1

(a) Product Rule.

(b) Mutually exclusive

(c) Ordered

(d) If there are k boxes and you place N objects into them, then at least one box will contain at least $\left\lceil \frac{N}{k} \right\rceil$ objects.

(e) $|A_1| + |A_2| + |A_3| + \cdots + |A_n|$ $\boldsymbol{if}$ $A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n$

(f) sample space.

(g) equally likely,

(h) $(xRy \wedge yRz) \rightarrow xRz$

2

(a) $\frac{10!}{(10-6)!}$ = 151200 ways

(b) $\frac{10!}{4!} - \frac{9!}{3!}$ = 90720 ways

(c) $\frac{8!}{5! \times (8-5)!} \times 6!$ = 40320 ways

(d) $\frac{8!}{4! \times (8-4)!} \times 6!$ = 50400 ways

(e) $\frac{8!}{4! \times (8-4)!} \times 5! \times 2! = 16800$ ways

3

(a) $\frac{11!}{(11-5)!} = 55440\ ways$

(b) $\frac{11!}{5! \times (11-5)!} = 462\ ways$

(c) $\left(\frac{11!}{2! \times (11-2)!}\right) \times \left(\frac{9!}{3! \times (9-3)!}\right) = 4620\ ways$

4

(a) $N = 2800, k = 9 \cdot 9 \cdot 8\ (non-repeating\ digits) = 648$

Then, by the pigeon hole principle, there will be at least $\left\lceil \frac{2800}{648} \right\rceil$ identical numbers.

(b) $\left\lceil \frac{N}{648} \right\rceil = 5 \rightarrow N = (5-1) \cdot 648 + 1 = 2593\ of\ the\ numbers.$

(c) Indeterminate using the pigeon hole principle.

5

(a) $= \sum_{k=0}^{8} \binom{8}{k} (x^2)^k (-6)^{8-k}$

$= \binom{8}{0}(x^2)^0(-6)^8 + \binom{8}{1}(x^2)^1(-6)^7 + \binom{8}{2}(x^2)^2(-6)^6 +$

$\binom{8}{3}(x^2)^3(-6)^5 + \binom{8}{4}(x^2)^4(-6)^4 + \binom{8}{5}(x^2)^5(-6)^3 + \binom{8}{6}(x^2)^6(-6)^2 +$

$\binom{8}{7}(x^2)^7(-6)^1 + \binom{8}{8}(x^2)^8(-6)^0$

$= \binom{8}{5}(-6)^3$

$= \frac{8!}{5!\,(8-5)!} \cdot (-6)^3 = 336 \cdot (-36)$

(b) $= \sum_{i=1}^{50} \binom{50}{k} 5^k \cdot 1^{k-1} - \sum_{i=1}^{2} \binom{50}{k} 5^k \cdot 1^{k-1}$

$= (1+5)^{50} - \left[\binom{50}{0} \cdot 5^0 + \binom{50}{1} \cdot 5^1 + \binom{50}{2} \cdot 5^2\right]$

$= (1+5)^{50} - [1 + 250 + 30625]$

$= 6^{50} - 30876$

6

(a) $\binom{1}{1} \cdot \binom{51}{4}$=249900 ways

(b) $There\ are\ 26\ red\ cards\ in\ the\ 52 -$
$card\ deck, and\ 5\ cards\ for\ poker. So\ there\ are\ \binom{26}{5}, or\ 65780\ ways.$

(c) There are only 13 diamond cards in the 52-cards deck, and the poker game has 5 cards. So there are $\binom{13}{5}, or$ 1287 ways.

(d) Same as part b, $\binom{26}{5}$, 65780 ways.

(e) Same as part b, $\binom{26}{5}$, 65780 ways.

7

(a) $P(first\ 2\ digits\ are\ 16) = \frac{8 \cdot 7 \cdot 6}{9 \cdot 9 \cdot 8 \cdot 7 \cdot 6} = \frac{1}{81}$

(b) $P(last\ digit\ is\ 0) = \frac{6}{9 \cdot 9 \cdot 8 \cdot 6} = \frac{1}{9 \cdot 9 \cdot 8} = \frac{1}{648}$

(c) $P([part\ a] \cup [part\ b]) = \frac{1}{81} - \frac{1}{648} = \frac{7}{648}$

8

(a) R is not reflexive since $(a, a) \notin R$

(b) R is symmetric since $(a, b) \in R \to (b, a) \in R$

(c) R is not transitive since $(a, b) \in R, and\ (b, c) \notin R \therefore (a, c) \notin R$

(d) $R \circ R$ means if a has chatted with b, and b has chatted with c, then a has chatted with c (transitive).

9

(a) R is reflexive since $a|a = 1$

(b) R is not symmetric since $a|b \to b\ will\ not\ divide\ a$

(c) R is antisymmetric since $(a, b) \in R, and\ (b, a) \in R \to (a = b)$

(d) R is transitive since $(a, b) \in R\ and\ (b, c) \in R \therefore (a, c) \in R$

(e) $R \circ R$, in this case, means if $a|b$ and $b|c$, then $a|(both\ b\ and\ c)$