MAT 243 ADDITIONAL PRACTICE UNIT 3

Exercise #1. Write the following statements in symbolic form, negate the statements and state them in English.

(1) Every dog has fleas.
Let x be in the universe of discourse of all dogs, and let the statement $F(x)$ read as "x has fleas."

(2) Every integer is a rational number and a real number.
Let x be in the universe of discourse of all integers, and let the statement $Q(x)$ read as "x is a rational number", and and let the statement $R(x)$ read as "x is a real number.",

(3) There is a polynomial that has no real roots.
Let g be in the universal discourse of all (real) positive polynomials. If a polynomial g has a real root, then there is some real number x such that $g(x) = 0$.

(4) A number n is a natural number if and only if it is an integer and nonnegative.
Let x be in the universe of discourse of all real numbers, and let the statement $N(x)$ read as "x is a natural number", and and let the statement $I(x)$ read as "x is an integerr.",

Exercise #2. Write the negation of each statement. It may help to rewrite the statement in an if ... then ... form.

(1) If it rains today, then I will stay home.
(2) I go to the pool only if it is a sunny day.
(3) $\forall x > 0\ \exists y < 0 (x + y = 5)$.
(4) $\forall x\ (y = x^2 + 2x - 8 < 0 \rightarrow -4 < x < 2)$.
(5) $\forall x\ (f(x) < 2 \vee f(x) > 5)$.

Exercise #1 solutions.

(1) Every dog has fleas.
Symbolic form: $\forall x\ F(x)$.
Negation: $\exists x \neg F(x)$.
Negation in English: Some dogs don't have fleas. Or, there is a dog that doesn't have fleas.

(2) Every integer is a rational number and a real number.
Symbolic form: $\forall x\ (Q(x) \wedge R(x)$
Negation: $\exists x\ (\neg Q(x) \vee \neg R(x)$
Negation in English: Some integers are either not a rational number or not a real number.

(3) There is a polynomial that has no real roots.
Let g be in the universal discourse of all (real) positive polynomials. If a polynomial g has a real root, then there is some real number x such that $g(x) = 0$.
Symbolic form: $\exists g\ \neg \exists x\ g(x) = 0 \equiv \exists g\ \forall x\ g(x) \neq 0$
Negation: $(\forall g \exists x\ f(x) = 0)$
Negation in English: Every real polynomial has at least one real root.

(4) A number n is a natural number if and only if it is an integer and nonnegative.
Let x be in the universe of discourse of all real numbers, and let the statement $N(x)$ read as "x is a natural number", and and let the statement $I(x)$ read as "x is an integerr.",
Symbolic form: $(\forall x\ (N(x) \leftrightarrow (I(x)) \wedge x \geq 0))$
Negation: $\exists x\ (N(x) \wedge (\neg I(x)) \vee x < 0)) \vee (\neg N(x) \wedge (I(x)) \wedge x \geq 0))$
Negation in English: There exists x such that either x is a natural number and it is not an integer or it is negative, or x is not a natural number and it is an integer and nonnegative.

Exercise #2 solutions. Write the negation of each statement. It may help to rewrite the statement in an if ... then ... form.

(1) If it rains today, then I will stay home.
Negation: It rains today, but I won't stay home.

(2) I go to the pool only if it is a sunny day.
Negation: I go to the pool and it is not a sunny day.

(3) $\forall x > 0 \exists y < 0 (x + y = 5)$.
Negation: $\exists x > 0\ \forall y < 0 (x + y \neq 5)$.

(4) $\forall x (y = x^2 + 2x - 8 < 0 \to -4 < x < 2)$.
Negation: $\exists x (y = x^2 + 2x - 8 < 0 \wedge (-4 \geq x \vee x \geq 2))$.

(5) $\forall x\ (f(x) < 2 \vee f(x) > 5)$.
Negation: $\exists x\ (2 \leq f(x) \leq 5)$.