Homework 4-2

Due: 11:59pm on Wednesday, November 12, 2014

To understand how points are awarded, read the Grading Policy for this assignment.

Centripetal Acceleration Explained

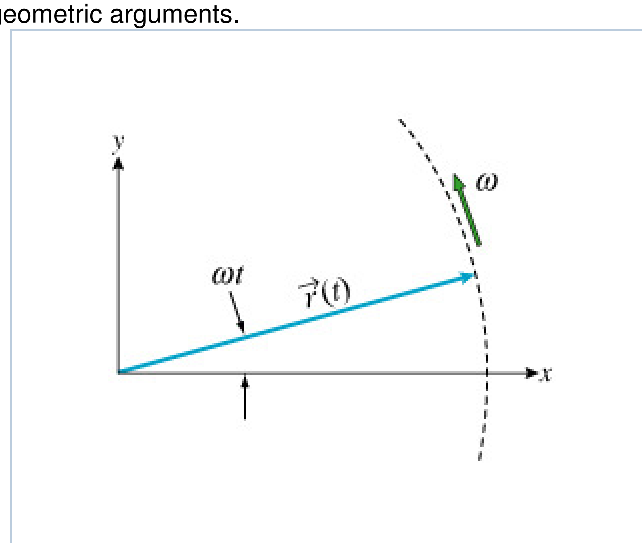
Learning Goal:

To understand that centripetal acceleration is the acceleration that causes motion in a circle.

Acceleration is the time derivative of velocity. Because velocity is a vector, it can change in two ways: the length (magnitude) can change and/or the direction can change. The latter type of change has a special name, the *centripetal acceleration*. In this problem we consider a mass moving in a circle of radius R with angular velocity ω,

$$\vec{r}(t) = R\left[\cos(\omega t)\,\hat{i} + \sin(\omega t)\,\hat{j}\right]$$
$$= R\cos(\omega t)\,\hat{i} + R\sin(\omega t)\,\hat{j}.$$

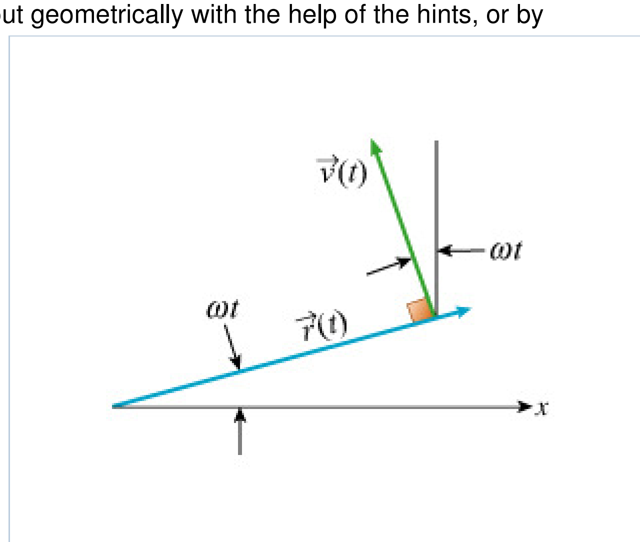
The main point of the problem is to compute the acceleration using geometric arguments.



Part A

What is the velocity of the mass at a time t? You can work this out geometrically with the help of the hints, or by differentiating the expression for $\vec{r}(t)$ given in the introduction.

Express this velocity in terms of R, ω, t, and the unit vectors $\hat{i}$ and $\hat{j}$.



Hint 1. Direction of the velocity

What is the angle between $\vec{r}(t=0)$ and $\vec{r}(t)$? As shown in the figure, this angle is directly related to the direction of the velocity vector. Keep in mind that when $t=0$, $\vec{r}(t=0)=R\hat{i}$.

Express your result in terms of quantities given in the problem introduction.

ANSWER:

θ = ωt

Hint 2. Speed

What is $v(t)$, the speed (magnitude of velocity) of the mass at time t?

Express $v(t)$ in terms of ω and R.

ANSWER:

$v(t)$ = ωR

ANSWER:

$\vec{v}(t)$ = $R\omega\cos(\omega t)\hat{j} - R\omega\sin(\omega t)\hat{i}$

Correct

Assume that the mass has been moving along its circular path for some time. You start timing its motion with a stopwatch when it crosses the positive *x* axis, an instant that corresponds to $t=0$. [Notice that when $t=0$, $\vec{r}(t=0)=R\hat{i}$.] For the remainder of this problem, assume that the time t is measured from the moment you start timing the motion. Then the time $-t$ refers to the moment a time t before you start your stopwatch.

Part B

What is the velocity of the mass at a time $-t$?

Express this velocity in terms of R, ω, t, and the unit vectors $\hat{i}$ and $\hat{j}$.

ANSWER:

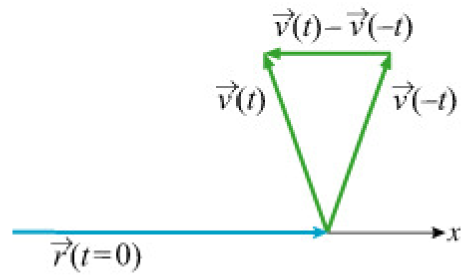
$\vec{v}(-t)$ = $R\omega\sin(\omega t)\hat{i} + R\omega\cos(\omega t)\hat{j}$

Correct

Part C

What is the average acceleration of the mass during the time interval from $-t$ to t?

Express this acceleration in terms of R, ω, t, and the unit vectors $\hat{i}$ and $\hat{j}$.



Hint 1. Average acceleration

The definition of average acceleration over the interval from t_1 to t_2 is

$$\vec{a}_{\rm avg}[t_1, t_2] = \frac{\vec{v}(t_2)-\vec{v}(t_1)}{t_2-t_1}.$$

ANSWER:

$\vec{a}_{\rm avg}[-t, t]$ = $\frac{-\left(2R\omega\sin\left(\omega t\right)\hat{i}\right)}{t+t}$

Correct

Part D

What is the magnitude of this acceleration in the limit of small t? In this limit, the average acceleration becomes the instantaneous acceleration.

Express your answer in terms of R and ω.

Hint 1. Expansion of $\sin(x)$

For small times t (or more precisely when $\omega t \ll 1$), what is the first term in the Taylor series expansion for $\sin(\omega t)$?

Express your answer in terms of ω and t.

Hint 1. Taylor series expansion

The Taylor series expansion of $\sin(x)$ is

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots.$$

ANSWER:

$\sin(\omega t)$ = ωt

ANSWER:

$a_{\mathrm{centrip}\eta\mathrm{l}}$ = $\omega^2 R$

Correct

Part E

Consider the following statements:

a. The centripetal acceleration might better be expressed as $-\omega^2 \vec{r}(t)$ because it is a vector.
b. The magnitude of the centripetal acceleration is v_{radial}^2/R.
c. The magnitude of the centripetal acceleration is $v_{\mathrm{tangential}}^2/R$.
d. A particle that is going along a path with *local* radius of curvature R at speed s experiences a centripetal acceleration $-s^2/R$.
e. If you are in a car turning left, the force you feel pushing you to the right is the force that causes the centripetal acceleration.

In these statements v_{radial} refers to the component of the velocity of an object in the direction toward or away from the origin of the coordinate system or the rotation axis. Conversely, $v_{\mathrm{tangential}}$ refers to the component of the velocity perpendicular to v_{radial}.

Identify the statement or statements that are *false*.

ANSWER:

- ○ a only
- ○ b only
- ○ c only
- ○ d only
- ○ e only
- ◉ b and e
- ○ c and e
- ○ d and e

Correct

That's right; the true statements are therefore:

- a. The centripetal acceleration might better be expressed as $-\omega^2 \vec{r}(t)$ because it is a vector.
- c. The magnitude of the centripetal acceleration is $v^2_{\text{tangential}}/R$.
- d. A particle that is going along a path with *local* radius of curvature R at speed s experiences an acceleration $-s^2/R$.

There is so much confusion about centripetal force that you should probably ban this term from your vocabulary and thought processes. If you are in a car turning left, your centripetal acceleration is to the left (i.e., inward) and some real force must be applied to you to give you this acceleration--typically this would be provided by friction with the seat. The force you "feel" pushing you to the right is not a real force but rather a "fictitious force" that is present if you are in an accelerating coordinate system (in this case the car). It is best to stick to inertial (i.e., nonaccelerating) coordinate systems when doing kinematics and dynamics (i.e., $\vec{F} = m\vec{a}$ calculations).

Position, Velocity, and Acceleration

Learning Goal:

To identify situations when position, velocity, and /or acceleration change, realizing that change can be in direction or magnitude.

If an object's position is described by a function of time, $\vec{r}(t)$ (measured from a nonaccelerating reference frame), then the object's velocity is described by the time derivative of the position, $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$, and the object's acceleration is described by the time derivative of the velocity, $\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt^2}$.

It is often convenient to discuss the average of the latter two quantities between times t_1 and t_2:

$$\vec{v}_{\text{avg}}(t_1, t_2) = \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$$

and

$$\vec{a}_{\text{avg}}(t_1, t_2) = \frac{\vec{v}(t_2) - \vec{v}(t_1)}{t_2 - t_1}.$$

Part A

You throw a ball. Air resistance on the ball is negligible. Which of the following functions change with time as the ball flies through the air?

Hint 1. The Pull of Gravity

The reason the ball comes back to your hand is that it is being pulled on by the Earth's gravity. This is the same reason that the ball feel's heavy when it's resting in your hand. Does the weight of the ball change at different heights, or is the pull of gravity constant throughout the ball's flight? What does this tell you about the acceleration of the ball?

ANSWER:

- only the position of the ball
- only the velocity of the ball
- only the acceleration of the ball
- the position and velocity of the ball
- the position and the velocity and acceleration of the ball

Correct

Part B

You are driving a car at 65 mph. You are traveling north along a straight highway. What could you do to give the car a nonzero acceleration?

Hint 1. What constitutes a nonzero acceleration?

The velocity of the car is described by a vector function, meaning it has both magnitude (65 mph) and direction (north). The car experiences a nonzero acceleration if you change either the magnitude of the velocity or the direction of the velocity.

ANSWER:

- Press the brake pedal.
- Turn the steering wheel.
- Either press the gas or the brake pedal.
- Either press the gas or the brake pedal or turn the steering wheel.

Correct

Part C

A ball is lodged in a hole in the floor near the outside edge of a merry-go-round that is turning at constant speed. Which kinematic variable or variables change with time, assuming that the position is measured from an origin at the center of the merry-go-round?

Hint 1. Change of a vector

A vector quantity has both magnitude and direction. The vector changes with time if *either* of these quantities changes with time.

ANSWER:

- the position of the ball only
- the velocity of the ball only
- the acceleration of the ball only
- both the position and velocity of the ball
- the position and velocity and acceleration of the ball

Correct

Part D

For the merry-go-round problem, do the magnitudes of the position, velocity, and acceleration vectors change with time?

Hint 1. Change of magnitude of a vector

A vector quantity has both magnitude and direction. The magnitude of a vector changes with time *only* if the length changes with time.

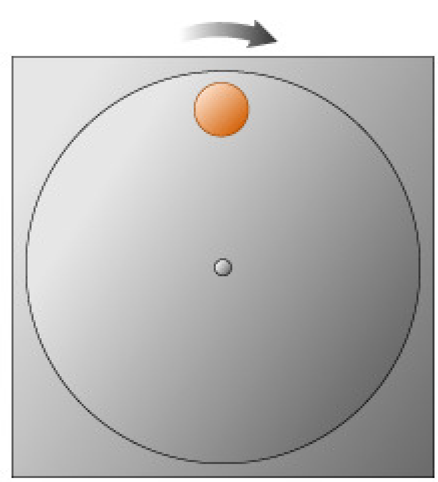
ANSWER:

- yes
- no

Correct

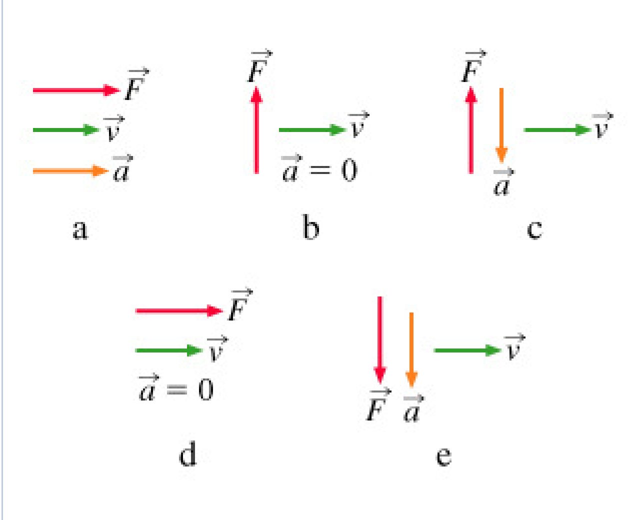
A Mass on a Turntable: Conceptual

A small metal cylinder rests on a circular turntable that is rotating at a constant rate, as illustrated in the diagram.



Part A

Which of the following sets of vectors best describes the velocity, acceleration, and net force acting on the cylinder at the point indicated in the diagram?



Hint 1. The direction of acceleration can be determined from Newton's second law

According to Newton's second law, the acceleration of an object has the same direction as the net force acting on that object.

ANSWER:

- a
- b
- c
- d
- e

Correct

Part B

Let R be the distance between the cylinder and the center of the turntable. Now assume that the cylinder is moved to a new location $R/2$ from the center of the turntable. Which of the following statements accurately describe the motion of the cylinder at the new location?

Check all that apply.

Hint 1. Find the speed of the cylinder

Find the speed v of the cylinder at the new location. Assume that the cylinder makes one complete turn in a period of time T.

Express your answer in terms of R and T.

ANSWER:

$v = \frac{\pi R}{T}$

Hint 2. Find the acceleration of the cylinder

Find the magnitude of the acceleration a of the cylinder at the new location. Assume that the cylinder makes one complete turn in a period of time T.

Express your answer in terms of R and T.

Hint 1. Centripetal acceleration

Recall that the acceleration of an object that moves in a circular path of radius r with constant speed v has magnitude given by

$$a = \frac{v^2}{r}.$$

Note that both the velocity and radius of the trajectory change when the cylinder is moved.

ANSWER:

$a = \frac{2\pi^2 R}{T^2}$

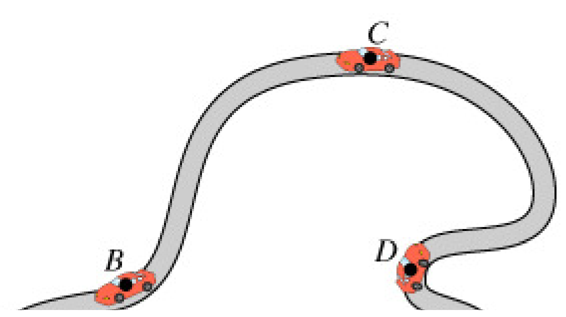
ANSWER:

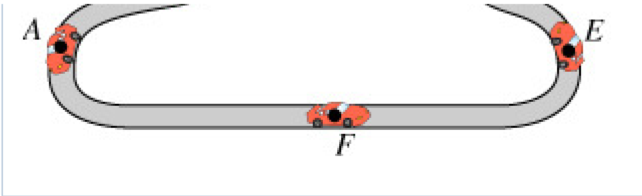
- [x] The speed of the cylinder has decreased.
- [ ] The speed of the cylinder has increased.
- [x] The magnitude of the acceleration of the cylinder has decreased.
- [ ] The magnitude of the acceleration of the cylinder has increased.
- [ ] The speed and the acceleration of the cylinder have not changed.

Correct

Accelerating along a Racetrack

A road race is taking place along the track shown in the figure . All of the cars are moving at constant speeds. The car at point F is traveling along a straight section of the track, whereas all the other cars are moving along curved segments of the track.





Part A

Let $\vec{v}_{\rm A}$ be the velocity of the car at point A. What can you say about the acceleration of the car at that point?

Hint 1. Acceleration along a curved path

Since acceleration is a vector quantity, an object moving at constant speed along a curved path has nonzero acceleration because the direction of its velocity $\vec{v}$ is changing, even though the magnitude of its velocity (the speed) is constant. Moreover, if the speed is constant, the object's acceleration is always perpendicular to the velocity vector $\vec{v}$ at each point along the curved path and is directed toward the center of curvature of the path.

ANSWER:

- The acceleration is parallel to $\vec{v}_{\rm A}$.
- The acceleration is perpendicular to $\vec{v}_{\rm A}$ and directed toward the inside of the track.
- The acceleration is perpendicular to $\vec{v}_{\rm A}$ and directed toward the outside of the track.
- The acceleration is neither parallel nor perpendicular to $\vec{v}_{\rm A}$.
- The acceleration is zero.

Correct

Part B

Let $\vec{v}_{\rm C}$ be the velocity of the car at point C. What can you say about the acceleration of the car at that point?

Hint 1. Acceleration along a curved path

Since acceleration is a vector quantity, an object moving at constant speed along a curved path has nonzero acceleration because the direction of its velocity $\vec{v}$ is changing, even though the magnitude of its velocity (the speed) is constant. Moreover, if the speed is constant, the object's acceleration is always perpendicular to the velocity vector $\vec{v}$ at each point along the curved path and is directed toward the center of curvature of the path.

ANSWER:

- The acceleration is parallel to $\vec{v}_{\rm C}$.
- The acceleration is perpendicular to $\vec{v}_{\rm C}$ and pointed toward the inside of the track.
- The acceleration is perpendicular to $\vec{v}_{\rm C}$ and pointed toward the outside of the track.
- The acceleration is neither parallel nor perpendicular to $\vec{v}_{\rm C}$.
- The acceleration is zero.

Correct

Part C

Let $\vec{v}_{\rm D}$ be the velocity of the car at point D. What can you say about the acceleration of the car at that point?

Hint 1. Acceleration along a curved path

Since acceleration is a vector quantity, an object moving at constant speed along a curved path has nonzero acceleration because the direction of its velocity $\vec{v}$ is changing, even though the magnitude of its velocity (the speed) is constant. Moreover, if the speed is constant, the object's acceleration is always perpendicular to the velocity vector $\vec{v}$ at each point along the curved path and is directed toward the center of curvature of the path.

ANSWER:

- The acceleration is parallel to $\vec{v}_{\rm D}$.
- The acceleration is perpendicular to $\vec{v}_{\rm D}$ and pointed toward the inside of the track.
- The acceleration is perpendicular to $\vec{v}_{\rm D}$ and pointed toward the outside of the track.
- The acceleration is neither parallel nor perpendicular to $\vec{v}_{\rm D}$.
- The acceleration is zero.

Correct

Part D

Let $\vec{v}_{\rm F}$ be the velocity of the car at point F. What can you say about the acceleration of the car at that point?

Hint 1. Acceleration along a straight path

The velocity of an object that moves along a straight path is always parallel to the direction of the path, and the object has a nonzero acceleration only if the magnitude of its velocity changes in time.

ANSWER:

- The acceleration is parallel to $\vec{v}_{\rm F}$.
- The acceleration is perpendicular to $\vec{v}_{\rm F}$ and pointed toward the inside of the track.
- The acceleration is perpendicular to $\vec{v}_{\rm F}$ and pointed toward the outside of the track.
- The acceleration is neither parallel nor perpendicular to $\vec{v}_{\rm F}$.
- The acceleration is zero.

Correct

Part E

Assuming that all cars have equal speeds, which car has the acceleration of the greatest magnitude, and which one has the acceleration of the least magnitude?

Give your answer as "X,Y" where X is the car with the greatest magnitude of acceleration and Y is the car with the least magnitude of accelation.

Hint 1. How to approach the problem

Recall that the magnitude of the acceleration of an object that moves at constant speed along a curved path is inversely proportional to the radius of curvature of the path.

ANSWER:

D,F

Correct

Part F

Assume that the car at point A and the one at point E are traveling along circular paths that have the same radius. If the car at point A now moves twice as fast as the car at point E, how is the magnitude of its acceleration related to that of car E.

Hint 1. Find the acceleration of the car at point E

Let r be the radius of the two curves along which the cars at points A and E are traveling. What is the magnitude $a_{\rm E}$ of the acceleration of the car at point E?

Express your answer in terms of the radius of curvature r and the speed $v_{\rm E}$ of car E.

Hint 1. Uniform circular motion

The magnitude a of the acceleration of an object that moves with constant speed v along a circular path of radius r is given by

$$a = \frac{v^2}{r}.$$

ANSWER:

$a_{\rm E}$ = $\frac{{v_E}^2}{r}$

Hint 2. Find the acceleration of the car at point A

If $v_A = 2v_E$, what is the acceleration $a_{\rm A}$ of the car at point A? Let r be the radius of the two curves along which the cars at points A and E are traveling.

Express your answer in terms of the speed $v_{\rm E}$ of the car at E and the radius r.

Hint 1. Uniform circular motion

The magnitude of the acceleration of an object that moves with constant speed v along a circular path of radius r is given by

$$a = \frac{v^2}{r}.$$

ANSWER:

$a_{\rm A}$ = $\frac{4{v_E}^2}{r}$

ANSWER:

- ○ The magnitude of the acceleration of the car at point A is twice that of the car at point E.
- ○ The magnitude of the acceleration of the car at point A is the same as that of the car at point E.
- ○ The magnitude of the acceleration of the car at point A is half that of the car at point E.
- ◉ The magnitude of the acceleration of the car at point A is four times that of the car at point E.

Correct

At the Test Track

You want to test the grip of the tires on your new race car. You decide to take the race car to a small test track to experimentally determine the coefficient of friction. The racetrack consists of a flat, circular road with a radius of 45 $\rm m$. The applet shows the result of driving the car around the track at various speeds.

Part A

What is μ_s, the coefficient of static friction between the tires and the track?

Express your answer to two significant figures.

Hint 1. How to approach the problem

You need to find the point at which the force of friction is just strong enough to keep the car on the circular track. Then, you can set the expression for the frictional force equal to the centripetal force needed to keep the car going in a circle. Solving this equation for μ_s gives the answer. Use m for the mass of the car in your calculations.

Hint 2. Use the applet to find the speed

When the car successfully goes around the track, without leaving the track at all, then the needed centripetal force must be less than or equal to the maximum possible static friction. When the car leaves the track during a lap, then the needed centripetal force must be greater than the maximum possible static friction. What is the lowest speed $v_{\rm min}$ at which the car leaves the track in the applet?

Express your answer in meters per second as an integer.

ANSWER:

$v_{\rm min}$ = 21 $\rm m/s$

Hint 3. Find an expression for μ_s

Find an expression for μ_s, the coefficient of static friction between the car's tires and the road. Use m for the mass of the car, g for the magnitude of the acceleration due to gravity, v for the speed at which the car is just about to leave the track, and r for the radius of the track.

Express your answer in terms of m, g, v, and r.

Hint 1. Expression for centripetal acceleration

The centripetal acceleration a required to move an object in a circular path of radius r at speed v is $a = v^2/r$.

Hint 2. Expression for the force of static friction

Recall that static friction will equal the force attempting to move an object unless the magnitude of that force exceeds $\mu_s n$, where n is the magnitude of the normal force and μ_s is the coefficient of static friction. Since the track is flat, the normal force is equal in magnitude to the weight of the car, giving $n = mg$. Thus, the magnitude of the maximum force of static friction is $\mu_s mg$.

ANSWER:

μ_s = $\frac{v^2}{gr}$

ANSWER:

μ_s = 0.91

Correct

PSS 8.1 Circular-Motion Problems

Learning Goal:

To practice Problem-Solving Strategy 8.1 for circular-motion problems.

A cyclist competes in a one-lap race around a flat, circular course of radius $140 \mathrm{m}$. Starting from rest and speeding up at a constant rate throughout the race, the cyclist covers the entire course in 60s . The mass of the bicycle (including the rider) is $76 \mathrm{kg}$. What is the magnitude of the net force $F_{\rm net}$ acting on the bicycle as it crosses the finish line?

PROBLEM-SOLVING STRATEGY 8.1 Circular-motion problems

MODEL: Make appropriate simplifying assumptions.

VISUALIZE: Draw a pictorial representation.

- Establish a coordinate system with the *r* axis pointing toward the center of the circle.
- Show important points in the motion on a sketch. Define symbols, and identify what the problem is trying to find.
- Identify the forces, and show them on a free-body diagram.

SOLVE: Newton's second law is

$$\left(F_{\rm net}\right)_r = \sum F_r = ma_r = \frac{mv^2}{r} = m\omega^2 r,$$

$$\left(F_{\rm net}\right)_t = \sum F_t = ma_t,$$

and

$$\left(F_{\rm net}\right)_z = \sum F_z = 0.$$

- Determine the force components from the free-body diagram. Be careful with signs.
- Solve for the acceleration, and then use kinematics to find velocities and positions.

ASSESS: Check that your result has the correct units, is reasonable, and answers the question.

Model

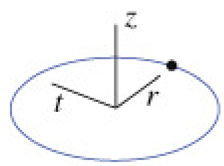
The cyclist moves in a circle at an increasing speed. This means that the cyclist has both centripetal and tangential acceleration. Moreover, the rate at which the cyclist's speed is increasing is constant. Thus, to simplify the problem, you can model the bicycle + rider as a particle in *nonuniform* circular motion and use constant-acceleration kinematics to work out your solution.

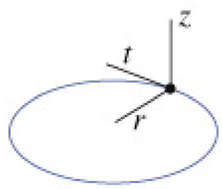
Visualize

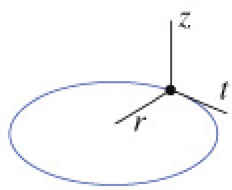
Part A

Which of the following sets of *rtz* coordinate axes is the most appropriate for this problem? The black dot represents the bicycle + rider at an arbitrary instant during the race.

ANSWER:







Correct

Unless otherwise stated, in circular-motion problems always use the usual convention in which the *t* axis points in the counterclockwise direction. Note that the *r* axis always points from the position of the cyclist to the center of the course, regardless where the cyclist is along the circular course. This means that the direction of the *r* axis changes as the cyclist moves.

Part B

Identify which of the following forces act on the bicycle + rider system, and sort them accordingly.

Drag the appropriate items to their respective bins.

ANSWER:

All attempts used; correct answer displayed

Since the motion is horizontal, Newton's second law requires the vertical component of the net force be zero, that is, $(F_{\rm net})_z = \sum F_z = 0$. This means that gravity will cancel out the normal force, and we don't need to worry about these vertical forces. As for the horizontal forces acting on the cyclist, both rolling friction and air resistance oppose the forward motion of the bicycle, so they must act along the tangential direction, opposite to the velocity vector. Static friction, instead, has two effects: It propels the bicycle tires forward (the tires push backward against the earth, and the earth pushes forward on the tires as friction) and prevents the bicycle from sliding sideways. So, it must have both a component along the tangential direction that provides the tangential acceleration and a component along the radial direction that provides the centripetal acceleration.

Note that although the effects of rolling friction and air resistance can be ignored, *static friction cannot be neglected*. If you neglect rolling friction and/or air resistance, you would simply end up with an overestimate of the cyclist's tangential acceleration. If you ignore static friction, instead, you would neglect the only force that provides the cyclist's centripetal acceleration, which is an essential element of circular motion.

Part C

Below is a top view of the circular course. The black dot represents the bicycle + rider at an arbitrary instant during the race. Assume the bicyclist is traveling around the track in the counterclockwise direction. To simplify the problem, also assume that rolling friction is negligible. (This is reasonable because the contact area between the bicycle tires and the ground is often very small.)
Draw a free-body diagram showing all the horizontal forces acting on the bicycle. Make a *reasonable* estimation of the direction of each force.

Draw the vectors starting at the black dot. The location and orientation of the vectors will be graded. The length of the vectors will not be graded.

Hint 1. How to estimate the direction of drag and static friction

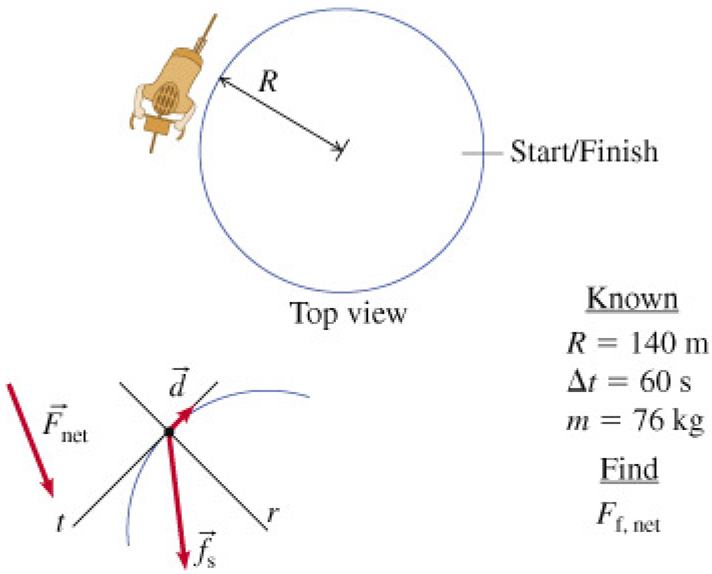
As explained in the previous part, air resistance (drag) opposes the forward motion of the bicycle, so $\vec{d}$ must act along the tangential direction, opposite to the velocity vector. Static friction, instead, has two effects. It propels the bicycle tires forward (the tires push backward against the earth, and the earth pushes forward on the tires as friction) and prevents the bicycle from sliding sideways. So, $\vec{f}_{\rm s}$ must have both a component along the tangential direction that provides the tangential acceleration and a component along the radial direction that provides the centripetal acceleration. Since the magnitudes and the exact direction of these forces are unknown, it is sufficient to determine the sign of their components to draw a reasonable free-body diagram. To do that, determine the signs of the components of the net force, which must have the same signs as the tangential and the centripetal acceleration. Then, draw $\vec{f}_{\rm s}$ and $\vec{d}$ so that their vector sum (the net force) has components with the required signs.

ANSWER:

Answer Requested

Now, you can complete your pictorial representation. Note that although the tangential acceleration (and therefore the tangential force) is constant through the race, the radial force F_r and acceleration a_r, as well as the angular velocity ω, are not constant. So, you should define unique symbols for these quantities at important points in the motion: For this problem, we'll use the subscript i to refer to values ($F_{\mathrm{i},r}$, $a_{\mathrm{i},r}$, ω_{i}) at the moment the race begins and the subscript f to refer to values ($F_{\mathrm{f},r}$, $a_{\mathrm{f},r}$, ω_{f}) at the moment the cyclist crosses the finish line. Keep in mind that you are trying to find the magnitude of the net force at the finish line, $F_{\mathrm{f,net}}$.

Your pictorial representation should look like this:



Solve

Part D

Find $F_{\mathrm{f,net}}$, the magnitude of the net force acting on the cyclist at the finish line.

Express your answer in newtons to two significant figures.

Hint 1. The net force in terms of its components

The magnitude of the net force is given by the usual formula for finding the magnitude of a vector in terms of its components:

$$F_{\mathrm{net}} = \sqrt{[(F_{\mathrm{net}})_r]^2 + [(F_{\mathrm{net}})_t]^2}.$$

To find the components of the net force acting on the cyclist at the finish line, use Newton's second law, as explained in the strategy above. That will require you to calculate the components of the cyclist's acceleration at the finish line.

Hint 2. Find the tangential acceleration

From the problem introduction, you know that the cyclist speeds up at a constant rate or, in other words, accelerates with constant tangential acceleration a_t. Use the appropriate kinematic equation for motion with constant acceleration to find a_t.

Express your answer in meters per second squared to three significant figures.

Hint 1. Motion with constant acceleration

For a particle in circular motion with constant tangential acceleration, it is convenient to use the equation for the angular displacement θ of the particle in terms of its angular velocity ω and angular acceleration α:

$$\theta_{\rm f} = \theta_{\rm i} + \omega_{\rm i}\Delta t + \frac{1}{2}\alpha(\Delta t)^2,$$

where the subscript i refers to initial values at time $t = 0$, and the subscript f refers to final values at time $t = \Delta t$.

In this problem, in making one complete circuit of the course, the cyclist's angular position changes by 2π. You also know that the cyclist starts from rest, so her initial angular velocity is zero ($\omega_{\rm i} = 0$).

Hint 2. Tangential and angular accelerations

Recall that for a particle in circular motion, the relationship between the particle's tangential and angular accelerations, a_t and α, respectively, is

$$a_t = r\alpha,$$

where r is the radius of the circular path.

ANSWER:

a_t = 0.489 $\rm m/s^2$

Hint 3. Find the radial acceleration

What is the cyclist's radial acceleration at the finish line, $a_{{\rm f},r}$? Note that the radial component of the cyclist's acceleration is simply the centripetal acceleration needed to keep the cyclist moving in a circle.

Express your answer in meters per second squared to three significant figures.

Hint 1. Centripetal acceleration

The centripetal acceleration of a particle moving in a circle of radius r is

$$a_r = \frac{v^2}{r} = \omega^2 r,$$

where v is the particle speed, and ω is its angular speed. Therefore, to find the cyclist's centripetal acceleration at the finish line, $a_{{\rm f},r}$, you will need to calculate the cyclist's speed, $v_{\rm f}$, or, alternatively, the cyclist's angular velocity, $\omega_{\rm f}$, at the finish line. In both cases, you will need to know the cyclist's constant tangential acceleration.

Hint 2. Find the final angular velocity

To find the cyclist's final angular velocity, $\omega_{\rm f}$, you can use the kinematic formula for motion with constant angular acceleration α:

$$\omega_{\rm f} = \omega_{\rm i} + \alpha(\Delta t).$$

Considering that the tangential acceleration, a_t, is related to the angular acceleration by the expression $a_t = r\alpha$, where r is the radius of the circular path, what is the final angular velocity $\omega_{\rm f}$ of the cyclist?

Express your answer in radians per second to three significant figures.

ANSWER:

$\omega_{\rm f}$ = 0.209 $\rm rad/s$

ANSWER:

$a_{{\rm f},r}$ = 6.14 $\rm m/s^2$

ANSWER:

$F_{\rm f,net}$ = 470 $\rm N$

Correct

Assess

Part E

To assess whether your calculations make sense, let's simplify the problem even further and assume air resistance is negligible. In this case, the net force acting on the bicyclist is equivalent to just the force of static friction, and your answer to Part D is the magnitude $f_{\rm s}$. Based on this value, what is the minimum coefficient of static friction $\mu_{\rm s}$ between the race track and the bicycle?

Express your answer numerically to two significant figures.

Hint 1. How to approach the problem

The magnitude $f_{\rm s}$ of the force of static friction is less than or equal to $\mu_s n$, where $\mu_{\rm s}$ is the coefficient of static friction and n is the magnitude of the normal force. Therefore, the minimum coefficient of static friction between the race track and the bicycle can be found by solving the equation $f_s = \mu_s n$, where you use your result from part D as $f_{\rm s}$.

You made use of the relation $(F_{\rm net})_z = \sum F_z = 0$ previously to justify ignoring gravity and the normal force in calculating the net force, because they had to cancel out. Now, use that relation to find the magnitude of the normal force.

ANSWER:

$\mu_s =$ 0.63

Answer Requested

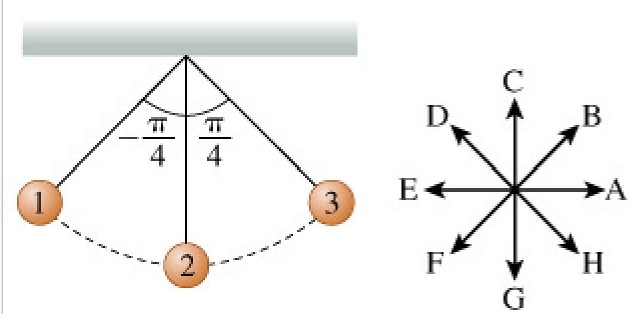
This is a reasonable value for the coefficient of static friction. Actual coefficients of static friction between race tracks and bicycle tires usually range from around 0.4 up to 0.7. If you found that your answer implied a coefficient of 1 or greater, then you would know that you had made a mistake. If you found that the minimum coefficient was very small, less than 0.01 for instance, then you might guess that you had made an error as well.

Direction of Acceleration of Pendulum

Learning Goal:

To understand that the direction of acceleration is in the direction of the *change* of the velocity, which is unrelated to the direction of the velocity.

The pendulum shown makes a full swing from $-\pi/4$ to $+\pi/4$. Ignore friction and assume that the string is massless. The eight labeled arrows represent directions to be referred to when answering the following questions.



Part A

Which of the following is a true statement about the acceleration of the pendulum bob, $\vec{a}$.

ANSWER:

- $\vec{a}$ is equal to the acceleration due to gravity.
- $\vec{a}$ is equal to the instantaneous rate of change in velocity.
- $\vec{a}$ is perpendicular to the bob's trajectory.
- $\vec{a}$ is tangent to the bob's trajectory.

Correct

Part B

What is the direction of $\vec{a}$ when the pendulum is at position 1?

Enter the letter of the arrow parallel to $\vec{a}$.

Hint 1. Velocity at position 1

What is the velocity of the bob when it is exactly at position 1?

ANSWER:

$v_1 =$ 0 $\mathrm{m/s}$

Hint 2. Velocity of bob after it has descended

What is the velocity of the bob just after it has descended from position 1?

ANSWER:

- very small and having a direction best approximated by arrow D
- very small and having a direction best approximated by arrow A
- very small and having a direction best approximated by arrow H
- The velocity cannot be determined without more information.

ANSWER:

H

Correct

Part C

What is the direction of $\vec{a}$ at the moment the pendulum passes position 2?

Enter the letter of the arrow that best approximates the direction of $\vec{a}$.

Hint 1. Instantaneous motion

At position 2, the instantaneous motion of the pendulum can be approximated as uniform circular motion. What is the direction of acceleration for an object executing uniform circular motion?

ANSWER:

C

Correct

We know that for the object to be traveling in a circle, some component of its acceleration must be pointing radially inward.

Part D

What is the direction of $\vec{a}$ when the pendulum reaches position 3?

Give the letter of the arrow that best approximates the direction of $\vec{a}$.

Hint 1. Velocity just before position 3

What is the velocity of the bob just before it reaches position 3?

ANSWER:

- (selected) very small and having a direction best approximated by arrow B
- very small and having a direction best approximated by arrow C
- very small and having a direction best approximated by arrow H
- The velocity cannot be determined without more information.

Hint 2. Velocity of bob at position 3

What is the velocity of the bob when it reaches position 3?

ANSWER:

$v_3 = 0 \;\; \mathrm{m/s}$

ANSWER:

F

Correct

Part E

As the pendulum *approaches* or *recedes from* which position(s) is the acceleration vector $\vec{a}$ almost parallel to the velocity vector $\vec{v}$.

ANSWER:

- position 2 only
- positions 1 and 2
- positions 2 and 3
- positions 1 and 3

Correct

Problem 8.59

In the absence of air resistance, a projectile that lands at the elevation from which it was launched achieves maximum range when launched at a 45° angle. Suppose a projectile of mass m is launched with speed v_0 into a headwind that exerts a constant, horizontal retarding force $\vec{F}_{\rm wind} = -F_{\rm wind}\hat{i}$.

Part A

Find an expression for the angle at which the range is maximum.

ANSWER:

- $\theta = tan^{-1}(mg/F)/2$
- $\theta = tan^{-1}(F/mg)$
- $\theta = sin^{-1}(mg/F)$
- $\theta = F/mg$

Correct

Part B

By what percentage is the maximum range of a 0.200$\rm kg$ ball reduced if $F_{\rm wind} =$ 0.100$\rm N$?

Express your answer with the appropriate units.

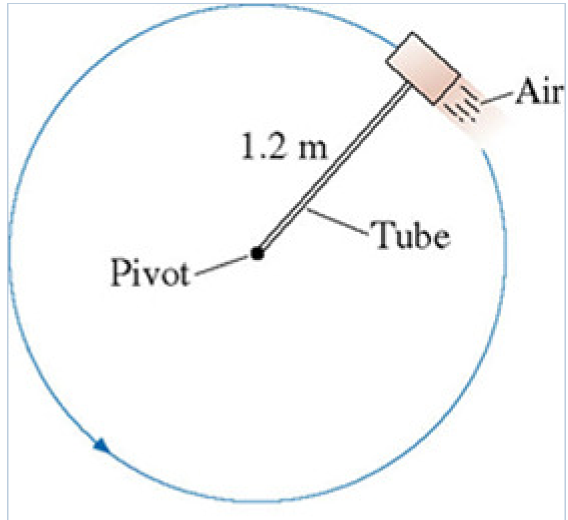
ANSWER:

4.97 $\%$

Correct

Problem 8.62

A 500g steel block rotates on a steel table while attached to a 1.20$\rm m$ -long hollow tube. Compressed air fed through the tube and ejected from a nozzle on the back of the block exerts a thrust force of 5.11$\rm N$ perpendicular to the tube. The maximum tension the tube can withstand without breaking is $50.0\mathrm{N}$. Assume the coefficient of kinetic friction between steel block and steel table is 0.60.



Part A

If the block starts from rest, how many revolutions does it make before the tube breaks?

Express your answer with the appropriate units.

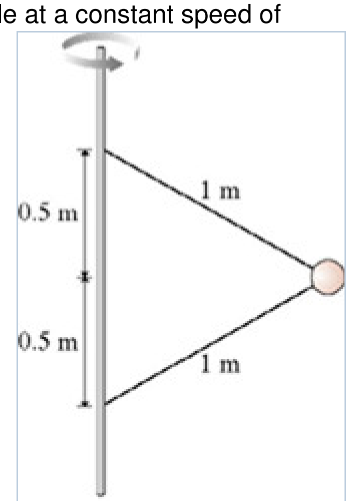
ANSWER:

1.83 rev

Answer Requested

Problem 8.63

Two wires are tied to the 300g sphere shown in figure. The sphere revolves in a horiIzontal circle at a constant speed of $8.50\mathrm{m/s}$.



Part A

What is the tension in the upper wire?

Express your answer with the appropriate units.

ANSWER:

17.4 N

Correct

Part B

What is the tension in the lower wire?

Express your answer with the appropriate units.

ANSWER:

11.5 N

Correct

Score Summary:

Your score on this assignment is 85.0%.
You received 8.5 out of a possible total of 10 points.