Hieu Pham

1

(a) The base of $f(x)$ is smaller than or equal to the base of $g(x)$.

(b) There is an integer c $such\ that\ b = ac$.

(c) There is an integer c $such\ that\ a = b + cm$.

(d) Co-prime.

2

(a) $f(x) = O(x^3)$ is the best estimate.
Reason: $\log n^6, \log n\ , and\ 17$ are constants, and will not be the dominating terms.

(b) $f(x) = O(1)$ is the best estimate.
Reason: Similar to previous problem. $g(x)$ is a function of x , there is no x term, and $g(x)$ = a product of some constants here. Therefore, $g(x) = O(1)$.

3

$4 \equiv 4\ mod\ 9$

$4^2 \equiv 7\ mod\ 9$

$4^3 \equiv 1\ mod\ 9, since\ (4^3 = 64 = (9 \times 7) + 1)$

$4^{1033} = 4^{((3)\times 344)+1} = 4^1 \times 4^{((3)\times 344)}$

If $a \equiv b\ mod\ n$

Then $a^k \equiv b^k\ mod\ n$

Therefore,

If $4^3 \equiv 1\ mod\ 9$

Then $4^{(3)\times 344} \equiv 1^{344}\ mod\ 9 \rightarrow 4^{1032} \equiv 1\ mod\ 9$

Also, if $a \equiv b\ mod\ n$

Then $\gamma a \equiv \gamma b\ mod\ n$

Since $4^{1032} \equiv 1\ mod\ 9 \rightarrow (4 \times 4^{1032}) \equiv (4 \times 1)\ mod\ 9 \rightarrow \mathbf{4^{1033} \equiv 4\ mod\ 9}$.

4

If $a \equiv b(mod\ 2m)$

Then $a = (b + p_1) \times (2m)$

$(a - b) = 2p_1m$

Similarly,

If $a \equiv b(mod\ m)$

Then $a = (b + p_2) \times m$

$(a - b) = p_2m \rightarrow (2p_1m = p_2m) \rightarrow (2p_1m - p_2m) = 0$

$(2p_1 - p_2)m = 0, m \neq 0$, so $(2p_1 - p_2) = 0 \rightarrow (p_2 = 2p_1)$

This statement is false because $p_2 \neq 2p_1$.

A counterexample shows that $10\big(mod\ (2 \times 3)\big) \neq 10\big(mod(3)\big)$ since $4 \neq 1$

5

Let $n = a_n10^n + \ a_{n-1}10^{n-1} + \cdots + a_310^3 + a_210^2 + a_110^1 + a_0$ be digits of n.

Also, $10 = 9 + 1$

$100 = 99 + 1$

$1000 = 999 + 1$

$10 \ldots n(zeros) = [9 \ldots (n-1)9] + 1$

$10^n = \{(k_n \times 9) + 1 : k_n = 1 \ldots (n-1)\ times\}$

Thus, $n = a_n(9[k_n \times 9] + 1) + \ a_{n-1}([k_{n-1} \times 9] + 1) + \cdots + a_3([k_3 \times 9] + 1) + a_2([k_2 \times 9] + 1) + a_1([k_1 \times 9] + 1) + a_0$

$n = 9(a_nk_n) + a_n + 9(a_{n-1}k_{n-1}) + a_{n-1} + \cdots + 9(a_3k_3) + a_3 + 9(a_2k_2) + a_2 + 9(a_1k_1) + a_1 + a_0$

$n = 9[a_nk_n + a_{n-1}k_{n-1} + \cdots + a_3k_3 + a_2k_2 + a_1k_1] + [a_n + a_{n-1} + \cdots + a_3 + a_2 + a_1 + a_0]$

$n = 9x + [a_n + a_{n-1} + \cdots + a_3 + a_2 + a_1 + a_0]\ where\ x = [a_nk_n + a_{n-1}k_{n-1} + \cdots + a_3k_3 + a_2k_2 + a_1k_1], which\ is\ an\ integer.$

Since $[a_n + a_{n-1} + \cdots + a_3 + a_2 + a_1 + a_0]$ is the sum of the digits, $[a_n + a_{n-1} + \cdots + a_3 + a_2 + a_1 + a_0]$ are also multiple of 9. (Premise)

Thus, $n = 9x + (multiple\ of\ 9) \rightarrow n$ is $multiple\ of\ 9$.

$\therefore n\ is\ divisible\ by\ 9$

6

The $gcd(847,161)\ is$ 7.

The Euklidian steps are as follows:

(1) 847 divided by 161 gives quotient 5 and remainder 42, since $867 = (5 \times 161) + 42$.

(2) 161 divided by 42 gives quotient 3 and remainder 35, since $161 = (3 \times 42) + 35$.

(3) 42 divided by 35 gives quotient 1 and remainder 7, since $42 = (1 \times 35) + 7$.

(4) 35 divided 7 gives quotient 5 and remainder 0, since $35 = (5 \times 7) + 0$.

The algorithm stops when the remainder = 0.

$\therefore gcd(847,161) = 7$