Chapter 3 Vectors

P3.1 $x = r\cos\theta = (5.50\text{ m})\cos 240° = (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$

$y = r\sin\theta = (5.50\text{ m})\sin 240° = (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$

P3.5 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

(a) The radius for this new point is

$$\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$$

and its angle is

$$\tan^{-1}\left(\frac{y}{-x}\right) = \boxed{180° - \theta}.$$

(b) $\sqrt{(-2x)^2 + (-2y)^2} = \boxed{2r}$ This point is in the third quadrant if (x, y) is in the first quadrant or in the fourth quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{180° + \theta}$.

(c) $\sqrt{(3x)^2 + (-3y)^2} = \boxed{3r}$ This point is in the fourth quadrant if (x, y) is in the first quadrant or in the third quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{-\theta}$.

***P3.14** We assume the floor is level. Take the *x* axis in the direction of the first displacement. $\boxed{\text{If both of the 90° turns are to the right or both to the left}}$, the displacements add like

$$40.0\text{ m }\hat{\mathbf{i}} + 15.0\text{ m }\hat{\mathbf{j}} - 20.0\text{ m }\hat{\mathbf{i}} = \left(20.0\,\hat{\mathbf{i}} + 15.0\,\hat{\mathbf{j}}\right)\text{m}$$

to give (a) displacement magnitude $(20^2 + 15^2)^{1/2}$ m = $\boxed{25.0\text{ m}}$

at (b) $\tan^{-1}(15/20)$ = $\boxed{36.9°.}$

$\boxed{\text{If one turn is right and the other is left}}$, the displacements add like

$$40.0\text{ m }\hat{\mathbf{i}} + 15.0\text{ m }\hat{\mathbf{j}} + 20.0\text{ m }\hat{\mathbf{i}} = \left(60.0\,\hat{\mathbf{i}} + 15.0\,\hat{\mathbf{j}}\right)\text{m}$$

to give (a) displacement magnitude $(60^2 + 15^2)^{1/2}$ m = $\boxed{61.8\text{ m}}$

at (b) $\tan^{-1}(15/60)$ = $\boxed{14.0°.\ \text{Just two answers are possible.}}$

P3.23 We have $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$:

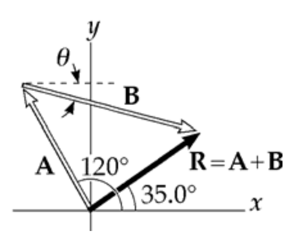

FIG. P3.23

$$A_x = 150\cos 120° = -75.0\text{ cm}$$
$$A_y = 150\sin 120° = 130\text{ cm}$$
$$R_x = 140\cos 35.0° = 115\text{ cm}$$
$$R_y = 140\sin 35.0° = 80.3\text{ cm}$$

Therefore,

$$\vec{\mathbf{B}} = [115-(-75)]\hat{\mathbf{i}} + [80.3-130]\hat{\mathbf{j}} = (190\hat{\mathbf{i}} - 49.7\hat{\mathbf{j}})\text{ cm}$$

$$\left|\vec{\mathbf{B}}\right| = \sqrt{190^2 + 49.7^2} = \boxed{196\text{ cm}}$$

$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7^\circ}.$$

P3.25 (a) $$\left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right) = \left(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\right) + \left(-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right) = \boxed{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}}}$$

(b) $$\left(\vec{\mathbf{A}} - \vec{\mathbf{B}}\right) = \left(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\right) - \left(-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right) = \boxed{4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}$$

(c) $$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \sqrt{2^2 + 6^2} = \boxed{6.32}$$

(d) $$\left|\vec{\mathbf{A}} - \vec{\mathbf{B}}\right| = \sqrt{4^2 + 2^2} = \boxed{4.47}$$

(e) $$\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\frac{6}{2}\right) = -71.6^\circ = \boxed{288^\circ}$$

$$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6^\circ}$$

P3.35 (a) $$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \boxed{\left(5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\text{ m}}$$

$$\left|\vec{\mathbf{C}}\right| = \sqrt{(5.00)^2 + (1.00)^2 + (3.00)^2}\text{ m} = \boxed{5.92\text{ m}}$$

(b) $$\vec{\mathbf{D}} = 2\vec{\mathbf{A}} - \vec{\mathbf{B}} = \boxed{\left(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}}\right)\text{ m}}$$

$$\left|\vec{\mathbf{D}}\right| = \sqrt{(4.00)^2 + (11.0)^2 + (15.0)^2}\text{ m} = \boxed{19.0\text{ m}}$$

P3.36 Let the positive x-direction be eastward, the positive y-direction be vertically upward, and the positive z-direction be southward. The total displacement is then

$$\vec{\mathbf{d}} = \left(4.80\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}}\right)\text{ cm} + \left(3.70\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm} = \left(4.80\hat{\mathbf{i}} + 8.50\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm}.$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2}\text{ cm} = \boxed{10.4\text{ cm}}$.

(b) Its angle with the y-axis follows from $\cos\theta = \frac{8.50}{10.4}$, giving $\boxed{\theta = 35.5^\circ}$.

P3.43 (a) $R_x = 40.0\cos 45.0° + 30.0\cos 45.0° = 49.5$

$R_y = 40.0\sin 45.0° - 30.0\sin 45.0° + 20.0 = 27.1$

$\vec{\mathbf{R}} = \boxed{49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}}$

(b) $\left|\vec{\mathbf{R}}\right| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$

$\theta = \tan^{-1}\left(\dfrac{27.1}{49.5}\right) = \boxed{28.7°}$

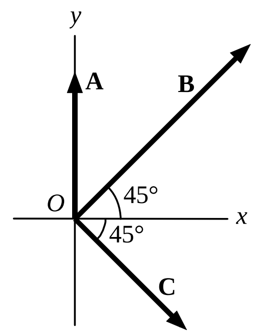

FIG. P3.43

P3.49 The position vector from the ground under the controller of the first airplane is

$$\vec{\mathbf{r}}_1 = (19.2\text{ km})(\cos 25°)\hat{\mathbf{i}} + (19.2\text{ km})(\sin 25°)\hat{\mathbf{j}} + (0.8\text{ km})\hat{\mathbf{k}}$$
$$= \left(17.4\hat{\mathbf{i}} + 8.11\hat{\mathbf{j}} + 0.8\hat{\mathbf{k}}\right)\text{ km}.$$

The second is at

$$\vec{\mathbf{r}}_2 = (17.6\text{ km})(\cos 20°)\hat{\mathbf{i}} + (17.6\text{ km})(\sin 20°)\hat{\mathbf{j}} + (1.1\text{ km})\hat{\mathbf{k}}$$
$$= \left(16.5\hat{\mathbf{i}} + 6.02\hat{\mathbf{j}} + 1.1\hat{\mathbf{k}}\right)\text{ km}.$$

Now the displacement from the first plane to the second is

$$\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 = \left(-0.863\hat{\mathbf{i}} - 2.09\hat{\mathbf{j}} + 0.3\hat{\mathbf{k}}\right)\text{ km}$$

with magnitude

$$\sqrt{(0.863)^2 + (2.09)^2 + (0.3)^2} = \boxed{2.29\text{ km}}.$$

P3.61 Since

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = 6.00\hat{\mathbf{j}},$$

we have

$$(A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} = 0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}$$

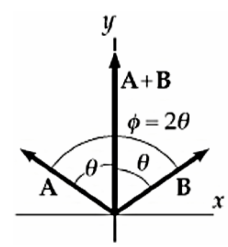

FIG. P3.61

giving

$$A_x + B_x = 0 \text{ or } A_x = -B_x \qquad [1]$$

and

$$A_y + B_y = 6.00. \qquad [2]$$

Since both vectors have a magnitude of 5.00, we also have

$$A_x^2 + A_y^2 = B_x^2 + B_y^2 = 5.00^2.$$

From $A_x = -B_x$, it is seen that

$$A_x^2 = B_x^2.$$

Therefore, $A_x^2 + A_y^2 = B_x^2 + B_y^2$ gives

$$A_y^2 = B_y^2 .$$

Then, $A_y = B_y$ and Equation [2] gives

$$A_y = B_y = 3.00 .$$

Defining θ as the angle between either $\vec{\mathbf{A}}$ or $\vec{\mathbf{B}}$ and the y axis, it is seen that

$$\cos\theta = \frac{A_y}{A} = \frac{B_y}{B} = \frac{3.00}{5.00} = 0.600 \text{ and } \theta = 53.1° .$$

The angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is then $\boxed{\phi = 2\theta = 106°}$.