Mathematical Induction

Use induction to prove that 3 divides $n^3 - n$ for all positive integers n.

Let $P(n)$ denote the proposition that $n^3 - n$ is divisible by 3 for all positive integers n.

BASIS STEP: $P(1)$ is true since 3 divides 0.

INDUCTIVE STEP: Let us assume $P(n)$ is true , that is $n^3 - n$ is divisible by 3 for an arbitrary positive integer n. This is our inductive hypothesis.

We have to show that $P(n+1)$ is true, that is $(n+1)^3 - (n+1)$ is divisible by 3 assuming the inductive hypothesis $P(n)$.

Proof: $(n+1)^3 - (n+1) = n^3 - n + 3(n^2 + n)$

$n^3 - n$ is divisible by 3 using the inductive hypothesis.

$3(n^2 + n)$ is divisible by 3 the definition of divisibility since $n^2 + n$ is an integer.

Thus, the sum $(n+1)^3 - (n+1) = n^3 - n + 3(n^2 + n)$ is also divisible by 3.

By the **Principle of Mathematical Induction** (Basis Step and Inductive Step together) $n^3 - n$ is divisible by 3 for all positive integers n.