Chapter 7 Energy of a System

P7.1 (a) $W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

P7.3 **METHOD ONE**

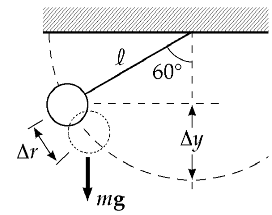

FIG. P7.3

Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then $\cos\theta = \cos(90° + \phi) = -\sin\phi$. The work done by the gravitational force on Batman is

$$W = \int_i^f F\cos\theta\, dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12\text{ m})\,d\phi$$

$$= -mg(12\text{ m})\int_0^{60°}\sin\phi\, d\phi = (-80\text{ kg})(9.8\text{ m/s}^2)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times10^3\text{ J}}$$

METHOD TWO

The force of gravity on Batman is $mg = (80\text{ kg})(9.8\text{ m/s}^2) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y-coordinate below the tree limb is
–12 m. His final y-coordinate is $(-12\text{ m})\cos 60° = -6\text{ m}$. His change in elevation is $-6\text{ m} - (-12\text{ m}) = 6\text{ m}$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}$$

P7.5

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right)\cdot\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = A_xB_x\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\right) + A_xB_y\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}\right) + A_xB_z\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}\right)$$

$$+A_yB_x\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}\right) + A_yB_y\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\right) + A_yB_z\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}\right)$$

$$+A_zB_x\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}\right) + A_zB_y\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{j}}\right) + A_zB_z\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}\right)$$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \boxed{A_xB_x + A_yB_y + A_zB_z}$$

P7.7 (a) $W = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}} = F_x x + F_y y = (6.00)(3.00)\ \text{N}\cdot\text{m} + (-2.00)(1.00)\ \text{N}\cdot\text{m} = \boxed{16.0\ \text{J}}$

(b) $$\theta = \cos^{-1}\left(\frac{\vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}}{F\Delta r}\right) = \cos^{-1}\frac{16}{\sqrt{\left((6.00)^2+(-2.00)^2\right)\left((3.00)^2+(1.00)^2\right)}} = \boxed{36.9^\circ}$$

P7.10 $\vec{\mathbf{A}} - \vec{\mathbf{B}} = \left(3.00\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}\right) - \left(-\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} + 5.00\hat{\mathbf{k}}\right)$

$\vec{\mathbf{A}} - \vec{\mathbf{B}} = 4.00\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6.00\hat{\mathbf{k}}$

$\vec{\mathbf{C}}\cdot\left(\vec{\mathbf{A}} - \vec{\mathbf{B}}\right) = \left(2.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\cdot\left(4.00\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6.00\hat{\mathbf{k}}\right) = 0 + (-2.00) + (+18.0) = \boxed{16.0}$

P7.14 $$W = \int_i^f \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \int_0^{5\text{ m}} \left(4x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}}\right)\ \text{N}\cdot dx\hat{\mathbf{i}}$$

$$\int_0^{5\text{ m}} (4\ \text{N/m})x\,dx + 0 = (4\ \text{N/m})\left.\frac{x^2}{2}\right|_0^{5\text{ m}} = \boxed{50.0\ \text{J}}$$

P7.16 (a) Spring constant is given by $F = kx$

$$k = \frac{F}{x} = \frac{(230\ \text{N})}{(0.400\ \text{m})} = \boxed{575\ \text{N/m}}$$

(b) Work $= F_{\text{avg}}x = \frac{1}{2}(230\ \text{N})(0.400\ \text{m}) = \boxed{46.0\ \text{J}}$

***P7.20** The spring exerts on each block an outward force of magnitude

$$|F_s| = kx = (3.85\ \text{N/m})(0.08\ \text{m}) = 0.308\ \text{N}$$

Take the $+x$ direction to the right. For the light block on the left, the vertical forces are given by $F_g = mg = (0.25\ \text{kg})(9.8\ \text{m/s}^2) = 2.45\ \text{N}$, $\sum F_y = 0$, $n - 2.45\ \text{N} = 0$, $n = 2.45\ \text{N}$. Similarly for the heavier block $n = F_g = (0.5\ \text{kg})(9.8\ \text{m/s}^2) = 4.9\ \text{N}$

(a) For the block on the left, $\sum F_x = ma_x$, $-0.308\ \text{N} = (0.25\ \text{kg})a$,

$a = \boxed{-1.23\ \text{m/s}^2}$. For the heavier block, $+0.308\ \text{N} = (0.5\ \text{kg})a$,

$a = \boxed{0.616\ \text{m/s}^2}$.

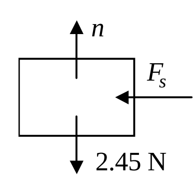

FIG. P7.20

(b) For the block on the left, $f_k = \mu_k n = 0.1(2.45\ \text{N}) = 0.245\ \text{N}$

$\sum F_x = ma_x$

$-0.308\ \text{m/s}^2 + 0.245\ \text{N} = (0.25\ \text{kg})a$

$a = \boxed{-0.252\ \text{m/s}^2 \quad \text{if the force of static friction is not too large.}}$

For the block on the right, $f_k = \mu_k n = 0.490 \text{ N}$. The maximum force of static friction would be larger, so no motion would begin and the acceleration is $\boxed{\text{zero}}$.

(c) Left block: $f_k = 0.462(2.45 \text{ N}) = 1.13 \text{ N}$. The maximum static friction force would be larger, so the spring force would produce no motion of this block or of the right-hand block, which could feel even more friction force. For both $a = \boxed{0}$.

P7.25 (a) The radius to the object makes angle θ with the horizontal, so its weight makes angle θ with the negative side of the x-axis, when we take the x-axis in the direction of motion tangent to the cylinder.

$$\sum F_x = ma_x$$
$$F - mg\cos\theta = 0$$
$$F = \boxed{mg\cos\theta}$$

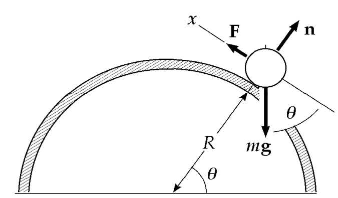

FIG. P7.25

(b) $W = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$

We use radian measure to express the next bit of displacement as $dr = Rd\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg\cos\theta R d\theta = mgR\sin\theta\Big|_0^{\pi/2}$$
$$W = mgR(1-0) = \boxed{mgR}$$

P7.29 (a) $K_A = \dfrac{1}{2}(0.600 \text{ kg})(2.00 \text{ m/s})^2 = \boxed{1.20 \text{ J}}$

(b) $\dfrac{1}{2}mv_B^2 = K_B$: $v_B = \sqrt{\dfrac{2K_B}{m}} = \sqrt{\dfrac{(2)(7.50)}{0.600}} = \boxed{5.00 \text{ m/s}}$

(c) $\sum W = \Delta K = K_B - K_A = \dfrac{1}{2}m\left(v_B^2 - v_A^2\right) = 7.50 \text{ J} - 1.20 \text{ J} = \boxed{6.30 \text{ J}}$

P7.31 $\vec{\mathbf{v}}_i = \left(6.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right) = \text{ m/s}$

(a) $v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{40.0} \text{ m/s}$

$K_i = \dfrac{1}{2}mv_i^2 = \dfrac{1}{2}(3.00 \text{ kg})\left(40.0 \text{ m}^2/\text{s}^2\right) = \boxed{60.0 \text{ J}}$

(b) $\vec{\mathbf{v}}_f = 8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$

$v_f^2 = \vec{\mathbf{v}}_f \cdot \vec{\mathbf{v}}_f = 64.0 + 16.0 = 80.0 \text{ m}^2/\text{s}^2$

$\Delta K = K_f - K_i = \dfrac{1}{2}m\left(v_f^2 - v_i^2\right) = \dfrac{3.00}{2}(80.0) - 60.0 = \boxed{60.0 \text{ J}}$

P7.36 (a) $v_f = 0.096\left(3\times 10^8\ \text{m/s}\right) = 2.88\times 10^7\ \text{m/s}$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}\left(9.11\times 10^{-31}\ \text{kg}\right)\left(2.88\times 10^7\ \text{m/s}\right)^2 = \boxed{3.78\times 10^{-16}\ \text{J}}$$

(b) $K_i + W = K_f$: $\quad 0 + F\Delta r\cos\theta = K_f$

$$F(0.028\ \text{m})\cos 0° = 3.78\times 10^{-16}\ \text{J}$$

$$F = \boxed{1.35\times 10^{-14}\ \text{N}}$$

(c) $\sum F = ma$; $\quad a = \dfrac{\sum F}{m} = \dfrac{1.35\times 10^{-14}\ \text{N}}{9.11\times 10^{-31}\ \text{kg}} = \boxed{1.48\times 10^{+16}\ \text{m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t \quad 2.88\times 10^7\ \text{m/s} = 0 + \left(1.48\times 10^{16}\ \text{m/s}^2\right)t$

$$t = \boxed{1.94\times 10^{-9}\ \text{s}}$$

Check: $\quad x_f = x_i + \dfrac{1}{2}\left(v_{xi} + v_{xf}\right)t$

$$0.028\ \text{m} = 0 + \frac{1}{2}\left(0 + 2.88\times 10^7\ \text{m/s}\right)t$$

$$t = 1.94\times 10^{-9}\ \text{s}$$

P7.39 $F_g = mg = (4.00\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 39.2\ \text{N}$

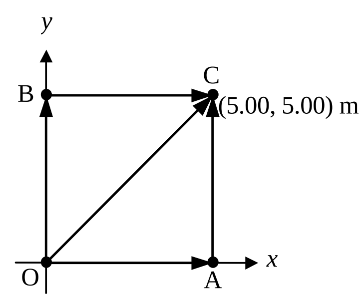

FIG. P7.39

(a) Work along OAC = work along OA + work along AC

$= F_g(\text{OA})\cos 90.0° + F_g(\text{AC})\cos 180°$

$= (39.2\ \text{N})(5.00\ \text{m}) + (39.2\ \text{N})(5.00\ \text{m})(-1)$

$= \boxed{-196\ \text{J}}$

(b) *W* along OBC = *W* along OB + *W* along BC

$= (39.2\ \text{N})(5.00\ \text{m})\cos 180° + (39.2\ \text{N})(5.00\ \text{m})\cos 90.0°$

$= \boxed{-196\ \text{J}}$

(c) Work along OC $= F_g(\text{OC})\cos 135°$

$= (39.2\ \text{N})\left(5.00\times\sqrt{2}\ \text{m}\right)\left(-\dfrac{1}{\sqrt{2}}\right) = \boxed{-196\ \text{J}}$

$\boxed{\text{The results should all be the same, since gravitational forces are conservative.}}$

P7.44 (a) $U = -\displaystyle\int_0^x \left(-Ax + Bx^2\right)dx = \boxed{\dfrac{Ax^2}{2} - \dfrac{Bx^3}{3}}$

(b) $\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F dx = \frac{A\left[\left(3.00^2\right)-\left(2.00\right)^2\right]}{2} - \frac{B\left[\left(3.00\right)^3-\left(2.00\right)^3\right]}{3} = \boxed{\frac{5.00}{2}A - \frac{19.0}{3}B}$

$\Delta K = \boxed{\left(-\frac{5.00}{2}A + \frac{19.0}{3}B\right)}$

P7.46 $F_x = -\frac{\partial U}{\partial x} = -\frac{\partial\left(3x^3y-7x\right)}{\partial x} = -\left(9x^2y-7\right) = 7-9x^2y$

$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial\left(3x^3y-7x\right)}{\partial y} = -\left(3x^3-0\right) = -3x^3$

Thus, the force acting at the point $(x,\ y)$ is $\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = \boxed{\left(7-9x^2y\right)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}}$.

P7.49 (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2}-L$ and the force it exerts is $k\left(\sqrt{x^2+L^2}-L\right)$ toward its fixed end. The *y* components of the two spring forces add to zero. Their *x* components add to

$$\vec{\mathbf{F}} = -2\hat{\mathbf{i}}k\left(\sqrt{x^2+L^2}-L\right)\frac{x}{\sqrt{x^2+L^2}} = \boxed{-2kx\hat{\mathbf{i}}\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)}.$$

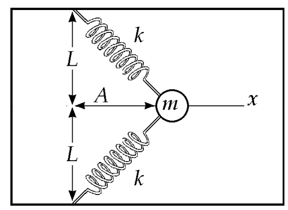

FIG. P7.49

(b) Choose $U = 0$ at $x = 0$. Then at any point the potential energy of the system is

$$U(x) = -\int_0^x F_x dx = -\int_0^x\left(-2kx + \frac{2kLx}{\sqrt{x^2+L^2}}\right)dx = 2k\int_0^x x dx - 2kL\int_0^x \frac{x}{\sqrt{x^2+L^2}}dx$$

$$U(x) = \boxed{kx^2 + 2kL\left(L-\sqrt{x^2+L^2}\right)}$$

(c) $U(x) = 40.0x^2 + 96.0\left(1.20-\sqrt{x^2+1.44}\right)$

For negative *x*, $U(x)$ has the same value as for positive *x*. The only equilibrium point (i.e., where $F_x = 0$) is $\boxed{x=0}$.

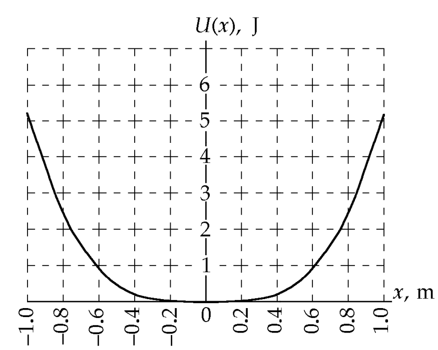

FIG. P7.49(c)

(d) $K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

$0 + 0.400\text{ J} + 0 = \frac{1}{2}(1.18\text{ kg})v_f^2 + 0$

$v_f = \boxed{0.823\text{ m/s}}$

P7.51 At start, $\vec{\mathbf{v}} = (40.0\text{ m/s})\cos 30.0°\hat{\mathbf{i}} + (40.0\text{ m/s})\sin 30.0°\hat{\mathbf{j}}$

At apex, $\vec{\mathbf{v}} = (40.0\text{ m/s})\cos 30.0°\hat{\mathbf{i}} + 0\hat{\mathbf{j}} = (34.6\text{ m/s})\hat{\mathbf{i}}$

And $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150\text{ kg})(34.6\text{ m/s})^2 = \boxed{90.0\text{ J}}$