ N} = m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}$$

where $\hat{\mathbf{a}}$ represents the direction of $\vec{\mathbf{a}}$

$$\left(-42.0\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\text{ N} = m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}$$

$$\sum \vec{\mathbf{F}} = \sqrt{(42.0)^2 + (1.00)^2}\text{ N} \text{ at } \tan^{-1}\left(\frac{1.00}{42.0}\right) \text{ below the } -x \text{ axis}$$

$$\sum \vec{\mathbf{F}} = 42.0\text{ N at } 181° = m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}$$

For the vectors to be equal, their magnitudes and their directions must be equal.

(a) Therefore $\boxed{\hat{\mathbf{a}} \text{ is at } 181°}$ counterclockwise from the x axis

(b) $m = \dfrac{42.0\text{ N}}{3.75\text{ m/s}^2} = \boxed{11.2\text{ kg}}$

(d) $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = 0 + \left(3.75\text{ m/s}^2\text{ at }181°\right)10.0\text{ s}$ so $\vec{\mathbf{v}}_f = 37.5\text{ m/s at }181°$

$\vec{\mathbf{v}}_f = 37.5\text{m/s }\cos 181°\hat{\mathbf{i}} + 37.5\text{m/s }\sin 181°\hat{\mathbf{j}}$ so $\vec{\mathbf{v}}_f = \boxed{\left(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}}\right)\text{ m/s}}$

(c) $\left|\vec{\mathbf{v}}_f\right| = \sqrt{37.5^2 + 0.893^2}\text{ m/s} = \boxed{37.5\text{ m/s}}$

P5.16 $v_x = \dfrac{dx}{dt} = 10t$, $v_y = \dfrac{dy}{dt} = 9t^2$

$a_x = \dfrac{dv_x}{dt} = 10$, $a_y = \dfrac{dv_y}{dt} = 18t$

At $t = 2.00\text{ s}$, $a_x = 10.0\text{ m/s}^2$, $a_y = 36.0\text{ m/s}^2$

$$\sum F_x = ma_x:\ 3.00\text{ kg}\left(10.0\text{ m/s}^2\right) = 30.0\text{ N}$$

$$\sum F_y = ma_y:\ 3.00\text{ kg}\left(36.0\text{ m/s}^2\right) = 108\text{ N}$$

$$\sum F = \sqrt{F_x^2 + F_y^2} = \boxed{112\text{ N}}$$

P5.21 See the solution for T_1 in Problem 5.20. The equation indicates that the tension is directly proportional to F_g. As $\theta_1 + \theta_2$ approaches zero (as the angle between the two upper ropes approaches 180°) the tension goes to infinity. Making the right-hand rope horizontal maximizes the tension in the left-hand rope, according to the proportionality of T_1 to $\cos\theta_2$.

***P5.23** (a) Isolate either mass

$$T + mg = ma = 0$$

$$|T| = |mg|$$

The scale reads the tension T, so

$$T = mg = 5.00\text{ kg}\left(9.80\text{ m/s}^2\right) = \boxed{49.0\text{ N}}$$

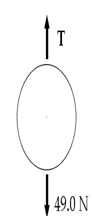

FIG. P5.23(a) and (b)

(b) The solution to part (a) is also the solution to (b).

(c) Isolate the pulley

$$\vec{\mathbf{T}}_2 + 2\vec{\mathbf{T}}_1 = 0$$

$$T_2 = 2|T_1| = 2mg = \boxed{98.0\text{ N}}$$

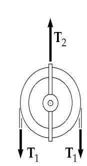

FIG. P5.23(c)

(d) $\sum \vec{\mathbf{F}} = \vec{\mathbf{n}} + \vec{\mathbf{T}} + m\vec{\mathbf{g}} = 0$

Take the component along the incline

$$n_x + T_x + mg_x = 0$$

or

$$0 + T - mg\sin 30.0° = 0$$

$$T = mg\sin 30.0° = \frac{mg}{2} = \frac{5.00(9.80)}{2}$$

$$= \boxed{24.5\ \text{N}}$$

FIG. P5.23(d)

P5.28 $m_1 = 2.00\ \text{kg}$, $m_2 = 6.00\ \text{kg}$, $\theta = 55.0°$

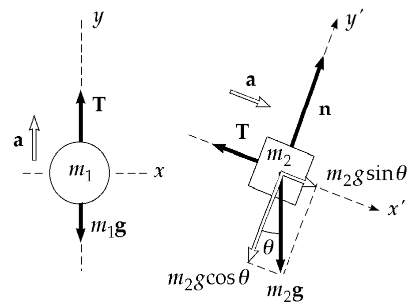

FIG. P5.28

(a) $$\sum F_x = m_2 g\sin\theta - T = m_2 a$$

and

$$T - m_1 g = m_1 a$$

$$a = \frac{m_2 g\sin\theta - m_1 g}{m_1 + m_2} = \boxed{3.57\ \text{m/s}^2}$$

(b) $$T = m_1(a+g) = \boxed{26.7\ \text{N}}$$

(c) Since $v_i = 0$, $v_f = at = (3.57\ \text{m/s}^2)(2.00\ \text{s}) = \boxed{7.14\ \text{m/s}}$.

P5.29 After it leaves your hand, the block's speed changes only because of one component of its weight:

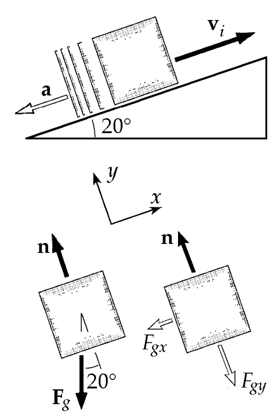

FIG. P5.29

$$\sum F_x = ma_x \qquad -mg\sin 20.0° = ma$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Taking $v_f = 0$, $v_i = 5.00\ \text{m/s}$, and $a = -g\sin(20.0°)$ gives

$$0 = (5.00)^2 - 2(9.80)\sin(20.0°)(x_f - 0)$$

or

$$x_f = \frac{25.0}{2(9.80)\sin(20.0°)} = \boxed{3.73\ \text{m}}$$

P5.31 Forces acting on 2.00 kg block:

$$T - m_1 g = m_1 a \qquad (1)$$

Forces acting on 8.00 kg block:

$$F_x - T = m_2 a \qquad (2)$$

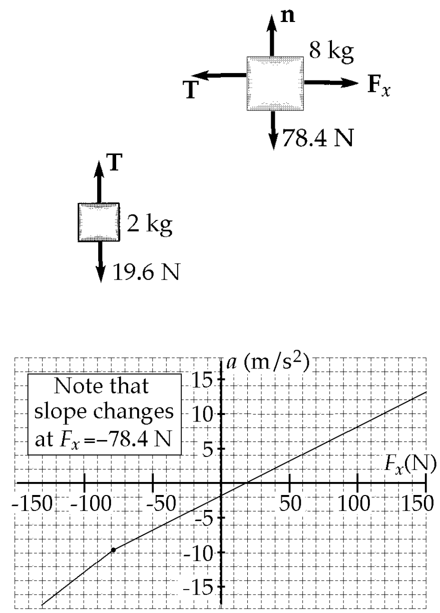

FIG. P5.31

(a) Eliminate T and solve for a:

$$a = \frac{F_x - m_1 g}{m_1 + m_2}$$

$$\boxed{a > 0 \text{ for } F_x > m_1 g = 19.6 \text{ N}}$$

(b) Eliminate a and solve for T:

$$T = \frac{m_1}{m_1 + m_2}\left(F_x + m_2 g\right)$$

$$\boxed{T = 0 \text{ for } F_x \le -m_2 g = -78.4 \text{ N}}$$

(c)

F_x, N	–100	–78.4	–50.0	0	50.0	100	
a_x, m/s^2		–12.5	–9.80	–6.96	–1.96	3.04	8.04

P5.41 $T - f_k = 5.00a$ (for 5.00 kg mass)

$9.00g - T = 9.00a$ (for 9.00 kg mass)

Adding these two equations gives:

$$9.00(9.80) - 0.200(5.00)(9.80) = 14.0a$$

$$a = 5.60 \text{ m/s}^2$$

$$\therefore T = 5.00(5.60) + 0.200(5.00)(9.80)$$

$$= \boxed{37.8 \text{ N}}$$

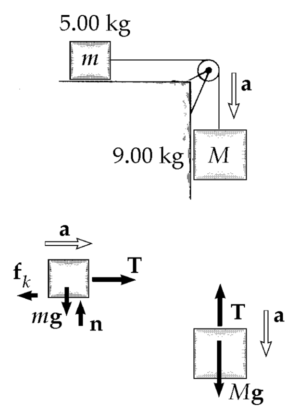

FIG. P5.41

P5.53 $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ gives the object's acceleration

$$\vec{\mathbf{a}} = \frac{\sum F}{m} = \frac{\left(8.00\hat{\mathbf{i}} - 4.00t\hat{\mathbf{j}}\right)\text{ N}}{2.00\text{ kg}}$$

$$\vec{\mathbf{a}} = \left(4.00\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(2.00\text{ m/s}^3\right)t\hat{\mathbf{j}} = \frac{d\vec{\mathbf{v}}}{dt}$$

Its velocity is

$$\int_{v_i}^{v} d\vec{\mathbf{v}} = \vec{\mathbf{v}} - \vec{\mathbf{v}}_i = \vec{\mathbf{v}} - 0 = \int_0^t \vec{\mathbf{a}}dt$$

$$\vec{\mathbf{v}} = \int_0^t \left[\left(4.00\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(2.00\text{ m/s}^3\right)t\hat{\mathbf{j}}\right]dt$$

$$\vec{\mathbf{v}} = \left(4.00t\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(1.00t^2\text{ m/s}^3\right)\hat{\mathbf{j}}$$

(a) We require $|\vec{\mathbf{v}}| = 15.0\text{ m/s}$, $|\vec{\mathbf{v}}|^2 = 225\text{ m}^2/\text{s}^2$

$$16.0t^2\text{ m}^2/\text{s}^4 + 1.00t^4\text{ m}^2/\text{s}^6 = 225\text{ m}^2/\text{s}^2$$

$$1.00t^4 + 16.0\text{ s}^2t^2 - 225\text{ s}^4 = 0$$

$$t^2 = \frac{-16.0 \pm \sqrt{(16.0)^2 - 4(-225)}}{2.00} = 9.00\text{ s}^2$$

$$t = \boxed{3.00\text{ s}}$$

Take $\vec{\mathbf{r}}_i = 0$ at $t = 0$. The position is

$$\vec{\mathbf{r}} = \int_0^t \vec{\mathbf{v}}dt = \int_0^t \left[\left(4.00t\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(1.00t^2\text{ m/s}^3\right)\hat{\mathbf{j}}\right]dt$$

$$\vec{\mathbf{r}} = \left(4.00\text{ m/s}^2\right)\frac{t^2}{2}\hat{\mathbf{i}} - \left(1.00\text{ m/s}^3\right)\frac{t^3}{3}\hat{\mathbf{j}}$$

at $t = 3\text{ s}$ we evaluate.

(b) $\vec{\mathbf{r}} = \boxed{\left(18.0\hat{\mathbf{i}} - 9.00\hat{\mathbf{j}}\right)\text{ m}}$

(c) So $|\vec{\mathbf{r}}| = \sqrt{(18.0)^2 + (9.00)^2}\text{ m} = \boxed{20.1\text{ m}}$

P5.61 (a) The crate is in equilibrium, just before it starts to move. Let the normal force acting on it be n and the friction force, f_s.

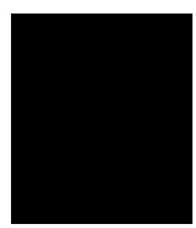

FIG. P5.61

Resolving vertically:

$$n = F_g + P\sin\theta$$

Horizontally:

$P\cos\theta = f_s$

But,

$$f_s \le \mu_s n$$

i.e.,

$$P\cos\theta \le \mu_s\left(F_g + P\sin\theta\right)$$

or

$$P\left(\cos\theta - \mu_s\sin\theta\right) \le \mu_s F_g$$

Divide by $\cos\theta$:

$$P\left(1-\mu_s\tan\theta\right) \le \mu_s F_g\sec\theta$$

Then

$$\boxed{P_{\text{minimum}} = \frac{\mu_s F_g\sec\theta}{1-\mu_s\tan\theta}}$$

(b) $$P = \frac{0.400\left(100\text{ N}\right)\sec\theta}{1-0.400\tan\theta}$$

$\theta(\text{deg})$	0.00	15.0	30.0	45.0	60.0
$P(N)$	40.0	46.4	60.1	94.3	260

If the angle were $68.2°$ or more, the expression for *P* would go to infinity and motion would become impossible.

P5.62 (a) Following the in-chapter example about a block on a frictionless incline, we have

$$a = g\sin\theta = \left(9.80\text{ m/s}^2\right)\sin 30.0°$$

$$\boxed{a = 4.90\text{ m/s}^2}$$

(b) The block slides distance *x* on the incline, with $\sin 30.0° = \dfrac{0.500\text{ m}}{x}$

$x = 1.00\text{ m}$: $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 0 + 2\left(4.90\text{ m/s}^2\right)\left(1.00\text{ m}\right)$

$v_f = \boxed{3.13\text{ m/s}}$ after time $t_s = \dfrac{2x_f}{v_f} = \dfrac{2\left(1.00\text{ m}\right)}{3.13\text{ m/s}} = 0.639\text{ s}$

(c) Now in free fall $y_f - y_i = v_{yi}t + \dfrac{1}{2}a_y t^2$:

$$-2.00 = \left(-3.13\text{ m/s}\right)\sin 30.0° t - \frac{1}{2}\left(9.80\text{ m/s}^2\right)t^2$$

$$\left(4.90\text{ m/s}^2\right)t^2 + \left(1.56\text{ m/s}\right)t - 2.00\text{ m} = 0$$

$$t = \frac{-1.56\text{ m/s} \pm \sqrt{\left(1.56\text{ m/s}\right)^2 - 4\left(4.90\text{ m/s}^2\right)\left(-2.00\text{ m}\right)}}{9.80\text{ m/s}^2}$$

Only the positive root is physical

$$t = 0.499\ \text{s}$$

$$x_f = v_x t = \left[(3.13\ \text{m/s})\cos 30.0°\right](0.499\ \text{s}) = \boxed{1.35\ \text{m}}$$

(d) total time $= t_s + t = 0.639\ \text{s} + 0.499\ \text{s} = \boxed{1.14\ \text{s}}$

(e) The mass of the block makes no difference.

P5.67 $\sum F = ma$

For m_1:

$$T = m_1 a$$

For m_2:

$$T - m_2 g = 0$$

Eliminating *T*,

$$a = \frac{m_2 g}{m_1}$$

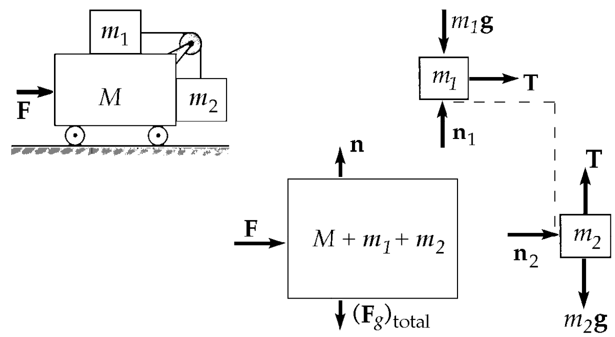

FIG. P5.67

For all 3 blocks:

$$F = (M + m_1 + m_2)a = \boxed{(M + m_1 + m_2)\left(\frac{m_2 g}{m_1}\right)}$$