# MAT 266 Test 1 Review

Techniques of integration and improper integrals, sections 5.5, 6.1-6.4 and 6.6 in *Essential Calculus, Early Transcendentals*, 2nd Edition, by James Stewart.

Find the following indefinite integrals (1-21).

1. $\int \frac{(\ln x)^5}{5x}\,dx$
2. $\int \frac{2\sin(x)}{1+\cos^2(x)}\,dx$
3. $\int \frac{3x+5}{5x^2-4x-1}\,dx$
4. $\int \frac{1}{x^2\sqrt{16-x^2}}\,dx$
5. $\int \frac{x^2}{\sqrt{5+x}}\,dx$
6. $\int 10ze^{3z}\,dz$
7. $\int t^6\ln(t)\,dt$
8. $\int \frac{2x^2+6}{(x+1)(x^2+1)}\,dx$
9. $\int \frac{x+1}{x^2+1}\,dx$
10. $\int 5x\cos(2x)\,dx$
11. $\int \frac{1}{x^2\sqrt{x^2-49}}\,dx$
12. $\int \frac{2x+3}{x(x^2-6x+9)}\,dx$
13. $\int \cos^2(3\theta)\,d\theta$
14. $\int \sec^7(x)\tan^3(x)\,dx$
15. $\int \sec^4(t)\tan^2(t)\,dt$
16. $\int \cos^3\left(\frac{x}{2}\right)dx$
17. $\int \sin^5(x)\cos^2(x)\,dx$
18. $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$
19. $\int \arctan(x)\,dx$
20. $\int \frac{10}{(9+x^2)^{3/2}}\,dx$
21. $\int \sin^{-1}(x)\,dx$

Find the exact value of the definite integrals (22-24).

22. $\int_0^2 (9-2t)^{3/2}\,dt$
23. $\int_0^2 \frac{e^x}{1+e^{2x}}\,dx$
24. $\int_0^3 x\sqrt{4-x}\,dx$

25. Find the average value of the given function on the given interval.

a) $f(x)=\cos\left(\frac{\pi t}{2}\right)$ on $[0,3]$ b) $f(x)=7e^{3x/5}$ on $[0,2]$ c) $f(x)=xe^{-x^2}$ on $[-2,0]$

Use the table of integrals in the back of your textbook to find the following indefinite integrals (26-28): (The rule for any problem requiring the use of a table will be provided on the test.)

26. $\int x^8\sqrt{4-x^6}\,dx$
27. $\int \frac{x^5}{\sqrt{49-4x^4}}\,dx$
28. $\int \sqrt{10e^x-e^{2x}}\,dx$

Write out the form of the partial fraction decomposition of the following functions.
Do not find the numerical coefficients. (29-31)

29. $\frac{4x^2}{(x+1)(x-2)^2}$
30. $\frac{8x^3+27}{x^3(x^2+x+1)^2}$
31. $\frac{2x^2+1}{16x^4-1}$

Algebraically, evaluate the following improper integrals or state why the integral diverges (32-40).

32. $\int_{-1}^{8} \frac{1}{\sqrt[3]{x}}\,dx$

33. $\int_{-3}^{2} \frac{1}{x^5}\,dx$

34. $\int_{-\infty}^{\infty} x^2 e^{-x^3}\,dx$

35. $\int_{2}^{\infty} \frac{3}{(x+4)^{3/2}}\,dx$

36. $\int_{-\infty}^{-3} \frac{1}{\sqrt{8-x}}\,dx$

37. $\int_{0}^{\pi/2} \tan(x)\,dx$

38. $\int_{0}^{1} \frac{\ln(x)}{\sqrt{x}}\,dx$

39. $\int_{2}^{\infty} e^{-x/2}\,dx$

40. $\int_{0}^{\infty} \frac{1}{x^2+7x+12}\,dx$

41. If $x = 5\tan(\theta)$ simplify the expression $4\theta - 16\sin(\theta)$ in terms of $x$.

42. Use the integral $\int \frac{\sqrt{x^2-9}}{x}\,dx$ to answer questions (i) and (ii):

i) choose the appropriate trig-substitution from the choices below.

a) $x = 3\sin(\theta)$ b) $x = 3\sec(\theta)$ c) $x = 3\tan(\theta)$ d) $x = 3\cos(\theta)$

ii) Using the trig-substitution from part (i), the resulting trigonometric integral in its simplified from is:

a) $\int \sin(\theta)\,d\theta$ b) $3\int \tan(\theta)\,d\theta$ c) $3\int \tan^2(\theta)\,d\theta$ d) $\frac{1}{3}\int \sec(\theta)\,d\theta$

43. Write the partial fraction decomposition of the rational function $\frac{2x-1}{(x+5)^2(x^2+4)}$.

a) $\frac{A}{(x+5)^2} + \frac{Bx+C}{x^2+4}$ b) $\frac{A}{(x+5)} + \frac{B}{(x+5)^2} + \frac{Cx+D}{x^2+4}$ c) $\frac{A}{(x+5)} + \frac{B}{(x+5)^2} + \frac{Cx}{x^2+4}$ d) $\frac{A}{(x+5)^2} + \frac{B}{x^2+4}$

44. Using the substitution, $u = 2x-1$, the integral $\int_3^5 x\sqrt{2x-1}\,dx$ is written as $k\int_a^b (u+1)\sqrt{u}\,du$. The proper values for $k$, $a$ and $b$ are given by $(k,a,b) =$

a) $\left(\frac{1}{4},2,3\right)$ b) $\left(\frac{1}{2},2,3\right)$ c) $\left(\frac{1}{2},5,9\right)$ d) $\left(\frac{1}{4},5,9\right)$ e) $(1,5,9)$

45. Given the integration rule below, determine $\int \sqrt{30x-9x^2}\;dx$.

rule: $\int \sqrt{2au-u^2}\;du = \frac{1}{2}(u-a)\sqrt{2au-u^2} + \frac{1}{2}a^2 \arccos\left(\frac{a-u}{a}\right) + \mathrm{C}$

a) $\frac{1}{6}(3x-5)\sqrt{30x-9x^2} + \frac{25}{6}\arccos\left(\frac{5-3x}{5}\right) + \mathrm{C}$

b) $\frac{1}{2}(3x-5)\sqrt{30x-9x^2} + \frac{225}{2}\arccos\left(\frac{5-3x}{5}\right) + \mathrm{C}$

c) $\frac{1}{2}(3x-5)\sqrt{30x-9x^2} + \frac{25}{2}\arccos\left(\frac{5-3x}{5}\right) + \mathrm{C}$

d) $\frac{3}{2}(3x-5)\sqrt{30x-9x^2} + \frac{75}{2}\arccos\left(\frac{5-3x}{5}\right) + \mathrm{C}$

e) None of the above

**Answers**

1) $\frac{(\ln x)^6}{30}+\mathrm{C}$ 2) $-2\arctan(\cos(x))+\mathrm{C}$ 3) $-\frac{11}{15}\ln|5x+1|+\frac{4}{3}\ln|x-1|+\mathrm{C}$

4) $-\frac{\sqrt{16-x^2}}{16x}+\mathrm{C}$ 5) $\frac{2}{5}(x+5)^{5/2}-\frac{20}{3}(x+5)^{3/2}+50\sqrt{x+5}+\mathrm{C}$
$=\frac{2}{15}\sqrt{x+5}\big(3x^2-20x+200\big)+\mathrm{C}$

6) $\frac{10z}{3}e^{3z}-\frac{10}{9}e^{3z}+\mathrm{C}=\frac{10}{9}e^{3z}\big(3z-1\big)+\mathrm{C}$ 7) $\frac{t^7\ln(t)}{7}-\frac{t^7}{49}+\mathrm{C}$

8) $4\ln|x+1|-\ln\left(x^2+1\right)+2\tan^{-1}(x)+\mathrm{C}=\ln\Big(\frac{(x+1)^4}{x^2+1}\Big)+2\arctan(x)+\mathrm{C}$

9) $\frac{1}{2}\ln\left(x^2+1\right)+\tan^{-1}(x)+\mathrm{C}$ 10) $\frac{5}{4}\cos(2x)+\frac{5}{2}x\sin(2x)+\mathrm{C}$ 11) $\frac{\sqrt{x^2-49}}{49x}+\mathrm{C}$

12) $-\frac{3}{x-3}+\frac{1}{3}\ln|x|-\frac{1}{3}\ln|x-3|+\ \mathrm{C}\ =-\frac{3}{x-3}+\frac{1}{3}\ln\left|\frac{x}{x-3}\right|+\mathrm{C}$ 13) $\frac{\theta}{2}+\frac{\sin(6\theta)}{12}+\mathrm{C}$

14) $\frac{\sec^9(x)}{9}-\frac{\sec^7(x)}{7}+\mathrm{C}$ 15) $\frac{\tan^5(t)}{5}+\frac{\tan^3(t)}{3}+\mathrm{C}$ 16) $2\sin\left(\frac{x}{2}\right)-\frac{2}{3}\sin^3\left(\frac{x}{2}\right)+\mathrm{C}$

17) $-\frac{\cos^7(x)}{7}+\frac{2\cos^5(x)}{5}-\frac{\cos^3(x)}{3}+\mathrm{C}$ 18) $\frac{(x^2+1)^{3/2}}{3}-\sqrt{x^2+1}+\mathrm{C}=\frac{1}{3}\sqrt{x^2+1}\big(x^2-2\big)+\mathrm{C}$

19) $x\tan^{-1}(x)-\frac{1}{2}\ln\left(x^2+1\right)+\mathrm{C}$ 20) $\frac{10x}{9\sqrt{9+x^2}}+\mathrm{C}$ 21) $x\arcsin(x)+\sqrt{1-x^2}+\mathrm{C}$

22) $\frac{243-25\sqrt{5}}{5}$ 23) $\arctan\left(e^2\right)-\frac{\pi}{4}$ 24) $\frac{94}{15}$

25) (a) $-\frac{2}{3\pi}$ (b) $\frac{35}{5}\left(e^{6/5}-1\right)$ (c) $\frac{e^{-4}-1}{4}$

26) $\frac{1}{12}x^3\left(x^6-2\right)\sqrt{4-x^6}+\frac{2}{3}\sin^{-1}\left(\frac{x^3}{2}\right)+\mathrm{C}$ $\big(u=x^3$ then formula #31$\big)$

27) $-\frac{x^2}{16}\sqrt{49-4x^4}+\frac{49}{32}\sin^{-1}\left(\frac{2x^2}{7}\right)+\mathrm{C}$ $\big(u=2x^2$ then formula #34$\big)$

28) $\sqrt{10e^x-e^{2x}}+5\cos^{-1}\left(\frac{5-e^x}{5}\right)+\mathrm{C}$ $\big(u=e^x$ then formula #115$\big)$

29) $\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$ 30) $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{Dx+E}{x^2+x+1}+\frac{Fx+G}{(x^2+x+1)^2}$ 31) $\frac{A}{2x+1}+\frac{B}{2x-1}+\frac{Cx+D}{4x^2+1}$

32) $\frac{9}{2}$ 33) diverges 34) diverges 35) $\sqrt{6}$ 36) diverges 37) diverges

38) $-4$ 39) $\frac{2}{e}$ 40) $\ln\left(\frac{4}{3}\right)$ 41) $4\tan^{-1}\left(\frac{x}{5}\right)-\frac{16x}{\sqrt{25+x^2}}$

42) (i) b (ii) c Note: $3\int\tan^2(\theta)d\theta=3\int\left(\sec^2(\theta)-1\right)d\theta=\ 3\tan(\theta)-3\theta+C$
$=\ \sqrt{x^2-9}-3\sec^{-1}\left(\frac{x}{3}\right)+C$ because $\sec(\theta)=\frac{x}{3}$

43) b 44) d 45) a