# Mathematical Induction

**Use induction to prove that**

**$\sum_{i=1}^{n} 2i$ =2+4+$6 + \cdots + 2\mathbf{n} = \mathbf{n}(\mathbf{n}+1)$ for all positive integers n.**

Let P(n) denote the proposition $\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{2i}$ =2+4+$\mathbf{6} + \cdots + \mathbf{2n} = \mathbf{n}(\mathbf{n}+\mathbf{1})$ , where $n$ is a positive integer.

**BASIS STEP**: P(1) is true, since $\sum_{\boldsymbol{i=1}}^{1} \boldsymbol{2i}$ =2 and 1 ·2=2

**INDUCTIVE STEP:** Let us assume P(n), that is
$\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{2i}$ =**2+4**$+\mathbf{6} + \cdots + \mathbf{2n} = \mathbf{n}(\mathbf{n}+\mathbf{1})$ is true for an arbitrary positive integer n.
This is our inductive hypothesis.

We have to show that the statement P(n+1),
$\sum_{\boldsymbol{i=1}}^{\boldsymbol{n+1}} \boldsymbol{2i} = \mathbf{2{+}4} + \mathbf{6} + \cdots + \mathbf{2n} + \mathbf{2}(\mathbf{n}+\mathbf{1}) = (\mathbf{n}+\mathbf{1})\big((\mathbf{n}+\mathbf{1})+\mathbf{1}\big) =$
$(\mathbf{n}+\mathbf{1})(\mathbf{n}+\mathbf{2})$ is true assuming the inductive hypothesis P(n).

**Proof:**

$$\sum_{i=1}^{n+1} 2i = \sum_{i=1}^{n} 2i + 2(n+1) = 2 + 4 + 6 + \cdots + 2n + 2(n+1) = n(n+1) + 2(n+1)$$

$$= (n+1)(n+2)$$

using the inductive hypothesis.

**By the Principle of Mathematical Induction (Basis Step and Inductive Step together)** $\sum_{i=1}^{n} 2i = 2+4+6+\cdots+2\mathrm{n} = \mathrm{n}(\mathrm{n}+1)$ for all positive integers $n$.