MAT 243 Rules of Inference Proof Practice

Solve the following problems, showing any necessary work. (Problems 1 and 2 should be easy, 3 and 4 what you would expect on a test, and 5 through 7 are more challenging.)

1. Prove $\neg r$, assuming the following:
   $q \lor p$
   $q \to \neg r$
   $\neg p$

2. Prove $(\neg(\neg p \land r) \land r) \to (\neg q \lor p)$, assuming the following:
   $((q \land r) \to (\neg q \lor p)) \lor p$
   $\neg p$
   $(\neg(\neg p \land r) \land r) \to (q \land r)$

3. Prove p, assuming the following:
   $q \to \neg r$
   s
   r
   $((r \land p) \lor q) \lor \neg s$

4. Prove $\neg r \lor \neg s$, assuming the following:
   $\neg((q \to p) \land p) \to \neg(\neg p \lor q)$
   $\neg p \lor q$
   $p \land (s \to \neg((q \to p) \land p))$

5. Prove r, assuming the following:
   t
   $\neg(\neg q \lor \neg p) \land q$
   $p \lor \neg q$
   $\neg(\neg q \land \neg p) \land (t \to (\neg(p \land \neg r) \land (p \to r)))$

6. Prove $\neg t$, assuming the following:
   q
   $s \to (\neg r \to (q \land ((\neg q \lor \neg t) \land \neg t)))$
   s
   $(\neg p \land \neg r) \lor \neg q$

7. Prove $\neg(r \lor \neg p) \to \neg t$, assuming the following:
   $r \lor q$
   $(r \to (s \to (\neg(r \lor \neg p) \to \neg t))) \lor \neg t$
   t
   $\neg q \to s$
   $\neg(\neg p \to \neg s)$
   $q \to (\neg p \to \neg s)$
   $\neg p \to \neg q$
   $\neg p$

Solutions

Note that there are other solutions possible (see the comment about problem #7), even if you avoid using "Logical Equivalences" in your proof.

1.

(1)	$q \vee p$	Assumption
(2)	$q \to \neg r$	Assumption
(3)	$\neg p$	Assumption
(4)	q	Disjunctive Syllogism (1), (3)
(5)	$\neg r$	Modus Ponens (2), (4)

2.

(1)	$((q \wedge r) \to (\neg q \vee p)) \vee p$	Assumption
(2)	$\neg p$	Assumption
(3)	$(\neg(\neg p \wedge r) \wedge r) \to (q \wedge r)$	Assumption
(4)	$(q \wedge r) \to (\neg q \vee p)$	Disjunctive Syllogism (1), (2)
(5)	$(\neg(\neg p \wedge r) \wedge r) \to (\neg q \vee p)$	Hypothetical Syllogism (3), (4)

3.

(1)	$q \to \neg r$	Assumption
(2)	s	Assumption
(3)	r	Assumption
(4)	$((r \wedge p) \vee q) \vee \neg s$	Assumption
(5)	$(r \wedge p) \vee q$	Disjunctive Syllogism (4), (2)
(6)	$\neg q$	Modus Tollens (1), (3)
(7)	$r \wedge p$	Disjunctive Syllogism (5), (6)
(8)	p	Simpificiation (7)

4. Notice that no r's appear in the assumptions! That means that the $\neg r$ in $\neg r \vee \neg s$ was added using Addition, and the real deduction you have to make is $\neg s$.

(1)	$\neg((q \to p) \wedge p) \to \neg(\neg p \vee q)$	Assumption
(2)	$\neg p \vee q$	Assumption
(3)	$p \wedge (s \to \neg((q \to p) \wedge p))$	Assumption
(4)	$\neg((q \to p) \wedge p)$	Modus Tollens (1), (2)
(5)	$s \to \neg((q \to p) \wedge p)$	Simplification (3)
(6)	$\neg s$	Modus Tollens (5), (4)
(7)	$\neg r \vee \neg s$	Addition (6)

5.

(1)	t	Assumption
(2)	$\neg(\neg q \vee \neg p) \wedge q$	Assumption
(3)	$p \vee \neg q$	Assumption
(4)	$\neg(\neg q \wedge \neg p) \wedge (t \to (\neg(p \wedge \neg r) \wedge (p \to r)))$	Assumption
(5)	$t \to (\neg(p \wedge \neg r) \wedge (p \to r))$	Simplification (4)
(6)	$\neg(p \wedge \neg r) \wedge (p \to r)$	Modus Ponens (5), (1)
(7)	$p \to r$	Simplification (6)
(8)	q	Simplification (2)
(9)	p	Disjunctive Syllogism (8), (3)
(10)	r	Modus Ponens (7), (9)

6.

(1)	q	Assumption
(2)	$s \to (\neg r \to (q \wedge ((\neg q \vee \neg t) \wedge \neg t)))$	Assumption
(3)	s	Assumption
(4)	$(\neg p \wedge \neg r) \vee \neg q$	Assumption
(5)	$\neg r \to (q \wedge ((\neg q \vee \neg t) \wedge \neg t))$	Modus Ponens (2), (3)
(6)	$\neg p \wedge \neg r$	Disjunctive Syllogism (4), (1)
(7)	$\neg r$	Simplification (6)
(8)	$q \wedge ((\neg q \vee \neg t) \wedge \neg t)$	Modus Ponens (5), (7)
(9)	$(\neg q \vee \neg t) \wedge \neg t$	Simplification (8)
(10)	$\neg t$	Simplification (9)

7. Note that step 10 can be replaced with Modus Ponens (7), (8). (Not all of the assumptions are necessary for the result to be true; this is a defect of my problem generator.)

(1)	$r \vee q$	Assumption
(2)	$(r \to (s \to (\neg(r \vee \neg p) \to \neg t))) \vee \neg t$	Assumption
(3)	t	Assumption
(4)	$\neg q \to s$	Assumption
(5)	$\neg(\neg p \to \neg s)$	Assumption
(6)	$q \to (\neg p \to \neg s)$	Assumption
(7)	$\neg p \to \neg q$	Assumption
(8)	$\neg p$	Assumption
(9)	$r \to (s \to (\neg(r \vee \neg p) \to \neg t))$	Disjunctive Syllogism (2), (3)
(10)	$\neg q$	Modus Tollens (6), (5)
(11)	r	Disjunctive Syllogism (1), (10)
(12)	$s \to (\neg(r \vee \neg p) \to \neg t)$	Modus Ponens (9), (11)
(13)	s	Modus Ponens (4), (10)
(14)	$\neg(r \vee \neg p) \to \neg t$	Modus Ponens (12), (13)