

- There are 25 prime numbers in the interval $[1,100]$. There are also 50 even numbers and 50 odd numbers. So, the probability for n is not a prime is $\frac{(100-25)}{100} = \frac{3}{4} = 0.75$. However, the probability that n is an odd number is $\frac{100-50}{100} = \frac{1}{2}$, so the probability for n to be both an odd and composite number is $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$, or 37.5%.
- There are 100 people, yet there are 52 people who own cats, making 48 people who own dogs. As shown in the Venn diagram below, the green circle. There are 59 people who own dogs, giving additional number to 11, meaning that these 11 people who must own both dogs and cats. This is the yellow portion of the Venn diagram. Consequently, the number of "cat only" owners got reduced by 11, or $52 - 11 = 41$, as shown in the light-purple portion of the Venn diagram below.



- Under normal circumstances, most flights, if not all, don't start from an airport of a city and fly to that city (or just circulating the sky for sight-seeing). Thus R is not reflexive.

There is a fair chance for a direct flight originating from a to b , and equally likely that there's another direct flight from b to a . Thus, R is symmetric.

Since the condition is: iff there is a direct flight from a to b , b may have flights to c , but the direct flight already stopped at b as the journey is done. Thus R is not transitive.

The meaning of the relation R^2 is a reflexive relation in this case. When the two airports R^2 related, they are themselves, or aRa .

- $R^2 = R \circ R$
 $R^2 = \{(1,2), (1,3), (2,3)\} \circ \{(1,2), (1,3), (2,3)\}$
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- $R^2 = R \circ R$ by the definition of composition.
 If $(x, y) \in R \circ R$, then $(\exists z \in R | (x, z) \in R)$ and $(\exists z \in R | (z, y) \in R^2)$.
 Similarly, If $(x, y) \in R^2$ or $(x, y) \in R \circ R$, then
 $(\exists z \in R | (z, x) \in R)$ and $(\exists z \in R | (y, z) \in R^2)$, which R contains the following ordered-pairs:

$$\{(x,z),(z,y),(z,x),(y,z)\}.$$

Then, $R^2 = R \circ R$

$$R^2 = \{(x,z),(z,y),(z,x),(y,z)\} \circ \{(x,z),(z,y),(z,x),(y,z)\}$$

$$R^2 = \{(x,y),(x,x),(z,z),(y,y),(y,x)\}$$

This shows that If $(x,y) \in R^2$ then $(y,x) \in R^2$

Thus it concludes that R^2 is symmetric.

6. $<^2 = \circ <$ by the definition of composition of $<$ with itself.

$$\text{Therefore, } <^2 = \{(a,b) | \exists c (a < c \cap c < b)\}$$

Thus, $(a, a+1) \in <$ and $(a, a+1) \notin <^2$