Answer to 3.6 problem 1(a)(b)

1(a) The RREF of the matrix is $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

A basis of the row space consists of the non zero rows of the RREF:

$$\{[1,0,2]^T, [0,1,0]^T\}$$

Since the pivots are in columns 1 and 2, a basis of the column space consists of the first and second column of the original matrix:

$$\{[1,2,4]^T, [3,1,7]^T\}$$

x_3 is a free variable. Letting $x_3 = s$ and solving for x_1 and x_2, yields

$$\begin{aligned} x_1 &= -2s \\ x_2 &= 0 \\ x_3 &= s \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = s \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

Thus $\{[-2,0,1]^T\}$ is a basis for the Nullspace of A.

Note: The rank of this matrix is 2 (dimension of column space = dimension of row space).
The nullity is 1 (dimension of Nullspace).

1(b) The RREF of the matrix is $\begin{bmatrix} 1 & 0 & 0 & -10/7 \\ 0 & 1 & 0 & -2/7 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

A basis of the row space consists of the non zero rows of the RREF:

$$\{[1,0,0,-10/7]^T, [0,1,0,-2/7]^T, [0,0,1,0]^T\}$$

Since the pivots are in columns 1, 2 and 3, a basis of the column space consists of the first, second and third column of the original matrix:

$$\{[-3,1,3]^T, [1,2,8]^T, [3,-1,4]^T\}$$

x_4 is a free variable. Letting $x_4 = s$ and solving for $x_{1,} x_2$ and x_3, yields

$$\begin{aligned} x_1 &= 10/7 s \\ x_2 &= 2/7 s \\ x_3 &= 0 \\ x_4 &= s \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = s \begin{bmatrix} 10/7 \\ 2/7 \\ 0 \\ 1 \end{bmatrix}$$

Thus $\{[10/7, 2/7, 0, 1]^T\}$ is a basis for the Nullspace of A.

Note: The rank of this matrix is 3 (dimension of column space = dimension of row space).
The nullity is 1 (dimension of Nullspace).