Mathematical Induction

Use induction to prove that $n^2 - 5n + 6 > 0$ for all positive integers $n > 3$.

Let $P(n)$ denote the proposition that $n^2 - 5n + 6 > 0$, where n is a positive integer, $n > 3$.

BASIS STEP: $P(4)$ is true since $2 = 4^2 - 5 \cdot 4 + 6 > 0$.

INDUCTIVE STEP: Let us assume $P(n)$, that is $n^2 - 5n + 6 > 0$ is true for an arbitrary positive integer $n > 3$. This is our inductive hypothesis.

We have to show that $P(n + 1)$, $(n + 1)^2 - 5(n + 1) + 6 > 0$ is also true assuming the inductive hypothesis $P(n)$.

Proof:

$$(n+1)^2-5(n+1)+6 = (n^2-5n+6)+(2n-5) > 0+1 > 0$$

since $n^2-5n+6>0$ by the inductive hypothesis and $2n-5>1$ when $n>3$.

By the **Principle of Mathematical Induction** (Basis Step and Inductive Step together) $n^2-5n+6>0$ for all positive integers $n>3$.