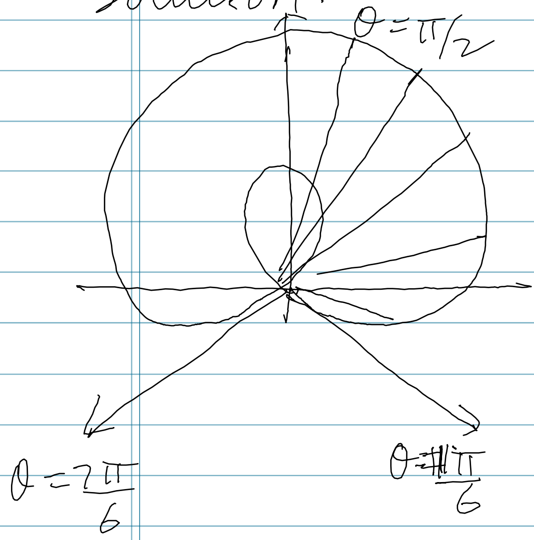
Example Find the area enclosed inside the outer loop of $r = 2 + 4\sin\theta$, $\theta \in [0, 2\pi)$

Solution.



$$r = 2 + 4\sin\theta = 0$$

$$\sin\theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6} \quad \text{on } [0, 2\pi)$$

For $-\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$

$$\frac{1}{2}(\text{Area}) = \frac{1}{2}\int_{-\pi/6}^{\pi/2} (2 + 4\sin\theta)^2 \, d\theta$$

$$= \frac{1}{2}\int_{-\pi/6}^{\pi/2} (4 + 16\sin\theta + 16\sin^2\theta) \, d\theta$$

$$= 2\int_{-\pi/6}^{\pi/2} (1 + 4\sin\theta + 4\sin^2\theta) \, d\theta$$

$$= 2\int_{-\pi/6}^{\pi/2} \left(1 + 4\sin\theta + 4\left(\frac{1 - \cos 2\theta}{2}\right)\right) d\theta$$

$$= 2\int_{-\pi/6}^{\pi/2} (3 + 4\sin\theta - 2\cos 2\theta) \, d\theta$$

$$= 2\left[3\theta - 4\cos\theta - \sin 2\theta\right]_{-\pi/6}^{\pi/2}$$

$$= 2\left[2\pi + \frac{3\sqrt{3}}{2}\right]$$

$$\text{Area} = 4\left[2\pi + \frac{3\sqrt{3}}{2}\right]$$