CS 1050B: Constructing Proofs

Problem Set 4 :

Due Wednesday, Oct 18th, after the class

1. **Rosen 5.1: 22**

   How many positive integers between 1000 and 9999 inclusive

   Answer:
   It will be useful to note first that there are exactly 9000 numbers in this range

   a) are divisible by 9?
   1000

   b) are even ?
   4500

   c) have distinct digits?
   $9 \cdot 9 \cdot 8 \cdot 7 = 4536$

   d) are not divisible by 3?
   6000

   e) are divisible by either 5 or 7?
   divisible by 5: 1800
   divisible by 7: 1286
   divisible by both 5 and 7: 257
   $1800 + 1286 - 257 = 2829$

   f) are not divisible by either 5 or 7?
   $9000 - 2829 = 6171$

   g) are divisible by 5 but not by 7?
   $1800 - 257 = 1543$

   h) are divisible by 5 and 7?
   257

2. **Rosen 5.1: 34**
   How many functions are there from the set $\{1, 2, \cdots, n\}$, where n is a positive integer, to the set $\{1, 0\}$?

   Answer: There are 2^n such functions, since there is a choice of 2 function values for each element of the domain.

3. **Rosen 5.2: 36**
   Prove that at a party where there are at least two people, there are two people who know the same number of other people there.

Proof: Let $K(x)$ be the number of other people at the party that person x knows. The possible values for $K(x)$ are 0, 1, $\cdots$, $n-1$, where $n \geq 2$ is the number of people at the party. We cannot apply the pigeonhole principle directly, since there are n pigeons and n pigeonholes. However, it is impossible for both 0 and $n-1$ to be in the range of K, since if one person knows everybody else, then nobody can know no one else (we assume that "knowing" is symmetric). Therefore the range of K has at most $n-1$ elements, whereas the domain has n elements, so K is not one-to-one , precisely what we wanted to prove.

4. **Rosen 5.3: 22**
How many permutations of the letters $ABCDEFGH$ contain

   a) the string ED?
   If ED is to be a substring, then we can think of that block of letters as one superletter, and the problem is to count permutations of seven items - the letters A, B, C, F, G, and H, and the superletter ED. Therefore the answer is $P(7,7) = 7! = 5040$

   b) the string CDE?
   $P(6,6) = 6! = 720$

   c) the strings BA and FGH?
   $P(5,5) = 5! = 120$

   d) the strings AB, DE, and GH?
   $P(5,5) = 5! = 120$

   e) the strings CAB and BED?
   If both CAB and BED are substrings, the $CABED$ has to be a substring. So we really just permuting four items: $CABED$, F, G and H. Therefore the answer is $P(4,4) = 4! = 24$.

   f) the strings BCA and ABF?
   There are no permutations with both of these substring, since B cannot be followed by both C and F at the same time.