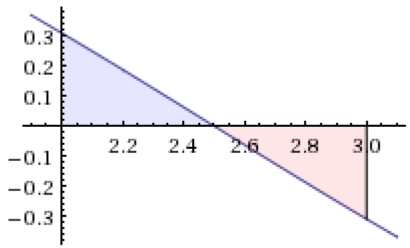
Chapter 7 . 6

Definite integral:

$$\int_{2.}^{3.} \cos\left(\frac{\pi x}{5}\right) dx = 0$$

Visual representation of the integral:



Riemann sums:

More cases

left sum	$\frac{0.309017+5.55112\times10^{-17}\cot\left(\frac{0.314159+0.\,i}{n}\right)}{n} = \left(1.76697\times10^{-16}+0.\,i\right)+\frac{0.309017}{n}+O\left(\left(\frac{1}{n}\right)^2\right)$

$\cot(x)$ is the cotangent function

Enable interactivity

Indefinite integral:

Approximate form

Step-by-step solution

$$\int \cos\left(\frac{\pi x}{5}\right) dx = \frac{5\sin\left(\frac{\pi x}{5}\right)}{\pi} + \text{constant}$$

$$F = kx$$

$$k = \frac{F}{x}$$

$$\frac{F_2}{x_2} = \frac{F_1}{x_1}$$

$$F_1 = 7$$

$$x_1 = 0.4$$

$$x_2 = 0.6$$

$$F_2 = ?$$

$$F_2 = x_2 \frac{F_1}{x_1} = \frac{(0.6).7}{0.4} = whatever.$$

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W= $\frac{1}{2}kx^2$ = $\frac{1}{2}Fx$

2J = [0.5]k(42cm-30cm)^2
2J=[0.5]k(12cm)^2
2J=[0.5]k(0.12)^2 $\rightarrow$ k = 277.778N/m

W_1= (0.5)k(Δx) where Δx = (35cm – 30cm), but must be converted to meters.

W_2= (0.5)k(Δx) where Δx = (40cm – 30cm), but must be converted to meters.

$W_F = W_2 – W_1$

Δs increment of chain
s measured in feet
density of chain 12.5/5= 25/10=2.5 lb/ft
f= 2.5 Δs
d=s
W=f*d=2.5Δs*s , s goes from 5 to 10. This becomes asΔs goes to 0

$\int_5^{10} 2.5s\, \mathrm{d}\, s$ =2.5{s^2/2} $\Big|_5^{10}$ =1.25{10^2-5^2}=1.25(75)=93.75

The first thing to notice is that you don't need to use calculus. We can assume the water is of constant density, so the work required will be equal to the work required to raise a point mass equal to the mass of the water to be pumped and located at its centre of gravity.

The water to be pumped has an elevation of 0.5 to 1 metre.
Thus the centre of gravity of the water to be pumped has an elevation of 0.75 metres, so the point mass has to be raised through a height of 0.25 metres.

Work done = potential energy gained = mgh

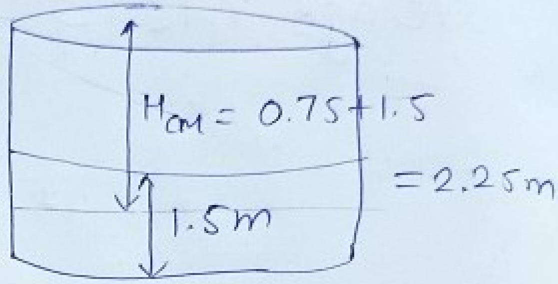
where m = mass, g = acceleration due to gravity and h = height

Mass = volume * density
Thus the mass of the water = 2 m * 1 m * 0.5 m * 1,000 kg/m^3
= 1,000 kg

g = 9.8 m/s^2

So the work required to pump half of the water out of the aquarium
= 1,000 kg * 9.8 m/s^2 * 0.25 m
= 2,450 Joules

Here,



mass of water

$= \pi \times 8^2 \times 1.5 \times 1000$

$= 301536$ Kg

$\therefore H_{CM} = 2.25$ m

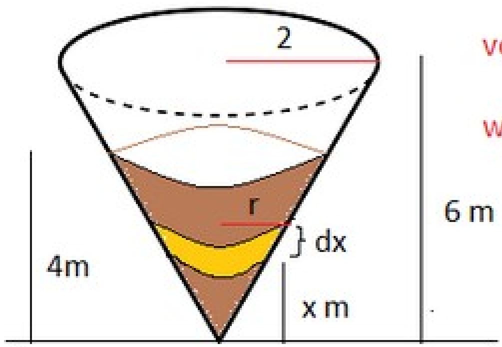
Work = $301536 \times 2.25 \times 9.8$

$= 6.648 \times 10^6$ J

b) $H_{CM} = 2.25 + 2 = 4.25$ m

Work done = $4.25 \times 301536 \times 9.8$

$= 1.255 \times 10^7$ J



volume of liquid element of width dx = pi*r*r*dx

where r is the radius of element.

By similar triangle property,
r/2 = x/6
=> r = x/3

Hence volume = pi*x*x*dx/9

and mass = density * volume
=> mass = 1090*pi*x*x*dx/9
=> mass = 380.48*x*x*dx

potential energy of element = m*g*x = 380.48*x*x*dx*9.8*x = 3728.7*x*x*x*dx
To pump this element we need to bring this element to the top of the cone.
Hence, final potential energy of element would be = 380.48*x*x*dx*9.8*6 = 22372.2*x*x*dx

Hence, work done to pump one element out = 22372.2*x*x*dx - 3728.7*x*x*x*dx

Hence total work done = integration of all such elements from x= 0 to 4m

$$= \int_0^4 22372.2*x*x*dx - \int_0^4 3728.7*x*x*x*dx$$

$$= \left[22372.2*x*x*x/3 - 3728.7*x*x*x*x/4 \right]_0^4$$

= (22372.2*64/3) - (3728.7*256/4)
= 238636.8 Joule

Technically you could just siphon the water out and not do any pump work.

But if you mount a pump on top of the tank, the work required to pump it out can be found by :

1. Calculate the total weight of the water in the tank = 2/3*Pi*r^3*62.5 = 16,362.5 pounds

2. Calculate the center of mass of the hemisphere from the circular top of the tank = 3*r/8 = 1.875 feet

Then the work required is 1.875 feet *16,362.5 pounds = 30679.6875 foot - pounds