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1

(a) An argument is valid iff the conclusion follows logically from the premises, i.e. if the conclusion must be true given that the premises are true.

(b) Modus Ponens.

(c) Fallacy.

(d) A theorem that is an immediate consequence (and perhaps special case) of another theorem.

2

(a)

1. $\neg r$ (premise).
2. s (premise).
3. $q \vee r$ (argument).
4. $q \to p$ (argument).
5. q (Disjunctive syllogism using lines 1, 3).
6. p (Modus ponens using lines 4, 5).
7. $p \wedge q$ (Conjunction and commutativity using lines 5, 6) [quod erat demonstrandum].

(b) Note: The word overweight is taken as "it is not the case that x is fit".

S(x) = "x is a swimmer", F(x) = "x is fit", Universe of discourse = all people.

$$\forall x\big(S(x) \to F(x)\big)$$

$$\neg F(Piroska)$$

$$\therefore\ \neg S(Piroska)$$

Yes, the argument is valid because of the quantifier and it is of the form $\forall x\big(P(x) \to Q(x)\big)$.

It is Universal Tollens.

3

(a) $Let\ P(x)\ be\ (x + 9 > x^2)$. Universe of discourse: x in the set of all real numbers ℝ.

$$\exists x \in \mathbb{R}, P(x)$$

This is trivial, as there is a real number x = 1.0, and *P(1.0)* is true.

(b) Universe of discourse: x in the set of all integers ℤ

$$\forall x \in \mathbb{Z}\ \exists y | x = y^2 - 1$$

Suppose y and x are arbitrary real numbers and y = $\sqrt{(x+1)}$ y. Then x = y^2 – 1.

4

Suppose *n* is an arbitrary even integer. By definition of even integer, the integer *n* is *even* if there exists an integer *k* such that *n* = 2*k* .
Similarly, Suppose *m* is an arbitrary even integer. By definition of even integer, the integer *m* is *even* if there exists an integer *k* such that *m* = 2*k.*
Then the sum of *m* and *n* is (*m* + *n*), which is equivalent to *(2k + 2k) = 4k.*
Since *4k = 2(2k),* which is a multiple of 2, *4k* is also an even number.
Thus, *(m + n) = 2(2k),* and by definition, *(m + n)* is even.
Therefore, if *m* and *n* are even integers, then their sum is also even. (Q. E. D).

5

1. p = "*m* is even", q = "*n* is even", Universe of discourse: the set of all integers ℤ
2. $(p \wedge q) \rightarrow (p \vee q)$ [premise]
3. $\neg(p \vee q) \equiv (\neg p \wedge \neg q)$ [Attempting contraposition]
4. $(\neg p \wedge \neg q) \equiv\ m\ is\ odd\ and\ n\ is\ odd$ [from line 3]
5. $\exists j \exists k (m = (2j+1)\ \wedge\ n = (2k+1))$ [$Definition\ of\ 2\ arbitrary\ odd\ integers$]
6. Then,
7. $(mn) = (2j+1)(2k+1)$
8. $= (4jk + 2j + 2k + 1)$
9. $= 2t + 1$ [$\exists t \in \mathbb{Z} | t = (2jk + j + k)$]
10. $\therefore (mn)\ is\ odd$ [Negation to proposition that (*mn*) is even]
11. Thus concludes the proof by contraposition.