

MAT 243 Online Written Homework Assignments for Week 1 (units 1-3)

Multiple Choice Questions:

1. Select the statement that is logically equivalent to “If you can dream it, then you can achieve it.”
 - a. You can’t achieve it unless you dream it.
 - b. You can achieve it only if you dream it.
 - c. Achieving it is a sufficient condition for dreaming it.
 - d. Dreaming it is a necessary condition for achieving it.
 - e. Achieving it is a necessary condition for dreaming it.
2. Given a domain of discourse of all people (meaning all men and all women), find the negation of “There is a woman who owns more than a billion dollars.”
 - a. All women own less than a billion dollars.
 - b. All women own at most a billion dollars.
 - c. There is a man who owns less than a billion dollars.
 - d. There is a man who owns at most a billion dollars.
 - e. There is a woman who owns less than a billion dollars.
3. Suppose $p = \exists x(R(x) \vee S(x))$ and $q = \exists xR(x) \vee \exists xS(x)$ where R and S are propositional functions on some domain of discourse. Then which of the following statements must be true?
 - a. $p \rightarrow q$
 - b. $q \rightarrow p$
 - c. $p \leftrightarrow q$
 - d. None of the above.
4. Let P be the predicate $P(x, y) \equiv$ “ x owns y ”. x represents people; y represents objects. Someone attempted to translate the following statements into equivalent formal expressions. Mark each answer “T” or “F” on the right side for True or False.
 - a. Josh doesn’t own the cat. . $P(\text{Josh}, \neg \text{cat})$
 - b. Everyone owns something. $\forall y \exists x P(x, y)$
 - c. Everything is owned by someone. $\forall x \exists y P(x, y)$
 - d. Some things are not owned by anyone. $\exists y \forall x \neg P(x, y)$
 - e. Some things are not owned by everyone. $\exists y \exists x \neg P(x, y)$

Format of the answer table for multiple choice:

1.	2.	3.	4.
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(your answer for #1-3 is one letter A-E, for #4 a string of five Ts and Fs, like "TTFFT".)

Free Response Questions

5. Write the following statements in "if.. then" form:

- a. "I'll live to the age of 120 only if I don't smoke."
- b. "Rain is necessary for rainbows."
- c. "Being able to see the stars is a sufficient condition for the sky being clear."
- d. "We will never go to Mars, unless we develop nuclear power sources."

6. Negate the following statement: $\exists x \forall y \exists z (xy \leq z)$. You have to simplify until the negation symbol has disappeared.

7. Translate the following statements into standard English and mark them true or false. The domain of discourse is implied, not stated.

Please ponder the meaning of "translation" before you start this problem. A true "translation" into English cannot contain variable names, and it must be immediately comprehensible to a person with only elementary math knowledge. A literal rendition where you simply replace the symbol \exists by "there exists" or other words to that effect, \forall by "for all" or equivalent, and the inequality symbols by "is greater (or equal)" is unacceptable and will earn zero credit.

- a. $\exists x \forall y (x \geq y)$
 - b. $\forall y \exists x (x > y)$
8. Demonstrate using logic algebra that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology. You do not need to identify the logical identities used by name, but you must show each step.