Logical Equivalences

Example 2

Show that $(\neg q \land (p \to q)) \to \neg p$ is a tautology using logical equivalences (Modus Tollens).

- $(\neg q \land (p \to q)) \to \neg p \equiv (\neg q \land (\neg p \lor q)) \to \neg p$: Conditional as a disjunction

- $(\neg q \land (\neg p \lor q)) \to \neg p \equiv ((\neg q \land \neg p) \lor (\neg q \land q)) \to \neg p$:Distributive Law

- $((\neg q \land \neg p) \lor (\neg q \land q)) \to \neg p \equiv ((\neg q \land \neg p) \lor F) \to \neg p$: $(\neg q \land q)$ *is a contradiction.*

- $((\neg q \wedge \neg p) \vee F) \rightarrow \neg p \equiv (\neg q \wedge \neg p) \rightarrow \neg p$

  Identity Laws

- $(\neg q \wedge \neg p) \rightarrow \neg p \equiv \neg(\neg q \wedge \neg p) \vee \neg p$

  Conditional as a disjunction

- $\neg(\neg q \wedge \neg p) \vee \neg p \equiv (q \vee p) \vee \neg p$

  De Morgan’s Law

- $(q \vee p) \vee \neg p \equiv q \vee (p \vee \neg p)$

  Associative Law

- $q \vee (p \vee \neg p) \equiv q \vee T$

$p \vee \neg p$ is a tautology

- $q \vee T \equiv T$

Domination Law