1. Suppose that m is part of the binomial expansion $(1-4)^{30}$.

   Then, consider the expansion of the binomial $(1-4)^{30}$,

$(1-4)^{30} = 1 - \binom{30}{1}4 + \binom{30}{2}4^2 - \binom{30}{3}4^3 + \binom{30}{4}4^4 - \binom{30}{5}4^5 + \binom{30}{6}4^6 - \cdots + \binom{30}{n-1}4^{(n-1)} - \binom{30}{n}4^n$

$3^{30} = 1 - 30 \times 4 + \frac{30 \times 29}{2} \times 16 - 4^2(m)$

$3^{30} = 1 - 120 + 6960 - 16m$

$16m = 6841 - 3^{30}$

The greatest common divisor of -205891132087808 and 16 is 16,

So $m = \frac{-205891132087808}{16} = \frac{16(-205891132087808)}{(16\times1)} = \frac{16}{16} \times (-12868195755488)$
$m = -12868195755488$

2. Minimum sample size = (2 x 4 x 3 x 2 x 3) = 144

3. $\frac{7!}{(7-5)!} = 2520$ Different ways. We have a 7-element set, S = {1,2,3,4,5,6,7}, having 7 items, and we want to find the number of ways 5 items can be selected (or ordered, or permutated).

4. People who do not own any dog: (100 – 55) = 45

   Number of people must be selected to guarantee that there will be at least 3 dog owners on the committee: 45 + 3 = 48

   People who do not own any cat: (100 – 60) = 40

   Number of people must be selected to guarantee that there will be at least

   2 dog owners: 45 + 2 = 47

   And at least 2 cat owners: 40 + 2 = 42

   On the committee.

   Number of people must be selected to guarantee that there will be at least

   2 owners of no dogs: 55 + 2 = 57

   And at least 2 owners of no cats: 60 + 2 = 62

5. $248832 = 2^{10} \times 3^5, m = 10, n = 5, so\ 248832 = 2^m \times 3^n$

Number of positive divisors: (m+1) x (n + 1) = 11 x 6 = 66

6. 50 = 2×25

50 = $2 \times 5 \times 5$

50 = $2^1 \times 5^2$

There are two distinct prime factors, 2 and 5 and there are 6 distinct factors: 1,2,5,10,25,50, and we want to choose 2 that will guarantee to have a product of 50, my choice will be 3 2-permutations for this set.

7. (A) Because Sweden is to be selected regardless of ways, consequently there are $\frac{192!}{(192-20)!}$ ways to guarantee that Sweden is included no matter what.

(B) There are 54 African countries, and 5 of them must be selected. Consequently only 49 countries in Africa are to be among the selection choices of 193 – 5 = 188. So do a $\binom{188}{20}$ (or $4386351623200771730501516 95$ ways) will include the 5 African diplomats.

(C) $\frac{193!}{(193-2)!} = 37056$ ways to shake all 193 diplomats' hands.