MAT 243 ONLINE WRITTEN HW 6

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(1) **(4 pts)** Fill in the blank in the statements below:
   (a) When making a sequence of choices in counting we are using the product Rule.
   (b) We are using the Sum Rule when choosing among mutually exclusive alternatives.
   (c) We are using Permutation when we have an ordered list of items.
   (d) The slightly more advanced version of the Pigeon Hole principal states that if there are k boxes, then blah blah blah
   (e) The general form of the Inclusion-exclusion formula says that $|A_1 \cup A_2 \cup .... \cup A_n| =$ _____
   (f) The set S of all possible outcomes is called the _____.
   (g) If all outcomes of an experiment E of a finite sample space S are _____ then $p(E) = \frac{|E|}{|S|}$.
   (h) A relation R on a set S is transitive if and only if _____

(2) **(15 pts)** Given 10 chips each of them of different colors (including red and white). For each of the following questions explain your answer. You should use factorial, combination, permutation or simple products whenever it is possible and you do not need to give the number it is equal to – just like it is done in the additional handout.

How many different ways can we put
   (a) 6 of the 10 chips in a line?
   (b) 6 of the 10 chips in a line such that the white chip must be used?
   (c) 6 of the 10 chips in a line such that the white chip is used and the red is not?
   (d) 6 of the 10 chips in a line such that both the red and the white chip is used ?
   (e) 6 of the 10 chips in a line such that both the red and the white chip is used and they are next to each other?

(3) **(10 pts)** You have \$15,000 to invest in the stock market and your financial advisor gives you a list of 11 possible stock options to choose from. How many different ways can you do that if you choose 5 stock options to invest
   (a) \$1000, \$2000, \$3000, \$4000 and \$5000 in the stock options? Justify your answer.
   (b) \$3000 in each of the 5 options? Justify your answer.
   (c) \$4500 in two and \$2000 in the other three options? Justify your answer.

(4) **(10 pts)** A printer is printing out 3-digit numbers between 100-999 such that the digits are not repeated.
   (a) If 2800 such numbers are printed out, at least how many of them will be identical? Justify your answer.
   (b) At least how many of the numbers should be printed, so that at least 5 of them will be identical? Justify your answer.
   (c) At least how many of the numbers should be printed, so that the number 243 is printed at least 6 times? Justify your answer.

(5) **(8 pts)** Use binomial theorem to find the following. Justify your answer.
   (a) The coefficient in front of the term x^{10} in $(x^2 - 6)^8$.
   (b)
$$\sum_{i=3}^{50} C(50, k)5^k =$$

(6) **(10 pts)** How many different 5-card poker hands
   (a) contain the Ace of diamond?
   (b) contain only red cards?
   (c) contain only diamonds?
   (d) contain only red cards or diamonds?
   (e) contain the Ace of diamond or only red cards or only diamonds?
   (Hint: use inclusion-exclusion principle)

   Justify your answers for full credit. You do not have to calculate the answer, just express the parts in a sum, difference and product form.

(7) **(6 pts)** Randomly pick a number from the set containing numbers between 10000-99999 such that the digits are **not repeated**. You do not have to calculate the answer, just give the fraction you obtain. You are welcome to express the parts of the fraction in a product form. Justify your answer:
   (a) What is the probability that its first two digits are 16?
   (b) What is the probability that its last digit is 0?
   (c) What is the probability that its first digits are 16 or its last digit is 0?

(8) **4 pts)** Suppose S is a set people, and R is the following relation on S: aRb if and only if a has chatted with b on the internet. Explain why if your answer is yes. Give a counter example if the answer is no. Use common sense.
   (a) Is R reflexive?
   (b) Is R symmetric?
   (c) Is R transitive?
   (d) What is the meaning of the relation R^2? Specifically, when are two people R^2 related? (Hint: $R^2 = R \circ R$)

(9) **(5 pts)** Suppose S is a set of positive integers, and R is the following relation on S: aRb if and only if a divides b. Explain why if your answer is yes. Give a counter example if the answer is no.
   (a) Is R reflexive?
   (b) Is R symmetric?
   (c) Is R antisymmetric?
   (d) Is R transitive?
   (e) What is the meaning of the relation R^2? Specifically, when are two integers R^2 related? (Hint: $R^2 = R \circ R$)