MAT 243 ONLINE WRITTEN HW 6 SOLUTIONS

(1) **(4 pts)** Fill in the blank in the statements below:

(a) When making a sequence of choices in counting we are using the Product Rule.

(b) We are using the Sum Rule when choosing among mutually exclusive alternatives.

(c) We are using Permutation when we have an ordered list of items.

(d) The slightly more advanced version of the Pigeon Hole principal states that if there are k boxes and you place N objects into them then at least one box will contain at least $\lceil \frac{N}{k} \rceil$ objects.

(2) **(15 pts)** Given 10 chips each of them of different colors (including red and white).

How many different ways can we put 6 of the 10 chips in a line

(a) with no restriction?

Answer: $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = P(10, 6)$ since the order matters.

(b) such that the red chip must be used?

Answer: $6 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 6 \cdot P(9, 5)$. 6 ways to place the red chip and $P(9, 5)$ ways to order the other 5 chips.

Second solution: total - number of ways without the red chip:

$$P(10, 6) - P(9, 6) = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 - 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = (10 - 4) \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$$

(c) such that the red chip is used and the white is not?

Answer: $6 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6 \cdot P(8, 5)$. 6 ways to place the red chip and $P(8, 5)$ ways to order the other 5 chips not including the white one.

Second solution: total without the white - number of ways without the red and the white chips:

$$P(9, 6) - P(8, 6) = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 - 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = (9 - 3) \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$$

(d) such that both the red and the white chip is used ?

Answer: $6 \cdot 5 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 6 \cdot 5 \cdot P(8,4)$. 6 ways to place the red chip, 5 ways to place the white chip and $P(8,4)$ ways to order the other 4 chips.

Second solution: total - number of ways without the white - number of ways without the red + number of ways without the red and the white chips:

$P(10,6) - 2\cdot$ answer from part b) + answer from part c)

(e) such that both the red and the white chip is used and they are next to each other?

Answer: $2 \cdot 5 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 2 \cdot 5 \cdot P(8,4)$. Stick the red and the white chips together into a "super chip". You can do it two different ways WR or RW. 5 ways to place the "super chip" and $P(8,4)$ ways to order the other 4 chips.

(3) **(10 pts)** You have \$15,000 to invest in the stock market and your financial advisor gives you a list of 12 possible stock options to choose from. How many different ways can you do that if you choose 5 stock options to invest

(a) \$1000, \$2000, \$3000, \$4000 and \$5000 in the stock options? Justify your answer.

Answer: $12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 = P(12,5)$ since the order matters due to different amount invested.

(b) \$3000 in each of the 5 options? Justify your answer.

Answer: $C(12,5)$ since the order does not matter due to same amount invested.

(c) \$6000 in two and \$1000 in the other three options? Justify your answer.

Answer: $C(12,2) \cdot C(10,3)$. First choose which 2 you invest \$6000 in and then choose which 3 you invest \$1000 in from the remaining 10 options. The order does not matter due to same amount invested.

(4) **(10 pts)** A printer is printing out 3-digit numbers between 100-999 such that the digits are not repeated.

(a) If 2000 such numbers are printed out, at least how many if them will be identical? Justify your answer.

$N = 2000$ is the number of objects and $k = 9 \cdot 9 \cdot 8 = 648$ is the number of different 3-digit numbers with no repeated digits which are the boxes. Then, by the GPHP at least $\lceil \frac{2000}{648} \rceil = 4$ will be identical.

Answer: At least 4 identical numbers will be printed.

(b) At least how many of the numbers should be printed, so that at least 3 of them will be identical? Justify your answer.

Using GPHP and the same boxes as in part a) $k = 648$. The question is the smallest N such that $\lceil \frac{N}{648} \rceil = 3$. Thus $N = (3-1)\cdot 648 + 1 = 1297$.

Answer: At least 1297 3-digit numbers with repeated digits should be printed.

(c) At least how many of the numbers should be printed, so that the number 243 is printed at least 6 times? Justify your answer.

Answer: Cannot be determined, the PHP does not guarantee that any of the specific values will be printed a certain number of times.

(5) **(8 pts)** A bag contains 100 apples, 100 bananas, 100 oranges and 100 pears. If someone randomly picks a fruit from the bag every second,

(a) how long will it be that she is sure to have at least a dozen of the same kind of fruit?

We will use GPHP again where $k = 4$ is the number of different fruits are the boxes. To answer the question first we need to find the smallest N such that $\lceil \frac{N}{4} \rceil = 12$. Thus $N = (12-1)\cdot 4 + 1 = 45$.

Answer: It will take 45 sec to be sure to have at least a dozen of the same kind of fruit.

(b) how long will it be that she is sure to have at least a dozen of bananas?

As the worst case scenario, she needs to take out all the other 3 fruits and 12 bananas to be sure to have at least 12 bananas. That means she needs to take out $3\cdot 100+12 = 312$ fruits.

Answer: It will take 5 minutes and 15 sec to be sure she has at least a dozen of bananas.

(6) **(8 pts)** Use then binomial theorem to find the following. Justify your answer.

(a) The coefficient in front of the term x^{10} in $(x^2-6)^8$.

Note $x^{10} = (x^2)^5$, thus $k = 5$ and $n = 8$. Thus the term in question is $C(8,5)(x^2)^5(-6)^3$.

Answer: The coefficient in the term is: $-C(8,5)(6)^3$

(b)

$$\sum_{k=3}^{50} C(50,k)5^k$$

Notice that the above sum is the same as

$$\sum_{k=3}^{50} C(50,k)5^k(1)^{n-k} = \sum_{k=0}^{50} C(50,k)5^k(1)^{n-k} - 1 - C(50,1)\cdot 5 - C(50,2)\cdot 5^2$$

The last three terms are corresponding to $k = 0, 1, 2$. Using the binomial theorem gives:

$$\sum_{k=0}^{50} C(50,k)5^k(1)^{n-k} = (5+1)^{50} = 6^{50}$$

Answer: $6^{50} - 1 - 50 \cdot 5 - C(50,2) \cdot 5^2$.