Some Notes on Proof Writing in MAT 243

All proofs you are expected to write in this class are informal, i.e. English-language essays that lead a reader from the assumptions, which must be clearly stated, to the conclusions, without unnecessary elements. A good proof is concise.

Sentences must be grammatically correct and have proper sentence structure. It is acceptable, even desirable, to use some mathematical symbols in there, but the English words plus the logical symbols must form a grammatically correct sentence.

Here is a counter-example:

"Since n is an even number -> n = 2k+1 for some k."

The conditional symbol -> can be read in various ways, but none of them make the above into a proper sentence:

"If since n is an even number then n = 2k+1 for some k."
"Since n is an even number implies n = 2k+1 for some k."

You could fix this by eliminating the word "since", or by replacing the arrow by a comma.

Grammatical correctness requires that sentences have proper punctuation, even when they end with an equation.

Good:
Since x=2, 2x=4.

Bad:
Since x=2
2x=4

Using the quantifiers symbolically in your proof writing is not wrong but looks awkward and does not improve readability. I would recommend you do not do this.

A major faux pas in proof writing is to pad your proofs with unnecessary statements, or to make your sentences unnecessarily verbose by spelling out the meaning of standard relational operators or sets in English.

Good:
Since $n \in \mathbb{N}$ ..

Bad:
Since n is an element of the set of natural numbers..

Good:
x=1 implies 2x=2.

Bad:
Since we know the equation x=1 to be true, it can be seen that 2x=2 must be true as well.

Good:
a<b implies a+1<b+1.

Bad:
Since the number a is less than the number b, we conclude logically that the number a plus one is less than the number b plus one.

It's not writing as many words as possible that will earn you a good grade on the proof papers, it is logical correctness and conciseness.

The logical framework is not optional!
(It really isn't.)

Sometimes, even after studying all of this, students still write "proofs" that are just a bunch of naked equations with all the logical "glue" missing.

Here is an example of such a bad proof of the statement that a multiple of an even integer is even.

"Proof":

$$n = 2k$$
$$nm = (2k)m = 2(km)$$

If you don't see what's wrong with that, then imagine you are a kid and you ask a parent to tell you the story of Little Red Riding Hood.

This is what the parent says: "Girl Wolf Grandmother"

That's the end of the "story". Surely, you would not find this acceptable. How is this even a story? It isn't. It's missing everything that makes the story the story- the premise, the characterization of the protagonists, the temporal sequence of events and the final outcome of the story.

The "proof" shown above is likewise missing almost everything that makes a proof a proof. What are n,k and m? What are the assumptions and what is the conclusion? How do we go from the assumptions to the conclusion?

These things need to be spelled out, or it's not a proof. So here is the corrected proof.

Correct proof:

Let n be an even integer. By definition of even integers, that means that $n = 2k$ for some integer k. Now let m be an arbitrary integer. By multiplying the previous equation by m, we obtain $n = (2k)m = 2(km)$. Since the number mn again satisfies the defining property of an even number, nm is even. Since the even number n and the integer m were arbitrary, we have proved that a product of any even number and any integer is even.