Constant Velocity Motion in One Dimension (along a straight line).

Introduction and Theory:

The basic concepts of kinematics used to describe the motion of an object are: **position**, **velocity**, and **acceleration**. Because space is three-dimensional, these quantities are vectors. In this lab, however, we will only consider one-dimensional motion along a line, for which we can simply use real numbers instead of vectors. Because vectors can be split into components, all the work we will do here will be valid for two-dimensional and three-dimensional motion if we view the position x as the x-component of the position vector , the velocity v as the x-component v_x of the velocity vector $\vec{\boldsymbol{v}}$, and the acceleration a as the x-component a_x of the acceleration vector $\vec{\boldsymbol{a}}$, and use a similar notation and formulas for the y- and z-components

Position is a vector drawn from the origin of the coordinate system to the actual position of the object.For motion in one dimension (along a straight line) the position at any time can be specified by a single value, x, with the unit of distance - meter. Positions to the right of the origin will be given a positive sign; positions to the left will be negative.

Displacement, , is a vector that points from an object's initial position toward its final position. Mathematically, $\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_{\mathrm{f}} - \vec{\boldsymbol{r}}_{\mathrm{i}}$, where $\vec{\boldsymbol{r}}_{\mathrm{f}}$ is the final position and $\vec{\boldsymbol{r}}_{\mathrm{i}}$ the initial position, and we follow the "delta-notation" convention that Δ-something = something final – something initial. From the definition it is also apparent that displacement has the same unit as position: meter in the SI system. The x-component of the displacement vector is called Δx and is given by $\Delta x = x_{\mathrm{f}} - x_{\mathrm{i}}$, where x_{f} is the final value of the x-coordinate and x_{i} its initial value. This means that if $\Delta x > 0$, the object has moved in the direction of the positive x-axis, whereas if $\Delta x < 0$, the object has moved in the direction of opposite to the positive x-axis.

From the above definition, **displacement** measures a change in position. Quite often we are interested in knowing how fast that position changes. For this, we introduce the concept of average velocity as:

$$\textit{Average velocity} = \frac{\textit{Displacement}}{\textit{Elapsed time}} \quad \text{or}$$

The x-component of the average velocity is, therefore,

$$v_{\mathrm{avg},x} = \frac{\Delta x}{\Delta t} \tag{1}$$

The unit of average velocity is the unit of position divided by the unit of time. In the SI system, this unit is m/s. One interesting aspect of this definition is that if the object moves and returns to the same point, its displacement is zero, and therefore its average velocity is zero. Still, the object may be moving very fast. A good example is an object spinning around an axis. We might see a point at the rim of the object moving very fast, but its average velocity would be zero if we choose the initial and final times to be such that the object just completes a revolution during the elapsed time. To describe these situations it is useful to introduce the concept of **distance** *d,* which is the length of the path along which the object moved. For a motion in a straight line it is the absolute magnitude of the difference between the final and the initial positions (For the above circular motion example it would the length of the circumference). As such distance is always positive. The SI unit for distance is meter. **Average Speed** is the rate at which distance is traveled. Average speed is given by distance traveled during a given elapsed time,

$$Average\ Speed = \frac{Distance}{Elapsed\ time} \quad \text{or} \quad v = \frac{d}{\Delta t}, \tag{2}$$

and is expressed in m/s. Average speed has always a positive value. Average speed indicates how fast the object is moving but it does not include any reference to the direction of motion. In the circular motion example above, the average speed is never zero, but the average velocity can be zero.

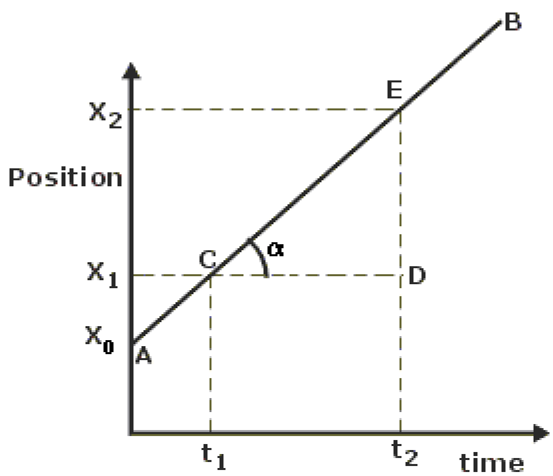
The discussion so far means that in order to understand motion during some elapsed time, we not only need the concepts of displacement and average velocity, but also distance and average speed, and these quantities are not related in any obvious way. The reason for this is that by specifying the initial and final positions, but nothing in-between, we are losing a lot of information. On the other hand, if we make the elapsed time infinitesimally small, we don't lose any information. The average velocity defined for an infinitesimally short elapsed time is called **instantaneous velocity** or simply **velocity**.

$$\vec{\boldsymbol{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\boldsymbol{r}}}{\Delta t} = \frac{d\vec{\boldsymbol{r}}}{dt} \tag{3}$$

where the last equality follows from the definition of derivative in calculus. Accordingly, the x-component of the instantaneous velocity vector is

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{4}$$

Similarly, we can define an instantaneous speed, or simply speed. This speed is just the magnitude of the velocity. In other words, when we go to the instantaneous limit, if we know the velocity we can tell the speed, whereas in the examples above, using average quantities, knowledge of the average velocity did not necessarily imply knowledge of the average speed.
Notice that from the definition of velocity, the sign of v_x tells us the direction of motion, but v_x does not contain any information about the location of the object along the axis, which is given by x. In other words, and object moving with velocity +6 m/s could be located at x = -4 m or at x = 15 m.
The most useful way of visualizing derivatives such as velocity is by making graphs. If a plot of x vs. t is made, the instantaneous velocity is the slope of the line tangent to the curve at time t. To simplify the notation, we can safely drop the subscript x and use v (instead of v_x) if we are only discussing one-dimensional motion.



$$\text{Slope} = \text{velocity} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{DE}{CD} = \tan\alpha$$

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If we know the position at all times, we can therefore compute the velocity at all times simply by calculating derivatives or drawing tangent lines. More interesting, however, is the reverse problem: if we know the initial position and the velocity at all times, what is the final position? This problem is solved using integral calculus, as discussed in the lecture. One particularly simply case is that of constant velocity. In that case it is easy to show (so easy that we don't really need calculus!) that the position is a linear function of time of form:

$$x(t) = x_0 + v(t-t_0)\ , \qquad (5)$$

where x_0 is the initial position at time t_0, and v the constant velocity of the object's motion.

Objectives:

- To gain a conceptual understanding of position, distance, displacement and velocity in one dimensional motion with constant velocity;
- To be able to model a cart's motion with graphs;
- To determine the relationship position vs time and velocity vs time for the constant velocity motion.

Equipment:

VirtualPhysicsLabs environment - frictionless cart and track with ultrasonic motion sensor; LoggerPro or Graphical Analysis software.

Procedure:

Go to the simulation website:
http://virtuallabs.ket.org/physics/

Log in and click the Labs tab. Scroll down the page with The Virtual Labs Collection to the Kinematics in One Dimension experiment. Read carefully the "Full description and detailed instructions" document to learn how to use the simulated track and all its features. Run the Kinematics in One Dimension lab and familiarize yourself with the set-up. With the mouse right-

click in the graph area change the Using Errorless Position Data option to Using the Realistic Position Data. Try out the different functions mentioned in the simulation description.
For example, let's try graphing. Click the ruler to make it appear above the track. Position the cart at 100 cm mark (in the middle of the track) and click on the motion sensor. After about 4 s click the sensor again to turn it off. Scroll the data table to check if the recorded position is correct. The graph and data table clear each time you turn on the sensor. Turn the motion sensor on again and move the cart all the way to one side and back to the other, watch the graph as it is created. Turn the sensor off and use your data table to note the minimum and maximum positions obtainable by the cart. Right click on the graph window and turn on the position and time grids.

In all future steps turn the motion sensor on just before you are ready to start the recording and turn it off immediately after you are done.

PART 1. Object moving away from the motion sensor.

Run 1. Make sure the track is level (press "Set θ = 0°), the "Recoil" feature is set to 0 and the "Brakes" are off. Select T_{max} = 15 s. Move the cart to the left end of track (close to the motion sensor). Start the motion sensor and push the cart relatively slowly to the right. If you have problems releasing the cart with a push, set the "V_0?" to # 5 and press "Go ➔" button situated next to V_0? . Stop the recording at 15 seconds.
Copy the data to clipboard and paste it into Logger Pro. Label the columns adequately and enter proper units. In the data table (or from the position vs time graph) choose two points far apart from each other (from the portion of your recording where the cart's position was changing).

a) In Table 1, clearly state the selected time values, t_1 and t_2, and the corresponding positions, x_1 and x_2. What distance did the cart travel between t_1 and t_2? What was the cart's displacement during $\Delta t = t_2 - t_1$, was it positive or negative? Calculate the average speed and the average velocity in time interval $\Delta t = t_2 - t_1$. Is the average velocity positive or negative?
b) Now select two consecutive data points of time and position, somewhere at the beginning of cart's movement. Report them in Table 1, and then calculate the instantaneous velocity at that instant.
c) Repeat the same calculation for a different instant of time, close to the right end of track but where the cart was still moving. Report all the data in Table 1.

In Discussion section compare the two instantaneous velocities to the average velocity? Can you conclude that during time interval $t_1 \rightarrow t_2$ the cart was moving with constant velocity? Provide the evidence to your answer.
On the position vs. time graph in GA highlight (drag around with the computer cursor) part of (the plot corresponding to the cart moving along the track and apply linear fit. The shaded area contains the selected data upon which the linear fit will be calculated. If a region is not selected, the linear fit will be calculated for all data displayed in the graph.

Hint: On the top tool bar, click the second icon from the right  or select Linear Fit in the pull down Analyze menu.

In Table 2, report the slope and y-intercept of position vs time graph. How does the slope value of the linear fit compare to your calculated average velocity? Write the equation of the cart's motion (5) using the value of the slope as the true value of the average velocity.
Add a new calculated column to the data table for the velocity. Remember to label the column and add proper units.

Hint: Pull down the Data menu and choose New calculated Column. A dialog window pops up where you get a chance to enter the name, the units and the equation. The velocity will be calculated as the rate of change of position, Δ position / Δ time. To do this enter in the equation box "delta" function from the Functions menu and put in parenthesis the correct variable (position) selected from the Variables list. Then type the division sign and again enter the delta function for the time column.

Display the velocity vs. time graph in a new graph window (Insert → Graph).
Carefully highlight the part of this plot in the time interval that corresponds to the one used to obtain the slope of the position vs time graph. The velocity vs time graph is a steady horizontal non-zero line. Apply statistics to the selected part.
Re-size both graph windows, position vs. time and velocity vs. time, to fit them on one page.

Hint: On the top tool bar, click the fourth icon from the right or select Statistics from the Analyze pull down menu.

In Table 3 report the mean value of velocity and standard deviation from the velocity vs time graph. How does the mean value from the displayed statistics compare to the slope of position vs. time plot?

Run 2. Bring the cart back to the left end of track and record a new run for a case when you push the cart a little harder, still away from the motion sensor (or use V_0? Set to # 6). Copy the new data and add it to the same Logger Pro file (as a new data set).

Hint: Choose New Data Set in the pull down Data menu. Scroll all the way up to the first row in the new data set and select the first time cell before pasting.

You can display the new data in the existing graph windows you've created for run 1. Just click the label of the vertical axis on the graph, select "More" and check the appropriate box in the Y-Axis Options window.
Report all further data in tables 4 and 5. Apply linear fit to the relevant portion of the new plot of position vs. time (where the cart was moving). From the new graph of position vs. time how can you tell that the cart was moving faster? From the new graph of position vs. time how can you conclude that the cart was moving with constant velocity? Apply linear fit to the relevant portion of the new plot of position vs. time (where the cart was moving). Make a plot velocity vs time as in run 1. What was the average (mean) velocity of the moving cart in run 2? How does its value compare to the average velocity of the first run? What graphical attribute signifies cart's speed? Write the equation of the cart's motion (5) using the value of the slope as the true value of the average velocity.

Save the Logger Pro file for future reference.
Organize your graph windows so they fit on one page, capture the screen and paste it into a Word file – you will have to attach/insert it into your lab report.

PART 2. Object moving toward the motion sensor.

Run 3. Position the cart at the right end of track, give it a light push toward the sensor (or use V_0? set to # 3) and record its motion. Open a new file in Logger Pro, copy/paste the position data. (Use option in KET "Copy to clipboard"). Complete steps a, b and c from run 1. Record all data in Table 6 ,7 and 8. Answer the questions listed in part 1 about the collected data in run 3. Write the equation of the cart's motion (5) using the value of the slope as the true value of the average velocity.

Run 4. Record yet another run with cart moving towards the motion sensor but faster than in run 3. If necessary you may use V_0? set to # 4. Copy the data to the same file in Logger Pro as a new data set. Add the new set of points to the position vs. time display. Record all the further data points in table 9 and 10. Apply linear fit to the relevant portion of the plot for run 4 and find cart's velocity. What graphical attribute signifies cart's speed? How can you deduct the direction of motion from the position vs. time plot? Write the equation of the cart's motion (5) using the value of the slope as the true value of the average velocity.

Save the Logger Pro file for future reference.
Organize your Logger Pro graph windows so they fit on one page, capture the screen and paste it into a Word file – you will have to attach it to your lab report.

PART 3. Matching position vs. time graphs.

Run 5. In KET virtual labs, click "Clear Line" to remove your previous graph. Select option 2 in the Match Graphs window. Increase the T_{max} to 25 seconds. First describe in details how would you need to move the cart along the track in order to reproduce the graph shown in the position vs. time display. Make it a part of the Discussion in your lab report. Then turn on the sensor and drag the cart in a fashion you had described. How well your recording matches the original graph?
Capture the screen and paste it to a Word file - you will have to attach it to your lab report.

PART 4. Motion with known velocity.
Run 6. Set the known velocity V_0 to – 20 cm/s (negative 20 cm/s). Position the cart in the middle of the track. Predict by calculation (use eqn. 1) the final position of the cart after 2 s from the moment it crosses the 80 cm mark on the track. Show all your calculations in Data Analysis. Turn on the motion sensor and press "Go ➔"" button situated next to V_0. Use data from the Data Table to check your prediction.
Capture the screen and paste it to a Word file - you will have to attach it to your lab report.