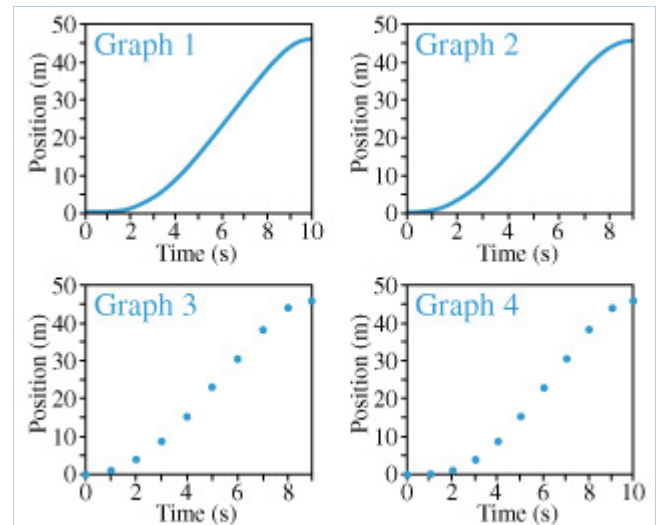


Homework 1-2**Due: 11:58pm on Tuesday, October 21, 2014**To understand how points are awarded, read the [Grading Policy](#) for this assignment.**Given Positions, Find Velocity and Acceleration****Learning Goal:**

To understand how to graph position, velocity, and acceleration of an object starting with a table of positions vs. time.

The table shows the x coordinate of a moving object. The position is tabulated at 1-s intervals. The x coordinate is indicated below each time. You should make the simplification that the acceleration of the object is bounded and contains no spikes.

time (s)	0	1	2	3	4	5	6	7	8	9
x (m)	0	1	4	9	16	24	32	40	46	48

Part AWhich graph best represents the function $x(t)$, describing the object's position vs. time?**Hint 1. Meaning of a bounded and nonspiky acceleration**A bounded and nonspiky acceleration results in a smooth graph of x vs. t .

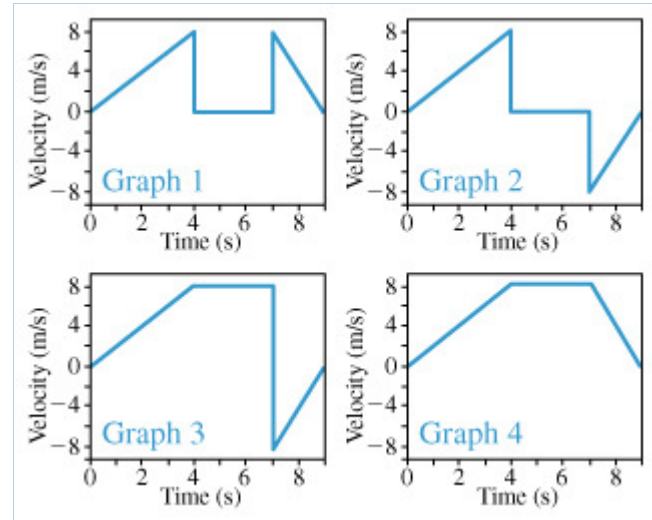
ANSWER:

- ☐ 1
- ☒ 2
- ☐ 3
- ☐ 4

Correct

Part B

Which of the following graphs best represents the function $v(t)$, describing the object's velocity as a function of time?



Hint 1. Find the velocity toward the end of the motion

Velocity is the time derivative of displacement. Given this, the velocity toward the end of the motion is _____.

ANSWER:

- ☐ positive and increasing
- ☒ positive and decreasing
- ☐ negative and increasing
- ☐ negative and decreasing

Hint 2. What are the implications of zero velocity?

Two of the possible velocity vs. time graphs indicate zero velocity between $t = 4$ and $t = 7$ s. What would the corresponding position vs. time graph look like in this region?

ANSWER:

- ☒ a horizontal line
- ☐ straight but sloping up to the right
- ☐ straight but sloping down to the right
- ☐ curved upward
- ☐ curved downward

Hint 3. Specify the characteristics of the velocity function

The problem states that "the acceleration of the object is bounded and contains no spikes." This means that the velocity _____.

ANSWER:

- ☐ has spikes
- ☒ has no discontinuities
- ☐ has no abrupt changes of slope
- ☐ is constant

ANSWER:

- ☐ 1
- ☐ 2
- ☐ 3
- ☒ 4

Correct

In principle, you could also just compute and plot the average velocity. The expression for the average velocity is

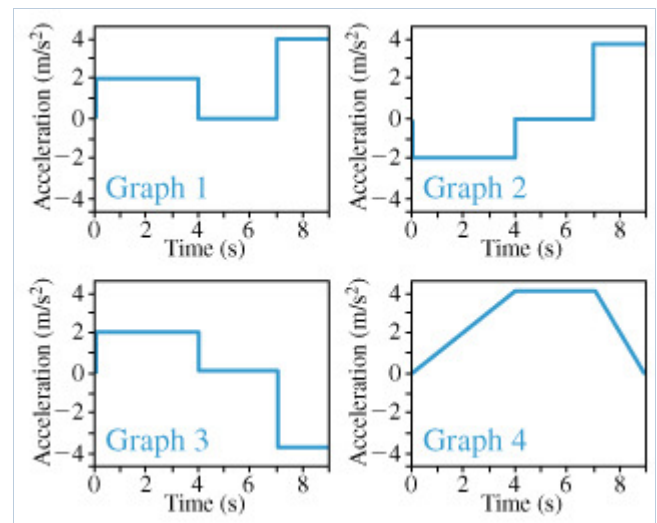
$$v_{\text{avg}}[t_1, t_2] = \frac{x(t_2) - x(t_1)}{t_2 - t_1}.$$

The notation $v_{\text{avg}}[t_1, t_2]$ emphasizes that this is not an instantaneous velocity, but rather an average over an interval. After you compute this, you must put a single point on the graph of velocity vs. time. The most accurate place to plot the average velocity is at the middle of the time interval over which the average was computed.

Also, you could work back and find the position from the velocity graph. The position of an object is the integral of its velocity. That is, the area under the graph of velocity vs. time from $t = 0$ up to time t must equal the position of the object at time t . Check that the correct velocity vs. time graph gives you the correct position according to this method.

Part C

Which of the following graphs best represents the function $a(t)$, describing the acceleration of this object?



Hint 1. Find the acceleration toward the end of the motion

Acceleration is the time derivative of velocity. Toward the end of the motion the acceleration is _____.

ANSWER:

- ☐ zero
- ☐ positive
- ☒ negative

Hint 2. Calculate the acceleration in the region of constant velocity

What is the acceleration a over the interval during which the object travels at constant speed?

Answer numerically in meters per second squared.

ANSWER:

$$a = 0 \text{ m/s}^2$$

Hint 3. Find the initial acceleration

Acceleration is the time derivative of velocity. Initially the acceleration is _____.

ANSWER:

- ☐ zero
- ☒ positive
- ☐ negative

ANSWER:

- ☐ 1
- ☐ 2
- ☒ 3
- ☐ 4

Correct

In one dimension, a linear increase or decrease in the velocity of an object over a given time interval implies constant acceleration over that particular time interval. You can find the magnitude of the acceleration using the formula for average acceleration over a time interval:

$$a_{\text{avg}}[t_1, t_2] = \frac{v(t_2) - v(t_1)}{t_2 - t_1}.$$

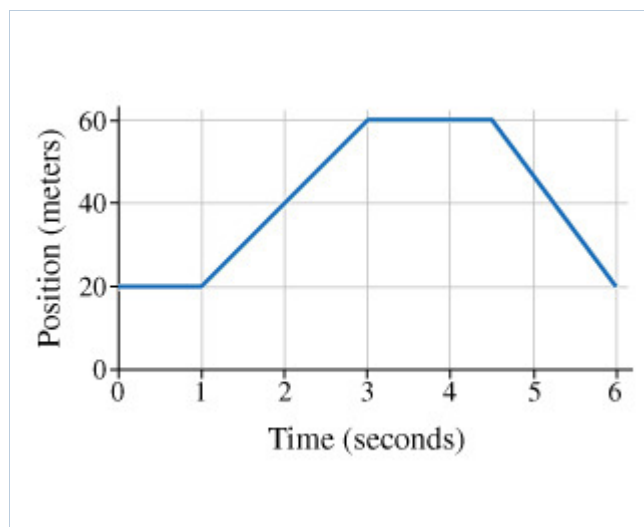
When the acceleration is constant over an extended interval, you can choose any value of t_1 and t_2 within the interval to compute the average.

± Average Velocity from a Position vs. Time Graph

Learning Goal:

To learn to read a graph of position versus time and to calculate average velocity.

In this problem you will determine the average velocity of a moving object from the graph of its position $x(t)$ as a function of time t . A traveling object might move at different speeds and in different directions during an interval of time, but if we ask at what *constant* velocity the object would have to travel to achieve the same displacement over the given time interval, that is what we call the object's *average velocity*. We will use the notation $v_{\text{ave}}[t_1, t_2]$ to indicate average velocity over the time interval from t_1 to t_2 . For instance, $v_{\text{ave}}[1, 3]$ is the average velocity over the time interval from $t = 1$ to $t = 3$.



Part A

Consulting the graph shown in the figure, find the object's average velocity over the time interval from 0 to 1 second.

Answer to the nearest integer.

Hint 1. Definition of average velocity

Average velocity is defined as the constant velocity at which an object would have to travel to achieve a given displacement (difference between final and initial positions, which can be negative) over a given time interval, from the initial time t_i to the final time t_f . The average velocity is therefore equal to the displacement divided by the given time interval. In symbolic form, average velocity is given by

$$v_{\text{ave}}[t_i, t_f] = \frac{x(t_f) - x(t_i)}{t_f - t_i}.$$

ANSWER:

$$v_{\text{ave}}[0, 1] = 0 \text{ m/s}$$

Correct

Part B

Find the average velocity over the time interval from 1 to 3 seconds.

Express your answer in meters per second to the nearest integer.

Hint 1. Find the change in position

The final and initial positions can be read off the y axis of the graph. What is the displacement during the time interval from 1 to 3 seconds?

Express your answer numerically, in meters

ANSWER:

$$x_f - x_i = 40 \text{ m}$$

Hint 2. Definition of average velocity

Average velocity is defined as the constant velocity at which an object would have to travel to achieve a given displacement (difference between final and initial positions, which can be negative) over a given time interval, from the initial time t_i to the final time t_f . The average velocity is therefore equal to the displacement divided by the given time interval. In symbolic form, average velocity is given by

$$v_{\text{ave}}[t_i, t_f] = \frac{x(t_f) - x(t_i)}{t_f - t_i}.$$

ANSWER:

$$v_{\text{ave}}[1, 3] = 20 \text{ m/s}$$

Correct

A note about instantaneous velocity. The instantaneous velocity at a certain moment in time is represented by the slope of the graph at that moment. For straight-line graphs, the (instantaneous) velocity remains constant over the interval, so the instantaneous velocity at any time during an interval is the same as the average velocity over that interval. For instance, in this case, the instantaneous velocity at any time from 1 to 3 seconds is the same as the average velocity of 20 m/s.

Part C

Now find $v_{\text{ave}}[0, 3]$.

Give your answer to three significant figures.

Hint 1. A note on the displacement

Since the object's position remains constant from time 0 to time 1, the object's displacement from 0 to 3 is the same as in Part B. However, the time interval has changed.

ANSWER:

$$v_{\text{ave}}[0, 3] = 13.3 \text{ m/s}$$

Correct

Note that $v_{\text{ave}}[0, 3]$ is not equal to the simple arithmetic average of $v_{\text{ave}}[0, 1]$ and $v_{\text{ave}}[1, 3]$, i.e., $\frac{v_{\text{ave}}[0, 1] + v_{\text{ave}}[1, 3]}{2}$, because they are averages for time intervals of different lengths.

Part D

Find the average velocity over the time interval from 3 to 6 seconds.

Express your answer to three significant figures.

Hint 1. Determine the displacement

What is the displacement?

Answer to the nearest integer.

ANSWER:

$$x(6.0) - x(3.0) = -40 \text{ m}$$

Hint 2. Determine the time interval

What is the time interval?

Answer to two significant figures.

ANSWER:

$$t_f - t_i = 3.0 \text{ s}$$

ANSWER:

$$v_{\text{ave}}[3.0, 6.0] = -13.3 \text{ m/s}$$

Correct**Part E**

Finally, find the average velocity over the whole time interval shown in the graph.

Express your answer to three significant figures.

Hint 1. Determine the displacement

What is the displacement?

Answer to the nearest integer.

ANSWER:

$$x(6.0) - x(0.0) = 0 \text{ m}$$

ANSWER:

$$v_{\text{ave}}[0.0, 6.0] = 0 \text{ m/s}$$

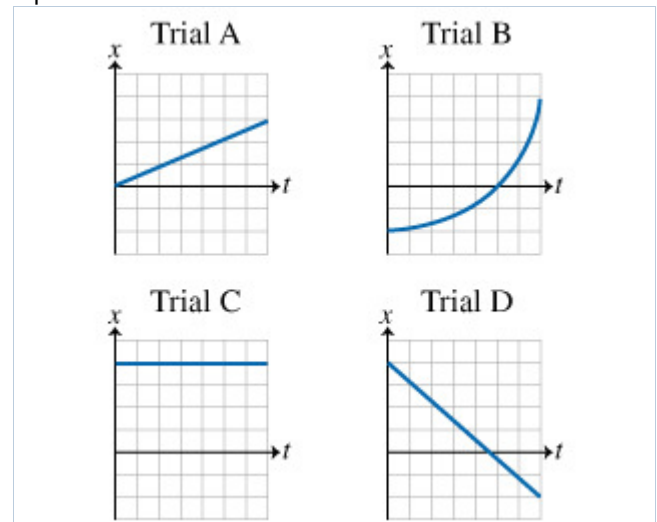
Correct

Note that though the *average* velocity is zero for this time interval, the *instantaneous* velocity (i.e., the slope of the graph) has several different values (positive, negative, zero) during this time interval.

Note as well that since average velocity over a time interval is defined as the change in position (displacement) in the given interval divided by the time, the object can travel a great distance (here 80 meters) and still have zero average velocity, since it ended up exactly where it started. Therefore, zero average velocity does not necessarily mean that the object was standing still the entire time!

Velocity from Graphs of Position versus Time

An object moves along the x axis during four separate trials. Graphs of position versus time for each trial are shown in the figure.

**Part A**

During which trial or trials is the object's velocity not constant?

Check all that apply.

Hint 1. Finding velocity from a position versus time graph

On a graph of coordinate x as a function of time t , the instantaneous velocity at any point is equal to the slope of the curve at that point.

Hint 2. Equation for slope

The slope of a line is its rise divided by the run:

$$\text{slope} = \frac{\Delta x}{\Delta t}.$$

ANSWER:

- ☐ Trial A
- ☒ Trial B
- ☐ Trial C
- ☐ Trial D

Correct

The graph of the motion during Trial B has a changing slope and therefore is not constant. The other trials all have graphs with constant slope and thus correspond to motion with constant velocity.

Part B

During which trial or trials is the magnitude of the average velocity the largest?

Check all that apply.

Hint 1. Definition of average velocity

Recall that

$$\text{average velocity} = \frac{\Delta(\text{position})}{\Delta(\text{time})} = \frac{\Delta x}{\Delta t}.$$

Then note that the question asks only about the magnitude of the velocity.

ANSWER:

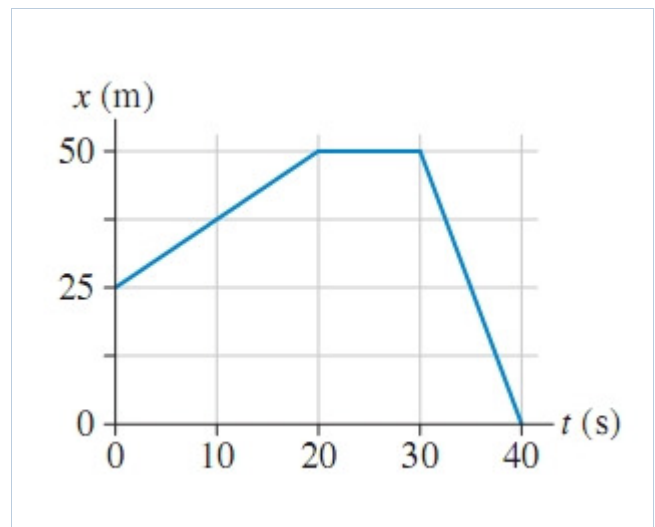
- ☐ Trial A
- ☒ Trial B
- ☐ Trial C
- ☒ Trial D

Correct

While Trial B and Trial D do not have the same average velocity, the only difference is the direction! The magnitudes are the same. Neither one is "larger" than the other, and it is only because of how we chose our axes that Trial B has a positive average velocity while Trial D has a negative average velocity. In Trial C the object does not move, so it has an average velocity of zero. During Trial A the object has a positive average velocity but its magnitude is less than that in Trial B and Trial D.

Problem 2.4

The figure is the position-versus-time graph of a jogger.



Part A

What is the jogger's velocity at $t = 10$ s?

Express your answer to two significant figures and include the appropriate units.

ANSWER:

$$v = 1.3 \frac{\text{m}}{\text{s}}$$

Correct

Part B

What is the jogger's velocity at $t = 25$ s?

Express your answer to two significant figures and include the appropriate units.

ANSWER:

$$v = 0 \frac{\text{m}}{\text{s}}$$

Correct

Part C

What is the jogger's velocity at $t = 35$ s?

Express your answer to two significant figures and include the appropriate units.

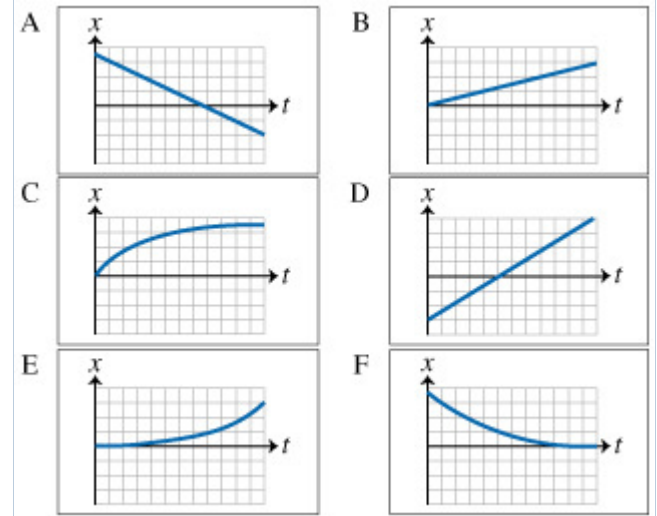
ANSWER:

$$v = -5.0 \frac{\text{m}}{\text{s}}$$

Correct

Position versus Time Graphs Conceptual Question

The motions described in each of the questions take place at an intersection on a two-lane road with a stop sign in each direction. For each motion, select the correct position versus time graph. For all of the motions, the stop sign is at the position $x = 0$, and east is the positive x direction.



Part A

A driver ignores the stop sign and continues driving east at constant speed.

Hint 1. Determining velocity from a position versus time graph

The slope on a position versus time graph is the rise (change in position) over the run (change in time). In physics, the ratio of change in position over change in time is defined as the velocity. Thus, the slope on a position versus time graph is the velocity of the object being graphed.

Hint 2. Driving east

Since east is defined as the positive x direction, a car traveling east must have a positive velocity. A positive velocity is represented as a positive slope on a position versus time graph.

Hint 3. Constant speed

Since velocity is represented by the slope on a position versus time graph, a car moving at constant speed must be represented by a line of constant slope.

ANSWER:

- ☐ A
- ☐ B
- ☐ C
- ☒ D
- ☐ E
- ☐ F

Correct

Part B

A driver ignores the stop sign and continues driving west at constant speed.

Hint 1. Determining velocity from a position versus time graph

The slope on a position versus time graph is the rise (change in position) over the run (change in time). In physics, the ratio of change in position over change in time is defined as the velocity. Thus, the slope on a position versus time graph is the velocity of the object being graphed.

Hint 2. Driving west

Since east is defined as the positive x direction, a car traveling west must have a negative velocity. A negative velocity is represented as a negative slope on a position versus time graph.

Hint 3. Constant speed

Since velocity is represented by the slope on a position versus time graph, a car moving at constant speed must be represented by a line of constant slope.

ANSWER:

- ☒ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ F

Correct

Part C

A driver, traveling west, slows and stops at the stop sign.

Hint 1. Determining velocity from a position versus time graph

The slope on a position versus time graph is the rise (change in position) over the run (change in time). In physics, the ratio of change in position over change in time is defined as the velocity. Thus, the slope on a position versus time graph is the velocity of the object being graphed.

Hint 2. Driving west

Since east is defined as the positive x direction, a car traveling west must have a negative velocity. A negative velocity is represented as a negative slope on a position versus time graph.

Hint 3. Acceleration

Since velocity is represented by the slope on a position versus time graph, a car that accelerates must be represented as a curve with changing slope. If a car slows, then the slope of the graph must approach zero. If a car's speed increases, the slope must become more positive or more negative (depending upon which direction it is moving).

ANSWER:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☒ F

Correct

Part D

A driver, after stopping at the stop sign, accelerates to the east.

Hint 1. Determining velocity from a position versus time graph

The slope on a position versus time graph is the rise (change in position) over the run (change in time). In physics, the ratio of change in position over change in time is defined as the velocity. Thus, the slope on a position versus time graph is the velocity of the object being graphed.

Hint 2. Driving east

Since east is defined as the positive x direction, a car traveling east must have a positive velocity. A positive velocity is represented as a positive slope on a position versus time graph.

Hint 3. Acceleration

Since velocity is represented by the slope on a position versus time graph, a car that accelerates must be represented as a curve with changing slope. If a car slows, then the slope of the graph must approach zero. If a car's speed increases, the slope must become more positive or more negative (depending upon which direction it is moving).

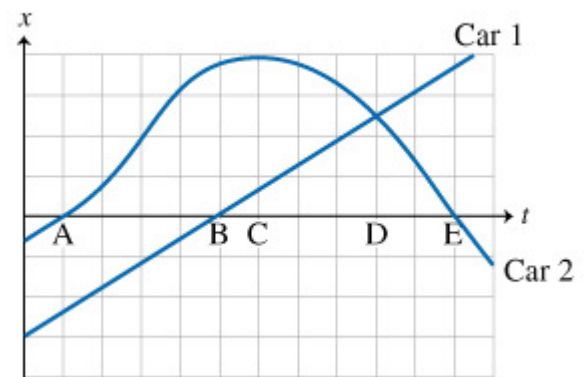
ANSWER:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☒ E
- ☐ F

Correct

Analyzing Position versus Time Graphs: Conceptual Question

Two cars travel on the parallel lanes of a two-lane road. The cars' motions are represented by the position versus time graph shown in the figure. Answer the questions using the times from the graph indicated by letters.



Part A

At which of the times do the two cars pass each other?

Hint 1. Two cars passing

Two objects can pass each other only if they have the same position at the same time.

ANSWER:

- ☐ A
- ☐ B
- ☐ C
- ☒ D
- ☐ E
- ☐ None
- ☐ Cannot be determined

Correct

Part B

Are the two cars traveling in the same direction when they pass each other?

ANSWER:

- ☐ yes
- ☒ no

Correct

Part C

At which of the lettered times, if any, does car #1 momentarily stop?

Hint 1. Determining velocity from a position versus time graph

The slope on a position versus time graph is the "rise" (change in position) over the "run" (change in time). In physics, the ratio of change in position over change in time is defined as the velocity. Thus, the slope on a position versus time graph is the velocity of the object being graphed.

ANSWER:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☒ none
- ☐ cannot be determined

Correct

Part D

At which of the lettered times, if any, does car #2 momentarily stop?

Hint 1. Determining velocity from a position versus time graph

The slope on a position versus time graph is the "rise" (change in position) over the "run" (change in time). In physics, the ratio of change in position over change in time is defined as the velocity. Thus, the slope on a position versus time graph is the velocity of the object being graphed.

ANSWER:

- ☐ A
- ☐ B
- ☒ C
- ☐ D
- ☐ E
- ☐ none
- ☐ cannot be determined

Correct

Part E

At which of the lettered times are the cars moving with nearly identical velocity?

Hint 1. Determining Velocity from a Position versus Time Graph

The slope on a position versus time graph is the "rise" (change in position) over the "run" (change in time). In physics, the ratio of change in position over change in time is defined as the velocity. Thus, the slope on a position versus time graph is the velocity of the object being graphed.

ANSWER:

- ☒ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ None
- ☐ Cannot be determined

Correct

Enhanced EOC: Problem 2.23

A particle moving along the x -axis has its position described by the function $x = (5.00 t^3 - 5.00 t + 5.00)$ m, where t is in s. At $t = 3.00$, what are the particle's (a) position, (b) velocity, and (c) acceleration?

You may want to review ( [pages 38 - 42](#)) .

For help with math skills, you may want to review:

[Differentiation of Polynomial Functions](#)

Part A

Express your answer with the appropriate units.

Hint 1. How to approach the problem

Evaluate the position at time $t = 3.00$ s.

ANSWER:

Correct

Part B

Express your answer with the appropriate units.

Hint 1. How to approach the problem

How do you determine the velocity as a function of time, $v(t)$, from the position, $x(t)$? What calculus operation do you have to perform?

Once you have $v(t)$, how do you determine v at a particular time?

ANSWER:

$$130 \frac{\text{m}}{\text{s}}$$

Correct

Part C

Express your answer with the appropriate units.

Hint 1. How to approach the problem

How do you determine the acceleration as a function of time, $a(t)$, from the velocity, $v(t)$? What calculus operation do you have to perform?

Once you have $a(t)$, how do you determine the acceleration at a particular time?

ANSWER:

$$90.0 \frac{\text{m}}{\text{s}^2}$$

Correct

Problem 2.3

Julie drives 100 mi to Grandmother's house. On the way to Grandmother's, Julie drives half the *distance* at 44.0mph and half the distance at 56.0mph . On her return trip, she drives half the *time* at 44.0mph and half the time at 56.0mph .

Part A

What is Julie's average speed on the way to Grandmother's house?

Express your answer with the appropriate units.

ANSWER:

$$49.3 \text{ mph}$$

Correct

Part B

What is her average speed on the return trip?

Express your answer with the appropriate units.

ANSWER:

50.0 mph

Correct

Running and Walking

Tim and Rick both can run at speed v_r and walk at speed v_w , with $v_r > v_w$. They set off together on a journey of distance D . Rick walks half of the distance and runs the other half. Tim walks half of the *time* and runs the other half.

Part A

How long does it take Rick to cover the distance D ?

Express the time taken by Rick in terms of v_r , v_w , and D .

Hint 1. Compute midpoint for Rick

Find the time that it takes Rick to walk the first half of the distance, that is, to travel a distance $D/2$ at speed v_w .

ANSWER:

$$t_{w,R} = \frac{D}{2v_w}$$

Hint 2. Compute running time for Rick

Now find the time Rick spends running.

ANSWER:

$$t_{r,R} = \frac{D}{2v_r}$$

Hint 3. What equation to use

Now just add the two times up and you're done.

ANSWER:

$$t_R = \frac{D}{v_r} + \frac{D}{v_w}$$

Correct

Part B

Find Rick's average speed for covering the distance D .

Express Rick's average speed in terms of v_r and v_w .

Hint 1. Calculate speed using D and time

You were given the total distance and have calculated the total time. Recall that average speed is equal to total distance traveled divided by the amount of time it took to travel this distance.

ANSWER:

$$v_{\text{ave,R}} = \frac{2v_r v_w}{v_r + v_w}$$

Correct

Part C

How long does it take Tim to cover the distance?

Express the time taken by Tim in terms of v_r , v_w , and D .

Hint 1. Calculate average speed

Tim walks at speed v_w half the time and runs at speed v_r for the other half.

Find Tim's average speed in terms of given quantities.

Hint 1. Solve a related problem

Consider a related problem. Tim walks for a time $t_T/2$ at speed v_w and runs for a time $t_T/2$ at speed v_r .

Find the total distance he travels in time t_T in terms of given quantities and t_T .

ANSWER:

$$d_T = \frac{v_w t_T}{2} + \frac{v_r t_T}{2}$$

ANSWER:

$$v_{\text{ave,T}} = \frac{v_w + v_r}{2}$$

ANSWER:

$$t_T = \frac{2D}{v_w + v_r}$$

Correct

Part D

Who covers the distance D more quickly?

Think logically, but without using the detailed answers in the previous parts.

Hint 1. Consider the relative positions at the midpoint

Imagine that both Rick and Tim do all of their walking before they start to run. Rick will start running when he has covered half of the total distance. When Tim reaches half of the total distance, will he already have started running?

ANSWER:

- ☐ Rick
- ☒ Tim
- ☐ Neither. They cover the distance in the same amount of time.

Correct

Part E

In terms of given quantities, by what amount of time, Δt , does Tim beat Rick?

It will help you check your answer if you simplify it algebraically and check the special case $v_r = v_w$.

Express the difference in time, Δt in terms of v_r , v_w , and D .

ANSWER:

$$\Delta t = \frac{D(v_w - v_r)^2}{2v_r v_w (v_r + v_w)}$$

Correct

Part F

In the special case that $v_r = v_w$, what would be Tim's margin of victory $\Delta t(v_r = v_w)$?

Hint 1. Think it through

If $v_r = v_w$, is the *any* difference between what Tim and Rick do?

ANSWER:

$$\Delta t(v_r = v_w) = 0$$

Correct

Problem 2.23

A particle moving along the x -axis has its position described by the function $x = (1.00 t^3 - 3.00 t + 5.00) \text{ m}$, where t is in s. At $t = 4.00$, what are the particle's (a) position, (b) velocity, and (c) acceleration?

Part A

Express your answer with the appropriate units.

ANSWER:

57.0 m

Correct

Part B

Express your answer with the appropriate units.

ANSWER:

45.0 $\frac{\text{m}}{\text{s}}$

Correct

Part C

Express your answer with the appropriate units.

ANSWER:

24.0 $\frac{\text{m}}{\text{s}^2}$

Correct

Score Summary:

Your score on this assignment is 100%.

You received 10 out of a possible total of 10 points.