Let A and B be sets.

Show that $P(A) \cup P(B) \subseteq P(A \cup B)$

Definition of subsets: $A \subseteq B \leftrightarrow \forall x(x \in A \rightarrow x \in B)$

We have to show that if X is an element of P(A) ∪ P(B) then X is an element of P(A∪B).

Show that: If X ∈ P(A) ∪ P(B) then X ∈ P(A∪B).

Let X be an arbitrary element of P(A) ∪ P(B).

Let X ∈ P(A) ∪ P(B)

Then X is an element of P(A) or of P(B).

Then X ∈ P(A) or X ∈ P(B)

Without of loss of generality we can assume the X is an element of P(A).

Let X ∈ P(A) .

If X is an element of P(A) then X is a subset of A.

If $X \in P(A)$ then $X \subseteq A$.

If X is a subset of A then X is a subset of A ∪ B.

If $X \subseteq A$ then $X \subseteq A \cup B$.

If X is a subset of A ∪ B then X is an element of P(A ∪ B).

If $X \subseteq A \cup B$ then $X \in P(A \cup B)$.