Proof Assignment for Final (Grading guide)

On this assignments, you are expected to write an informal proof, i.e. an argument with complete, grammatically correct English sentences. Correct logical structure is paramount for a full score. Prove the following theorems using induction. (See the following examples. You will have only one induction problem in the final.)

1. (20 points) **Use induction** to prove that for all positive integers n, $n \geq 1$

$$\sum_{i=0}^{n-1} 2^i = 2^n - 1$$

Let P(n) denote the proposition $\sum_{i=0}^{n-1} 2^i = 2^n - 1$, where n is a positive integer.
Basis step: P(1) is true since $\sum_{i=0}^{1-1} 2^i = 2^0 = 1 \ and \ 2^1 - 1 = 1$ (5 points)
Inductive step: Let us assume P(n): $\sum_{i=0}^{n-1} 2^i = 2^n - 1$ is true for an arbitrary positive integer k. (4 points)
We have to show that P(n+1): $\sum_{i=0}^{n+1-1} 2^i = 2^{n+1} - 1$ is true. (5 points)
Proof: $\sum_{i=0}^{n+1-1} 2^i = \sum_{i=0}^{n} 2^i = \sum_{i=0}^{n-1} 2^i + 2^{\mathrm{n}} = 2^n - 1 + 2^n$ $= 2 * 2^n - 1 = 2^{n+1} - 1$ using the inductive hypothesis.
(6 points)

By the Principle of Mathematical Induction
$\sum_{i=0}^{n-1} 2^i = 2^n - 1$ is true for all positive integers $n \geq 1$.

2. (20 points) **Use induction** to prove that for all positive integers n, $n \geq 4$, $n! > 2^n$.

Let P(n) denote the proposition $n! > 2^n$, where n is a positive integer $n \geq 4$.

BASIS STEP: P(4) is true since 4!=24> $2^4 = 16$. (5 points)

INDUCTIVE STEP: Let us assume P(n), that is ($n! > 2^n$) is true for an arbitrary positive integer $n \geq 4$. This is our inductive hypothesis. (4 points)

We have to show that P(n+1), ($(n+1)! > 2^{n+1}$) is also true assuming the inductive hypothesis P(n). (5 points)

Proof: (6 points)

$(\boldsymbol{n}+\mathbf{1})! = (\boldsymbol{n}+\mathbf{1}) \cdot \boldsymbol{n}! > (\boldsymbol{n}+\mathbf{1}) \cdot \mathbf{2}^{\boldsymbol{n}}$ using the inductive hypothesis and $(n+1) \cdot 2^n > 2 \cdot 2^n$=$2^{n+1}$, when $k \geq 4$.

By the Principle of Mathematical Induction (Basis Step and Inductive Step together) $n! > 2^n$ for all positive integers $n \geq 4$