Chapter 13 Universal Gravitation

***P13.6** (a) The Sun-Earth distance is 1.496×10^{11} m and the Earth-Moon distance is 3.84×10^{8} m, so the distance from the Sun to the Moon during a solar eclipse is

$$1.496\times10^{11}\ \text{m} - 3.84\times10^{8}\ \text{m} = 1.492\times10^{11}\ \text{m}$$

The mass of the Sun, Earth, and Moon are $M_S = 1.99\times10^{30}$ kg

$M_E = 5.98\times10^{24}$ kg

and $M_M = 7.36\times10^{22}$ kg

We have $F_{SM} = \dfrac{Gm_1m_2}{r^2} = \dfrac{\left(6.67\times10^{-11}\right)\left(1.99\times10^{30}\right)\left(7.36\times10^{22}\right)}{\left(1.492\times10^{11}\right)^2} = \boxed{4.39\times10^{20}\ \text{N}}$

(b) $$F_{EM} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\right)\left(7.36\times10^{22}\right)}{\left(3.84\times10^{8}\right)^2} = \boxed{1.99\times10^{20}\ \text{N}}$$

(c) $$F_{SE} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\right)\left(5.98\times10^{24}\right)}{\left(1.496\times10^{11}\right)^2} = \boxed{3.55\times10^{22}\ \text{N}}$$

(d) The force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon. In a sense, the Moon orbits the Sun more than it orbits the Earth. The Moon's path is everywhere concave toward the Sun. Only by subtracting out the solar orbital motion of the Earth-Moon system do we see the Moon orbiting the center of mass of this system.

P13.9 $a = \dfrac{MG}{\left(4R_E\right)^2} = \dfrac{9.80\ \text{m/s}^2}{16.0} = \boxed{0.613\ \text{m/s}^2}$ toward the Earth.

P13.11 $$g = \frac{GM}{R^2} = \frac{G\rho\left(4\pi R^3/3\right)}{R^2} = \frac{4}{3}\pi G\rho R$$

If $$\frac{g_M}{g_E} = \frac{1}{6} = \frac{4\pi G\rho_M R_M/3}{4\pi G\rho_E R_E/3}$$

then $$\frac{\rho_M}{\rho_E} = \left(\frac{g_M}{g_E}\right)\left(\frac{R_E}{R_M}\right) = \left(\frac{1}{6}\right)(4) = \boxed{\frac{2}{3}}$$

P13.14 By Kepler's Third Law, $T^2 = ka^3$ (a = semi-major axis)

For any object orbiting the Sun, with T in years and a in A.U., $k = 1.00$. Therefore, for Comet Halley

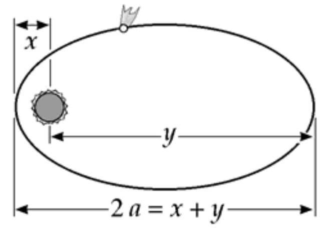

FIG. P13.14

$$(75.6)^2 = (1.00)\left(\frac{0.570 + y}{2}\right)^3$$

The farthest distance the comet gets from the Sun is

$y = 2(75.6)^{2/3} - 0.570 = \boxed{35.2 \text{ A.U.}}$ (out around the orbit of Pluto).

P13.15 $T^2 = \dfrac{4\pi^2 a^3}{GM}$ (Kepler's third law with $m << M$)

$$M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2\left(4.22\times10^8 \text{ m}\right)^3}{\left(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.77\times 86\,400 \text{ s}\right)^2} = \boxed{1.90\times10^{27} \text{ kg}}$$

(approximately 316 Earth masses)

P13.21 $\vec{\mathbf{g}} = \dfrac{Gm}{l^2}\hat{\mathbf{i}} + \dfrac{Gm}{l^2}\hat{\mathbf{j}} + \dfrac{Gm}{2l^2}\left(\cos 45.0°\hat{\mathbf{i}} + \sin 45.0\hat{\mathbf{j}}\right)$

so $\vec{\mathbf{g}} = \dfrac{GM}{l^2}\left(1 + \dfrac{1}{2\sqrt{2}}\right)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$ or

$$\vec{\mathbf{g}} = \boxed{\frac{Gm}{l^2}\left(\sqrt{2} + \frac{1}{2}\right) \text{ toward the opposite corner}}$$

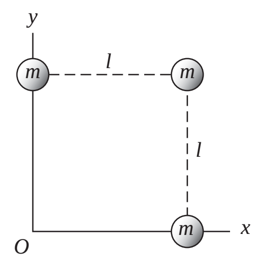

FIG. P13.21

P13.22 (a)

$$F = \frac{GMm}{r^2} = \frac{\left(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left[100\left(1.99\times10^{30} \text{ kg}\right)\left(10^3 \text{ kg}\right)\right]}{\left(1.00\times10^4 \text{ m} + 50.0 \text{ m}\right)^2} = \boxed{1.31\times10^{17} \text{ N}}$$

(b) $$\Delta F = \frac{GMm}{r_{\text{front}}^2} - \frac{GMm}{r_{\text{back}}^2}$$

$$\Delta g = \frac{\Delta F}{m} = \frac{GM\left(r_{\text{back}}^2 - r_{\text{front}}^2\right)}{r_{\text{front}}^2 r_{\text{back}}^2}$$

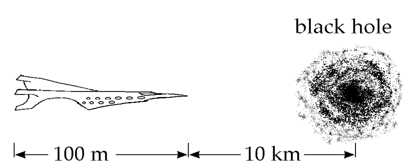

FIG. P13.22

$$\Delta g = \frac{\left(6.67\times10^{-11}\right)\left[100\left(1.99\times10^{30}\right)\right]\left[\left(1.01\times10^4 \text{ m}\right)^2 - \left(1.00\times10^4 \text{ m}\right)^2\right]}{\left(1.00\times10^4 \text{ m}\right)^2\left(1.01\times10^4 \text{ m}\right)^2}$$

$$\Delta g = \boxed{2.62\times10^{12} \text{ N/kg}}$$

P13.24 (a)

$$U=-\frac{GM_Em}{r}=-\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(100\ \text{kg})}{(6.37+2.00)\times10^6\ \text{m}}=\boxed{-4.77\times10^9\ \text{J}}$$

(b), (c) Planet and satellite exert forces of equal magnitude on each other, directed downward on the satellite and upward on the planet.

$$F=\frac{GM_Em}{r^2}=\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(100\ \text{kg})}{\left(8.37\times10^6\ \text{m}\right)^2}=\boxed{569\ \text{N}}$$

P13.25 (a) $$\rho=\frac{M_S}{\frac{4}{3}\pi r_E^2}=\frac{3\left(1.99\times10^{30}\ \text{kg}\right)}{4\pi\left(6.37\times10^6\ \text{m}\right)^3}=\boxed{1.84\times10^9\ \text{kg/m}^3}$$

(b) $$g=\frac{GM_S}{r_E^2}=\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)}{\left(6.37\times10^6\ \text{m}\right)^2}=\boxed{3.27\times10^6\ \text{m/s}^2}$$

(c)

$$U_g=-\frac{GM_Sm}{r_E}=-\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)(1.00\ \text{kg})}{6.37\times10^6\ \text{m}}=\boxed{-2.08\times10^{13}\ \text{J}}$$

P13.31

$$\frac{1}{2}mv_i^2+GM_Em\left(\frac{1}{r_f}-\frac{1}{r_i}\right)=\frac{1}{2}mv_f^2$$

$$\frac{1}{2}v_i^2+GM_E\left(0-\frac{1}{R_E}\right)=\frac{1}{2}v_f^2$$

or $$v_f^2=v_1^2-\frac{2GM_E}{R_E}$$

and $$v_f=\left(v_1^2-\frac{2GM_E}{R_E}\right)^{1/2}$$

$$v_f=\left[\left(2.00\times10^4\right)^2-1.25\times10^8\right]^{1/2}=\boxed{1.66\times10^4\ \text{m/s}}$$

***P13.32** $$E_{\text{tot}}=-\frac{GMm}{2r}$$

$$\Delta E=\frac{GMm}{2}\left(\frac{1}{r_i}-\frac{1}{r_f}\right)=\frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)}{2}\frac{10^3\ \text{kg}}{10^3\ \text{m}}\left(\frac{1}{6\,370+100}-\frac{1}{6\,370+200}\right)$$

$$\Delta E=4.69\times10^8\ \text{J}=\boxed{469\ \text{MJ}}$$

Both in the original orbit and in the final orbit, the total energy is negative, with an absolute value equal to the positive kinetic energy. The potential energy is negative and twice as large as the total energy. As the satellite is lifted from the lower to the higher orbit, the gravitational energy increases, the kinetic energy decreases, and the total energy increases. The value of each becomes closer to zero. Numerically, the gravitational energy increases by 938 MJ, the kinetic energy decreases by 469 MJ, and the total energy increases by 469 MJ.

P13.33 To obtain the orbital velocity, we use

$$\sum F = \frac{mMG}{R^2} = \frac{mv^2}{R}$$

or $$v = \sqrt{\frac{MG}{R}}$$

We can obtain the escape velocity from

$$\frac{1}{2}mv_{esc}^2 = \frac{mMG}{R}$$

or $$v_{esc} = \sqrt{\frac{2MG}{R}} = \boxed{\sqrt{2}v}$$

***P13.34**

Gravitational screening does not exist. The presence of the satellite has no effect on the force the planet exerts on the rocket.

The rocket is in a potential well at Ganymede's surface with energy

$$U_1 = -\frac{Gm_1m_2}{r} = -\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2 m_2\left(1.495\times10^{23}\ \text{kg}\right)}{\text{kg}^2\left(2.64\times10^6\ \text{m}\right)}$$

$$U_1 = -3.78\times10^6 m_2\ \ \text{m}^2/\text{s}^2$$

The potential well from Jupiter at the distance of Ganymede is

$$U_2 = -\frac{Gm_1m_2}{r} = -\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2 m_2\left(1.90\times10^{27}\ \text{kg}\right)}{\text{kg}^2\left(1.071\times10^9\ \text{m}\right)}$$

$$U_2 = -1.18\times10^8 m_2\ \ \text{m}^2/\text{s}^2$$

To escape from both requires

$$\frac{1}{2}m_2v_{esc}^2 = +\left(3.78\times10^6 + 1.18\times10^8\right)m_2\ \ \text{m}^2/\text{s}^2$$

$$v_{esc} = \sqrt{2\times1.22\times10^8\ \ \text{m}^2/\text{s}^2} = \boxed{15.6\ \text{km/s}}$$

P13.41 Let m represent the mass of the spacecraft, r_E the radius of the Earth's orbit, and x the distance from Earth to the spacecraft.

The Sun exerts on the spacecraft a radial inward force of $$F_s = \frac{GM_sm}{(r_E - x)^2}$$

while the Earth exerts on it a radial outward force of $$F_E = \frac{GM_Em}{x^2}$$

The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year.

Thus,

$$F_S - F_E = \frac{GM_S m}{(r_E - x)^2} - \frac{GM_E m}{x^2} = \frac{mv^2}{(r_E - x)} = \frac{m}{(r_E - x)}\left[\frac{2\pi(r_E - x)}{T}\right]^2$$

which reduces to $$\frac{GM_S}{(r_E - x)^2} - \frac{GM_E}{x^2} = \frac{4\pi^2(r_E - x)}{T^2} \qquad (1)$$

Cleared of fractions, this equation would contain powers of *x* ranging from the fifth to the zeroth. We do not solve it algebraically. We may test the assertion that *x* is between 1.47×10^9 m and 1.48×10^9 m by substituting both of these as trial solutions, along with the following data: $M_S = 1.991 \times 10^{30}$ kg, $M_E = 5.983 \times 10^{24}$ kg, $r_E = 1.496 \times 10^{11}$ m, and $T = 1.000 \text{ yr} = 3.156 \times 10^7$ s.

With $x = 1.47 \times 10^9$ m substituted into equation (1), we obtain

$$6.052 \times 10^{-3} \text{ m/s}^2 - 1.85 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

or $$5.868 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

With $x = 1.48 \times 10^9$ m substituted into the same equation, the result is

$$6.053 \times 10^{-3} \text{ m/s}^2 - 1.82 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2$$

or $$5.870\,9 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2$$

Since the first trial solution makes the left-hand side of equation (1) slightly less than the right hand side, and the second trial solution does the opposite, the true solution is determined as between the trial values. To three-digit precision, it is 1.48×10^9 m.

As an equation of fifth degree, equation (1) has five roots. The Sun-Earth system has five Lagrange points, all revolving around the Sun synchronously with the Earth. The SOHO and ACE satellites are at one. Another is beyond the far side of the Sun. Another is beyond the night side of the Earth. Two more are on the Earth's orbit, ahead of the planet and behind it by 60°. Plans are under way to gain perspective on the Sun by placing a spacecraft at one of these two co-orbital Lagrange points. The Greek and Trojan asteroids are at the co-orbital Lagrange points of the Jupiter-Sun system.

P13.47 From the walk, $2\pi r = 25\,000$ m. Thus, the radius of the planet is

$$r = \frac{25\,000 \text{ m}}{2\pi} = 3.98 \times 10^3 \text{ m}$$

From the drop:

$$\Delta y = \frac{1}{2}gt^2 = \frac{1}{2}g(29.2 \text{ s})^2 = 1.40 \text{ m}$$

so,

$$g = \frac{2(1.40 \text{ m})}{(29.2 \text{ s})^2} = 3.28 \times 10^{-3} \text{ m/s}^2 = \frac{MG}{r^2}$$

$$\therefore M = \boxed{7.79 \times 10^{14} \text{ kg}}$$

P13.51 (a) At infinite separation $U = 0$ and at rest $K = 0$. Since energy of the two-planet system is conserved we have,

$$0 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{Gm_1 m_2}{d} \qquad (1)$$

The initial momentum of the system is zero and momentum is conserved.

Therefore,

$$0 = m_1 v_1 - m_2 v_2 \qquad (2)$$

Combine equations (1) and (2):

$$\boxed{v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}} \quad \text{and}$$

$$\boxed{v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}}$$

Relative velocity

$$v_r = v_1 - (-v_2) = \boxed{\sqrt{\frac{2G(m_1+m_2)}{d}}}$$

(b) Substitute given numerical values into the equation found for v_1 and v_2 in part (a) to find

$$v_1 = 1.03\times10^4 \text{ m/s} \quad \text{and} \quad v_2 = 2.58\times10^3 \text{ m/s}$$

Therefore,

$$K_1 = \frac{1}{2}m_1 v_1^2 = \boxed{1.07\times10^{32}\text{ J}} \quad \text{and}$$

$$K_2 = \frac{1}{2}m_2 v_2^2 = \boxed{2.67\times10^{31}\text{ J}}$$

P13.56 (a) From the data about perigee, the energy of the satellite-Earth system is

$$E = \frac{1}{2}mv_p^2 - \frac{GM_E m}{r_p} = \frac{1}{2}(1.60)(8.23\times10^3)^2 - \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.60)}{7.02\times10^6}$$

or

$$E = \boxed{-3.67\times10^7\text{ J}}$$

(b) $L = mvr\sin\theta = mv_p r_p \sin 90.0° = (1.60\text{ kg})(8.23\times10^3\text{ m/s})(7.02\times10^6\text{ m})$

$= \boxed{9.24\times10^{10}\text{ kg}\cdot\text{m}^2/\text{s}}$

(c) Since both the energy of the satellite-Earth system and the angular momentum of the Earth are conserved,

at apogee we must have $\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = E$

and $$mv_a r_a \sin 90.0° = L$$

Thus,

$$\frac{1}{2}(1.60)v_a^2 - \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.60)}{r_a} = -3.67\times10^7\text{ J}$$

and $$(1.60\text{ kg})v_a r_a = 9.24\times10^{10}\text{ kg}\cdot\text{m}^2/\text{s}$$

Solving simultaneously,

$$\frac{1}{2}(1.60)v_a^2 - \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.60)(1.60)v_a}{9.24\times10^{10}} = -3.67\times10^7$$

which reduces to $$0.800v_a^2 - 11\,046v_a + 3.672\,3\times10^7 = 0$$

so $$v_a = \frac{11\,046 \pm \sqrt{(11\,046)^2 - 4(0.800)(3.672\,3\times10^7)}}{2(0.800)}$$

This gives $v_a = 8\,230\text{ m/s}$ or $\boxed{5\,580\text{ m/s}}$. The smaller answer refers to the velocity at the apogee while the larger refers to perigee.

Thus,

$$r_a = \frac{L}{mv_a} = \frac{9.24\times10^{10}\text{ kg}\cdot\text{m}^2/\text{s}}{(1.60\text{ kg})(5.58\times10^3\text{ m/s})} = \boxed{1.04\times10^7\text{ m}}$$

(d) The major axis is $2a = r_p + r_a$, so the semi-major axis is

$$a = \frac{1}{2}(7.02\times10^6\text{ m} + 1.04\times10^7\text{ m}) = \boxed{8.69\times10^6\text{ m}}$$

(e) $$T = \sqrt{\frac{4\pi^2 a^3}{GM_E}} = \sqrt{\frac{4\pi^2(8.69\times10^6\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}}$$

$$T = 8\,060\text{ s} = \boxed{134\text{ min}}$$

***P13.59** The gravitational forces the particles exert on each other are in the *x* direction. They do not affect the velocity of the center of mass. Energy is conserved for the pair of particles in a reference frame coasting along with their center of mass, and momentum conservation means that the identical particles move toward each other with equal speeds in this frame:

$$U_{gi} + K_i + K_i = U_{gf} + K_f + K_f$$

$$-\frac{Gm_1m_2}{r_i}+0=-\frac{Gm_1m_2}{r_f}+\tfrac{1}{2}m_1v^2+\tfrac{1}{2}m_2v^2$$

$$-\frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(1000\ \mathrm{kg})^2}{20\ \mathrm{m}}=-\frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(1000\ \mathrm{kg})^2}{2\ \mathrm{m}}+2\left(\tfrac{1}{2}\right)(1000\ \mathrm{kg})v^2$$

$$\left(\frac{3.00\times10^{-5}\ \mathrm{J}}{1000\ \mathrm{kg}}\right)^{1/2}=v=1.73\times10^{-4}\ \mathrm{m/s}$$

Then their vector velocities are $\boxed{(800+1.73\times10^{-4})\,\hat{\mathbf{i}}\ \mathrm{m/s}\text{ and }(800-1.73\times10^{-4})\,\hat{\mathbf{i}}\ \mathrm{m/s}.}$ for the trailing particle and the leading particle, respectively