MAT 243 ACTIVITY

Example : Find the mistake(s) in the following proofs to show that the product of two odd integers is odd.

Suggested Proof 1: Assume that n is an arbitrary odd integer and m is an arbitrary odd integer.
By definition of odd, $n = 2k + 1$ and $m = 2k + 1$ for some integer k.
Then $mn = (2k + 1)(2k + 1) = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$.
Let $t = 2k^2 + 2k$. Since k is an integer, t is also an integer.
Thus, $mn = 2t + 1$ and, by definition of odd, mn is odd. □

Mistake made: Using the same variable for two different things

correction: By definition of odd, $n = 2k + 1$ for some integer k, $m = 2l + 1$ for some integer l.

Suggested Proof 2: Assume that n is an arbitrary odd integer and m is an arbitrary odd integer.
By definition of odd, $n = 2k + 1$ and $m = 2l + 1$.
Then $mn = (2k + 1)(2l + 1) = 4kl + 2k + 2l + 1 = 2(2kl + k + l) + 1$.
Let $t = 2kl + k + l$. Since k and l are integers, t is also an integer.
Thus, $mn = 2t + 1$ and, by definition of odd, mn is odd. □

Mistake made: Stating a definition incorrectly

correction: By definition of odd, $n = 2k + 1$ for some integer k, $m = 2l + 1$ for some integer l.

Suggested Proof 3: Assume that n is an arbitrary odd integer and m is an arbitrary odd integer.
By definition of odd, $n = 2k + 1$ for some integer k and $m = 2l + 1$ for some integer l.
Then $mn = (2k + 1)(2l + 1) = 4kl + 2k + 2l + 1$.
Thus, mn is odd. □

Mistake made: jumping to the conclusion too early

correction: Follow the last steps of the previous proof.

Suggested Proof 4: Assume that n is an arbitrary odd integer and m is an arbitrary odd integer.
When we multiply together any two odd integers, the product is always odd.
Thus, mn is odd. □

Mistake made: Assuming what you need to show

Suggested Proof 5: Assume that n is an arbitrary odd integer and m is an arbitrary odd integer.
By definition of odd, $n = 2k + 1$ for any integer k and $m = 2l + 1$ for any integer l .
Then $mn = (2k + 1)(2l + 1) = 4kl + 2k + 2l + 1 = 2(2kl + k + l) + 1$.
Let $t = 2kl + k + l$. Since k and l are integers, t is also an integer.
Thus, $mn = 2t + 1$ and, by definition of odd, mn is odd. $\square$

Mistake made: Mixing up *some* and *any* in the definition

Suggested Proof 6: Assume that n is an arbitrary odd integer and m is an arbitrary odd integer.
If n is odd then $n = 2k + 1$ for some integer k and if m is odd then $m = 2l + 1$ for any integer l .
Then $mn = (2k + 1)(2l + 1) = 4kl + 2k + 2l + 1 = 2(2kl + k + l) + 1$.
Let $t = 2kl + k + l$. Since k and l are integers, t is also an integer.
Thus, $mn = 2t + 1$ and, by definition of odd, mn is odd. $\square$

Mistake made: Using *if ... then ...* incorrectly instead of the definition