Chapter 8 Conservation of Energy

P8.3

$$U_i + K_i = U_f + K_f:$$

$$mgh + 0 = mg(2R) + \frac{1}{2}mv^2$$

$$g(3.50R) = 2g(R) + \frac{1}{2}v^2$$

$$\boxed{v = \sqrt{3.00gR}}$$

$$\sum F = m\frac{v^2}{R}: \qquad n + mg = m\frac{v^2}{R}$$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00\left(5.00\times 10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)$$

$$= \boxed{0.098\,0\ \text{N downward}}$$

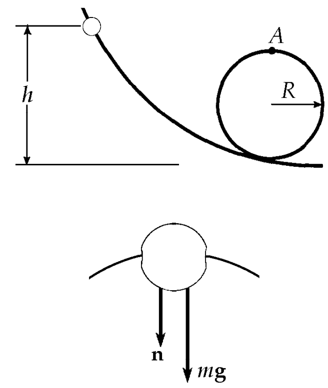


FIG. P8.3

P8.4 (a) $(\Delta K)_{A\to B} = \sum W = W_g = mg\Delta h = mg(5.00 - 3.20)$

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = m(9.80)(1.80)$$

$$v_B = \boxed{5.94\ \text{m/s}}$$

Similarly, $v_C = \sqrt{v_A^2 + 2g(5.00 - 2.00)} = \boxed{7.67\ \text{m/s}}$

(b) $W_g\big|_{A\to C} = mg(3.00\ \text{m}) = \boxed{147\ \text{J}}$

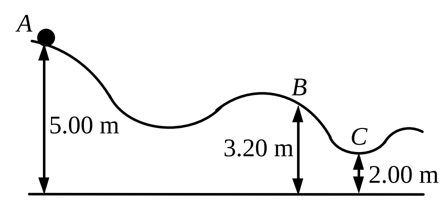


FIG. P8.4

P8.10 (a) $K_i + U_{gi} = K_f + U_{gf}$

$$\frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + mgy_f$$

$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

Note that we have used the Pythagorean theorem to express the original kinetic energy in terms of the velocity components. Kinetic energy itself does not have components.

Now $v_{xi} = v_{xf}$, so for the first ball

$$y_f = \frac{v_{yi}^2}{2g} = \frac{(1\,000\sin 37.0°)^2}{2(9.80)} = \boxed{1.85\times 10^4\ \text{m}}$$

and for the second

$$y_f = \frac{(1\,000)^2}{2(9.80)} = \boxed{5.10\times 10^4\ \text{m}}$$

(b) The total energy of each is constant with value

$$\frac{1}{2}(20.0\text{ kg})(1\,000\text{ m/s})^2 = \boxed{1.00\times 10^7\text{ J}}$$

P8.11 (a) For a 5-m cord the spring constant is described by $F = kx$, $mg = k(1.5\text{ m})$. For a longer cord of length L the stretch distance is longer so the spring constant is smaller in inverse proportion:

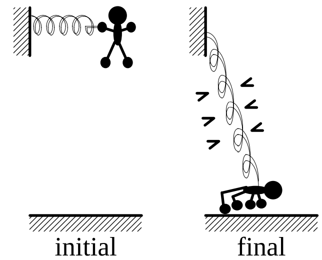


FIG. P8.11(a)

$$k = \frac{5\text{ m}}{L}\frac{mg}{1.5\text{ m}} = 3.33mg/L$$

$$(K+U_g+U_s)_i = (K+U_g+U_s)_f$$

$$0+mgy_i+0 = 0+mgy_f+\frac{1}{2}kx_f^2$$

$$mg(y_i-y_f) = \frac{1}{2}kx_f^2 = \frac{1}{2}3.33\frac{mg}{L}x_f^2$$

here $y_i - y_f = 55\text{ m} = L + x_f$

$$55.0\text{ m}L = \frac{1}{2}3.33(55.0\text{ m}-L)^2$$

$$55.0\text{ m}L = 5.04\times 10^3\text{ m}^2 - 183\text{ m}L + 1.67L^2$$

$$0 = 1.67L^2 - 238L + 5.04\times 10^3 = 0$$

$$L = \frac{238\pm\sqrt{238^2-4(1.67)(5.04\times 10^3)}}{2(1.67)} = \frac{238\pm 152}{3.33} = \boxed{25.8\text{ m}}$$

only the value of L less than 55 m is physical.

(b)

$$k = 3.33\frac{mg}{25.8\text{ m}} \qquad x_{\max} = x_f = 55.0\text{ m} - 25.8\text{ m} = 29.2\text{ m}$$

$$\sum F = ma \qquad +kx_{\max} - mg = ma$$

$$3.33\frac{mg}{25.8\text{ m}}29.2\text{ m} - mg = ma$$

$$a = 2.77g = \boxed{27.1\text{ m/s}^2}$$

P8.13 $\sum F_y = ma_y$: $n - 392\text{ N} = 0$

$$n = 392\text{ N}$$

$$f_k = \mu_k n = (0.300)(392\text{ N}) = 118\text{ N}$$

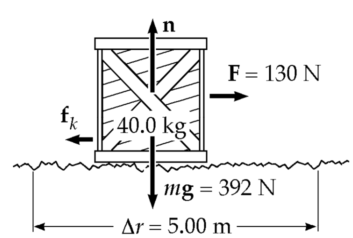


FIG. P8.13

(a) $W_F = F\Delta r\cos\theta = (130)(5.00)\cos 0° = \boxed{650\text{ J}}$

(b) $\Delta E_{\text{int}} = f_k\Delta x = (118)(5.00) = \boxed{588\text{ J}}$

(c) $W_n = n\Delta r\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\Delta r\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W_{\text{other}} - \Delta E_{\text{int}}$

$\frac{1}{2}mv_f^2 - 0 = 650\text{ J} - 588\text{ J} + 0 + 0 = \boxed{62.0\text{ J}}$

(f) $v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(62.0\text{ J})}{40.0\text{ kg}}} = \boxed{1.76\text{ m/s}}$

P8.18 (a) $U_f = K_i - K_f + U_i$

$U_f = 30.0 - 18.0 + 10.0 = \boxed{22.0\text{ J}}$

$\boxed{E = 40.0\text{ J}}$

(b) Yes, $\Delta E_{\text{mech}} = \Delta K + \Delta U$ is not equal to zero, some nonconservative force or forces must act. For conservative forces $\Delta K + \Delta U = 0$.

P8.19 $U_i + K_i + \Delta E_{\text{mech}} = U_f + K_f$:

$m_2gh - fh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$

$f = \mu n = \mu m_1 g$

$m_2gh - \mu m_1gh = \frac{1}{2}(m_1 + m_2)v^2$

$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$

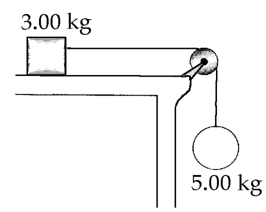


FIG. P8.19

$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(1.50\text{ m})[5.00\text{ kg} - 0.400(3.00\text{ kg})]}{8.00\text{ kg}}} = \boxed{3.74\text{ m/s}}$

P8.21 (a) $\Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = -\frac{1}{2}mv_i^2 = \boxed{-160\text{ J}}$

(b) $\Delta U = mg(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$

(c) The mechanical energy converted due to friction is 86.5 J

$f = \frac{86.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$

(d) $f = \mu_k n = \mu_k mg\cos 30.0° = 28.8\text{ N}$

$\mu_k = \frac{28.8\text{ N}}{(5.00\text{ kg})(9.80\text{ m/s}^2)\cos 30.0°} = \boxed{0.679}$

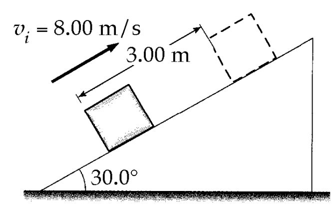


FIG. P8.21

P8.25 (a) The object moved down distance $1.20\text{ m}+x$. Choose $y=0$ at its lower point.

$$K_i+U_{gi}+U_{si}+\Delta E_{\text{mech}}=K_f+U_{gf}+U_{sf}$$

$$0+mgy_i+0+0=0+0+\frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$

$$0=(160\text{ N/m})x^2-(14.7\text{ N})x-17.6\text{ J}$$

$$x=\frac{14.7\text{ N}\pm\sqrt{(-14.7\text{ N})^2-4(160\text{ N/m})(-17.6\text{ N}\cdot\text{m})}}{2(160\text{ N/m})}$$

$$x=\frac{14.7\text{ N}\pm 107\text{ N}}{320\text{ N/m}}$$

The negative root tells how high the object will rebound if it is instantly glued to the spring. We want

$$x=\boxed{0.381\text{ m}}$$

(b) From the same equation,

$$(1.50\text{ kg})(1.63\text{ m/s}^2)(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$

$$0=160x^2-2.44x-2.93$$

The positive root is $x=\boxed{0.143\text{ m}}$.

(c) The equation expressing the energy version of the nonisolated system model has one more term:

$$mgy_i-f\Delta x=\frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m}+x)-0.700\text{ N}(1.20\text{ m}+x)=\frac{1}{2}(320\text{ N/m})x^2$$

$$17.6\text{ J}+14.7\text{ N}x-0.840\text{ J}-0.700\text{ N}x=160\text{ N/m}\,x^2$$

$$160x^2-14.0x-16.8=0$$

$$x=\frac{14.0\pm\sqrt{(14.0)^2-4(160)(-16.8)}}{320}$$

$$x=\boxed{0.371\text{ m}}$$

P8.36 (a) Burning 1 lb of fat releases energy

$$1\text{ lb}\left(\frac{454\text{ g}}{1\text{ lb}}\right)\left(\frac{9\text{ kcal}}{1\text{ g}}\right)\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right)=1.71\times10^7\text{ J}$$

The mechanical energy output is

$$(1.71\times10^7\text{ J})(0.20)=nF\Delta r\cos\theta$$

Then

$$3.42\times10^6\text{ J}=nmg\Delta y\cos0^\circ$$

$$3.42\times10^6\text{ J}=n(50\text{ kg})(9.8\text{ m/s}^2)(80\text{ steps})(0.150\text{ m})$$
$$3.42\times10^6\text{ J}=n(5.88\times10^3\text{ J})$$

where the number of times she must climb the steps is $n=\dfrac{3.42\times10^6\text{ J}}{5.88\times10^3\text{ J}}=\boxed{582}$.
This method is impractical compared to limiting food intake.

(b) Her mechanical power output is

$$\mathcal{P}=\frac{W}{t}=\frac{5.88\times10^3\text{ J}}{65\text{ s}}=\boxed{90.5\text{ W}}=90.5\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right)=\boxed{0.121\text{ hp}}$$

P8.37 (a) $(K+U_g)_A=(K+U_g)_B$

$$0+mgy_A=\frac{1}{2}mv_B^2+0 \qquad v_B=\sqrt{2gy_A}=\sqrt{2(9.8\text{ m/s}^2)6.3\text{ m}}=\boxed{11.1\text{ m/s}}$$

(b) $$a_c=\frac{v^2}{r}=\frac{(11.1\text{ m/s})^2}{6.3\text{ m}}=\boxed{19.6\text{ m/s}^2\text{ up}}$$

(c) $$\sum F_y=ma_y \qquad +n_B-mg=ma_c$$
$$n_B=76\text{ kg}(9.8\text{ m/s}^2+19.6\text{ m/s}^2)=\boxed{2.23\times10^3\text{ N up}}$$

(d) We compute the amount of chemical energy converted into mechanical energy as
$W=F\Delta r\cos\theta=2.23\times10^3\text{ N}(0.450\text{ m})\cos0^\circ=\boxed{1.01\times10^3\text{ J}}$

(e) $$(K+U_g+U_{chemical})_B=(K+U_g)_D$$
$$\frac{1}{2}mv_B^2+0+1.01\times10^3\text{ J}=\frac{1}{2}mv_D^2+mg(y_D-y_B)$$
$$\frac{1}{2}76\text{ kg}(11.1\text{ m/s})^2+1.01\times10^3\text{ J}=\frac{1}{2}76\text{ kg }v_D^2+76\text{ kg}(9.8\text{ m/s}^2)6.3\text{ m}$$
$$\sqrt{\frac{(5.70\times10^3\text{ J}-4.69\times10^3\text{ J})2}{76\text{ kg}}}=v_D=\boxed{5.14\text{ m/s}}$$

(f) $(K+U_g)_D=(K+U_g)_E$ where E is the apex of his motion

$$\frac{1}{2}mv_D^2+0=0+mg(y_E-y_D) \qquad y_E-y_D=\frac{v_D^2}{2g}=\frac{(5.14\text{ m/s})^2}{2(9.8\text{ m/s}^2)}=\boxed{1.35\text{ m}}$$

(g) Consider the motion with constant acceleration between takeoff and touchdown. The time is the positive root of

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-2.34 \text{ m} = 0 + 5.14 \text{ m/s}\,t + \frac{1}{2}\left(-9.8 \text{ m/s}^2\right)t^2$$

$$4.9t^2 - 5.14t - 2.34 = 0$$

$$t = \frac{5.14 \pm \sqrt{5.14^2 - 4(4.9)(-2.34)}}{9.8} = \boxed{1.39 \text{ s}}$$

P8.44

$$\mathcal{P}\Delta t = W = \Delta K = \frac{(\Delta m)v^2}{2}$$

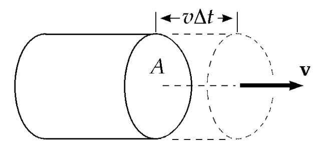


FIG. P8.44

The density is $\rho = \dfrac{\Delta m}{\text{vol}} = \dfrac{\Delta m}{A\Delta x}$

Substituting this into the first equation and solving for $\mathcal{P}$, since $\dfrac{\Delta x}{\Delta t} = v$, for a constant speed, we get

$$\boxed{\mathcal{P} = \frac{\rho A v^3}{2}}$$

Also, since $\mathcal{P} = Fv$,

$$\boxed{F = \frac{\rho A v^2}{2}}$$

Our model predicts the same proportionalities as the empirical equation, and gives $D = 1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

P8.49

$$v = 100 \text{ km/h} = 27.8 \text{ m/s}$$

The retarding force due to air resistance is

$$R = \frac{1}{2}D\rho A v^2 = \frac{1}{2}(0.330)\left(1.20 \text{ kg/m}^3\right)\left(2.50 \text{ m}^2\right)\left(27.8 \text{ m/s}\right)^2 = 382 \text{ N}$$

Comparing the energy of the car at two points along the hill,

$$K_i + U_{gi} + \Delta E = K_f + U_{gf}$$

or

$$K_i + U_{gi} + \Delta W_e - R(\Delta s) = K_f + U_{gf}$$

where ΔW_e is the work input from the engine. Thus,

$$\Delta W_e = R(\Delta s) + (K_f - K_i) + (U_{gf} - U_{gi})$$

Recognizing that $K_f = K_i$ and dividing by the travel time Δt gives the required power input from the engine as

$$\mathcal{P} = \left(\frac{\Delta W_e}{\Delta t}\right) = R\left(\frac{\Delta s}{\Delta t}\right) + mg\left(\frac{\Delta y}{\Delta t}\right) = Rv + mgv\sin\theta$$

$$\mathcal{P} = (382\text{ N})(27.8\text{ m/s}) + (1\,500\text{ kg})(9.80\text{ m/s}^2)(27.8\text{ m/s})\sin 3.20°$$

$$\mathcal{P} = \boxed{33.4\text{ kW} = 44.8\text{ hp}}$$

P8.52 m = mass of pumpkin

R = radius of silo top

$$\sum F_r = ma_r \Rightarrow n - mg\cos\theta = -m\frac{v^2}{R}$$

When the pumpkin first loses contact with the surface, $n = 0$.

Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

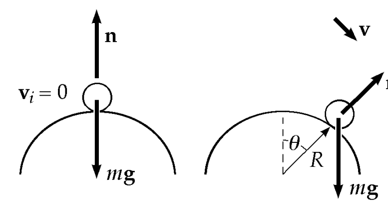


FIG. P8.52

Choose $U_g = 0$ in the $\theta = 90.0°$ plane. Then applying conservation of energy for the pumpkin-Earth system between the starting point and the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

Using the result from the force analysis, this becomes

$$\frac{1}{2}mRg\cos\theta + mgR\cos\theta = mgR$$

, which reduces to

$\cos\theta = \dfrac{2}{3}$, and gives $\theta = \cos^{-1}(2/3) = \boxed{48.2°}$

as the angle at which the pumpkin will lose contact with the surface.

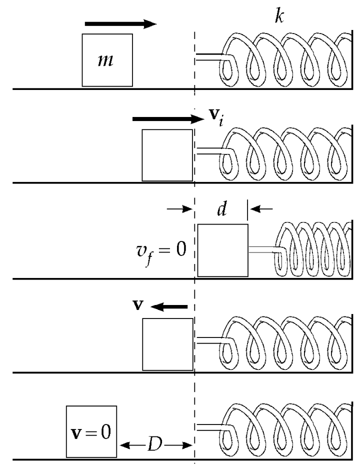
P8.54 (a) Between the second and the third picture, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$

$$\frac{1}{2}(50.0\text{ N/m})d^2 + 0.250(1.00\text{ kg})(9.80\text{ m/s}^2)d - \frac{1}{2}(1.00\text{ kg})(3.00\text{ m/s})^2 = 0$$

$$d = \frac{[-2.45 \pm 21.35]\text{ N}}{50.0\text{ N/m}} = \boxed{0.378\text{ m}}$$

(b) Between picture two and picture four, $\Delta E_{\text{mech}} = \Delta K + \Delta U$



$$-f(2d) = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$$

$$v = \sqrt{(3.00\ \text{m/s})^2 - \frac{2}{(1.00\ \text{kg})}(2.45\ \text{N})(2)(0.378\ \text{m})}$$

$$= \boxed{2.30\ \text{m/s}}$$

(c) For the motion from picture two to picture five, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(D + 2d) = -\frac{1}{2}(1.00\ \text{kg})(3.00\ \text{m/s})^2$$

$$D = \frac{9.00\ \text{J}}{2(0.250)(1.00\ \text{kg})(9.80\ \text{m/s}^2)} - 2(0.378\ \text{m}) = \boxed{1.08\ \text{m}}$$

FIG. P8.54

P8.55

$$\Delta E_{\text{mech}} = -f\Delta x$$

$$E_f - E_i = -f \cdot d_{BC}$$

$$\frac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$$

$$\mu = \frac{mgh - \frac{1}{2}kx^2}{mgd_{BC}} = \boxed{0.328}$$

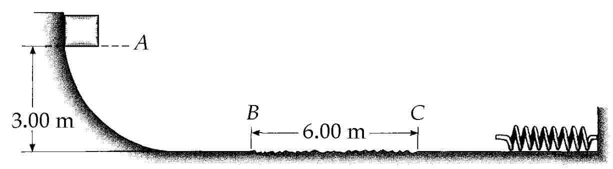


FIG. P8.55

P8.56 Let λ represent the mass of each one meter of the chain and T represent the tension in the chain at the table edge. We imagine the edge to act like a frictionless and massless pulley.

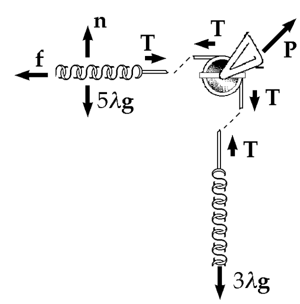


FIG. P8.56

(a) For the five meters on the table with motion impending,

$$\sum F_y = 0: \qquad +n - 5\lambda g = 0$$

$$n = 5\lambda g$$

$$f_s \le \mu_s n = 0.6(5\lambda g) = 3\lambda g$$

$$\sum F_x = 0: \qquad +T - f_s = 0 \quad T = f_s$$

$$T \le 3\lambda g$$

The maximum value is barely enough to support the hanging segment according to

$$\sum F_y = 0: \qquad +T - 3\lambda g = 0 \qquad T = 3\lambda g$$

so it is at this point that the chain starts to slide.

(b) Let x represent the variable distance the chain has slipped since the start.

Then length $(5-x)$ remains on the table, with now

$$\sum F_y = 0: \qquad +n-(5-x)\lambda g = 0 \qquad n=(5-x)\lambda g$$

$$f_k = \mu_k n = 0.4(5-x)\lambda g = 2\lambda g - 0.4x\lambda g$$

Consider energies of the chain-Earth system at the initial moment when the chain starts to slip, and a final moment when $x=5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8-\frac{3}{2}=6.5\text{ m}$.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f:$$

$$0+\left(m_1 g y_1 + m_2 g y_2\right)_i - \int_i^f f_k dx = \left(\frac{1}{2}mv^2 + mgy\right)_f$$

$$(5\lambda g)8 + (3\lambda g)6.5 - \int_0^5 (2\lambda g - 0.4x\lambda g)dx = \frac{1}{2}(8\lambda)v^2 + (8\lambda g)4$$

$$40.0g + 19.5g - 2.00g\int_0^5 dx + 0.400g\int_0^5 x\,dx = 4.00v^2 + 32.0g$$

$$27.5g - 2.00gx\Big|_0^5 + 0.400g\frac{x^2}{2}\Big|_0^5 = 4.00v^2$$

$$27.5g - 2.00g(5.00) + 0.400g(12.5) = 4.00v^2$$

$$22.5g = 4.00v^2$$

$$v = \sqrt{\frac{(22.5\text{ m})(9.80\text{ m/s}^2)}{4.00}} = \boxed{7.42\text{ m/s}}$$

P8.62 (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.

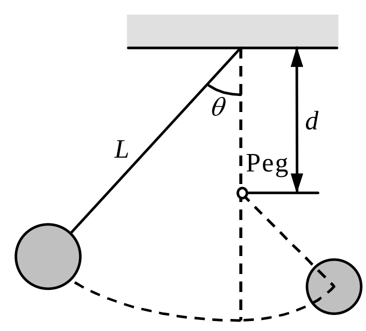


FIG. P8.62

(b) Relative to the point of suspension,

$$U_i = 0, \quad U_f = -mg\left[d-(L-d)\right]$$

From this we find that

$$-mg(2d-L) + \frac{1}{2}mv^2 = 0$$

Also for centripetal motion,

$$mg = \frac{mv^2}{R} \quad \text{where } R = L-d$$

Upon solving, we get $\boxed{d = \frac{3L}{5}}$.

P8.64 (a) At the top of the loop the car and riders are in free fall:

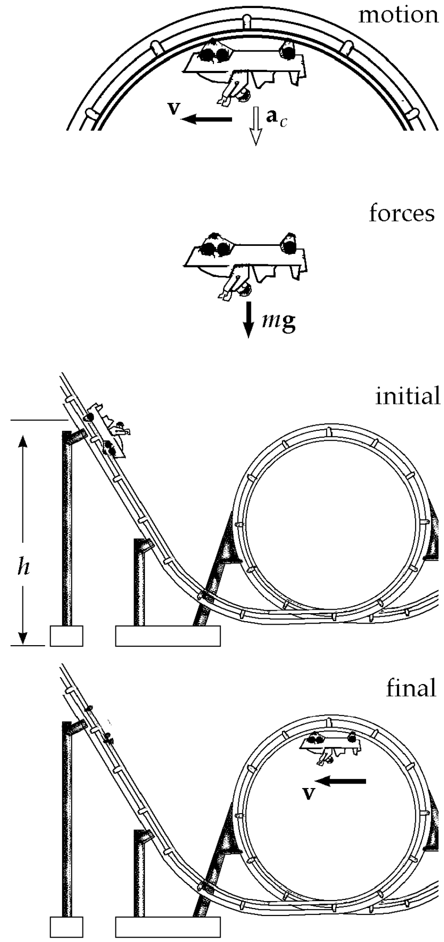


FIG. P8.64

$$\sum F_y = ma_y:$$

$$mg \text{ down} = \frac{mv^2}{R} \text{ down}$$

$$v = \sqrt{Rg}$$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$$K_i + U_{gi} = K_f + U_{gf}:$$

$$0 + mgh = \frac{1}{2}mv^2 + mg(2R)$$

$$gh = \frac{1}{2}Rg + g(2R)$$

$$\boxed{h = 2.50R}$$

(b) Let h now represent the height $\geq 2.5\,R$ of the release point. At the bottom of the loop we have

$$mgh = \frac{1}{2}mv_b^2 \quad \text{or} \quad v_b^2 = 2gh$$

$$\sum F_y = ma_y:$$

$$n_b - mg = \frac{mv_b^2}{R}(\text{up})$$

$$n_b = mg + \frac{m(2gh)}{R}$$

At the top of the loop,

$$mgh = \frac{1}{2}mv_t^2 + mg(2R)$$

$$v_t^2 = 2gh - 4gR$$

$$\sum F_y = ma_y: \quad -n_t - mg = -\frac{mv_t^2}{R}$$

$$n_t = -mg + \frac{m}{R}(2gh - 4gR)$$

$$n_t = \frac{m(2gh)}{R} - 5mg$$

Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m(2gh)}{R} - \frac{m(2gh)}{R} + 5mg = \boxed{6mg}$$