Chapter 10 Rotation of a Rigid Object About a Fixed Axis

P10.3 (a) $\alpha = \frac{\omega - \omega_i}{t} = \frac{12.0 \text{ rad/s}}{3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$

(b) $\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2}(4.00 \text{ rad/s}^2)(3.00 \text{ s})^2 = \boxed{18.0 \text{ rad}}$

P10.5 $\omega_i = \frac{100 \text{ rev}}{1.00 \text{ min}}\left(\frac{1 \text{ min}}{60.0 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = \frac{10\pi}{3} \text{ rad/s}, \; \omega_f = 0$

(a) $t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - (10\pi/3)}{-2.00} \text{ s} = \boxed{5.24 \text{ s}}$

(b) $\theta_f = \bar{\omega} t = \left(\frac{\omega_f + \omega_i}{2}\right) t = \left(\frac{10\pi}{6} \text{ rad/s}\right)\left(\frac{10\pi}{6} \text{ s}\right) = \boxed{27.4 \text{ rad}}$

P10.9 $\omega = 5.00 \text{ rev/s} = 10.0\pi \text{ rad/s}$. We will break the motion into two stages: (1) a period during which the tub speeds up and (2) a period during which it slows down.

While speeding up, $\theta_1 = \bar{\omega} t = \frac{0 + 10.0\pi \text{ rad/s}}{2}(8.00 \text{ s}) = 40.0\pi \text{ rad}$

While slowing down, $\theta_2 = \bar{\omega} t = \frac{10.0\pi \text{ rad/s} + 0}{2}(12.0 \text{ s}) = 60.0\pi \text{ rad}$

So, $\theta_{\text{total}} = \theta_1 + \theta_2 = 100\pi \text{ rad} = \boxed{50.0 \text{ rev}}$

P10.10 (a) $v = r\omega$; $\omega = \frac{v}{r} = \frac{45.0 \text{ m/s}}{250 \text{ m}} = \boxed{0.180 \text{ rad/s}}$

(b) $a_r = \frac{v^2}{r} = \frac{(45.0 \text{ m/s})^2}{250 \text{ m}} = \boxed{8.10 \text{ m/s}^2 \text{ toward the center of track}}$

P10.14 (a) $\omega = \frac{v}{r} = \frac{25.0 \text{ m/s}}{1.00 \text{ m}} = \boxed{25.0 \text{ rad/s}}$

(b) $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2(\Delta\theta)} = \frac{(25.0 \text{ rad/s})^2 - 0}{2[(1.25 \text{ rev})(2\pi \text{ rad/rev})]} = \boxed{39.8 \text{ rad/s}^2}$$

(c) $\Delta t = \frac{\Delta\omega}{\alpha} = \frac{25.0 \text{ rad/s}}{39.8 \text{ rad/s}^2} = \boxed{0.628 \text{ s}}$

P10.21 (a) $I = \sum_j m_j r_j^2$

In this case,

$$r_1 = r_2 = r_3 = r_4$$
$$r = \sqrt{(3.00\text{ m})^2 + (2.00\text{ m})^2} = \sqrt{13.0}\text{ m}$$
$$I = \left[\sqrt{13.0}\text{ m}\right]^2 [3.00 + 2.00 + 2.00 + 4.00]\text{ kg}$$
$$= \boxed{143\text{ kg}\cdot\text{m}^2}$$

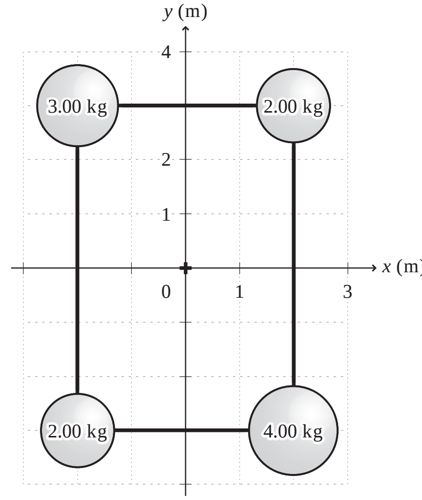

FIG. P10.21

(b) $K_R = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(143\text{ kg}\cdot\text{m}^2\right)(6.00\text{ rad/s})^2$

$= \boxed{2.57 \times 10^3\text{ J}}$

***P10.22** $m_1 = 4.00\text{ kg}$, $r_1 = |y_1| = 3.00\text{ m}$

$m_2 = 2.00\text{ kg}$, $r_2 = |y_2| = 2.00\text{ m}$

$m_3 = 3.00\text{ kg}$, $r_3 = |y_3| = 4.00\text{ m}$

$\omega = 2.00\text{ rad/s}$ about the x-axis

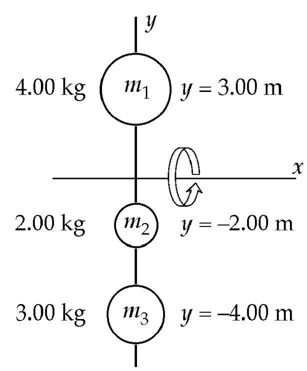

FIG. P10.22

(a)
$$I_x = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$
$$I_x = 4.00(3.00)^2 + 2.00(2.00)^2 + 3.00(4.00)^2 = \boxed{92.0\text{ kg}\cdot\text{m}^2}$$
$$K_R = \frac{1}{2} I_x \omega^2 = \frac{1}{2}(92.0)(2.00)^2 = \boxed{184\text{ J}}$$

(b)
$$v_1 = r_1\omega = 3.00(2.00) = \boxed{6.00\text{ m/s}}$$
$$K_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2}(4.00)(6.00)^2 = 72.0\text{ J}$$
$$v_2 = r_2\omega = 2.00(2.00) = \boxed{4.00\text{ m/s}}$$
$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2}(2.00)(4.00)^2 = 16.0\text{ J}$$
$$v_3 = r_3\omega = 4.00(2.00) = \boxed{8.00\text{ m/s}}$$
$$K_3 = \frac{1}{2} m_3 v_3^2 = \frac{1}{2}(3.00)(8.00)^2 = 96.0\text{ J}$$
$$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = \boxed{184\text{ J}} = \frac{1}{2} I_x \omega^2$$

(c) The kinetic energies computed in parts (a) and (b) are the same. Rotational kinetic energy can be viewed as the total translational kinetic energy of the particles in the rotating object.

P10.23 $I = Mx^2 + m(L-x)^2$

$$\frac{dI}{dx} = 2Mx - 2m(L-x) = 0 \text{ (for an extremum)}$$

$$\therefore x = \frac{mL}{M+m}$$

$\dfrac{d^2I}{dx^2} = 2m + 2M$; therefore I is at a minimum when the axis of rotation passes through $x = \dfrac{mL}{M+m}$ which is also the center of mass of the system. The moment of inertia about an axis passing through x is

$$I_{CM} = M\left[\frac{mL}{M+m}\right]^2 + m\left[1 - \frac{m}{M+m}\right]^2 L^2 = \frac{Mm}{M+m}L^2 = \mu L^2$$

where

$$\mu = \frac{Mm}{M+m}$$

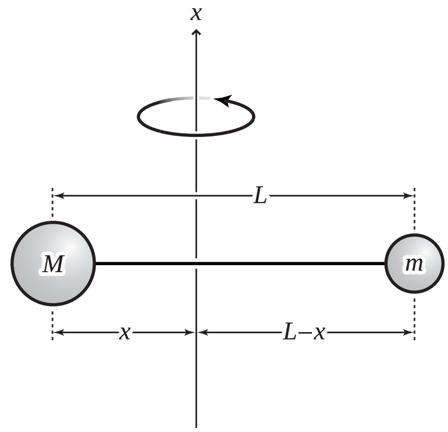

FIG. P10.23

P10.26 We assume the rods are thin, with radius much less than L. Call the junction of the rods the origin of coordinates, and the axis of rotation the z-axis.

For the rod along the y-axis, $I = \frac{1}{3}mL^2$ from the table.

For the rod parallel to the z-axis, the parallel-axis theorem gives

$$I = \frac{1}{2}mr^2 + m\left(\frac{L}{2}\right)^2 \cong \frac{1}{4}mL^2$$

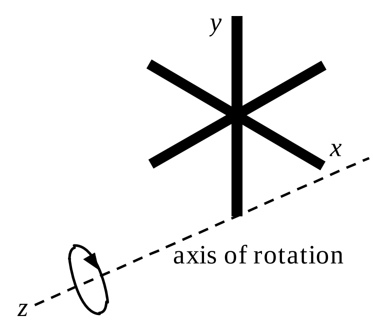

FIG. P10.26

In the rod along the x-axis, the bit of material between x and $x + dx$ has mass $\left(\frac{m}{L}\right)dx$ and is at distance $r = \sqrt{x^2 + \left(\frac{L}{2}\right)^2}$ from the axis of rotation. The total rotational inertia is:

$$I_{\text{total}} = \frac{1}{3}mL^2 + \frac{1}{4}mL^2 + \int_{-L/2}^{L/2}\left(x^2 + \frac{L^2}{4}\right)\left(\frac{m}{L}\right)dx$$

$$= \frac{7}{12}mL^2 + \left(\frac{m}{L}\right)\frac{x^3}{3}\Bigg|_{-L/2}^{L/2} + \frac{mL}{4}x\Bigg|_{-L/2}^{L/2}$$

$$= \frac{7}{12}mL^2 + \frac{mL^2}{12} + \frac{mL^2}{4} = \boxed{\frac{11mL^2}{12}}$$

Note: The moment of inertia of the rod along the x axis can also be calculated from the parallel-axis theorem as $\frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2$.

P10.27 Treat the tire as consisting of three parts. The two sidewalls are each treated as a hollow cylinder of inner radius 16.5 cm, outer radius 30.5 cm, and height 0.635 cm. The tread region is treated as a hollow cylinder of inner radius 30.5 cm, outer radius 33.0 cm, and height 20.0 cm.

Use $I = \frac{1}{2}m\left(R_1^2 + R_2^2\right)$ for the moment of inertia of a hollow cylinder.

Sidewall:

$$m = \pi\left[(0.305\text{ m})^2 - (0.165\text{ m})^2\right]\left(6.35\times10^{-3}\text{ m}\right)\left(1.10\times10^3\text{ kg/m}^3\right) = 1.44\text{ kg}$$

$$I_{\text{side}} = \frac{1}{2}(1.44\text{ kg})\left[(0.165\text{ m})^2 + (0.305\text{ m})^2\right] = 8.68\times10^{-2}\text{ kg}\cdot\text{m}^2$$

Tread:

$$m = \pi\left[(0.330\text{ m})^2 - (0.305\text{ m})^2\right](0.200\text{ m})\left(1.10\times10^3\text{ kg/m}^3\right) = 11.0\text{ kg}$$

$$I_{\text{tread}} = \frac{1}{2}(11.0\text{ kg})\left[(0.330\text{ m})^2 + (0.305\text{ m})^2\right] = 1.11\text{ kg}\cdot\text{m}^2$$

Entire Tire:

$$I_{\text{total}} = 2I_{\text{side}} + I_{\text{tread}} = 2\left(8.68\times10^{-2}\text{ kg}\cdot\text{m}^2\right) + 1.11\text{ kg}\cdot\text{m}^2 = \boxed{1.28\text{ kg}\cdot\text{m}^2}$$

P10.30 We consider the cam as the superposition of the original solid disk and a disk of negative mass cut from it. With half the radius, the cut-away part has one-quarter the face area and one-quarter the volume and one-quarter the mass M_0 of the original solid cylinder:

$$M_0 - \frac{1}{4}M_0 = M \qquad\qquad M_0 = \frac{4}{3}M$$

By the parallel-axis theorem, the original cylinder had moment of inertia

$$I_{\text{CM}} + M_0\left(\frac{R}{2}\right)^2 = \frac{1}{2}M_0R^2 + M_0\frac{R^2}{4} = \frac{3}{4}M_0R^2$$

The negative-mass portion has $I = \frac{1}{2}\left(-\frac{1}{4}M_0\right)\left(\frac{R}{2}\right)^2 = -\frac{M_0R^2}{32}$. The whole cam has

$$I = \frac{3}{4}M_0R^2 - \frac{M_0R^2}{32} = \frac{23}{32}M_0R^2 = \frac{23}{32}\frac{4}{3}MR^2 = \frac{23}{24}MR^2 \text{ and}$$

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{23}{24}MR^2\omega^2 = \boxed{\frac{23}{48}MR^2\omega^2}$$

P10.33 $\sum\tau = 0.100\text{ m}(12.0\text{ N}) - 0.250\text{ m}(9.00\text{ N}) - 0.250\text{ m}(10.0\text{ N}) = \boxed{-3.55\text{ N}\cdot\text{m}}$

The thirty-degree angle is unnecessary information.

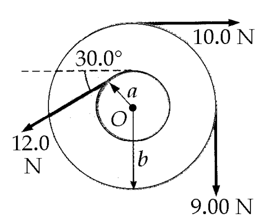

FIG. P10.33

P10.37 For m_1,

$$\sum F_y = ma_y: \quad +n - m_1 g = 0$$

$$n_1 = m_1 g = 19.6 \text{ N}$$

$$f_{k1} = \mu_k n_1 = 7.06 \text{ N}$$

$$\sum F_x = ma_x: \quad -7.06 \text{ N} + T_1 = (2.00 \text{ kg})a \qquad (1)$$

For the pulley,

$$\sum \tau = I\alpha: \quad -T_1 R + T_2 R = \frac{1}{2} MR^2 \left(\frac{a}{R}\right)$$

$$-T_1 + T_2 = \frac{1}{2}(10.0 \text{ kg})a$$

$$-T_1 + T_2 = (5.00 \text{ kg})a \qquad (2)$$

For m_2, $\quad +n_2 - m_2 g\cos\theta = 0$

$$n_2 = 6.00 \text{ kg}\left(9.80 \text{ m/s}^2\right)(\cos 30.0°)$$

$$= 50.9 \text{ N}$$

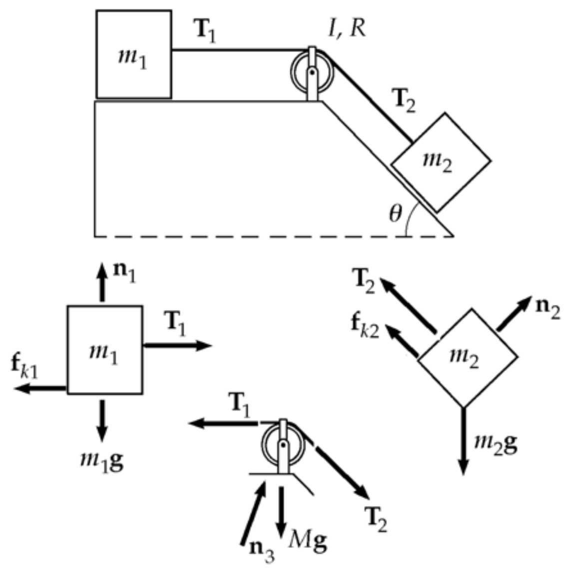

FIG. P10.37

$$f_{k2} = \mu_k n_2$$

$$= 18.3 \text{ N}: \quad -18.3 \text{ N} - T_2 + m_2 \sin\theta = m_2 a$$

$$-18.3 \text{ N} - T_2 + 29.4 \text{ N} = (6.00 \text{ kg})a \qquad (3)$$

(a) Add equations (1), (2), and (3):

$$-7.06 \text{ N} - 18.3 \text{ N} + 29.4 \text{ N} = (13.0 \text{ kg})a$$

$$a = \frac{4.01 \text{ N}}{13.0 \text{ kg}} = \boxed{0.309 \text{ m/s}^2}$$

(b) $T_1 = 2.00 \text{ kg}\left(0.309 \text{ m/s}^2\right) + 7.06 \text{ N} = \boxed{7.67 \text{ N}}$

$T_2 = 7.67 \text{ N} + 5.00 \text{ kg}\left(0.309 \text{ m/s}^2\right) = \boxed{9.22 \text{ N}}$

P10.38 $I = \frac{1}{2}mR^2 = \frac{1}{2}(100 \text{ kg})(0.500 \text{ m})^2 = 12.5 \text{ kg}\cdot\text{m}^2$

$$\omega_i = 50.0 \text{ rev/min} = 5.24 \text{ rad/s}$$

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 5.24 \text{ rad/s}}{6.00 \text{ s}} = -0.873 \text{ rad/s}^2$$

$$\tau = I\alpha = 12.5 \text{ kg}\cdot\text{m}^2\left(-0.873 \text{ rad/s}^2\right) = -10.9 \text{ N}\cdot\text{m}$$

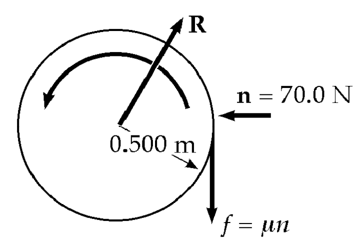

FIG. P10.38

The magnitude of the torque is given by $fR = 10.9 \text{ N}\cdot\text{m}$, where f is the force of friction.

Therefore,

$$f = \frac{10.9 \text{ N}\cdot\text{m}}{0.500 \text{ m}} \quad \text{and} \quad f = \mu_k n$$

yields $\quad \mu_k = \frac{f}{n} = \frac{21.8 \text{ N}}{70.0 \text{ N}} = \boxed{0.312}$

P10.53 (a) $K_{\text{trans}} = \frac{1}{2}mv^2 = \frac{1}{2}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{500\text{ J}}$

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right) = \frac{1}{4}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{250\text{ J}}$

(c) $K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750\text{ J}}$

P10.56 $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$ where $\omega = \frac{v}{R}$ since no slipping occurs.

Also, $U_i = mgh$, $U_f = 0$, and $v_i = 0$

Therefore, $\frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2 = mgh$

Thus, $v^2 = \frac{2gh}{\left[1 + \left(I/mR^2\right)\right]}$

For a disk, $I = \frac{1}{2}mR^2$

So $v^2 = \frac{2gh}{1+\frac{1}{2}}$ or $\boxed{v_{\text{disk}} = \sqrt{\frac{4gh}{3}}}$

For a ring, $I = mR^2$ so $v^2 = \frac{2gh}{2}$ or $\boxed{v_{\text{ring}} = \sqrt{gh}}$

Since $v_{\text{disk}} > v_{\text{ring}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

***P10.62** (a) We consider the elevator-sheave-counterweight-Earth system, including n passengers, as an isolated system and apply the conservation of mechanical energy. We take the initial configuration, at the moment the drive mechanism switches off, as representing zero gravitational potential energy of the system.

Therefore, the initial mechanical energy of the system is

$$E_i = K_i + U_i = (1/2)\, m_e v^2 + (1/2)\, m_c v^2 + (1/2) I_s \omega^2$$

$$= (1/2)\, m_e v^2 + (1/2)\, m_c v^2 + (1/2)[(1/2) m_s r^2](v/r)^2$$

$$= (1/2)\, [m_e + m_c v^2 + (1/2) m_s]\, v^2$$

The final mechanical energy of the system is entirely gravitational because the system is momentarily at rest:

$$E_f = K_f + U_f = 0 + m_e g d - m_c g d$$

where we have recognized that the elevator car goes up by the same distance d that the counter- weight goes down. Setting the initial and final energies of the system equal to each other, we have

$$(1/2)\, [m_e + m_c + (1/2) m_s]\, v^2 = (m_e - m_c)\, gd$$

$(1/2)$ [800 kg + n 80 kg + 950 kg + 140 kg]$(3\text{ m/s})^2$ = (800 kg + n 80 kg – 950 kg)(9.8

m/s^2) d

$\boxed{d = [1890 + 80n](0.459\text{ m})/(80n - 150)}$

(b) $d = [1890 + 80\times2](0.459\text{ m})/(80\times2 - 150) = \boxed{94.1\text{ m}}$

(c) $d = [1890 + 80\times12](0.459\text{ m})/(80\times12 - 150) = \boxed{1.62\text{ m}}$

(d) $d = [1890 + 80\times0](0.459\text{ m})/(80\times0 - 150) = \boxed{-5.79\text{ m}}$

(e) The rising car will coast to a stop only for $n \ge 2$. For $n = 0$ or $n = 1$, the car would accelerate
upward if released.

(f) The graph looks roughly like one branch of a hyperbola. It comes down steeply from 94.1 m for n
= 2, flattens out, and very slowly approaches 0.459 m as n becomes large.

(g) The radius of the sheave is not necessary. It divides out in the expression $(1/2)I\omega^2 = (1/4)m_{\text{sheave}}v^2$.

(h) In this problem, as often in everyday life, energy conservation refers to minimizing use of
electric energy or fuel. In physical theory, energy conservation refers to the constancy of the total
energy of an isolated system, without regard to the different prices of energy in different forms.

(i) The result of applying $\sum F = ma$ and $\sum \tau = I\alpha$ to elevator car, counterweight, and sheave, and
adding up the resulting equations is

$\boxed{(800\text{ kg} + n\,80\text{ kg} - 950\text{ kg})(9.8\text{ m/s}^2) = [800\text{ kg} + n\,80\text{ kg} + 950\text{ kg} + 140\text{ kg}]a}$

$\boxed{a = (9.80\text{ m/s}^2)(80n - 150)/(1\,890 + 80n)}$ downward

P10.69 τ_f will oppose the torque due to the hanging object:

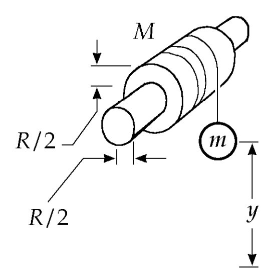

FIG. P10.69

$$\sum \tau = I\alpha = TR - \tau_f: \qquad \tau_f = TR - I\alpha \qquad (1)$$

Now find T, I and α in given or known terms and substitute into equation (1).

$$\sum F_y = T - mg = -ma: \qquad T = m(g - a) \qquad (2)$$

also $\Delta y = v_i t + \dfrac{at^2}{2}$ $\qquad a = \dfrac{2y}{t^2} \qquad (3)$

and

$$\alpha = \frac{a}{R} = \frac{2y}{Rt^2} \qquad (4)$$

$$I = \frac{1}{2}M\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8}MR^2 \qquad (5)$$

Substituting (2), (3), (4), and (5) into (1), we find

$$\tau_f = m\left(g - \frac{2y}{t^2}\right)R - \frac{5}{8}\frac{MR^2(2y)}{Rt^2} = \boxed{R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5}{4}\frac{My}{t^2}\right]}$$

P10.75 (a) Let R_E represent the radius of the Earth. The base of the building moves east at $v_1 = \omega R_E$ where ω is one revolution per day. The top of the building moves east at $v_2 = \omega(R_E + h)$. Its eastward speed relative to the ground is $v_2 - v_1 = \omega h$. The object's time of fall is given by $\Delta y = 0 + \frac{1}{2}gt^2$, $t = \sqrt{\frac{2h}{g}}$. During its fall the object's eastward motion is unimpeded so its deflection distance is $\Delta x = (v_2 - v_1)t = \omega h\sqrt{\frac{2h}{g}} = \boxed{\omega h^{3/2}\left(\frac{2}{g}\right)^{1/2}}$

(b) $$\frac{2\pi \text{ rad}}{86\,400 \text{ s}}(50 \text{ m})^{3/2}\left(\frac{2 \text{ s}^2}{9.8 \text{ m}}\right)^{1/2} = \boxed{1.16 \text{ cm}}$$

(c) The deflection is only 0.02% of the original height, so it is negligible in many practical cases.

P10.77 $$\sum F = T - Mg = -Ma: \quad \sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$$

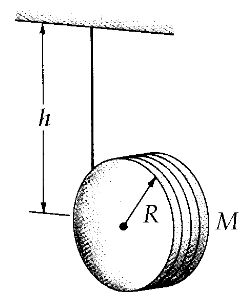

FIG. P10.77

(a) Combining the above two equations we find

$$T = M(g - a)$$

and $$a = \frac{2T}{M}$$

thus $$T = \boxed{\frac{Mg}{3}}$$

(b) $$a = \frac{2T}{M} = \frac{2}{M}\left(\frac{Mg}{3}\right) = \boxed{\frac{2}{3}g}$$

(c) $$v_f^2 = v_i^2 + 2a(x_f - x_i) \qquad v_f^2 = 0 + 2\left(\frac{2}{3}g\right)(h - 0)$$

$$v_f = \boxed{\sqrt{\frac{4gh}{3}}}$$

For comparison, from conservation of energy for the system of the disk and the Earth
we have

$$U_{gi} + K_{\text{rot } i} + K_{\text{trans } i} = U_{gf} + K_{\text{rot } f} + K_{\text{trans } f}:$$

$$Mgh + 0 + 0 = 0 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 + \frac{1}{2}Mv_f^{\,2}$$

$$v_f = \sqrt{\frac{4gh}{3}}$$

P10.79 (a) $\Delta K_{\text{rot}} + \Delta K_{\text{trans}} + \Delta U = 0$

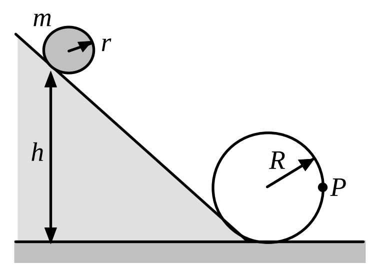

FIG. P10.79

Note that initially the center of mass of the sphere is a distance $h + r$ above the bottom of the loop; and as the mass reaches the top of the loop, this distance above the reference level is $2R - r$. The conservation of energy requirement gives

$$mg(h+r) = mg(2R-r) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For the sphere $I = \frac{2}{5}mr^2$ and $v = r\omega$ so that the expression becomes

$$gh + 2gr = 2gR + \frac{7}{10}v^2 \qquad (1)$$

Note that $h = h_{\text{min}}$ when the speed of the sphere at the top of the loop satisfies the condition

$$\sum F = mg = \frac{mv^2}{(R-r)} \text{ or } v^2 = g(R-r)$$

Substituting this into Equation (1) gives

$$h_{\text{min}} = 2(R-r) + 0.700(R-r) \text{ or } \boxed{h_{\text{min}} = 2.70(R-r) = 2.70R}$$

(b) When the sphere is initially at $h = 3R$ and finally at point *P*, the conservation of energy equation gives

$$mg(3R+r) = mgR + \frac{1}{2}mv^2 + \frac{1}{5}mv^2, \text{ or}$$

$$v^2 = \frac{10}{7}(2R+r)g$$

Turning clockwise as it rolls without slipping past point *P*, the sphere is slowing down with counterclockwise angular acceleration caused by the torque of an upward force f of static friction. We have $\sum F_y = ma_y$ and $\sum \tau = I\alpha$ becoming $f - mg = -m\alpha r$ and $fr = \left(\frac{2}{5}\right)mr^2\alpha$.

Eliminating f by substitution yields $\alpha = \frac{5g}{7r}$ so that $\sum F_y = \boxed{-\frac{5}{7}mg}$

$$\sum F_x = -n = -\frac{mv^2}{R-r} = -\frac{(10/7)(2R+r)}{R-r}mg = \boxed{\frac{-20mg}{7}} \quad \text{(since } R >> r\text{)}$$

P10.83 (a) $\sum F_x = F + f = Ma_{CM}$

$\sum \tau = FR - fR = I\alpha$

$FR - (Ma_{CM} - F)R = \frac{Ia_{CM}}{R}$ $\qquad \boxed{a_{CM} = \frac{4F}{3M}}$

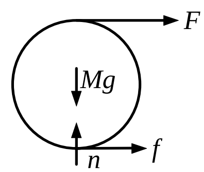

FIG. P10.83

(b) $f = Ma_{CM} - F = M\left(\frac{4F}{3M}\right) - F = \boxed{\frac{1}{3}F}$

(c) $v_f^2 = v_i^2 + 2a(x_f - x_i)$

$v_f = \boxed{\sqrt{\frac{8Fd}{3M}}}$

P10.84 Call f_t the frictional force exerted by each roller backward on the plank. Name as f_b the rolling resistance exerted backward by the ground on each roller. Suppose the rollers are equally far from the ends of the plank.

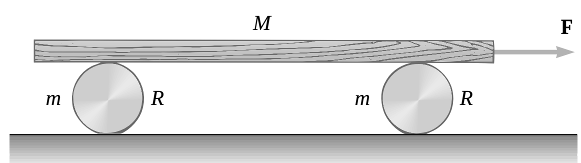

FIG. P10.84

For the plank,

$$\sum F_x = ma_x \quad 6.00\text{ N} - 2f_t = (6.00\text{ kg})a_p$$

The center of each roller moves forward only half as far as the plank. Each roller has acceleration $\frac{a_p}{2}$ and angular acceleration

$$\frac{a_p/2}{(5.00\text{ cm})} = \frac{a_p}{(0.100\text{ m})}$$

Then for each,

$$\sum F_x = ma_x \quad +f_t - f_b = (2.00\text{ kg})\frac{a_p}{2}$$

$$\sum \tau = I\alpha \qquad f_t(5.00\text{ cm}) + f_b(5.00\text{ cm}) = \frac{1}{2}(2.00\text{ kg})(5.00\text{ cm})^2 \frac{a_p}{10.0\text{ cm}}$$

So $\quad f_t + f_b = \left(\frac{1}{2}\text{ kg}\right)a_p$

Add to eliminate f_b:

$$2f_t = (1.50\text{ kg})a_p$$

(a) And $6.00\text{ N} - (1.50\text{ kg})a_p = (6.00\text{ kg})a_p$

$$a_p = \frac{(6.00\text{ N})}{(7.50\text{ kg})} = \boxed{0.800\text{ m/s}^2}$$

For each roller, $a = \frac{a_p}{2} = \boxed{0.400\text{ m/s}^2}$

(b) Substituting back, $2f_t = (1.50\text{ kg})0.800\text{ m/s}^2$

$$f_t = \boxed{0.600\text{ N}}$$

$$0.600\text{ N} + f_b = \frac{1}{2}\text{ kg}\left(0.800\text{ m/s}^2\right)$$

$$f_b = -0.200\text{ N}$$

The negative sign means that the horizontal force of ground on each roller is $\boxed{0.200\text{ N forward}}$ rather than backward as we assumed.

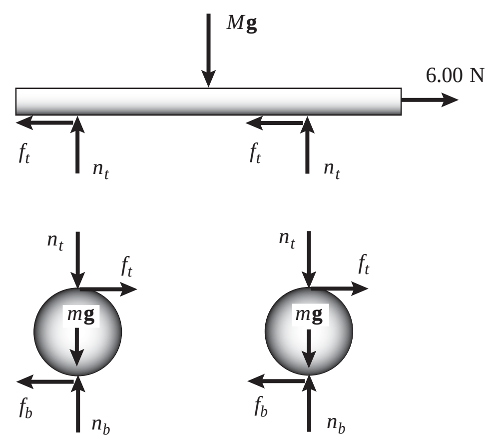

FIG. P10.84(b)

P10.85 $\sum F_x = ma_x$ reads $-f + T = ma$. If we take torques around the center of mass, we can use $\sum \tau = I\alpha$, which reads $+fR_2 - TR_1 = I\alpha$. For rolling without slipping, $\alpha = \frac{a}{R_2}$. By substitution,

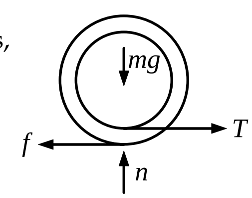

FIG. P10.85

$$fR_2 - TR_1 = \frac{Ia}{R_2} = \frac{I}{R_2 m}(T - f)$$

$$fR_2^2 m - TR_1R_2m = IT - If$$

$$f\left(I + mR_2^2\right) = T\left(I + mR_1R_2\right)$$

$$f = \left(\frac{I + mR_1R_2}{I + mR_2^2}\right)T$$

Since the answer is positive, the friction force is confirmed to be $\boxed{\text{to the left}}$.