Chapter 14 Fluid Mechanics

P14.1 $M = \rho_{\text{iron}} V = \left(7\,860 \text{ kg/m}^3\right)\left[\frac{4}{3}\pi\left(0.015\,0 \text{ m}\right)^3\right]$

$M = \boxed{0.111 \text{ kg}}$

P14.3 $P = \frac{F}{A} = \frac{50.0(9.80)}{\pi\left(0.500\times10^{-2}\right)^2} = \boxed{6.24\times10^6 \text{ N/m}^2}$

P14.4 The Earth's surface area is $4\pi R^2$. The force pushing inward over this area amounts to

$$F = P_0 A = P_0\left(4\pi R^2\right)$$

This force is the weight of the air:

$$F_g = mg = P_0\left(4\pi R^2\right)$$

so the mass of the air is

$$m = \frac{P_0\left(4\pi R^2\right)}{g} = \frac{\left(1.013\times10^5 \text{ N/m}^2\right)\left[4\pi\left(6.37\times10^6 \text{ m}\right)^2\right]}{9.80 \text{ m/s}^2} = \boxed{5.27\times10^{18} \text{ kg}}$$

P14.6 (a) $P = P_0 + \rho g h = 1.013\times10^5 \text{ Pa} + \left(1\,024 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)\left(1\,000 \text{ m}\right)$

$P = \boxed{1.01\times10^7 \text{ Pa}}$

(b) The gauge pressure is the difference in pressure between the water outside and the air inside the submarine, which we suppose is at 1.00 atmosphere.

$$P_{\text{gauge}} = P - P_0 = \rho g h = 1.00\times10^7 \text{ Pa}$$

The resultant inward force on the porthole is then

$$F = P_{\text{gauge}} A = 1.00\times10^7 \text{ Pa}\left[\pi\left(0.150 \text{ m}\right)^2\right] = \boxed{7.09\times10^5 \text{ N}}$$

P14.7 $F_g = 80.0 \text{ kg}\left(9.80 \text{ m/s}^2\right) = 784 \text{ N}$

When the cup barely supports the student, the normal force of the ceiling is zero and the cup is in equilibrium.

$$F_g = F = PA = \left(1.013\times10^5 \text{ Pa}\right)A$$

$$A = \frac{F_g}{P} = \frac{784}{1.013\times10^5} = \boxed{7.74\times10^{-3} \text{ m}^2}$$

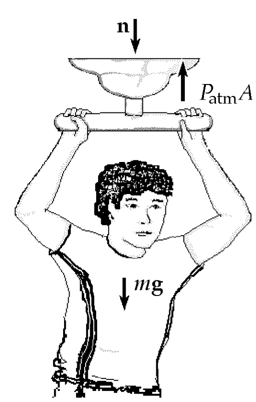


FIG. P14.7

P14.11 The pressure on the bottom due to the water is $P_b = \rho g z = 1.96 \times 10^4 \text{ Pa}$

So, $F_b = P_b A = \boxed{5.88 \times 10^6 \text{ N down}}$

On each end, $F = P_{average} A = 9.80 \times 10^3 \text{ Pa}\left(20.0 \text{ m}^2\right) = \boxed{196 \text{ kN outward}}$

On the side, $F = P_{average} A = 9.80 \times 10^3 \text{ Pa}\left(60.0 \text{ m}^2\right) = \boxed{588 \text{ kN outward}}$

P14.16 (a) Using the definition of density, we have

$$h_w = \frac{m_{\text{water}}}{A_2 \rho_{\text{water}}} = \frac{100 \text{ g}}{5.00 \text{ cm}^2\left(1.00 \text{ g/cm}^3\right)} = \boxed{20.0 \text{ cm}}$$

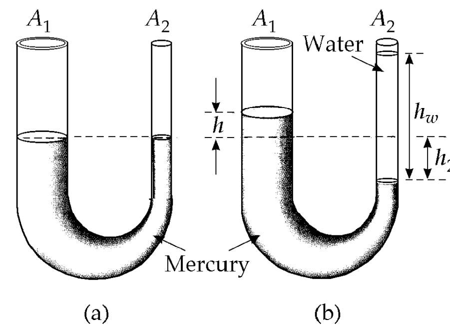


FIG. P14.16

(b) Sketch (b) at the right represents the situation after the water is added. A volume $\left(A_2 h_2\right)$ of mercury has been displaced by water in the right tube. The additional volume of mercury now in the left tube is $A_1 h$. Since the total volume of mercury has not changed,

$$A_2 h_2 = A_1 h \qquad \text{or} \qquad h_2 = \frac{A_1}{A_2} h$$

(1)

At the level of the mercury–water interface in the right tube, we may write the absolute pressure as:

$$P = P_0 + \rho_{\text{water}} g h_w$$

The pressure at this same level in the left tube is given by

$$P = P_0 + \rho_{\text{Hg}} g\left(h + h_2\right) = P_0 + \rho_{\text{water}} g h_w$$

which, using equation (1) above, reduces to

$$\rho_{\text{Hg}} h\left[1 + \frac{A_1}{A_2}\right] = \rho_{\text{water}} h_w$$

or $h = \dfrac{\rho_{\text{water}} h_w}{\rho_{\text{Hg}}\left(1 + A_1 / A_2\right)}$

Thus, the level of mercury has risen a distance of

$$h = \frac{\left(1.00 \text{ g/cm}^3\right)\left(20.0 \text{ cm}\right)}{\left(13.6 \text{ g/cm}^3\right)\left(1 + 10.0/50.0\right)} = \boxed{0.490 \text{ cm}} \quad \text{above the original level.}$$

P14.17 $\Delta P_0 = \rho g \Delta h = -2.66 \times 10^3 \text{ Pa}: \quad P = P_0 + \Delta P_0 = \left(1.013 - 0.026\,6\right) \times 10^5 \text{ Pa} = \boxed{0.986 \times 10^5 \text{ Pa}}$

P14.20 (a) The balloon is nearly in equilibrium:

$$\sum F_y = m a_y \Rightarrow B - \left(F_g\right)_{\text{helium}} - \left(F_g\right)_{\text{payload}} = 0$$

or $\rho_{\text{air}} g V - \rho_{\text{helium}} g V - m_{\text{payload}} g = 0$

This reduces to

$$m_{\text{payload}} = \left(\rho_{\text{air}} - \rho_{\text{helium}}\right) V = \left(1.29 \text{ kg/m}^3 - 0.179 \text{ kg/m}^3\right)\left(400 \text{ m}^3\right)$$

$$m_{\text{payload}} = \boxed{444 \text{ kg}}$$

(b) Similarly,

$$m_{\text{payload}} = \left(\rho_{\text{air}} - \rho_{\text{hydrogen}}\right)V = \left(1.29 \text{ kg/m}^3 - 0.089\,9 \text{ kg/m}^3\right)\left(400 \text{ m}^3\right)$$

$$m_{\text{payload}} = \boxed{480 \text{ kg}}$$

The surrounding air does the lifting, nearly the same for the two balloons.

P14.27 (a) According to Archimedes,

$$B = \rho_{\text{water}} V_{\text{water}}\, g = \left(1.00 \text{ g/cm}^3\right)\left[20.0 \times 20.0 \times (20.0 - h)\right] g$$

But $B = \text{Weight of block} = mg = \rho_{\text{wood}} V_{\text{wood}}\, g = \left(0.650 \text{ g/cm}^3\right)(20.0 \text{ cm})^3 g$

$$0.650(20.0)^3 g = 1.00(20.0)(20.0)(20.0 - h)g$$

$20.0 - h = 20.0(0.650)$ so $h = 20.0(1 - 0.650) = \boxed{7.00 \text{ cm}}$

(b) $B = F_g + Mg$ where $M = \text{mass of lead}$

$$1.00(20.0)^3 g = 0.650(20.0)^3 g + Mg$$

$$M = (1.00 - 0.650)(20.0)^3 = 0.350(20.0)^3 = 2\,800 \text{ g} = \boxed{2.80 \text{ kg}}$$

P14.37 Flow rate $Q = 0.012\,0 \text{ m}^3/\text{s} = v_2 A_2$

$$v_2 = \frac{Q}{A_2} = \frac{0.012\,0 \text{ m}^3/\text{s}}{\pi(0.011 \text{ m})^2} = \boxed{31.6 \text{ m/s}}$$

P14.49 In the reservoir, the gauge pressure is $\Delta P = \dfrac{2.00 \text{ N}}{2.50 \times 10^{-5} \text{ m}^2} = 8.00 \times 10^4 \text{ Pa}$

From the equation of continuity: $A_1 v_1 = A_2 v_2$

$\left(2.50 \times 10^{-5} \text{ m}^2\right)v_1 = \left(1.00 \times 10^{-8} \text{ m}^2\right)v_2$ $\quad v_1 = \left(4.00 \times 10^{-4}\right)v_2$

Thus, v_1^2 is negligible in comparison to v_2^2.

Then, from Bernoulli's equation: $(P_1 - P_2) + \frac{1}{2}\rho v_1^2 + \rho g y_1 = \frac{1}{2}\rho v_2^2 + \rho g y_2$

P14.51 When the balloon comes into equilibrium, we must have

$$\sum F_y = B - F_{g,\text{ balloon}} - F_{g,\text{ He}} - F_{g,\text{ string}} = 0$$

$F_{g,\text{ string}}$ is the weight of the string above the ground, and B is the buoyant force. Now

$$F_{g,\text{ balloon}} = m_{\text{balloon}}\, g$$

$$F_{g,\text{ He}} = \rho_{\text{He}} V g$$

$$B = \rho_{\text{air}} V g$$

and $F_{g,\text{ string}} = m_{\text{string}} \dfrac{h}{L} g$

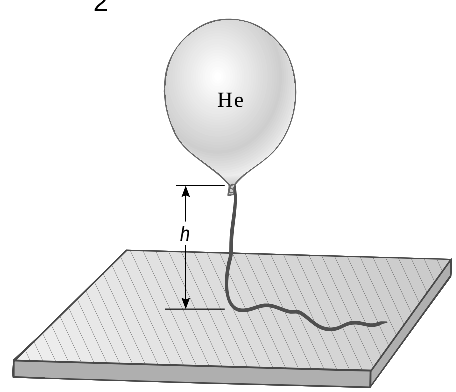


FIG. P14.51

Therefore, we have

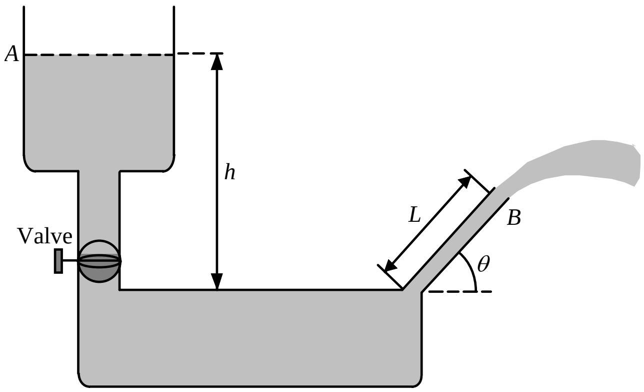
$$\rho_{\text{air}}Vg - m_{\text{balloon}}g - \rho_{\text{He}}Vg - m_{\text{string}}\frac{h}{L}g = 0$$

or $\quad h = \dfrac{(\rho_{\text{air}} - \rho_{\text{He}})V - m_{\text{balloon}}}{m_{\text{string}}}L$

giving $\quad h = \dfrac{(1.29-0.179)(\text{kg/m}^3)\left(4\pi(0.400\text{ m})^3/3\right) - 0.250\text{ kg}}{0.050\,0\text{ kg}}(2.00\text{ m}) = \boxed{1.91\text{ m}}$

P14.52 Consider the diagram and apply Bernoulli's equation to points *A* and *B*, taking $y = 0$ at the level of point *B*, and recognizing that v_A is approximately zero. This gives:

$$P_A + \frac{1}{2}\rho_w(0)^2 + \rho_w g(h - L\sin\theta)$$
$$= P_B + \frac{1}{2}\rho_w v_B^2 + \rho_w g(0)$$



Now, recognize that $P_A = P_B = P_{\text{atmosphere}}$ since both points are open to the atmosphere (neglecting variation of atmospheric pressure with altitude). Thus, we obtain

FIG. P14.52

$$v_B = \sqrt{2g(h - L\sin\theta)} = \sqrt{2(9.80\text{ m/s}^2)\left[10.0\text{ m} - (2.00\text{ m})\sin 30.0°\right]}$$
$$v_B = 13.3\text{ m/s}$$

Now the problem reduces to one of projectile motion with $v_{yi} = v_B\sin 30.0° = 6.64\text{ m/s}$. Then, $v_{yf}^2 = v_{yi}^2 + 2a(\Delta y)$ gives at the top of the arc (where $y = y_{\max}$ and $v_{yf} = 0$)

$$0 = (6.64\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(y_{\max} - 0)$$

or $y_{\max} = \boxed{2.25\text{ m (above the level where the water emerges)}}$.

P14.55 At equilibrium, $\sum F_y = 0$: $\quad B - F_{\text{spring}} - F_{g,\text{ He}} - F_{g,\text{ balloon}} = 0$

giving $\quad F_{\text{spring}} = kL = B - (m_{\text{He}} + m_{\text{balloon}})g$

But $\quad B = \text{weight of displaced air} = \rho_{\text{air}}Vg$

and $\quad m_{\text{He}} = \rho_{\text{He}}V$

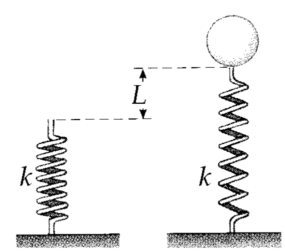


FIG. P14.55

Therefore, we have: $\quad kL = \rho_{\text{air}}Vg - \rho_{\text{He}}Vg - m_{\text{balloon}}g$

or $\quad L = \dfrac{(\rho_{\text{air}} - \rho_{\text{He}})V - m_{\text{balloon}}}{k}g$

From the data given,

$$L=\frac{\left(1.29\ \text{kg/m}^3-0.180\ \text{kg/m}^3\right)5.00\ \text{m}^3-2.00\times10^{-3}\ \text{kg}}{90.0\ \text{N/m}}\left(9.80\ \text{m/s}^2\right)$$

Thus, this gives $L=\boxed{0.604\ \text{m}}$

P14.66 Let *s* stand for the edge of the cube, *h* for the depth of immersion, ρ_{ice} stand for the density of the ice, ρ_w stand for density of water, and ρ_a stand for density of the alcohol.

(a) According to Archimedes's principle, at equilibrium we have

$$\rho_{\text{ice}}gs^3=\rho_w ghs^2\Rightarrow h=s\frac{\rho_{\text{ice}}}{\rho_w}$$

With $\rho_{\text{ice}}=0.917\times10^3\ \text{kg/m}^3$

$\rho_w=1.00\times10^3\ \text{kg/m}^3$

and $s=20.0\ \text{mm}$

we get $h=20.0(0.917)=18.34\ \text{mm}\approx\boxed{18.3\ \text{mm}}$

(b) We assume that the top of the cube is still above the alcohol surface. Letting h_a stand for the thickness of the alcohol layer, we have

$$\rho_a gs^2h_a+\rho_w gs^2h_w=\rho_{\text{ice}}gs^3 \qquad \text{so} \qquad h_w=\left(\frac{\rho_{\text{ice}}}{\rho_w}\right)s-\left(\frac{\rho_a}{\rho_w}\right)h_a$$

With $\rho_a=0.806\times10^3\ \text{kg/m}^3$

and $h_a=5.00\ \text{mm}$

we obtain $h_w=18.34-0.806(5.00)=14.31\ \text{mm}\approx\boxed{14.3\ \text{mm}}$

(c) Here $h'_w=s-h'_a$, so Archimedes's principle gives

$$\rho_a gs^2h'_a+\rho_w gs^2\left(s-h'_a\right)=\rho_{\text{ice}}gs^3\Rightarrow\rho_a h'_a+\rho_w\left(s-h'_a\right)=\rho_{\text{ice}}s$$

$$h'_a=s\frac{\left(\rho_w-\rho_{\text{ice}}\right)}{\left(\rho_w-\rho_a\right)}=20.0\frac{(1.000-0.917)}{(1.000-0.806)}=8.557\approx\boxed{8.56\ \text{mm}}$$

P14.67 Energy for the fluid-Earth system is conserved.

$$(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f \qquad 0+\frac{mgL}{2}+0=\frac{1}{2}mv^2+0$$

$$v=\sqrt{gL}=\sqrt{2.00\text{ m}\left(9.8\text{ m/s}^2\right)}=\boxed{4.43\text{ m/s}}$$

P14.68 (a) The flow rate, Av, as given may be expressed as follows:

$$\frac{25.0\text{ liters}}{30.0\text{ s}}=0.833\text{ liters/s}=833\text{ cm}^3/\text{s}$$

The area of the faucet tap is $\pi\text{ cm}^2$, so we can find the velocity as

$$v=\frac{\text{flow rate}}{A}=\frac{833\text{ cm}^3/\text{s}}{\pi\text{ cm}^2}=265\text{ cm/s}=\boxed{2.65\text{ m/s}}$$

(b) We choose point 1 to be in the entrance pipe and point 2 to be at the faucet tap. $A_1v_1=A_2v_2$ gives $v_1=0.295\text{ m/s}$. Bernoulli's equation is:

$$P_1-P_2=\frac{1}{2}\rho\left(v_2^2-v_1^2\right)+\rho g\left(y_2-y_1\right)$$

and gives

$$P_1-P_2=\frac{1}{2}\left(10^3\text{ kg/m}^3\right)\left[\left(2.65\text{ m/s}\right)^2-\left(0.295\text{ m/s}\right)^2\right]+\left(10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)\left(2.00\text{ m}\right)$$

or $\quad P_{\text{gauge}}=P_1-P_2=\boxed{2.31\times10^4\text{ Pa}}$

P14.71 (a) For diverging stream lines that pass just above and just below the hydrofoil we have

$$P_t+\rho g y_t+\frac{1}{2}\rho v_t^2=P_b+\rho g y_b+\frac{1}{2}\rho v_b^2$$

Ignoring the buoyant force means taking $y_t\approx y_b$

$$P_t+\frac{1}{2}\rho\left(nv_b\right)^2=P_b+\frac{1}{2}\rho v_b^2$$

$$P_b-P_t=\frac{1}{2}\rho v_b^2\left(n^2-1\right)$$

The lift force is $\left(P_b-P_t\right)A=\frac{1}{2}\rho v_b^2\left(n^2-1\right)A$

(b) For liftoff,

$$\frac{1}{2}\rho v_b^2\left(n^2-1\right)A = Mg$$

$$v_b = \left(\frac{2Mg}{\rho\left(n^2-1\right)A}\right)^{1/2}$$

The speed of the boat relative to the shore must be nearly equal to this speed of the water below the hydrofoil relative to the boat.

(c)

$$v^2\left(n^2-1\right)A\rho = 2Mg$$

$$A = \frac{2(800\text{ kg})9.8\text{ m/s}^2}{(9.5\text{ m/s})^2\left(1.05^2-1\right)1\,000\text{ kg/m}^3} = \boxed{1.70\text{ m}^2}$$