Homework 3-2

Due: 11:59pm on Tuesday, November 4, 2014

To understand how points are awarded, read the Grading Policy for this assignment.

Tactics Box 5.3 Drawing a Free-Body Diagram

Learning Goal:

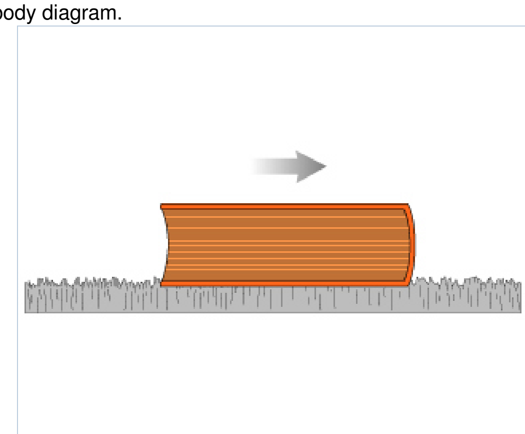
To practice Tactics Box 5.3 Drawing a Free-Body Diagram.

A free-body diagram is a diagram that represents the object as a particle and shows all of the forces acting on the object. Learning how to draw such a diagram is a very important skill in solving physics problems. This tactics box explains the essential steps to construct a correct free-body diagram.

TACTICS BOX 5.3 Drawing a free-body diagram

1. Identify all forces acting on the object. This step was described in Tactics Box 5.2.
2. Draw a coordinate system. Use the axes defined in your pictorial representation. If those axes are tilted, for motion along an incline, then the axes of the free-body diagram should be similarly tilted.
3. Represent the object as a dot at the origin of the coordinate axes. This is the particle model.
4. Draw vectors representing each of the identified forces. This was described in Tactics Box 5.1. Be sure to label each force vector.
5. Draw and label the *net force* vector $\vec{F}_{\mathrm{net}}$. Draw this vector beside the diagram, not on the particle. Or, if appropriate, write $\vec{F}_{\mathrm{net}} = \vec{0}$. Then, check that $\vec{F}_{\mathrm{net}}$ points in the same direction as the acceleration vector $\vec{a}$ on your motion diagram.

Apply these steps to the following problem: Your physics book is sliding on the carpet. Draw a free-body diagram.



Part A

Which forces are acting on the book?

Check all that apply.

Hint 1. How to identify all forces acting on the object

These are the steps outlined in Tactics Box 5.2 that will help you to identify all forces acting on the object whose motion you wish to study:

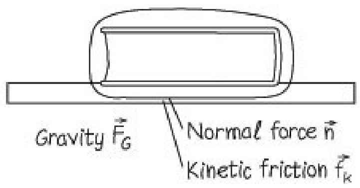
1. Identify the object of interest. This is the object whose motion you wish to study.
2. Draw a picture of the situation. Show the object of interest and all other objects—such as ropes, springs, or surfaces—that touch it.
3. Draw a closed curve around the object. Only the object of interest is inside the curve; everything else is outside.
4. Locate every point on the boundary of this curve where other objects touch the object of interest. These are the points where *contact forces* are exerted on the object.
5. Name and label each contact force acting on the object. There is at least one force at each point of contact; there may be more than one. When necessary, use subscripts to distinguish forces of the same type.
6. Name and label each long-range force acting on the object. For now, the only long-range force is the gravitational force.

ANSWER:

- [ ] drag
- [x] kinetic friction
- [x] gravity
- [ ] spring force
- [x] normal force
- [ ] static friction
- [ ] tension

Correct

As explained in Tactics Box 5.2, to identify all the forces acting on the object it helps to sketch the situation. So, draw a closed curve around the object of interest, and mark and label each contact force and long-range force acting on it. In this problem, your sketch might look like this:



Part B

Draw the most appropriate set of coordinate axes for this problem.

The orientation of your vectors will be graded.

ANSWER:

Correct

Part C

In the diagram below, the book is represented by a black dot at the origin of the coordinate axes, in accord with the particle model. Use this diagram to draw a free-body diagram for this problem that shows all of the forces identified in Part A. Make certain all vectors have the correct orientation, and choose their magnitudes consistent with the expected direction of the net force.

The net force vector, $\vec{F}_{\rm net}$, has been provided for you in this item. When you are finished drawing the force vectors identified in Part A, $\vec{F}_{\rm net}$ should point in the correct direction.

Draw each force vector with its tail centered at the black dot. The location and orientation of your vectors will be graded.

Hint 1. The relationship between the net force and the acceleration

If you apply Newton's second law to this problem, you will find that the net force $\vec{F}_{\rm net}$ should point in the same direction as the book's acceleration.

Hint 2. Find the direction of the book's acceleration

What is the direction of the book's acceleration? Recall that the book is moving to the right on a surface where friction cannot be neglected.

ANSWER:

- to the right
- to the left
- upward
- downward

ANSWER:

All attempts used; correct answer displayed

Free-Body Diagrams and Newton's Laws

When solving problems involving forces and Newton's laws, the following summary of things to do will start your mind thinking about getting involved in the problem at hand.

Problem Solving: Free-Body Diagrams and Newton's Laws

1. Draw a sketch of the situation.
2. Consider only one object (at a time), and draw a free-body diagram for that body, showing all the forces acting on that body. Do not show any forces that the body exerts on other bodies. If several bodies are involved, draw a free-body diagram for each body separately, showing all the forces acting on that body.
3. Newton's second law involves vectors, and it is usually important to resolve vectors into components. Choose an *x* and *y* axis in a way that simplifies the calculation.
4. For each body, Newton's second law can be applied to the x and y components separately. That is the *x* component of the net force on that body will be related to the *x* component of that body's acceleration: $\Sigma F_x = ma_x$, and similarly for the *y* direction.
5. Solve the equation or equations for the unknown(s).

Apply these steps

Use the steps outlined above to find the magnitude of the acceleration a of a chair and the magnitude of the normal force F_N acting on the chair: Yusef pushes a chair of mass m = 55.0kg across a carpeted floor with a force $\vec{F}_{\mathrm{p}}$ (the subscript 'p' here is lowercase and throughout the question) of magnitude F_{p} = 148N directed at θ = 35.0$\mathrm{degrees}$ below the horizontal . The magnitude of the kinetic frictional force between the carpet and the chair is F_{k} = 106N .

Part A

Identify and sketch all the external forces acting on the chair. Because the chair can be represented as a point particle of mass m, draw the forces with their tails centered on the black dot in the middle of the chair. Be certain to draw your forces so that they have the correct orientation.

Draw the vectors starting at the black dot. The location and orientation of the vectors will be graded. The length of the vectors will not be graded.

ANSWER:

Correct

Part B

Which set of coordinate axes is the most convenient to use in this problem?

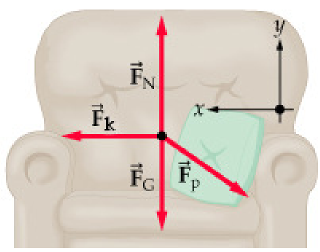
Hint 1. Determine the direction of the acceleration

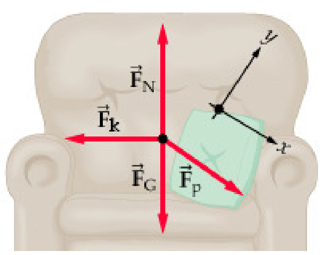
Indicate the direction of the chair's acceleration $\vec{a}$ on the figure below.

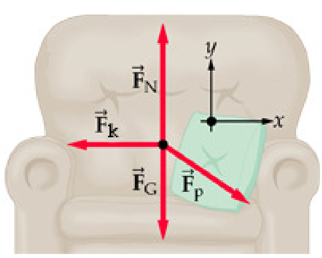
The orientation of the vector will be graded. The location and length of the vector will not be graded.

ANSWER:

ANSWER:







Correct

Now that you have selected a coordinate system, you should resolve the forces into *x* and *y* components so that you can apply Newton's second law to each coordinate direction independently.

Part C

Use the component form of Newton's second law to write an expression for the *x* component of the net force, ΣF_x.

Express your answer in terms of some or all of the variables: F_G, F_N, F_p, θ, and F_k .

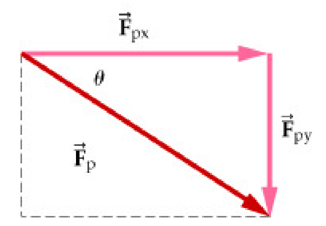
Hint 1. Find the *x* component of the pushing force

What is $F_{\rm px}$, the *x* component of the force $\vec{F}_{\rm p}$ exerted by Yusef?

Express your answer in terms of the variables $F_{\rm p}$ and θ.

Hint 1. Right angle triangle trigonometry

Keep in mind that the cosine of an angle is defined as the length of the adjacent leg divided by the length of the hypoteneuse. The sine of an angle is the length of the opposite leg divided by the length of the hypoteneuse. Apply these definitions to the triangle formed by $\vec{F}_{\rm p}$, θ, $F_{\rm px}$,and $F_{\rm py}$.



ANSWER:

$F_{\rm px}$ = $F_p \cos(\theta)$

ANSWER:

ΣF_x = $F_p \cos(\theta) - F_k \quad = ma_x$

All attempts used; correct answer displayed

Part D

Use the component form of Newton's second law to write an expression for the *y* component of the net force, ΣF_y.

Express your answer in terms of some or all of the variables: F_G, F_N, $F_{\rm p}$, θ, and $F_{\rm k}$.

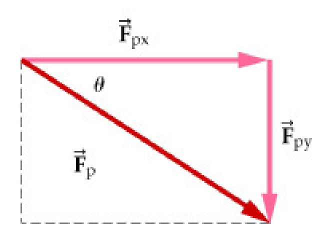
Hint 1. Find the *y* component of the pushing force

What is $F_{\rm py}$, the *y* component of the force $\vec{F}_{\rm p}$ exerted by Yusef?

Express your answer in terms of the variables $F_{\rm p}$ and θ.

Hint 1. Right angle triangle trigonometry

Keep in mind that the cosine of an angle is defined as the length of the adjacent leg divided by the length of the hypoteneuse. The sine of an angle is the length of the opposite leg divided by the length of the hypoteneuse. Apply these definitions to the triangle formed by $\vec{F}_{\rm p}$, θ, $F_{\rm px}$,and $F_{\rm py}$.



ANSWER:

F_{py} = $-F_p \sin(\theta)$

ANSWER:

ΣF_y = $F_N - F_G - F_p \sin(\theta) \quad = ma_y$

All attempts used; correct answer displayed

You have created two equations that describe the motion of the chair:

$$\Sigma F_x = F_{\text{p}} \cos\theta - F_{\text{k}} = ma_x$$

and

$$\Sigma F_y = F_{\text{N}} - F_{\text{G}} - F_{\text{p}} \sin\theta = ma_y$$

Solve these equations to find F_N and a.

Part E

What is the magnitude of the acceleration a of the chair? What is the magnitude of the normal force F_N acting on the chair?

Express your answers, separated by a comma, in meters per second squared and newtons to three significant figures.

Hint 1. Find the component of the acceleration in the *y* direction

The chair only moves in the postive *x* direction. What is a_y?

Express your answer in meters per second squared.

ANSWER:

a_y = 0 $\mathrm{m/s^2}$

Hint 2. Find the weight of the chair

What is the weight of the chair?

Express the weight in newtons to three significant figures.

Hint 1. An equation for weight

Recall that the weight of an object of mass m is given by mg, where 9.8$\mathrm{m/s^2}$ is the acceleration due to gravity.

ANSWER:

F_G = 539 N

ANSWER:

a, F_N = 0.277,624 $\mathrm{m/s^2, N}$

All attempts used; correct answer displayed

A free-body diagram is a useful way to begin all problems involving forces. This drawing will help you to easily identify the most appropriate coordinate axes and to resolve any 2 dimensional vectors into components. Then you can apply Newton's second law to each coordinate direction to set up equations which will allow you to solve for any unknown quantities.

Video Tutor: Tension in String between Hanging Weights

First, launch the video below. You will be asked to use your knowledge of physics to predict the outcome of an experiment. Then, close the video window and answer the question at right. You can watch the video again at any point.

Part A

Consider the video tutorial you just watched. Suppose that we duplicate this experimental setup in an elevator. What will the spring scale read if the elevator is moving upward at constant speed?

Hint 1. How to approach the problem

What does the phrase "at constant speed" imply about the acceleration of the system?

ANSWER:

- 0 N
- More than 18 N
- Less than 18 N but greater than 0 N
- 18 N

Correct

Since the elevator is not accelerating, the reading on the scale is the same as in the video.

Problem 5.55

Part A

A bag of groceries is on the seat of your car as you stop for a stop light. The bag does not slide. Draw a free-body diagram for the bag.

Draw the force vectors with their tails at the dot. The orientation of your vectors will be graded. The exact length of your vectors will not be graded but the relative length of one to the other will be graded.

ANSWER:

Correct

PSS 6.1 Equilibrium Problems

Learning Goal:

To practice Problem-Solving Strategy 6.1 for equilibrium problems.

A pair of students are lifting a heavy trunk on move-in day. Using two ropes tied to a small ring at the center of the top of the trunk, they pull the trunk straight up at a constant velocity $\vec{v}$. Each rope makes an angle θ with respect to the vertical. The gravitational force acting on the trunk has magnitude F_G.

Find the tension T in each rope.

PROBLEM-SOLVING STRATEGY 6.1 Equilibrium problems

MODEL: Make simplifying assumptions.

VISUALIZE:

- Establish a coordinate system, define symbols, and identify what the problem is asking you to find. This is the process of translating words into symbols.
- Identify all forces acting on the object, and show them on a free-body diagram.
- These elements form the *pictorial representation* of the problem.

SOLVE: The mathematical representation is based on Newton's first law:

$$\vec{F}_{\rm net} = \sum_i \vec{F}_i = \vec{0}.$$

The vector sum of the forces is found directly from the free-body diagram.

ASSESS: Check if your result has the correct units, is reasonable, and answers the question.

Model

The trunk is moving at a constant velocity. This means that you can model it as a particle in dynamic equilibrium and apply the strategy above. Furthermore, you can ignore the masses of the ropes and the ring because it is reasonable to assume that their combined weight is much less than the weight of the trunk.

Visualize

Part A

The most convenient coordinate system for this problem is one in which the *y* axis is vertical and the ropes both lie in the *xy* plane, as shown below.

Identify the forces acting on the trunk, and then draw a free-body diagram of the trunk in the diagram below. The black dot represents the trunk as it is lifted by the students.

Draw the vectors starting at the black dot. The location and orientation of the vectors will be graded. The length of the vectors will not be graded.

ANSWER:

Correct

Part B

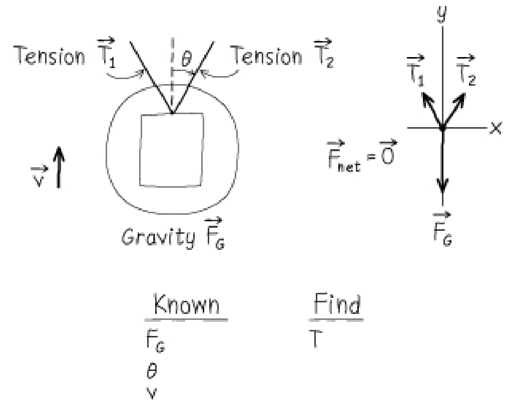
In the free-body diagram drawn in the previous part, different symbols are used to represent the tensions in the two ropes. This notation could be simplified by identifying a useful relationship between these two forces. Which of the following statements properly describes the relationship between the magnitude T_1 of the tension force in rope 1 and the magnitude T_2 of the tension force in rope 2?

ANSWER:

- $T_1 > T_2$, because the first rope attached must hold the full weight of the trunk before the second rope is attached.
- $T_1 < T_2$, because rope 1 is shorter than rope 2.
- $T_1 = T_2$, because two ropes attached to the same object should have the same tension.
- $T_1 = T_2$, because the ropes attach to the trunk at the same point and at the same angle.

Correct

This is a type of reasoning, used often in physics, called a *symmetry argument*. Since the ropes are in identical situations, except for one being the mirror image of the other, they have to possess identical tensions. Since the two tension forces have equal magnitude, just use T to denote the magnitude of the tension force in either rope. With the information you have gathered here, you can build a complete pictorial representation:



Solve

Part C

Find the force of tension T in each rope.

Express your answer in terms of some or all of the variables θ, v, and F_G.

Hint 1. How to approach the problem

Since we are told that the trunk moves up at a constant velocity, we say that it is in dynamic equilibrium. When an object is in static or dynamic equilibrium, Newton's first law states that no net force acts upon the object. As stated in the strategy above, the mathematical representation of Newton's first law is $\sum_i \vec{F_i} = \vec{0}$. This is a vector equation that corresponds to two separate equations when written in component form: one equation in the *x* direction, $\sum_i (F_i)_x = 0$, and one equation in the *y* direction, $\sum_i (F_i)_y = 0$.

In this particular problem, even though some of the forces acting on the system are two-dimensional, it is not necessary to set up Newton's first law in both directions to find the tension in the ropes. The symmetry argument that allowed us to state that the two tensions have the same magnitude has reduced the number of unknowns to one, T. So, to find T it is sufficient to set up and solve Newton's first law in just one direction.

Hint 2. Set up Newton's first law in the *y* direction

Since the ropes make the same angle with the vertical, and the two tensions have the same magnitude, the *y* component of the tension in both ropes must be the same; label it T_y.

Which of the following equations is the correct form of Newton's first law for the *y* component of the net force acting on the trunk?

ANSWER:

- $T_y - F_G = 0$
- $2T_y - F_G = 0$
- $2T_y + F_G = 0$
- $T_y + F_G = 0$
- $T_y \cos\theta - F_G = 0$

Hint 3. Relate the *y* component to the total tension in one rope

Find T_y, the magnitude of the *y* component of the tension.

Express your answer in terms of T and θ.

ANSWER:

T_y = $T\cos(\theta)$

ANSWER:

T = $\frac{F_G}{2\cos(\theta)}$

All attempts used; correct answer displayed

Assess

Part D

To assess whether your results make sense, sort the following situations according to whether the tension in the ropes increases, decreases, or is unchanged as a result of the change mentioned in each picture. In each case, assume that all the conditions, other than those mentioned in each picture, remain the same as in the situation described in the problem introduction. Use your intuition, not your math skills, to find your answers.

Drag the appropriate items to their respective bins.

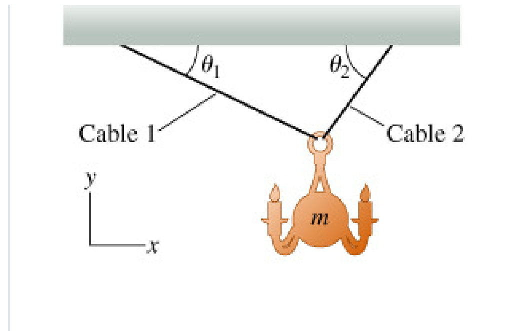
ANSWER:

Correct

Now, use your math skills. Look at your expression for T from Part C. How does T change if θ increases or decreases? How does T change if the gravitational force on the trunk has a larger magnitude, that is, if the trunk is heavier? Your answer from Part C says that T is directly proportional to F_G and inversely proportional to $\cos\theta$. This means your mathematical expression for T correctly predicts what your intuition has suggested. Your calculations do make sense!

Hanging Chandelier

A chandelier with mass m is attached to the ceiling of a large concert hall by two cables. Because the ceiling is covered with intricate architectural decorations (not indicated in the figure, which uses a humbler depiction), the workers who hung the chandelier couldn't attach the cables to the ceiling directly above the chandelier. Instead, they attached the cables to the ceiling near the walls. Cable 1 has tension T_1 and makes an angle of θ_1 with the ceiling. Cable 2 has tension T_2 and makes an angle of θ_2 with the ceiling.



Part A

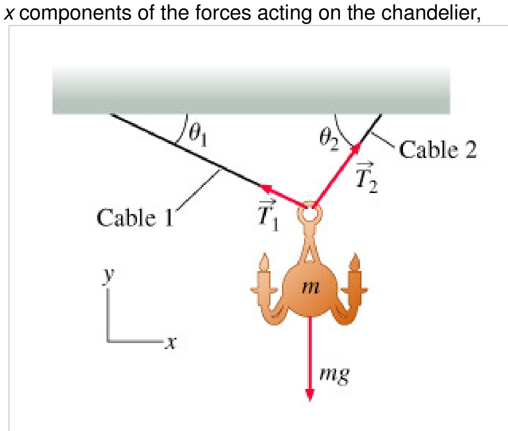
Find an expression for T_1, the tension in cable 1, that does not depend on T_2.

Express your answer in terms of some or all of the variables m, θ_1, and θ_2, as well as the magnitude of the acceleration due to gravity g. You must use parentheses around θ_1 and θ_2, when they are used as arguments to any trigonometric functions in your answer.

Hint 1. Find the sum of forces in the *x* direction

The chandelier is static; hence the vector forces on it sum to zero. Type in the sum of the *x* components of the forces acting on the chandelier, using the coordinate system shown.

Express your answer in terms of some or all of the variables m, T_1, T_2, θ_1, and θ_2. You must use parentheses around θ_1 and θ_2, when they are used as arguments to any trigonometric functions in your answer.



ANSWER:

$\sum F_x = 0 =$ $T_2\cos(\theta_2) - T_1\cos(\theta_1)$

Hint 2. Find the sum of forces in the *y* direction

Now type the corresponding equation relating the *y* components of the forces acting on the chandelier, again using the coordinate system shown.

Express your answer in terms of some or all of the variables m, T_1, T_2, θ_1, and θ_2, as well as the magnitude of the acceleration due to gravity g. You must use parentheses around θ_1 and θ_2, when they are used as arguments to any trigonometric functions in your answer.

ANSWER:

$\sum F_y = 0 =$ $T_1\sin(\theta_1) + T_2\sin(\theta_2) - mg$

Hint 3. Putting it all together

There are two unknowns in this problem, T_1 and T_2. Each of the previous two hints leads you to an equation involving these two unknowns. Eliminate T_2 from this pair of equations and solve for T_1.

ANSWER:

$T_1 = \dfrac{mg\cos(\theta_2)}{\sin(\theta_1+\theta_2)}$

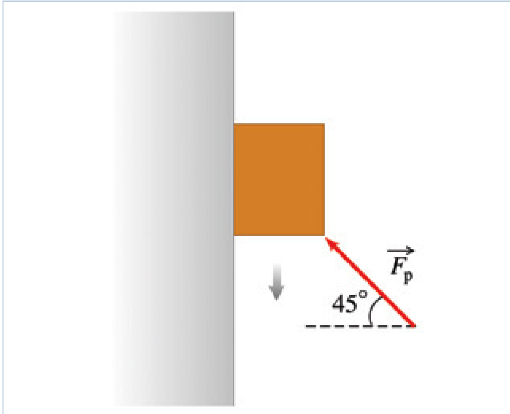
All attempts used; correct answer displayed

PSS 6.2 Dynamics Problems

Learning Goal:

To practice Problem-Solving Strategy 6.2 for dynamics problems.

A box of mass $3.1\mathrm{kg}$ slides down a rough vertical wall. The gravitational force on the box is $30\mathrm{N}$. When the box reaches a speed of $2.5\mathrm{m/s}$, you start pushing on one edge of the box at a 45° angle (*use degrees* in your calculations throughout this problem) with a constant force of magnitude F_p = $23\mathrm{N}$, as shown in figure. There is now a frictional force between the box and the wall of magnitude $13\mathrm{N}$.
How fast is the box sliding 2.5s after you started pushing on it?



PROBLEM-SOLVING STRATEGY 6.2 Dynamics problems

MODEL: Make simplifying assumptions.

VISUALIZE: Draw a pictorial representation.

- Show important points in the motion with a sketch, establish a coordinate system, define symbols, and identify what the problem is trying you to find. This is the process of translating words into symbols.
- Use a motion diagram to determine the object's acceleration vector $\vec{a}$.
- Identify all forces acting on the object, and show them on a free-body diagram.
- It's OK to go back and forth between these steps as you visualize the situation.

SOLVE: The mathematical representation is based on Newton's second law:

$$\vec{F}_\mathrm{net} = \sum_i \vec{F}_i = m\vec{a}.$$

The vector sum of the forces is found directly from the free-body diagram. Depending on the problem, either

- Solve for the acceleration, and then use kinematics to find velocities and positions; or
- Use kinematics to determine the acceleration, and then solve for unknown forces.

ASSESS: Check that your result has the correct units, is reasonable, and answers the question.

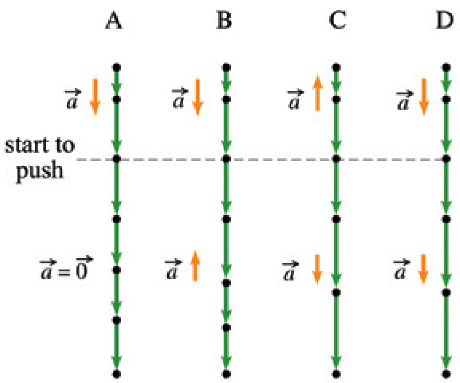
Model

Start by making a simplifying assumption: We will assume all the forces acting on the box are constant, so now you can model the box as a particle moving with a constant acceleration.

Visualize

Part A

Using our simplified model, in which we know that the forces are constant (but we don't know what their magnitudes are), which of the following motion diagrams would be a *reasonable* representation of the motion of the box?



Check all that apply.

ANSWER:

- [x] A
- [x] B
- [ ] C
- [x] D

All attempts used; correct answer displayed

Before you start pushing on the box, the box is acted upon by only the gravitational force that pulls the box downward. The resulting acceleration vector is directed downward. It should be noted that there is no frictional force at this point, since the normal force acting on the box from the vertical wall is zero at this point.
The important point in the box's motion is the moment when you start pushing on it. At that moment, three possible scenarios may occur. The box may continue to slide down the wall at an increasing speed, but with a smaller acceleration than before because of the opposing effect of the pushing force and frictional force. In this case, $\vec{a}$ would point downward (diagram D). Alternatively, if the opposing effect of the pushing force and frictional force were such that the box's speed would decrease, the box would be slowing down as it moves downward and $\vec{a}$ would point upward (diagram B). Somewhere in between these two cases, a third scenario arises in which the net force on the box is zero and the box slides down the wall at a constant speed (diagram A). Only the direction and magnitude of the net force can tell you which of these scenarios is the correct one for this problem. Keep in mind that in all cases the motion is always vertical, meaning that the horizontal component of the acceleration must be zero.

Part B

Still using our simplified model (in which we do not know the magnitudes of the forces), draw a free-body diagram showing all the forces acting on the box *after* you start pushing on it. The positive y axis is taken to be upward. The black dot represents the box. Since our model is about having constant forces of unknown magnitude, you do not need to draw the vectors to scale, but your final diagram should be physically *reasonable*.

Draw the vectors starting at the black dot. The location and orientation of the vectors will be graded. The relative lengths of the vectors will not be graded.

ANSWER:

Correct

If all the vectors in your free-body diagram were drawn with their correct relative lengths, their vector sum $\vec{F}_{\rm net} = \sum_i \vec{F}_i$ would provide some quantitative information about the box's acceleration. Since this is not the case here, you cannot obtain any further information about the box's acceleration without first performing some calculations based on Newton's second law.

Note that in the diagram above, a coordinate system was established so that the positive *x* axis points to the right and the positive *y* axis points upward. This is the same coordinate system used throughout the rest of this problem. Keep in mind that you are trying to find the box's speed 2.5s after starting to push on it.

Solve

Part C

Find the box's speed $v_{\rm f}$ at 2.5s after you first started pushing on it.

Express your answer in meters per second to three significant figures.

Hint 1. How to approach the problem

This is a one-dimensional kinematics problem. You are given the box's initial speed and need to calculate its final speed after a certain period of time. You know that motion occurs only in the vertical direction, so there's no need to write down any equation in the *x* direction. All you need is a_y, the *y* component of the box's acceleration, which can be calculated by applying Newton's second law in the y direction.

Hint 2. Set up Newton's second law in the *y* direction

Newton's second law states that $(F_{\rm net})_y$, the *y* component of the net force acting on an object, is equal to the *y* component of the object's acceleration multiplied by its mass, that is, $(F_{\rm net})_y = ma_y$.

Using the coordinate system shown in Part B, enter an expression for $(F_{\rm net})_y$ in terms of the forces acting on the box. Use f, F_G, and n for the magnitudes of the friction force, the gravitational force, and the normal force, respectively; use $F_{\rm p}$ for the magnitude of the pushing force.

Express your answer in terms of some or all of the variables f, n, F_G, and $F_{\rm p}$.

Hint 1. Find $F_{\rm py}$, the *y* component of the pushing force

Enter an expression for the *y* component of the pushing force, $F_{\rm py}$. Recall that you push on the box at a 45° angle. Use $F_{\rm p}$ for the magnitude of the pushing force.

Express your answer in terms of $F_{\rm p}$, the magnitude of the pushing force.

ANSWER:

$F_{\rm py}$ = $F_p \sin(45)$

Correct

ANSWER:

$(F_{\rm net})_y = \sum_i (F_i)_y =$ = $f + F_p \sin(45) - F_G$ $= ma_y$

Correct

Now, solve for a_y.

Hint 3. Determine which kinematic equation to use

Which of the following equations is the most appropriate one to calculate the box's speed 2.5s after you first started pushing on it?

ANSWER:

- $y_{\rm f} = y_{\rm i} + v_{{\rm i}y}\Delta t + \frac{1}{2}a_y(\Delta t)^2$
- $v_{{\rm f}y}^2 = v_{{\rm i}y}^2 + 2a_y(y_{\rm f} - y_{\rm i})$
- $v_{{\rm f}y} = v_{{\rm i}y} + a_y\Delta t$

Correct

ANSWER:

$v_{\rm f}$ = 3.09 $\rm m/s$

All attempts used; correct answer displayed

Assess

Part D

Assuming that the angle at which you push on the edge of the box is again 45°, with what magnitude of force $F_{\rm p}$ should you push if the box were to slide down the wall at a constant velocity? Note that, in general, the magnitude of the friction force will change if you change the magnitude of the pushing force. Thus, for this part, assume that the magnitude of the friction force is $f = 0.566F_p$.

Express your answer in newtons to three significant figures.

Hint 1. Dynamic equilibrium

If the box slides down the wall at a constant velocity, its acceleration must be zero. This is the condition for dynamical equilibrium, or $\vec{F}_{\rm net} = \vec{0}$. Set up again Newton's second law in the *y* direction as you did in the previous part, keeping in mind that now the magnitude $F_{\rm p}$ of the pushing force is unknown. Use the equilibrium condition $(F_{\rm net})_y = 0$, and solve for $F_{\rm p}$.

ANSWER:

$F_{\rm p}$ = 23.6 $\rm N$

Correct

Your results make sense. If you push with a force of magnitude $23\mathrm{N}$, as described in Part C, the box will continue to speed up because the *y* component of the net force, and therefore the *y* component of the box's acceleration, remains negative (i.e., it points downward). In this case, the effect of the pushing force is simply to reduce the magnitude of the box's acceleration. Then, to reduce the box's acceleration further to zero, you need to push with a force of magnitude $F_p > 23\mathrm{N}$, as you have just calculated. Note that if you pushed even harder, the acceleration will become positive, causing the box to slow down and possibly come to a stop. These are the three scenarios whose motion diagrams were identified in Part A.

Contact Forces Introduced

Learning Goal:

To introduce contact forces (normal and friction forces) and to understand that, except for friction forces under certain circumstances, these forces must be determined from: net Force = ma.

Two solid objects cannot occupy the same space at the same time. Indeed, when the objects touch, they exert repulsive *normal* forces on each other, as well as *frictional* forces that resist their slipping relative to each other. These *contact forces* arise from a complex interplay between the electrostatic forces between the electrons and ions in the objects and the laws of quantum mechanics. As two surfaces are pushed together these forces increase exponentially over an atomic distance scale, easily becoming strong enough to distort the bulk material in the objects if they approach too close. In everyday experience, contact forces are limited by the deformation or acceleration of the objects, rather than by the fundamental interatomic forces. Hence, we can conclude the following:

The magnitude of contact forces is determined by $\sum \vec{F} = m\vec{a}$, that is, by the other forces on, and acceleration of, the contacting bodies. The only exception is that the frictional forces cannot exceed μn (although they can be smaller than this or even zero).

Normal and friction forces

Two types of contact forces operate in typical mechanics problems, the *normal* and *frictional* forces, usually designated by n and f (or F_{fric}, or something similar) respectively. These are the components of the overall contact force: n perpendicular to and f parallel to the plane of contact.

Kinetic friction when surfaces slide

When one surface is sliding past the other, experiments show three things about the friction force (denoted f_{k}):

1. The frictional force opposes the relative motion at the point of contact,
2. f_{k} is proportional to the normal force, and
3. the ratio of the magnitude of the frictional force to that of the normal force is fairly constant over a wide range of speeds.

The constant of proportionality is called the *coefficient of kinetic friction*, often designated μ_{k}. As long as the sliding continues, the frictional force is then

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} n \text{ (valid when the surfaces slide by each other).}$$

Static friction when surfaces don't slide

When there is no relative motion of the surfaces, the frictional force can assume *any* value from zero up to a maximum $\mu_{\mathrm{s}} n$, where μ_{s} is the *coefficient of static friction*. Invariably, μ_{s} is larger than μ_{k}, in agreement with the observation that when a force is large enough that something breaks loose and starts to slide, it often accelerates.

The frictional force for surfaces with no relative motion is therefore

$$f_{\mathrm{s}} \le \mu_{\mathrm{s}} n \text{ (valid when the contacting surfaces have no relative motion).}$$

The actual magnitude and direction of the static friction force are such that it (together with other forces on the object) causes the object to remain motionless with respect to the contacting surface as long as the static friction force required does not exceed $\mu_{\mathrm{s}} n$. The equation $f_{\mathrm{s}} = \mu_{\mathrm{s}} n$ is valid *only* when the surfaces are on the verge of sliding.

Part A

When two objects slide by one another, which of the following statements about the force of friction between them, is true?

ANSWER:

- The frictional force is always equal to $\mu_k n$.
- The frictional force is always less than $\mu_k n$.
- The frictional force is determined by other forces on the objects so it can be either equal to or less than $\mu_k n$.

Correct

Part B

When two objects are in contact with no relative motion, which of the following statements about the frictional force between them, is true?

ANSWER:

- The frictional force is always equal to $\mu_s n$.
- The frictional force is always less than $\mu_s n$.
- The frictional force is determined by other forces on the objects so it can be either equal to or less than $\mu_s n$.

Correct

For static friction, the actual magnitude and direction of the friction force are such that it, together with any other forces present, will cause the object to have the observed acceleration. The magnitude of the force cannot exceed $\mu_s n$. If the magnitude of static friction needed to keep acceleration equal to zero exceeds $\mu_s n$, then the object will slide subject to the resistance of kinetic friction. Do *not* automatically assume that $f_s = \mu_s n$ unless you are considering a situation in which the magnitude of the static friction force is as large as possible (i.e., when determining at what point an object will just begin to slip). Whether the actual magnitude of the friction force is 0, less than $\mu_s n$, or equal to $\mu_s n$ depends on the magnitude of the other forces (if any) as well as the acceleration of the object through $\sum \vec{F} = m\vec{a}$.

Part C

When a board with a box on it is slowly tilted to larger and larger angle, common experience shows that the box will at some point "break loose" and start to accelerate down the board.

The box begins to slide once the component of gravity acting parallel to the board F_g just begins to exceeds the maximum force of static friction. Which of the following is the most general explanation for why the box accelerates down the board?

ANSWER:

- The force of kinetic friction is smaller than that of maximum static friction, but F_g remains the same.
- Once the box is moving, F_g is smaller than the force of maximum static friction but larger than the force of kinetic friction.
- Once the box is moving, F_g is larger than the force of maximum static friction.
- When the box is stationary, F_g equals the force of static friction, but once the box starts moving, the sliding reduces the normal force, which in turn reduces the friction.

Correct

At the point when the box finally does "break loose," you know that the component of the box's weight that is parallel to the board just exceeds $\mu_s n$ (i.e., this component of gravitational force on the box has just reached a magnitude such that the force of static friction, which has a maximum value of $\mu_s n$, can no longer oppose it.) For the box to then accelerate, there must be a net force on the box along the board. Thus, the component of the box's weight parallel to the board must be greater than the force of kinetic friction. Therefore the force of kinetic friction $\mu_k n$ must be less than the force of static friction $\mu_s n$ which implies $\mu_k < \mu_s$, as expected.

Part D

Consider a problem in which a car of mass M is on a road tilted at an angle θ. The normal force

Select the best answer.

ANSWER:

- $n = Mg$
- $n = Mg\cos(\theta)$
- $n = \frac{Mg}{\cos(\theta)}$
- is found using $\sum \vec{F} = M\vec{a}$

Correct

The key point is that contact forces must be determined from Newton's equation. In the problem described above, there is not enough information given to determine the normal force (e.g., the acceleration is unknown). Each of the answer options is valid under some conditions ($\theta = 0$, the car is sliding down an icy incline, or the car is going around a banked turn), but in fact none is likely to be correct if there are other forces on the car or if the car is accelerating. Do not memorize values for the normal force valid in different problems--you must determine $\vec{n}$ from $\sum \vec{F} = m\vec{a}$.

Static Friction and Frictional Force Ranking Task

Below are six crates at rest on level surfaces. The crates have different masses and the frictional coefficients [given as (μ_s, μ_k)] between the crates and the surfaces differ. The same external force is applied to each crate, but none of the crates move.

Part A

Rank the crates on the basis of the frictional force acting on them.

Rank from largest to smallest. To rank items as equivalent, overlap them.

Hint 1. The static friction relationship

For coefficient of static friction, μ_s, and normal force of magnitude n, the magnitude of the force of static friction, f_s, is given by the relationship

$$f_s \ \leq \ \mu_s n.$$

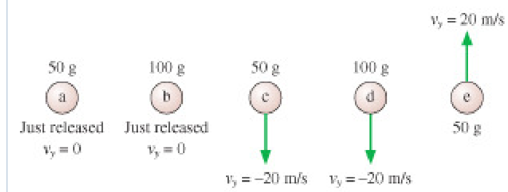
Note that the equation is an inequality. The force of static friction will have a magnitude equal to the magnitude of the net force that would cause motion, as long as that net force has a magnitude less than or equal to $\mu_s n$. Since none of the crates are moving, you can conclude that the magnitude of the net force does not exceed $\mu_s n$ for any of the crates.

ANSWER:

All attempts used; correct answer displayed

Conceptual Question 6.18

Five balls move through the air as shown in **the figure** . All five have the same size and shape. Air resistance is not negligible.



Part A

Rank in order, from largest to smallest, the magnitudes of the accelerations a_a to a_e. Some may be equal.

Rank from largest to smallest. To rank items as equivalent, overlap them.

ANSWER:

All attempts used; correct answer displayed

Problem 6.64

At $t = 0$, an object of mass m is at rest at $x = 0$ on a horizontal, frictionless surface. Starting at $t = 0$, a horizontal force $F_x = F_0 e^{-t/T}$ is exerted on the object.

Part A

Find an expression for the object's velocity at an arbitrary later time t.

Express your answer in terms of the variables F_0, m, T, and t.

ANSWER:

$v_x = \frac{F_0 T}{m}\left(1 - e^{\frac{-t}{T}}\right)$

Answer Requested

Part B

What is the object's velocity after a very long time has elapsed?

Express your answer in terms of the variables F_0, m, and T.

ANSWER:

$v_x = \frac{F_0 T}{m}$

Correct

Problem 6.72

A block of mass m is at rest at the origin at $t = 0$. It is pushed with constant force F_0 from $x = 0$ to $x = L$ across a horizontal surface whose coefficient of kinetic friction is $\mu_k = \mu_0(1 - x/L)$. That is, the coefficient of friction decreases from μ_0 at $x = 0$ to zero at $x = L$.

Part A

We would like to know the velocity of the block when it reaches some position x. Finding this requires an integration. However, acceleration is defined as a derivative with respect to time, which leads to integrals with respect to time, but the force is given as a function of position. To get around this, use the chain rule to find an alternative definition for the acceleration a_x that can be written in terms of v_x and $\frac{dv_x}{dx}$. This is a purely mathematical exercise; it has nothing to do with the forces given in the problem statement.

Express your answer in terms of the variables v_x and $\frac{dv_x}{dx}$.

ANSWER:

$a_x = v_x \frac{dv_x}{dx}$

Answer Requested

Part B

Now use the result of Part A to find an expression for the block's velocity when it reaches position $x = L$.

Express your answer in terms of the variables L, F_0, m, μ_0, and appropriate constants.

ANSWER:

$v_x(L) = \sqrt{L\left(\frac{2F_0}{m} - \mu_0 g\right)}$

Answer Requested

Score Summary:

Your score on this assignment is 54.0%.
You received 6.47 out of a possible total of 12 points.