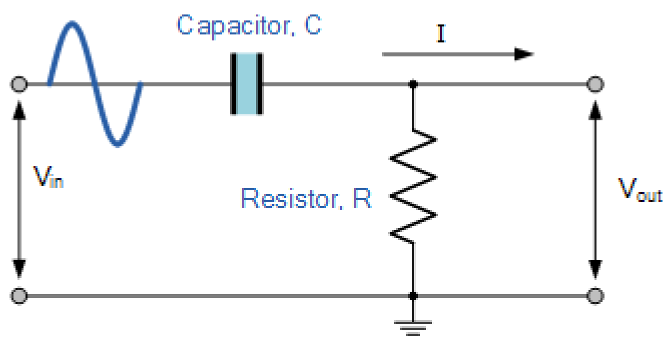
High Pass Filters

A **High Pass Filter** or **HPF**, is the exact opposite to that of the previously seen **Low Pass filter** circuit, as now the two components have been interchanged with the output signal (Vout) being taken from across the resistor as shown.

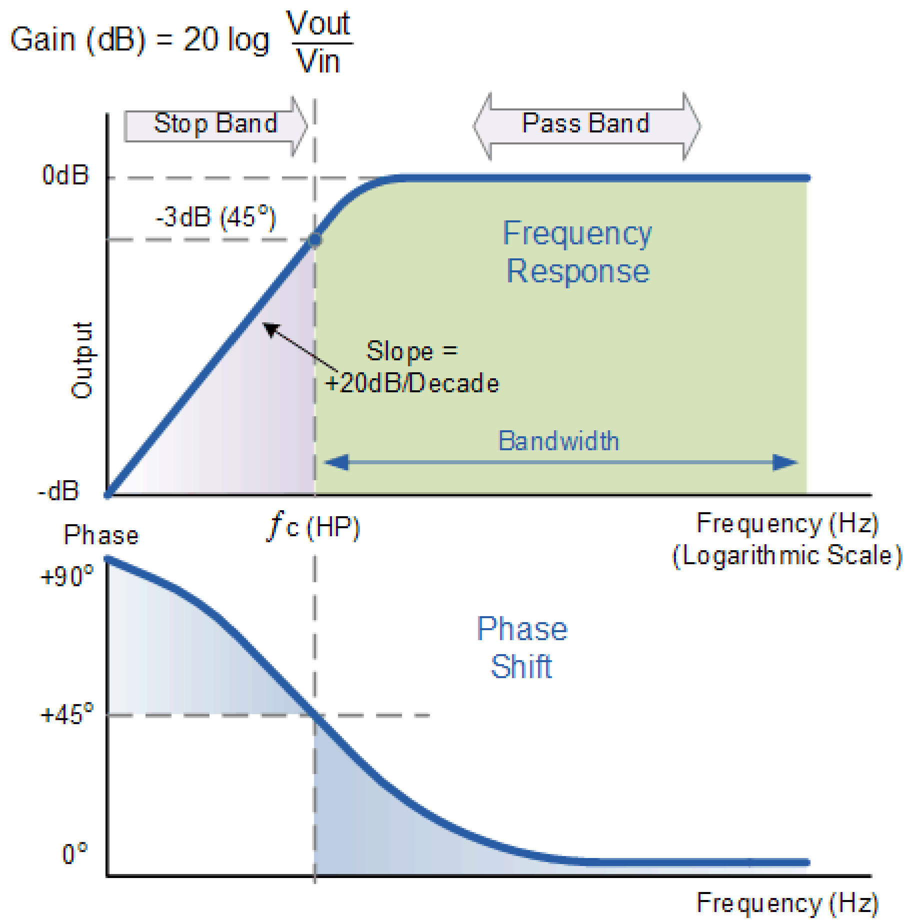
Where as the low pass filter only allowed signals to pass below its cut-off frequency point, *ƒ*c, the passive high pass filter circuit as its name implies, only passes signals above the selected cut-off point, *ƒ*c eliminating any low frequency signals from the waveform. Consider the circuit below.

The High Pass Filter Circuit



In this circuit arrangement, the reactance of the capacitor is very high at low frequencies so the capacitor acts like an open circuit and blocks any input signals at Vin until the cut-off frequency point (*ƒ*c) is reached. Above this cut-off frequency point the reactance of the capacitor has reduced sufficiently as to now act more like a short circuit allowing all of the input signal to pass directly to the output as shown below in the High Pass Frequency Response Curve.

Frequency Response of a 1st Order High Pass Filter.



The **Bode Plot** or Frequency Response Curve above for a High Pass filter is the exact opposite to that of a low pass filter. Here the signal is attenuated or damped at low frequencies with the output increasing at +20dB/Decade (6dB/Octave) until the frequency reaches the cut-off point (*ƒ*c) where again R = Xc. It has a response curve that extends down from infinity to the cut-off frequency, where the output voltage amplitude is $1/\sqrt{2} = 70.7\%$ of the input signal value or -3dB (20 log (Vout/Vin)) of the input value.

Also we can see that the phase angle (Φ) of the output signal **LEADS** that of the input and is equal to **+45°** at frequency *ƒ*c. The frequency response curve for a high pass filter implies that the filter can pass all signals out to infinity. However in practice, the high pass filter response does not extend to infinity but is limited by the electrical characteristics of the components used.

The cut-off frequency point for a first order high pass filter can be found using the same equation as that of the low pass filter, but the equation for the phase shift is modified slightly to account for the positive phase angle as shown below.

Cut-off Frequency and Phase Shift

$$fc = \frac{1}{2\pi RC}$$

$$Phase\ Shift\ \ \phi = arctan\ \frac{1}{2\pi fRC}$$

The circuit gain, Av which is given as Vout/Vin (magnitude) and is calculated as:

$$A_V = \frac{V_{OUT}}{V_{IN}} = \frac{R}{\sqrt{R^2 + XC^2}} = \frac{R}{Z}$$

$$\text{at low } f:\ Xc \to \infty,\ \ \text{Vout} = 0$$
$$\text{at high } f:\ Xc \to 0,\ \ \text{Vout} = \text{Vin}$$

High Pass Filter Example No1.

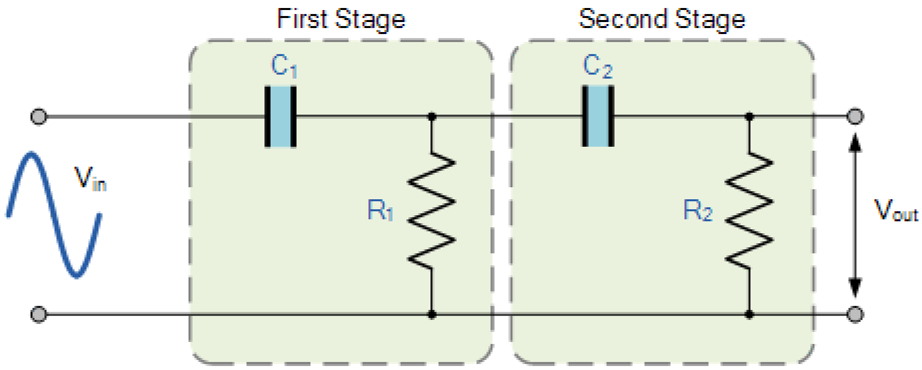
Calculate the cut-off or "breakpoint" frequency (fc) for a simple **high pass filter** consisting of an 82pF capacitor connected in series with a 240kΩ resistor.

$$fc = \frac{1}{2\pi RC} = \frac{1}{2\pi\ x\ 240{,}000 x 82 x 10^{-12}} = 8{,}087\ Hz\ or\ 8kHz$$

Second-order Low Pass Filter

Again as with low pass filters, high pass filter stages can be cascaded together to form a second-order (two-pole) filter as shown.

Second-order High Pass Filter



The above circuit uses two first-order high pass filters connected or cascaded together to form a second-order or two-pole filter network. Then a first-order high pass filter can be converted into a second-order type by simply using an additional RC network, the same as for the 2nd-order low pass filter. The resulting second-order high pass filter circuit will have a slope of -40dB/decade (-12dB/octave).

As with the low pass filter, the cut-off frequency, fc is determined by both the resistors and capacitors as follows.

$$f_C = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} \quad \text{Hz}$$

High Pass Filter Summary

The **High Pass Filter** is the exact opposite to the low pass filter. This filter has no output voltage from DC (0Hz), up to a specified cut-off frequency (fc) point. This lower cut-off frequency point is 70.7% or **-3dB** (dB = -20log Vout/Vin) of the voltage gain allowed to pass.

The frequency range "below" this cut-off point fc is generally known as the **Stop Band** while the frequency range "above" this cut-off point is generally known as the **Pass Band**.

The cut-off frequency, corner frequency or -3dB point of a high pass filter can be found using the standard formula of: fc = 1/(2πRC). The phase angle of the resulting output signal at fc is **+45°**. Generally, the high pass filter is less distorting than its equivalent low pass filter due to the higher operating frequencies.

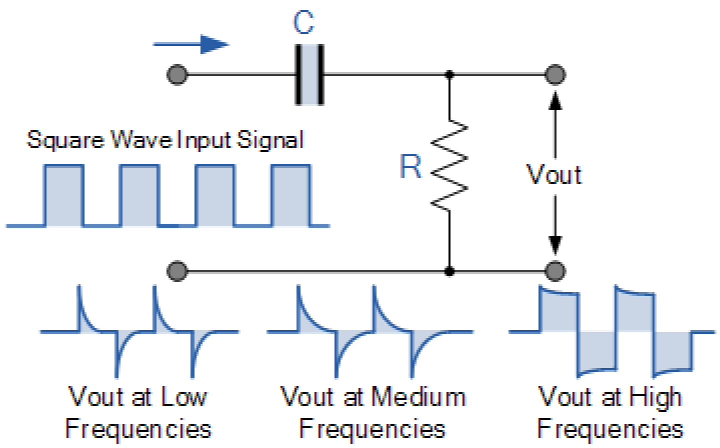
A very common application of a passive high pass filter, is in audio amplifiers as a coupling capacitor between two audio amplifier stages and in speaker systems to direct the higher frequency signals to the smaller "tweeter" type speakers while blocking the lower bass signals or are also used as filters to reduce any low frequency noise or “rumble” type distortion. When used like this in audio applications the high pass filter is sometimes called a “low-cut”, or “bass cut” filter.

The output voltage Vout depends upon the time constant and the frequency of the input signal as seen previously. With an AC sinusoidal signal applied to the circuit it behaves as a simple 1st Order high pass filter. But if we change the input signal to that of a “square wave” shaped signal that has an almost vertical step input, the response of the circuit changes dramatically and produces a circuit known commonly as an **Differentiator**.

The RC Differentiator

Up until now the input waveform to the filter has been assumed to be sinusoidal or that of a sine wave consisting of a fundamental signal and some harmonics operating in the frequency domain giving us a frequency domain response for the filter. However, if we feed the **High Pass Filter** with a **Square Wave** signal operating in the time domain giving an impulse or step response input, the output waveform will consist of short duration pulse or spikes as shown.

The RC Differentiator Circuit



Each cycle of the square wave input waveform produces two spikes at the output, one positive and one negative and whose amplitude is equal to that of the input. The rate of decay of the spikes depends upon the time constant, (RC) value of both components, (t = R x C) and the value of the input frequency. The output pulses resemble more and more the shape of the input signal as the frequency increases.

Low Pass Filter Introduction

Basically, an electrical filter is a circuit that can be designed to modify, reshape or reject all unwanted frequencies of an electrical signal and accept or pass only those signals wanted by the circuits designer. In other words they "filter-out" unwanted signals and an ideal filter will separate and pass sinusoidal input signals based upon their frequency.

In low frequency applications (up to 100kHz), passive filters are generally constructed using simple RC (Resistor-Capacitor) networks, while higher frequency filters (above 100kHz) are usually made from RLC (Resistor-Inductor-Capacitor) components.

Passive Filters are made up of passive components such as resistors, capacitors and inductors and have no amplifying elements (transistors, op-amps, etc) so have no signal gain, therefore their output level is always less than the input.

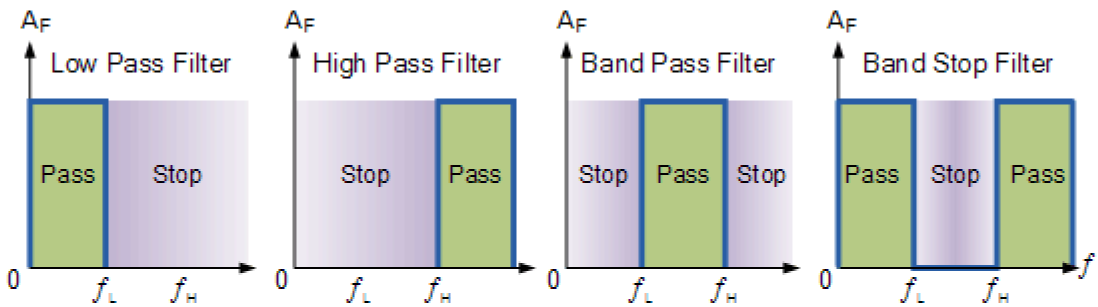
Filters are so named according to the frequency range of signals that they allow to pass through them, while blocking or "attenuating" the rest. The most commonly used filter designs are the:

- 1. The Low Pass Filter – the low pass filter only allows low frequency signals from 0Hz to its cut-off frequency, ƒc point to pass while blocking those any higher.
- 2. The High Pass Filter – the high pass filter only allows high frequency signals from its cut-off frequency, ƒc point and higher to infinity to pass through while blocking those any lower.
- 3. The Band Pass Filter – the band pass filter allows signals falling within a certain frequency band setup between two points to pass through while blocking both the lower and higher frequencies either side of this frequency band.

Simple First-order passive filters (1st order) can be made by connecting together a single resistor and a single capacitor in series across an input signal, (Vin) with the output of the filter, (Vout) taken from the junction of these two components. Depending on which way around we connect the resistor and the capacitor with regards to the output signal determines the type of filter construction resulting in either a **Low Pass Filter** or a **High Pass Filter**.

As the function of any filter is to allow signals of a given band of frequencies to pass unaltered while attenuating or weakening all others that are not wanted, we can define the amplitude response characteristics of an ideal filter by using an ideal frequency response curve of the four basic filter types as shown.

Ideal Filter Response Curves



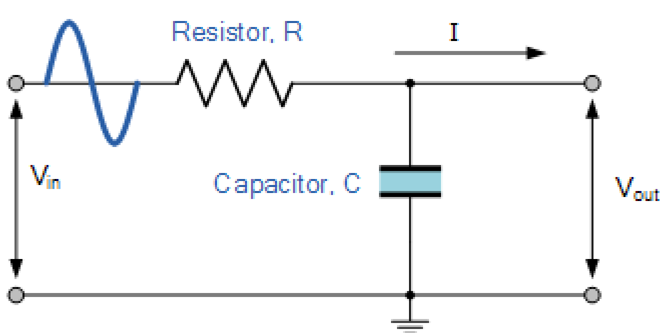
Filters can be divided into two distinct types: active filters and passive filters. Active filters contain amplifying devices to increase signal strength while passive do not contain amplifying devices to strengthen the signal. As there are two passive components within a passive filter design the output signal has a smaller amplitude than its corresponding input signal, therefore passive RC filters attenuate the signal and have a gain of less than one, (unity).

A Low Pass Filter can be a combination of capacitance, inductance or resistance intended to produce high attenuation above a specified frequency and little or no attenuation below that frequency. The frequency at which the transition occurs is called the "cutoff" frequency. The simplest low pass filters consist of a resistor and capacitor but more sophisticated low pass filters have a combination of series inductors and parallel capacitors. In this tutorial we will look at the simplest type, a passive two component RC low pass filter.

The Low Pass Filter

A simple passive **RC Low Pass Filter** or **LPF**, can be easily made by connecting together in series a single Resistor with a single Capacitor as shown below. In this type of filter arrangement the input signal (Vin) is applied to the series combination (both the Resistor and Capacitor together) but the output signal (Vout) is taken across the capacitor only. This type of filter is known generally as a "first-order filter" or "one-pole filter", why first-order or single-pole?, because it has only "one" reactive component, the capacitor, in the circuit.

RC Low Pass Filter Circuit



As mentioned previously in the Capacitive Reactance tutorial, the reactance of a capacitor varies inversely with frequency, while the value of the resistor remains constant as the frequency changes. At low frequencies the capacitive reactance, (Xc) of the capacitor will be very large compared to the resistive value of the resistor, R. This means that the voltage potential, Vc across the capacitor will be much larger than the voltage drop, Vr developed across the resistor. At high frequencies the reverse is true with Vc being small and Vr being large due to the change in the capacitive reactance value.

While the circuit above is that of an RC Low Pass Filter circuit, it can also be classed as a frequency variable potential divider circuit similar to the one we looked at in the Resistors tutorial. In that tutorial we used the following equation to calculate the output voltage for two single resistors connected in series.

$$V_{out} = V_{in} \times \frac{R_2}{R_1 + R_2}$$

where: $R_1 + R_2 = R_T$, the total resistance of the circuit

We also know that the capacitive reactance of a capacitor in an AC circuit is given as:

$$X_C = \frac{1}{2\pi f C} \text{ in Ohm's}$$

Opposition to current flow in an AC circuit is called **impedance**, symbol Z and for a series circuit consisting of a single resistor in series with a single capacitor, the circuit impedance is calculated as:

$$Z = \sqrt{R^2 + X_C^2}$$

Then by substituting our equation for impedance above into the resistive potential divider equation gives us:

RC Potential Divider Equation

$$V_{out} = V_{in} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = V_{in} \frac{X_C}{Z}$$

So, by using the potential divider equation of two resistors in series and substituting for impedance we can calculate the output voltage of an RC Filter for any given frequency.

Low Pass Filter Example No1

A **Low Pass Filter** circuit consisting of a resistor of 4k7Ω in series with a capacitor of 47nF is connected across a 10v sinusoidal supply. Calculate the output voltage (Vout) at a frequency of 100Hz and again at frequency of 10,000Hz or 10kHz.

Voltage Output at a Frequency of 100Hz.

$$Xc = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 100 \times 47 \times 10^{-9}} = 33{,}863\Omega$$

$$V_{OUT} = V_{IN} \times \frac{Xc}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{33863}{\sqrt{4700^2 + 33863^2}} = 9.9v$$

Voltage Output at a Frequency of 10,000Hz (10kHz).

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10{,}000 \times 47 \times 10^{-9}} = 338.6\Omega$$

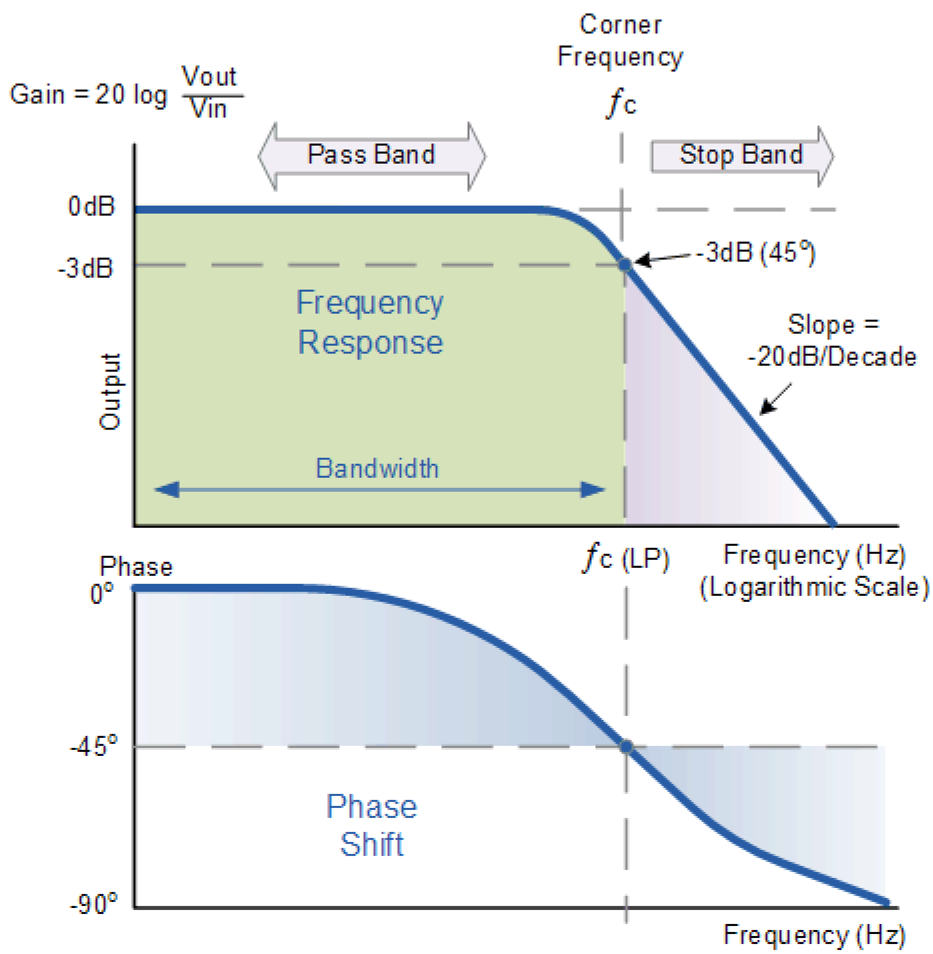
$$V_{OUT} = V_{IN} \times \frac{X_C}{\sqrt{R^2 + X_C^2}} = 10 \times \frac{338.6}{\sqrt{4700^2 + 338.6^2}} = 0.718v$$

Frequency Response

We can see from the results above, that as the frequency applied to the RC network increases from 100Hz to 10kHz, the voltage dropped across the capacitor and therefore the output voltage (Vout) from the circuit decreases from 9.9v to 0.718v.

By plotting the networks output voltage against different values of input frequency, the **Frequency Response Curve** or **Bode Plot** function of the low pass filter circuit can be found, as shown below.

Frequency Response of a 1st-order Low Pass Filter



The Bode Plot shows the **Frequency Response** of the filter to be nearly flat for low frequencies and all of the input signal is passed directly to the output, resulting in a gain of nearly 1, called unity, until it reaches its **Cut-off Frequency** point (fc). This is because the reactance of the capacitor is high at low frequencies and blocks any current flow through the capacitor.

After this cut-off frequency point the response of the circuit decreases to zero at a slope of -20dB/ Decade or (-6dB/Octave) "roll-off". Note that the angle of the slope, this -20dB/ Decade roll-off will always be the same for any RC combination.

Any high frequency signals applied to the low pass filter circuit above this cut-off frequency point will become greatly attenuated, that is they rapidly decrease. This happens because at very high frequencies the reactance of the capacitor becomes so low that it gives the effect of a short circuit condition on the output terminals resulting in zero output.

Then by carefully selecting the correct resistor-capacitor combination, we can create a RC circuit that allows a range of frequencies below a certain value to pass through the circuit unaffected while any frequencies applied to the circuit above this cut-off point to be attenuated, creating what is commonly called a **Low Pass Filter**.

For this type of "Low Pass Filter" circuit, all the frequencies below this cut-off, fc point that are unaltered with little or no attenuation and are said to be in the filters **Pass band** zone. This pass band zone also represents the **Bandwidth** of the filter. Any signal frequencies above this point cut-off point are generally said to be in the filters **Stop band** zone and they will be greatly attenuated.

This "Cut-off", "Corner" or "Breakpoint" frequency is defined as being the frequency point where the capacitive reactance and resistance are equal, R = Xc = 4k7Ω. When this occurs the output signal is attenuated to 70.7% of the input signal value or **-3dB** (20 log (Vout/Vin)) of the input. Although R = Xc, the output is **not** half of the input signal. This is because it is equal to the vector sum of the two and is therefore 0.707 of the input.

As the filter contains a capacitor, the Phase Angle (Φ) of the output signal **LAGS** behind that of the input and at the -3dB cut-off frequency (fc) and is -45^o out of phase. This is due to the time taken to charge the plates of the capacitor as the input voltage changes, resulting in the output voltage (the voltage across the capacitor) "lagging" behind that of the input signal. The higher the input frequency applied to the filter the more the capacitor lags and the circuit becomes more and more "out of phase".

The cut-off frequency point and phase shift angle can be found by using the following equation:

Cut-off Frequency and Phase Shift

$$fc = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 4700 \times 47 \times 10^{-9}} = 720\text{Hz}$$

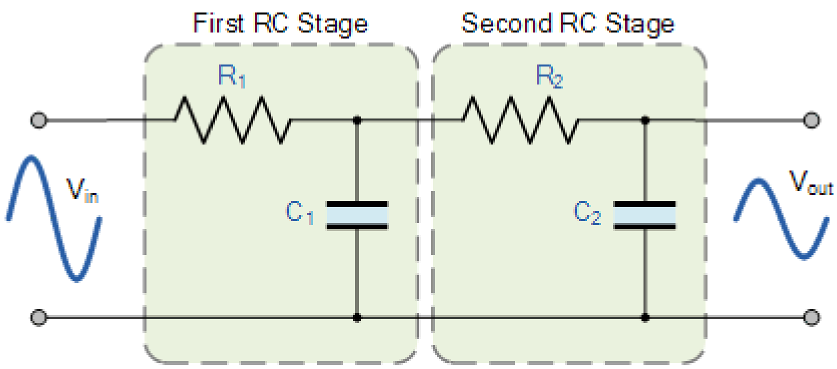
$$\text{Phase Shift } \varphi = -\arctan(2\pi f RC)$$

Then for our simple example of a "**Low Pass Filter**" circuit above, the cut-off frequency (fc) is given as 720Hz with an output voltage of 70.7% of the input voltage value and a phase shift angle of -45^o.

Second-order Low Pass Filter

Thus far we have seen that simple first-order RC low pass filters can be made by connecting a single resistor in series with a single capacitor. This single-pole arrangement gives us a roll-off slope of -20dB/decade attenuation of frequencies above the cut-off point at f_{-3dB} . However, sometimes in filter circuits this -20dB/decade (-6dB/octave) angle of the slope may not be enough to remove an unwanted signal then two stages of filtering can be used as shown.

Second-order Low Pass Filter



The above circuit uses two passive first-order low pass filters connected or "cascaded" together to form a second-order or two-pole filter network. Therefore we can see that a first-order low pass filter can be converted into a second-order type by simply adding an additional RC network to it and the more RC stages we add the higher becomes the order of the filter. If a number (n) of such RC stages are cascaded together, the resulting RC filter circuit would be known as an "n^{th}-order" filter with a roll-off slope of "n x -20dB/decade".

So for example, a second-order filter would have a slope of -40dB/decade (-12dB/octave), a fourth-order filter would have a slope of -80dB/decade (-24dB/octave) and so on. This means that, as the order of the filter is increased, the roll-off slope becomes steeper and the actual stop band response of the filter approaches its ideal stop band characteristics.

Second-order filters are important and widely used in filter designs because when combined with first-order filters any higher-order n^{th}-value filters can be designed using them. For example, a third order low-pass filter is formed by connecting in series or cascading together a first and a second-order low pass filter.

But there is a downside too cascading together RC filter stages. Although there is no limit to the order of the filter that can be formed, as the order increases, the gain and accuracy of the final filter declines. When identical RC filter stages are cascaded together, the output gain at the required cut-off frequency (fc) is reduced (attenuated) by an amount in relation to the number of filter stages used as the roll-off slope increases. We can define the amount of attenuation at the selected cut-off frequency using the following formula.

Passive Low Pass Filter Gain at *ƒ*c

$$\left(\frac{1}{\sqrt{2}}\right)^n$$

where "n" is the number of filter stages.

So for a second-order passive low pass filter the gain at the corner frequency *ƒ*c will be equal to 0.7071 x 0.7071 = 0.5Vin (-6dB), a third-order passive low pass filter will be equal to 0.353Vin (-9dB), fourth-order will be 0.25Vin (-12dB) and so on. The corner frequency, *ƒ*c for a second-order passive low pass filter is determined by the resistor/capacitor (RC) combination and is given as.

2nd-Order Filter Corner Frequency

$$f_C = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} \quad \text{Hz}$$

In reality as the filter stage and therefore its roll-off slope increases, the low pass filters -3dB corner frequency point and therefore its pass band frequency changes from its original calculated value above by an amount determined by the following equation.

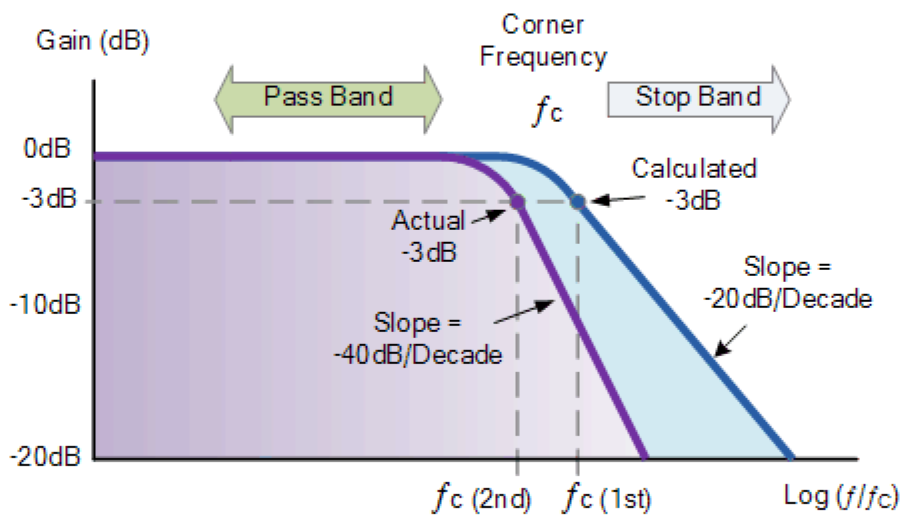
2nd-Order Low Pass Filter -3dB Frequency

$$f_{(-3dB)} = f_C\sqrt{2^{\left(\frac{1}{n}\right)} - 1}$$

where *ƒ*c is the calculated cut-off frequency, n is the filter order and f_{-3dB} is the new -3dB pass band frequency as a result in the increase of the filters order.

Then the frequency response (bode plot) for a second-order low pass filter assuming the same -3dB cut-off point would look like:

Frequency Response of a 2nd-order Low Pass Filter



In practice, cascading passive filters together to produce larger-order filters is difficult to implement accurately as the dynamic impedance of each filter order affects its neighbouring network. However, to reduce the loading effect we can make the impedance of each following stage 10x the previous stage, so R2 = 10 x R1 and C2 = 1/10th C1. Second-order and above filter networks are generally used in the feedback circuits of op-amps, making what are commonly known as Active Filters or as a phase-shift network in RC Oscillator circuits.

Low Pass Filter Summary

Low Pass Filter has a constant output voltage from D.C. (0Hz), up to a specified Cut-off frequency, ($ƒc$) point. This cut-off frequency point is 0.707 or **-3dB** (dB = -20log Vout/Vin) of the voltage gain allowed to pass.

The frequency range “below” this cut-off point $ƒc$ is generally known as the **Pass Band** as the input signal is allowed to pass through the filter. The frequency range “above” this cut-off point is generally known as the **Stop Band** as the input signal is blocked or stopped from passing through.

A simple 1st order low pass filter can be made using a single resistor in series with a single non-polarized capacitor (or any single reactive component) across an input signal Vin, whilst the output signal Vout is taken from across the capacitor.

The cut-off frequency or -3dB point, can be found using the standard formula, $ƒc = 1/(2\pi RC)$. The phase angle of the output signal at $ƒc$ and is -45^{o} for a Low Pass Filter.

The gain of the filter or any filter for that matter, is generally expressed in **Decibels** and is a function of the output value divided by its corresponding input value and is given as:

$$Gain\ in\ dB = 20 \log \frac{Vout}{Vin}$$

Applications of passive Low Pass Filters are in audio amplifiers and speaker systems to direct the lower frequency bass signals to the larger bass speakers or to reduce any high frequency noise or "hiss" type distortion. When used like this in audio applications the low pass filter is sometimes called a "high-cut", or "treble cut" filter.

If we were to reverse the positions of the resistor and capacitor in the circuit so that the output voltage is now taken from across the resistor, we would have a circuit that produces an output frequency response curve similar to that of a High Pass Filter, and this is discussed in the next tutorial.

Time Constant

Until now we have been interested in the frequency response of a low pass filter and that the filters cut-off frequency (fc) is the product of the resistance (R) and the capacitance (C) in the circuit with respect to some specified frequency point and that by altering any one of the two components alters this cut-off frequency point by either increasing it or decreasing it.

We also know that the phase shift of the circuit lags behind that of the input signal due to the time required to charge and then discharge the capacitor as the sine wave changes. This combination of R and C produces a charging and discharging effect on the capacitor known as its **Time Constant** (τ) of the circuit as seen in the RC Circuit tutorials giving the filter a response in the time domain.

The time constant, **tau** (τ), is related to the cut-off frequency fc as.

$$\tau = RC = \frac{1}{2\pi fc}$$

or expressed in terms of the cut-off frequency, fc as.

$$fc = \frac{1}{2\pi RC} \ or \ \frac{1}{2\pi\tau}$$

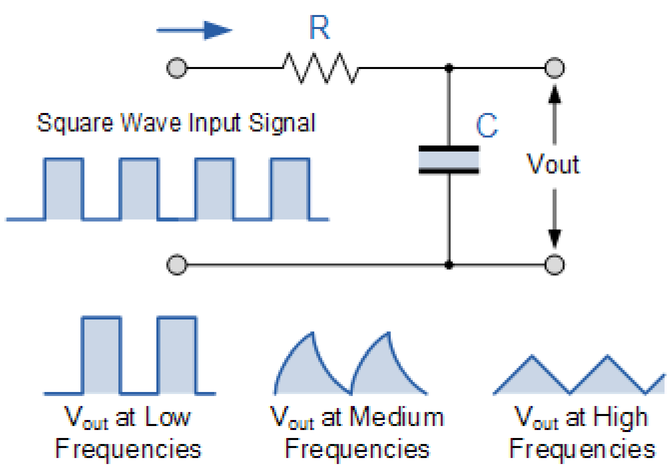
The output voltage, Vout depends upon the time constant and the frequency of the input signal. With a sinusoidal signal that changes smoothly over time, the circuit behaves as a simple 1st order low pass filter as we have seen above. But what if we were to change the input signal to

that of a “square wave” shaped “ON/OFF” type signal that has an almost vertical step input, what would happen to our filter circuit now. The output response of the circuit would change dramatically and produce another type of circuit known commonly as an
Integrator.

The RC Integrator

The **Integrator** is basically a low pass filter circuit operating in the time domain that converts a square wave “step” response input signal into a triangular shaped waveform output as the capacitor charges and discharges. A **Triangular** waveform consists of alternate but equal, positive and negative ramps. As seen below, if the RC time constant is long compared to the time period of the input waveform the resultant output waveform will be triangular in shape and the higher the input frequency the lower will be the output amplitude compared to that of the input.

The RC Integrator Circuit



This then makes this type of circuit ideal for converting one type of electronic signal to another for use in wave-generating or wave-shaping circuits.

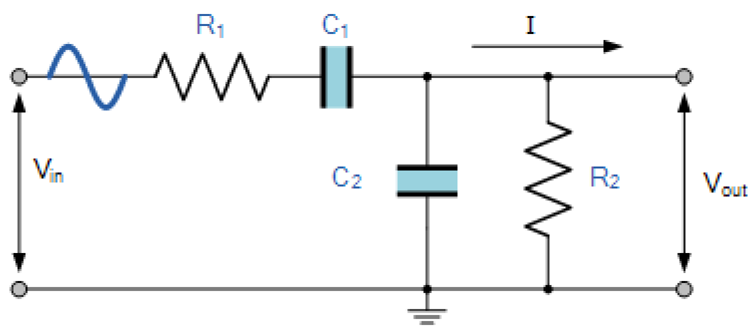
Band Pass Filters

The cut-off frequency or *ƒ*c point in a simple RC passive filter can be accurately controlled using just a single resistor in series with a non-polarized capacitor, and depending upon which way around they are connected either a low pass or a high pass filter is obtained.

One simple use for these types of Passive Filters is in audio amplifier applications or circuits such as in loudspeaker crossover filters or pre-amplifier tone controls. Sometimes it is necessary to only pass a certain range of frequencies that do not begin at 0Hz, (DC) or end at some high frequency point but are within a certain frequency band, either narrow or wide.

By connecting or "cascading" together a single **Low Pass Filter** circuit with a **High Pass Filter** circuit, we can produce another type of passive RC filter that passes a selected range or "band" of frequencies that can be either narrow or wide while attenuating all those outside of this range. This new type of passive filter arrangement produces a frequency selective filter known commonly as a **Band Pass Filter** or **BPF** for short.

Band Pass Filter Circuit



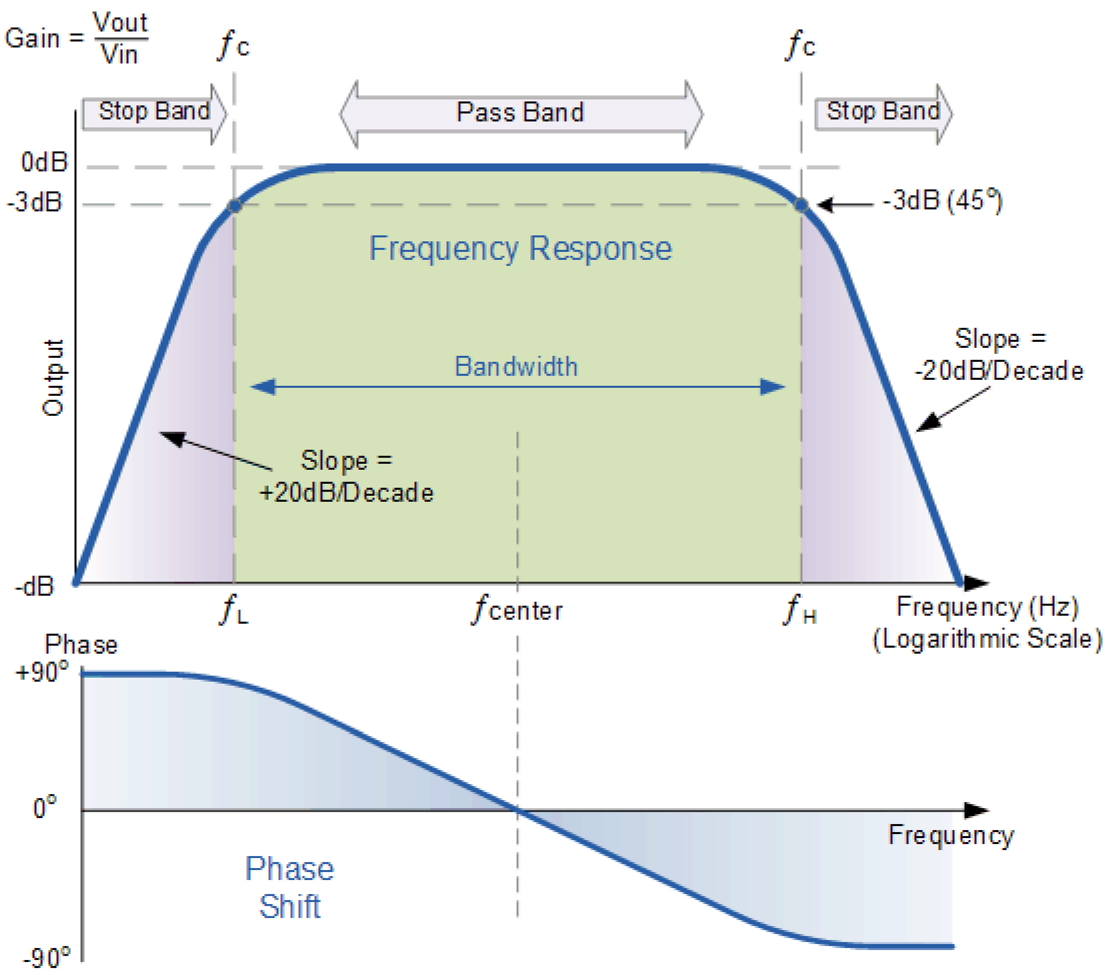
Unlike a low pass filter that only pass signals of a low frequency range or a high pass filter which pass signals of a higher frequency range, a **Band Pass Filters** passes signals within a certain "band" or "spread" of frequencies without distorting the input signal or introducing extra noise. This band of frequencies can be any width and is commonly known as the filters **Bandwidth**.

Bandwidth is commonly defined as the frequency range that exists between two specified frequency cut-off points (*ƒ*c), that are 3dB below the maximum centre or resonant peak while attenuating or weakening the others outside of these two points.

Then for widely spread frequencies, we can simply define the term "bandwidth", BW as being the difference between the lower cut-off frequency (fc_{LOWER}) and the higher cut-off frequency (fc_{HIGHER}) points. In other words, $BW = f_H - f_L$. Clearly for a pass band filter to function correctly, the cut-off frequency of the low pass filter must be higher than the cut-off frequency for the high pass filter.

The “ideal” **Band Pass Filter** can also be used to isolate or filter out certain frequencies that lie within a particular band of frequencies, for example, noise cancellation. Band pass filters are known generally as second-order filters, (two-pole) because they have “two” reactive component, the capacitors, within their circuit design. One capacitor in the low pass circuit and another capacitor in the high pass circuit.

Frequency Response of a 2nd Order Band Pass Filter.



The **Bode Plot** or frequency response curve above shows the characteristics of the band pass filter. Here the signal is attenuated at low frequencies with the output increasing at a slope of +20dB/Decade (6dB/Octave) until the frequency reaches the “lower cut-off” point f_L. At this frequency the output voltage is again $1/\sqrt{2}$ = 70.7% of the input signal value or **-3dB** (20 log (Vout/Vin)) of the input.

The output continues at maximum gain until it reaches the “upper cut-off” point f_H where the output decreases at a rate of -20dB/Decade (6dB/Octave) attenuating any high frequency signals. The point of maximum output gain is generally the geometric mean of the two -3dB value between the lower and upper cut-off points and is called the "Centre Frequency" or "Resonant Peak" value fr. This geometric mean value is calculated as being $f r^2 = f_{(UPPER)} \times f_{(LOWER)}$.

A band pass filter is regarded as a second-order (two-pole) type filter because it has "two" reactive components within its circuit structure, then the phase angle will be twice that of the previously seen first-order filters, ie, **180°**. The phase angle of the output signal **LEADS** that of the input by **+90°** up to the centre or resonant frequency, fr point were it becomes "zero" degrees (0°) or "in-phase" and then changes to **LAG** the input by **-90°** as the output frequency increases.

The upper and lower cut-off frequency points for a band pass filter can be found using the same formula as that for both the low and high pass filters, For example.

$$fc = \frac{1}{2\pi RC}\ Hz$$

Then clearly, the width of the pass band of the filter can be controlled by the positioning of the two cut-off frequency points of the two filters.

Band Pass Filter Example No1.

A second-order **band pass filter** is to be constructed using RC components that will only allow a range of frequencies to pass above 1kHz (1,000Hz) and below 30kHz (30,000Hz). Assuming that both the resistors have values of 10kΩ´s, calculate the values of the two capacitors required.

The High Pass Filter Stage.

The value of the capacitor C1 required to give a cut-off frequency f_L of 1kHz with a resistor value of 10kΩ is calculated as:

$$C = \frac{1}{2\pi fc.R} = \frac{1}{2\pi x 1{,}000 x 10{,}000} = 15.8nF$$

Then, the values of R1 and C1 required for the high pass stage to give a cut-off frequency of 1.0kHz are: R1 = 10kΩ´s and C1 = 15nF.

The Low Pass Filter Stage.

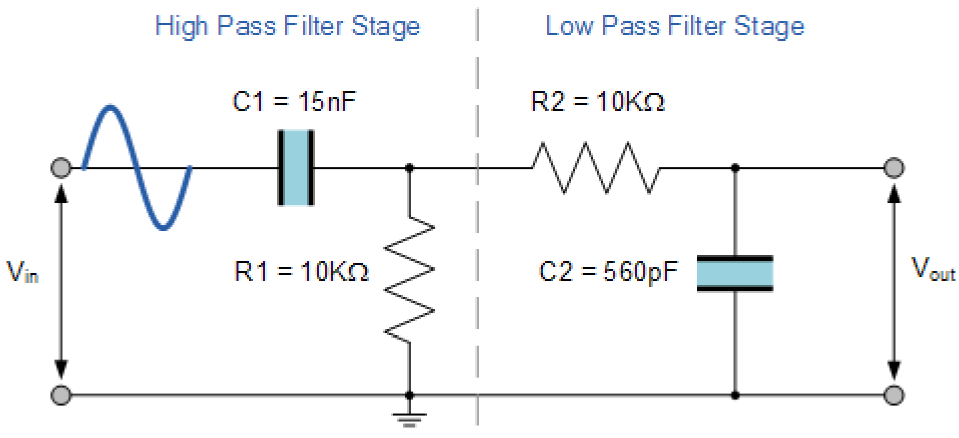
The value of the capacitor C2 required to give a cut-off frequency f_H of 30kHz with a resistor value of 10kΩ is calculated as:

$$C = \frac{1}{2\pi fc.R} = \frac{1}{2\pi x 30{,}000 x 10{,}000} = 510\, pF$$

Then, the values of R2 and C2 required for the low pass stage to give a cut-off frequency of 30kHz are, R = 10kΩ´s and C = 510pF. However, the nearest preferred value of the calculated capacitor value of 510pF is 560pF so this is used instead.

With the values of both the resistances R1 and R2 given as 10kΩ, and the two values of the capacitors C1 and C2 found for both the high pass and low pass filters as 15nF and 560pF respectively, then the circuit for our simple passive **Band Pass Filter** is given as.

Completed Band Pass Filter Circuit



Band Pass Filter Resonant Frequency

We can also calculate the "Resonant" or "Centre Frequency" (fr) point of the band pass filter were the output gain is at its maximum or peak value. This peak value is not the arithmetic average of the upper and lower -3dB cut-off points as you might expect but is in fact the "geometric" or mean value. This geometric mean value is calculated as being $fr^2 = fc_{(UPPER)} \times fc_{(LOWER)}$ for example:

Centre Frequency Equation

$$fr = \sqrt{f_L \; x \; f_H}$$

- Where, f_r is the resonant or centre frequency

- f_L is the lower -3dB cut-off frequency point
- f_H is the upper -3db cut-off frequency point

and in our simple example above, the calculated cut-off frequencies were found to be f_L = 1,060 Hz and f_H = 28,420 Hz using the filter values.

Then by substituting these values into the above equation gives a central resonant frequency of:

$$fr = \sqrt{f_L \; x \; f_H} = \sqrt{1{,}060 \; x \; 28{,}420} = 5{,}48\,kHz$$

Band Pass Filter Summary

A **Band pass Filter** can be made by cascading together a **Low Pass Filter** and a **High Pass Filter** with the frequency range between the lower and upper -3dB cut-off points being know as the filters "Bandwidth".

The centre or resonant frequency point is the geometric mean of the lower and upper cut-off points. At this centre frequency the output signal is at its maximum and the phase shift of the output signal is the same as the input signal.

The amplitude of the output signal from a band pass filter or any passive RC filter for that matter, will always be less than that of the input signal. In other words a passive filter is also an attenuator giving a voltage gain of less than 1 (Unity). To provide an output signal with a voltage gain greater than unity, some form of amplification is required within the design of the circuit.

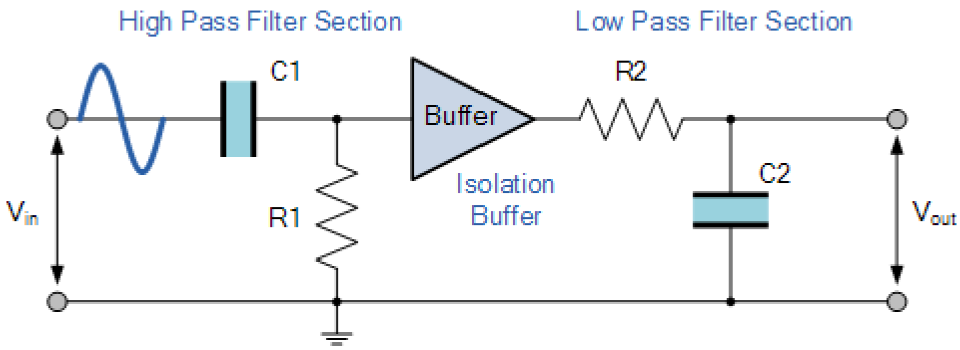
A **Passive Band Pass Filter** is classed as a second-order type filter because it has two reactive components within its design, the capacitors. It is made up from two single RC filter circuits that are each first-order filters themselves.

If more filters are cascaded together the resulting circuit will be known as an "n^{th}-order" filter where the "n" stands for the number of individual reactive components and therefore poles within the filter circuit. For example, filters can be a 2^{nd}-order, 4^{th}-order, 10^{th}-order, etc.

The higher the filters order the steeper will be the slope at n times -20dB/decade. However, a single capacitor value made by combining together two or more individual capacitors is still one capacitor.

Our example above shows the output frequency response curve for an "ideal" band pass filter with constant gain in the pass band and zero gain in the stop bands. In practice the frequency response of this Band Pass Filter circuit would not be the same as the input reactance of the high pass circuit would affect the frequency response of the low pass circuit (components connected in series or parallel) and vice versa. One way of overcoming this would be to provide some form of electrical isolation between the two filter circuits as shown below.

Buffering Individual Filter Stages



One way of combining amplification and filtering into the same circuit would be to use an Operational Amplifier or Op-amp, and examples of these are given in the Operational Amplifier section. In the next tutorial we will look at filter circuits which use an operational amplifier within their design to not only to introduce gain but provide isolation between stages. These types of filter arrangements are generally known as **Active Filters**.

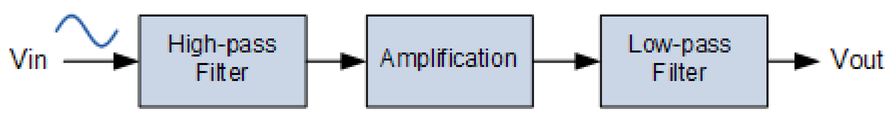
Active Band Pass Filter

As we saw previously in the Passive Band Pass Filter tutorial, the principal characteristic of a **Band Pass Filter** or any filter for that matter, is its ability to pass frequencies relatively unattenuated over a specified band or spread of frequencies called the "Pass Band".

For a low pass filter this pass band starts from 0Hz or DC and continues up to the specified cut-off frequency point at -3dB down from the maximum pass band gain. Equally, for a high pass filter the pass band starts from this -3dB cut-off frequency and continues up to infinity or the maximum open loop gain for an Active Filter.

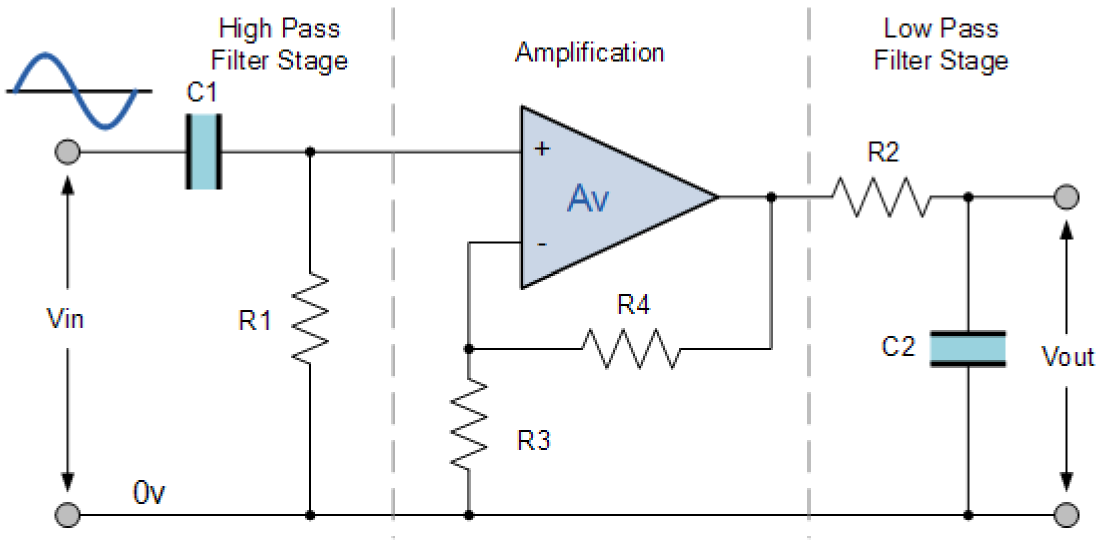
However, the **Active Band Pass Filter** is slightly different in that it is a frequency selective filter circuit used in electronic systems to separate a signal at one particular frequency, or a range of signals that lie within a certain "band" of frequencies from signals at all other frequencies. This band or range of frequencies is set between two cut-off or corner frequency points labelled the "lower frequency" (f_L) and the "higher frequency" (f_H) while attenuating any signals outside of these two points.

Simple **Active Band Pass Filter** can be easily made by cascading together a single Low Pass Filter with a single High Pass Filter as shown.

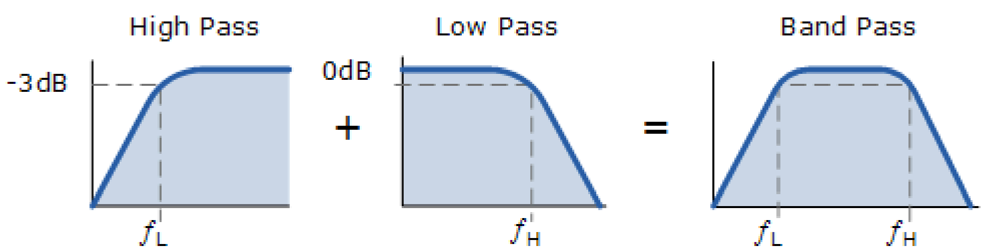


The cut-off or corner frequency of the low pass filter (LPF) is higher than the cut-off frequency of the high pass filter (HPF) and the difference between the frequencies at the -3dB point will determine the "bandwidth" of the band pass filter while attenuating any signals outside of these points. One way of making a very simple **Active Band Pass Filter** is to connect the basic passive high and low pass filters we look at previously to an amplifying op-amp circuit as shown.

Active Band Pass Filter Circuit



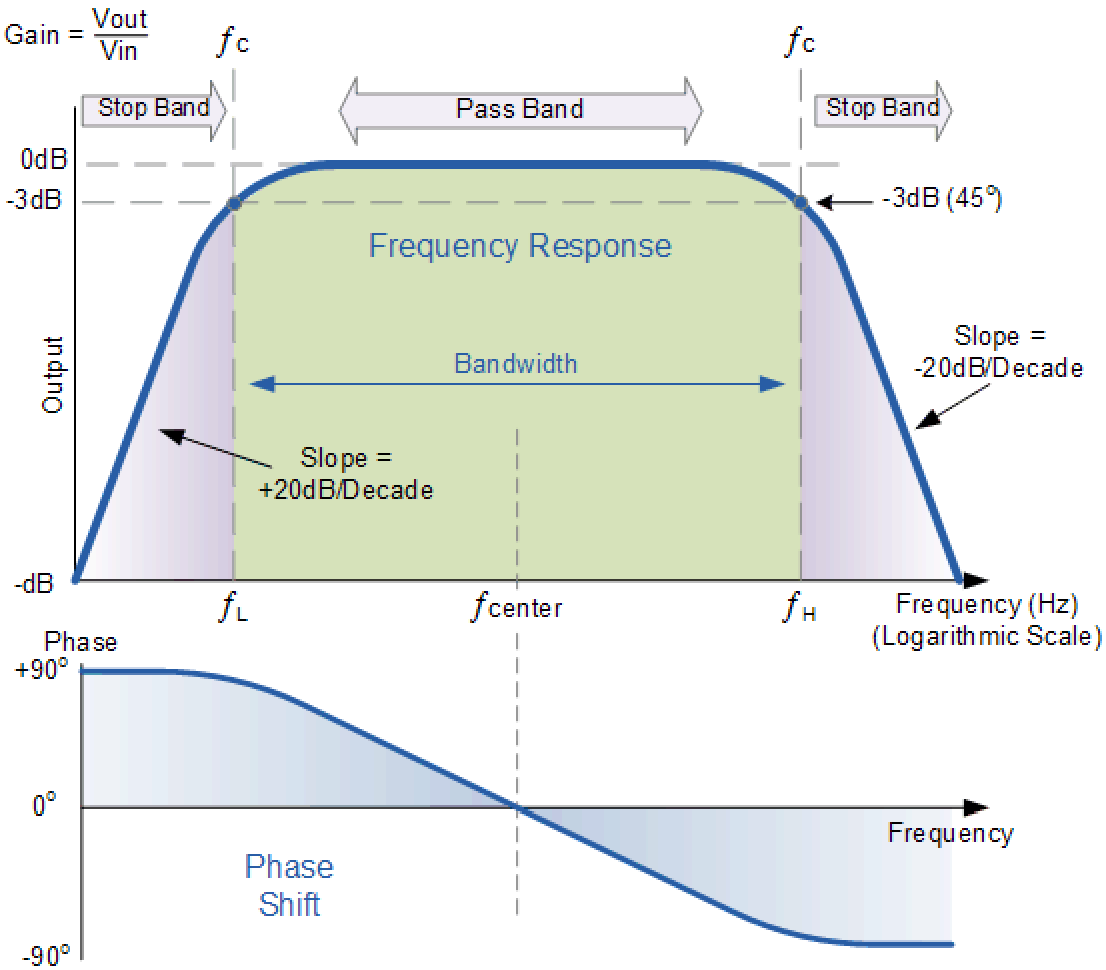
This cascading together of the individual low and high pass passive filters produces a low "Q-factor" type filter circuit which has a wide pass band. The first stage of the filter will be the high pass stage that uses the capacitor to block any DC biasing from the source. This design has the advantage of producing a relatively flat asymmetrical pass band frequency response with one half representing the low pass response and the other half representing high pass response as shown.



The higher corner point (f_H) as well as the lower corner frequency cut-off point (f_L) are calculated the same as before in the standard first-order low and high pass filter circuits. Obviously, a reasonable separation is required between the two cut-off points to prevent any interaction between the low pass and high pass stages. The amplifier also provides isolation between the two stages and defines the overall voltage gain of the circuit.

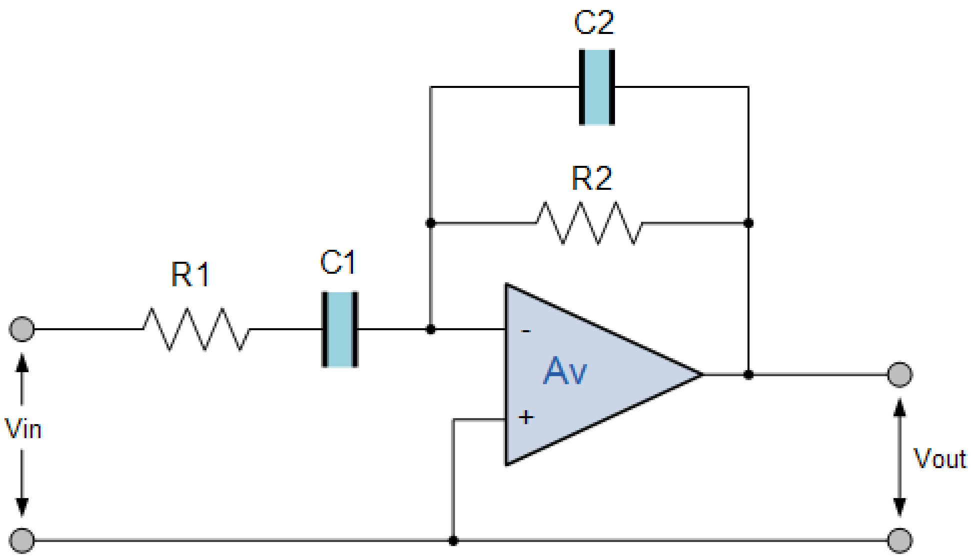
The bandwidth of the filter is therefore the difference between these upper and lower -3dB points. For example, if the -3dB cut-off points are at 200Hz and 600Hz then the bandwidth of the filter would be given as: Bandwidth (BW) = 600 – 200 = 400Hz. The normalised frequency response and phase shift for an active band pass filter will be as follows.

Active Band Pass Frequency Response



While the above passive tuned filter circuit will work as a band pass filter, the pass band (bandwidth) can be quite wide and this may be a problem if we want to isolate a small band of frequencies. Active band pass filter can also be made using inverting operational amplifier. So by rearranging the positions of the resistors and capacitors within the filter we can produce a much better filter circuit as shown below. For an active band pass filter, the lower cut-off -3dB point is given by f_{C2} while the upper cut-off -3dB point is given by f_{C1}.

Inverting Band Pass Filter Circuit



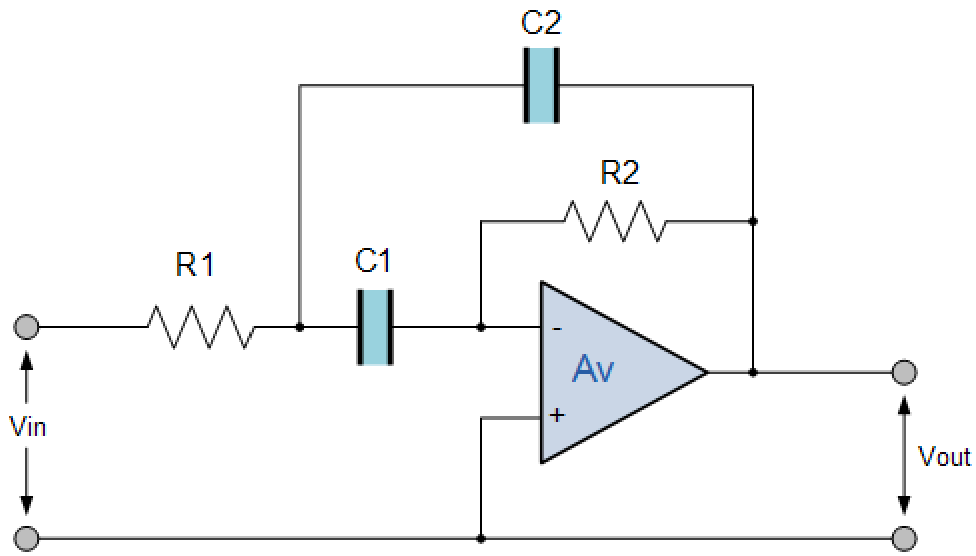
$$Voltage\ Gain = -\frac{R_2}{R_1},\quad fc_1 = \frac{1}{2\pi R_1 C_1},\quad fc_2 = \frac{1}{2\pi R_2 C_2}$$

This type of band pass filter is designed to have a much narrower pass band. The centre frequency and bandwidth of the filter is related to the values of R1, R2, C1 and C2. The output of the filter is again taken from the output of the op-amp.

Multiple Feedback Band Pass Active Filter

We can improve the band pass response of the above circuit by rearranging the components again to produce an infinite-gain multiple-feedback (IGMF) band pass filter. This type of active band pass design produces a “tuned” circuit based around a negative feedback active filter giving it a high “Q-factor” (up to 25) amplitude response and steep roll-off on either side of its centre frequency. Because the frequency response of the circuit is similar to a resonance circuit, this center frequency is referred to as the resonant frequency, (fr). Consider the circuit below.

Infinite Gain Multiple Feedback Active Filter



This active band pass filter circuit uses the full gain of the operational amplifier, with multiple negative feedback applied via resistor, R_2 and capacitor C_2. Then we can define the characteristics of the IGMF filter as follows:

$$f_r = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \qquad Q_{BP} = \frac{f_r}{BW_{(3dB)}} = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$$

$$\text{Maximum Gain, } (Av) = -\frac{R_2}{2R_1} = -2Q^2$$

We can see then that the relationship between resistors, R_1 and R_2 determines the band pass "Q-factor" and the frequency at which the maximum amplitude occurs, the gain of the circuit will be equal to $-2Q^2$. Then as the gain increases so to does the selectivity. In other words, high gain – high selectivity.

Active Band Pass Filter Example No1

An active band pass filter that has a gain Av of one and a resonant frequency, fr of 1kHz is constructed using an infinite gain multiple feedback filter circuit. Calculate the values of the components required to implement the circuit.

Firstly, we can determine the values of the two resistors, R_1 and R_2 required for the active filter using the gain of the circuit to find Q as follows.

$$Av = 1 = -2Q^2 \quad \therefore\ Q_{BP} = \sqrt{\frac{1}{2}} = 0.7071$$

$$Q = 0.7071 = \frac{1}{2}\sqrt{\frac{R_2}{R_1}} \quad \therefore\ \frac{R_2}{R_1} = \left(\frac{0.7071}{\frac{1}{2}}\right)^2 = 2$$

Then we can see that a value of Q = 0.7071 gives a relationship of resistor, R_2 being twice the value of resistor R_1. Then we can choose any suitable value of resistances to give the required ratio of two. Then resistor $R_1 = 10k\Omega$ and $R_2 = 20k\Omega$.

The center or resonant frequency is given as 1kHz. Using the new resistor values obtained, we can determine the value of the capacitors required assuming that $C = C_1 = C_2$.

$$f_r = 1{,}000\text{Hz} = \frac{1}{2\pi C\sqrt{R_1 R_2}}$$

$$\therefore\ C = \frac{1}{2\pi f_r\sqrt{R_1 R_2}} = \frac{1}{2\pi 1000\sqrt{10{,}000 \times 20{,}000}} = 11.2\text{nF}$$

The closest standard value is 10nF.

Resonant Frequency Point

The actual shape of the frequency response curve for any passive or active band pass filter will depend upon the characteristics of the filter circuit with the curve above being defined as an "ideal" band pass response. An active band pass filter is a **2nd Order** type filter because it has "two" reactive components (two capacitors) within its circuit design.

As a result of these two reactive components, the filter will have a peak response or **Resonant Frequency** (fr) at its "center frequency", fc. The center frequency is generally calculated as being the geometric mean of the two -3dB frequencies between the upper and the lower cut-off points with the resonant frequency (point of oscillation) being given as:

$$fr = \sqrt{f_L \ x \ f_H}$$

- Where:
- f_r is the resonant or Center Frequency
- f_L is the lower -3dB cut-off frequency point
- f_H is the upper -3db cut-off frequency point

and in our simple example above the resonant center frequency of the active band pass filter is given as:

$$fr = \sqrt{200 \ x \ 600} = \sqrt{120{,}000} = 346 \, Hz$$

The "Q" or Quality Factor

In a **Band Pass Filter** circuit, the overall width of the actual pass band between the upper and lower -3dB corner points of the filter determines the **Quality Factor** or **Q-point** of the circuit. This **Q Factor** is a measure of how "Selective" or "Un-selective" the band pass filter is towards a given spread of frequencies. The lower the value of the Q factor the wider is the bandwidth of the filter and consequently the higher the Q factor the narrower and more "selective" is the filter.

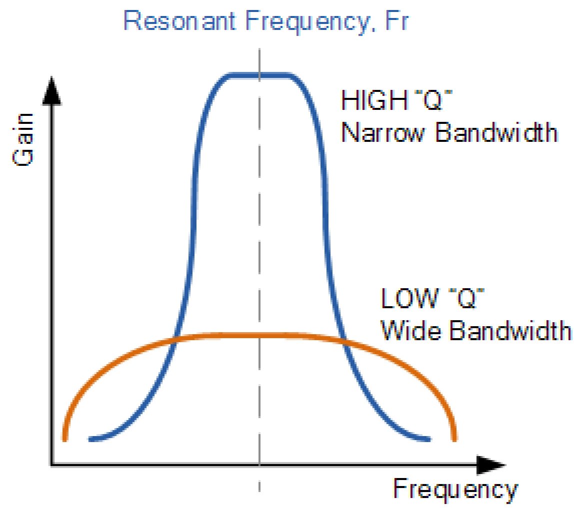
The **Quality Factor, Q** of the filter is sometimes given the Greek symbol of **Alpha**, (α) and is known as the **alpha-peak frequency** where:

$$\alpha = \frac{1}{Q}$$

As the quality factor of an active band pass filter (Second-order System) relates to the "sharpness" of the filters response around its centre resonant frequency (*fr*) it can also be thought of as the "Damping Factor" or "Damping Coefficient" because the more damping the filter has the flatter is its response and likewise, the less damping the filter has the sharper is its response. The damping ratio is given the Greek symbol of **Xi**, (ξ) where:

$$\xi = 2\alpha$$

The “Q” of a band pass filter is the ratio of the **Resonant Frequency**, (*ƒ*r) to the **Bandwidth**, (BW) between the upper and lower -3dB frequencies and is given as:



$$Q = \frac{\text{Resonant Frequency}}{\text{Bandwidth}}$$

Then for our simple example above the quality factor “**Q**” of the band pass filter is given as:

346Hz / 400Hz = **0.865**. Note that **Q** is a ratio and has no units.

When analysing Active Filters, generally a normalised circuit is considered which produces an “ideal” frequency response having a rectangular shape, and a transition between the pass band and the stop band that has an abrupt or very steep roll-off slope. However, these ideal responses are not possible in the real world so we use approximations to give us the best frequency response possible for the type of filter we are trying to design.

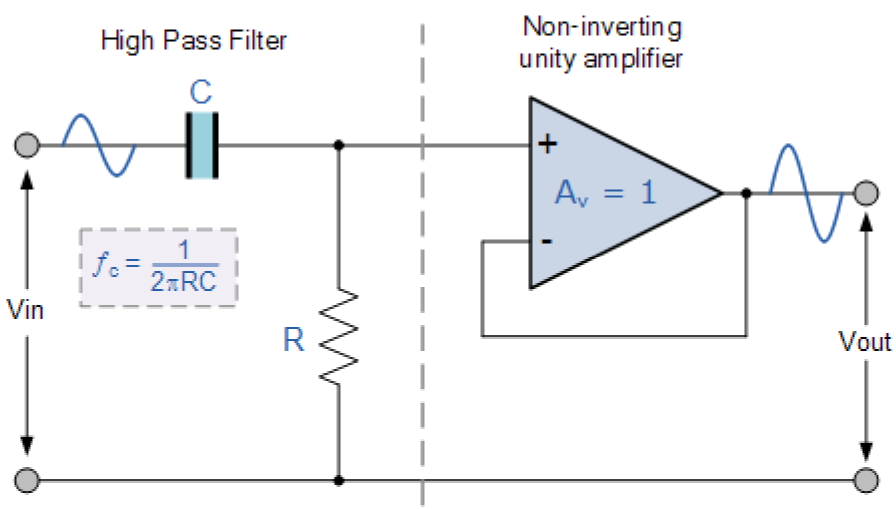
Probably the best known filter approximation for doing this is the Butterworth or maximally-flat response filter. In the next tutorial we will look at higher order filters and use Butterworth approximations to produce filters that have a frequency response which is as flat as mathematically possible in the pass band and a smooth transition or roll-off rate.

Active High Pass Filters

The basic electrical operation of an **Active High Pass Filter** (HPF) is exactly the same as we saw for its equivalent RC passive high pass filter circuit, except this time the circuit has an operational amplifier or op-amp included within its filter design providing amplification and gain control.

Like the previous active low pass filter circuit, the simplest form of an *active high pass filter* is to connect a standard inverting or non-inverting operational amplifier to the basic RC high pass passive filter circuit as shown.

First Order Active High Pass Filter

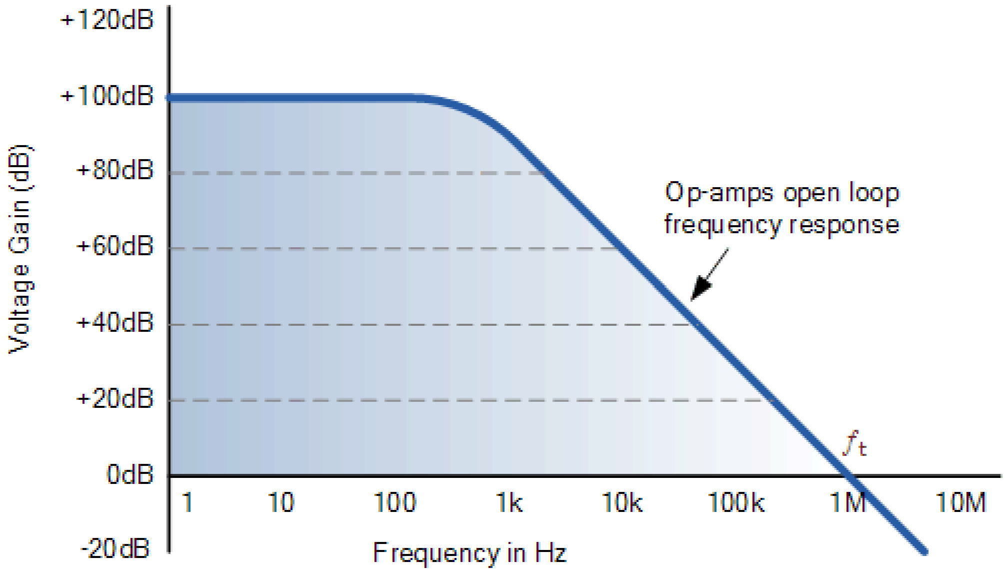


Technically, there is no such thing as an **active high pass filter**. Unlike Passive High Pass Filters which have an "infinite" frequency response, the maximum pass band frequency response of an Active High Pass Filter is limited by the open-loop characteristics or bandwidth of the operational amplifier being used, making them appear as if they are band pass filters with a high frequency cut-off determined by the selection of op-amp and gain.

In the Operational Amplifier tutorial we saw that the maximum frequency response of an op-amp is limited to the Gain/Bandwidth product or open loop voltage gain (A_V) of the operational amplifier being used giving it a bandwidth limitation, where the closed loop response of the op amp intersects the open loop response.

A commonly available operational amplifier such as the uA741 has a typical "open-loop" (without any feedback) DC voltage gain of about 100dB maximum reducing at a roll off rate of -20dB/Decade (-6db/Octave) as the input frequency increases. The gain of the uA741 reduces until it reaches unity gain, (0dB) or its “transition frequency” (ft) which is about 1MHz. This causes the op-amp to have a frequency response curve very similar to that of a first-order low pass filter and this is shown below.

Frequency response curve of a typical Operational Amplifier.



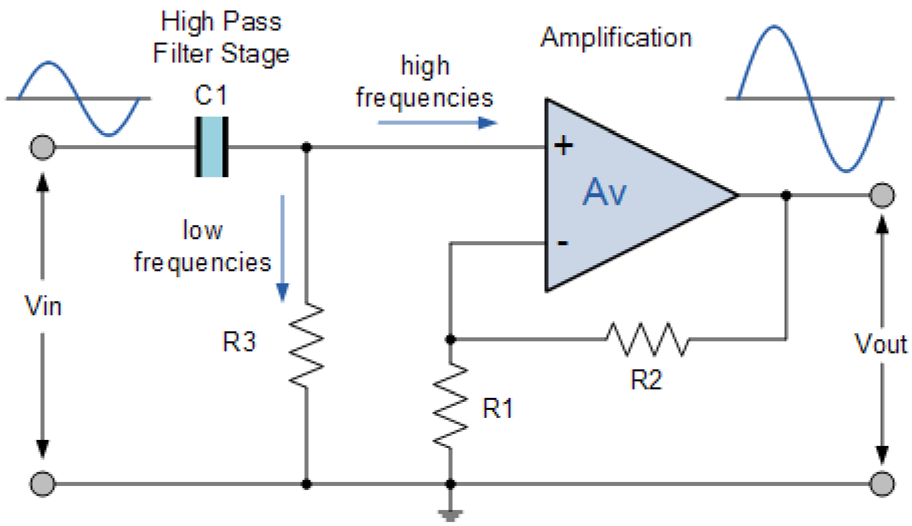
Then the performance of a “high pass filter” at high frequencies is limited by this unity gain crossover frequency which determines the overall bandwidth of the open-loop amplifier. The gain-bandwidth product of the op-amp starts from around 100kHz for small signal amplifiers up to about 1GHz for high-speed digital video amplifiers and op-amp based active filters can achieve very good accuracy and performance provided that low tolerance resistors and capacitors are used.

Under normal circumstances the maximum pass band required for a closed loop active high pass or band pass filter is well below that of the maximum open-loop transition frequency. However, when designing active filter circuits it is important to choose the correct op-amp for the circuit as the loss of high frequency signals may result in signal distortion.

Active High Pass Filter

A first-order (single-pole) **Active High Pass Filter** as its name implies, attenuates low frequencies and passes high frequency signals. It consists simply of a passive filter section followed by a non-inverting operational amplifier. The frequency response of the circuit is the same as that of the passive filter, except that the amplitude of the signal is increased by the gain of the amplifier and for a non-inverting amplifier the value of the pass band voltage gain is given as 1 + R2/R1, the same as for the low pass filter circuit.

Active High Pass Filter with Amplification



This *first-order high pass filter*, consists simply of a passive filter followed by a non-inverting amplifier. The frequency response of the circuit is the same as that of the passive filter, except that the amplitude of the signal is increased by the gain of the amplifier.

For a non-inverting amplifier circuit, the magnitude of the voltage gain for the filter is given as a function of the feedback resistor (R2) divided by its corresponding input resistor (R1) value and is given as:

Gain for an Active High Pass Filter

$$\text{Voltage Gain, } (Av) = \frac{Vout}{Vin} = \frac{A_F\left(\frac{f}{fc}\right)}{\sqrt{1+\left(\frac{f}{fc}\right)^2}}$$

- Where:
- A_F = the Pass band Gain of the filter, (1 + R2/R1)
- f = the Frequency of the Input Signal in Hertz, (Hz)
- fc = the Cut-off Frequency in Hertz, (Hz)

Just like the low pass filter, the operation of a high pass active filter can be verified from the frequency gain equation above as:

- 1. At very low frequencies, $f < fc$ $\dfrac{Vout}{Vin} < A_F$
- 2. At the cut-off frequency, $f = fc$ $\dfrac{Vout}{Vin} = \dfrac{A_F}{\sqrt{2}} = 0.707\,A_F$
- 3. At very high frequencies, $f > fc$ $\dfrac{Vout}{Vin} \cong A_F$

Then, the **Active High Pass Filter** has a gain A_F that increases from 0Hz to the low frequency cut-off point, f_C at 20dB/decade as the frequency increases. At f_C the gain is $0.707A_F$, and after f_C all frequencies are pass band frequencies so the filter has a constant gain A_F with the highest frequency being determined by the closed loop bandwidth of the op-amp.

When dealing with filter circuits the magnitude of the pass band gain of the circuit is generally expressed in *decibels* or *dB* as a function of the voltage gain, and this is defined as:

Magnitude of Voltage Gain in (dB)

$$Av(dB) = 20\log_{10}\left(\frac{Vout}{Vin}\right)$$

$$\therefore\ -3dB = 20\log_{10}\left(0.707\,\frac{Vout}{Vin}\right)$$

For a first-order filter the frequency response curve of the filter increases by 20dB/decade or 6dB/octave up to the determined cut-off frequency point which is always at -3dB below the maximum gain value. As with the previous filter circuits, the lower cut-off or corner frequency (fc) can be found by using the same formula:

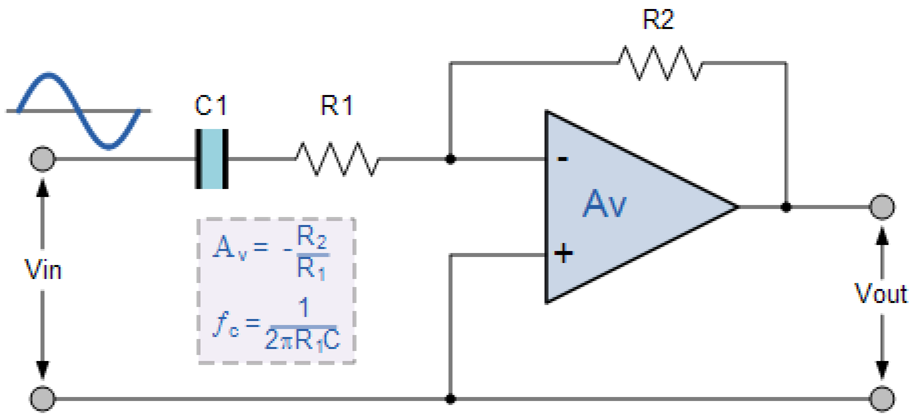
$$fc = \frac{1}{2\pi RC}\ Hz$$

The corresponding phase angle or phase shift of the output signal is the same as that given for the passive RC filter and **leads** that of the input signal. It is equal to **+45°** at the cut-off frequency fc value and is given as:

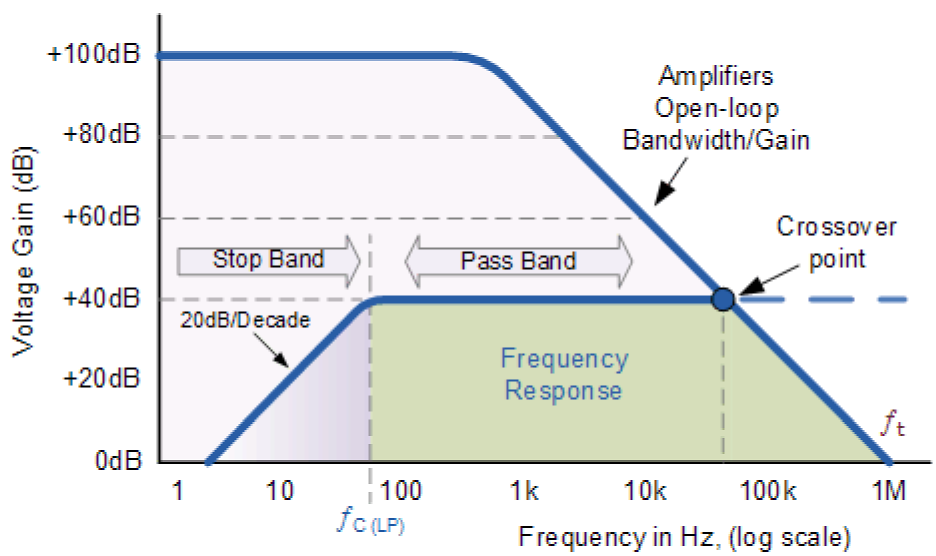
$$\textit{Phase Shift}\ \ \phi = \tan^{-1}\left(\frac{1}{2\pi fRC}\right)$$

A simple first-order active high pass filter can also be made using an inverting operational amplifier configuration as well, and an example of this circuit design is given along with its corresponding frequency response curve. A gain of 40dB has been assumed for the circuit.

Inverting Operational Amplifier Circuit



Frequency Response Curve



Active High Pass Filter Example No1

A first order active high pass filter has a pass band gain of two and a cut-off corner frequency of 1kHz. If the input capacitor has a value of 10nF, calculate the value of the cut-off frequency determining resistor and the gain resistors in the feedback network. Also, plot the expected frequency response of the filter.

With a cut-off corner frequency given as 1kHz and a capacitor of 10nF, the value of R will therefore be:

$$R = \frac{1}{2\pi f_C C} = \frac{1}{2\pi.1000.10\times10^{-9}} = 15.92\text{k}\Omega$$

or 16kΩ's to the nearest preferred value.

The pass band gain of the filter, A_F is given as being, 2.

$$A_F = 1 + \frac{R_2}{R_1}, \quad \therefore 2 = 1 + \frac{R_2}{R_1} \quad \text{and} \quad \frac{R_2}{R_1} = 1$$

As the value of resistor, R_2 divided by resistor, R_1 gives a value of one. Then, resistor R_1 must be equal to resistor R_2, since the pass band gain, $A_F = 2$. We can therefore select a suitable value for the two resistors of say, 10kΩ's each for both feedback resistors.

So for a high pass filter with a cut-off corner frequency of 1kHz, the values of R and C will be, 10kΩ's and 10nF respectively. The values of the two feedback resistors to produce a pass band gain of two are given as: $R_1 = R_2 = 10\text{k}\Omega$'s

The data for the frequency response bode plot can be obtained by substituting the values obtained above over a frequency range from 100Hz to 100kHz into the equation for voltage gain:

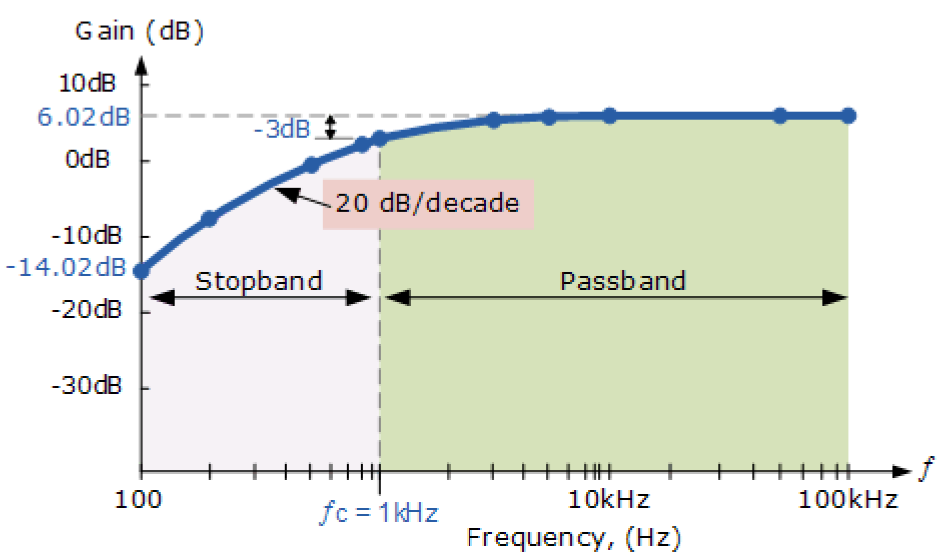
$$\text{Voltage Gain, } (Av) = \frac{Vout}{Vin} = \frac{A_F\left(\frac{f}{fc}\right)}{\sqrt{1+\left(\frac{f}{fc}\right)^2}}$$

This then will give us the following table of data.

Frequency, *f* (Hz)	Voltage Gain (*Vo / Vin*)	Gain, (dB) 20log(*Vo / Vin*)
100	0.20	-14.02
200	0.39	-8.13
500	0.89	-0.97
800	1.25	1.93
1,000	1.41	3.01
3,000	1.90	5.56
5,000	1.96	5.85
10,000	1.99	5.98
50,000	2.00	6.02
100,000	2.00	6.02

The frequency response data from the table above can now be plotted as shown below. In the stop band (from 100Hz to 1kHz), the gain increases at a rate of 20dB/decade. However, in the pass band after the cut-off frequency, f_C = 1kHz, the gain remains constant at 6.02dB. The upper-frequency limit of the pass band is determined by the open loop bandwidth of the operational amplifier used as we discussed earlier. Then the bode plot of the filter circuit will look like this.

The Frequency Response Bode-plot for our example.

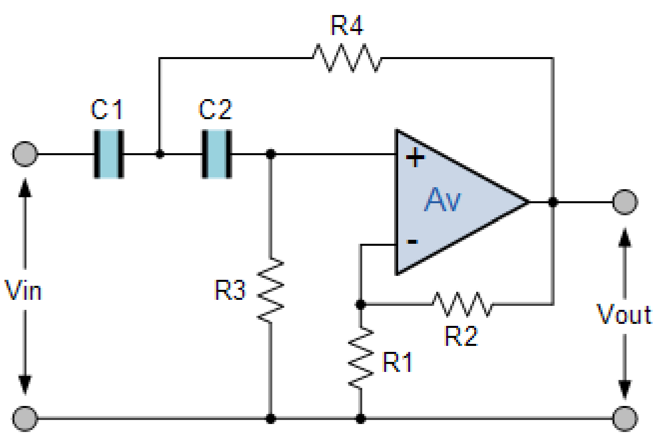


Applications of **Active High Pass Filters** are in audio amplifiers, equalizers or speaker systems to direct the high frequency signals to the smaller tweeter speakers or to reduce any low frequency noise or “rumble” type distortion. When used like this in audio applications the active high pass filter is sometimes called a “Treble Boost” filter.

Second-order High Pass Active Filter

As with the passive filter, a first-order high pass active filter can be converted into a second-order high pass filter simply by using an additional RC network in the input path. The frequency response of the second-order high pass filter is identical to that of the first-order type except that the stop band roll-off will be twice the first-order filters at 40dB/decade (12dB/octave). Therefore, the design steps required of the second-order active high pass filter are the same.

Second-order Active High Pass Filter Circuit



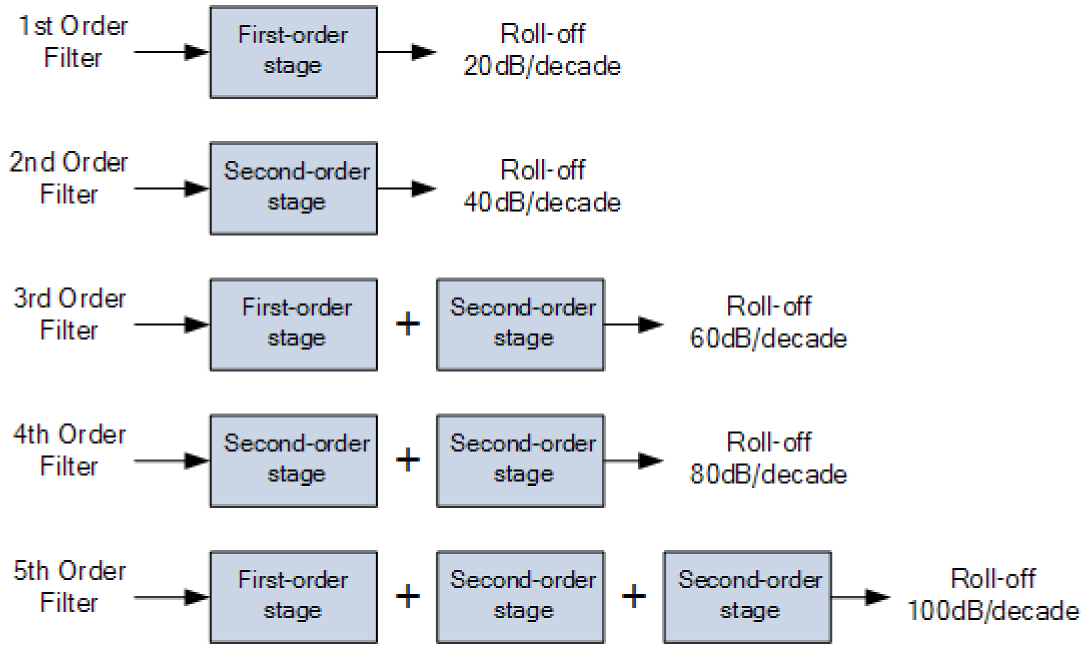
$$A_V = 1 + \frac{R_2}{R_1}$$

$$f_C = \frac{1}{2\pi\sqrt{R_3 R_4 C_1 C_2}}$$

Higher-order [High Pass Active Filters](), such as third, fourth, fifth, etc are formed simply by cascading together first and second-order filters. For example, a third order high pass filter is formed by cascading in series first and second order filters, a fourth-order high pass filter by cascading two second-order filters together and so on.

Then an **Active High Pass Filter** with an even order number will consist of only second-order filters, while an odd order number will start with a first-order filter at the beginning as shown.

Cascading Active High Pass Filters



Although there is no limit to the order of a filter that can be formed, as the order of the filter increases so to does its size. Also, its accuracy declines, that is the difference between the actual stop band response and the theoretical stop band response also increases.

If the frequency determining resistors are all equal, R1 = R2 = R3 etc, and the frequency determining capacitors are all equal, C1 = C2 = C3 etc, then the cut-off frequency for any order of filter will be exactly the same. However, the overall gain of the higher-order filter is fixed because all the frequency determining components are equal.

In the next tutorial about filters, we will see that Active Band Pass Filters, can be constructed by cascading together a high pass and a low pass filter.

Active Filters

In the RC Passive Filter tutorials, we saw how a basic first-order filter circuits, such as the low pass and the high pass filters can be made using just a single resistor in series with a non-polarized capacitor connected across a sinusoidal input signal.

We also noticed that the main disadvantage of Passive Filters is that the amplitude of the output signal is less than that of the input signal, ie, the gain is never greater than unity and that the load impedance affects the filters characteristics.

With passive filter circuits containing multiple stages, this loss in signal amplitude called “Attenuation” can become quiet severe. One way of restoring or controlling this loss of signal is by using amplification through the use of **Active Filters**.

As their name implies, **Active Filters** contain active components such as operational amplifiers, transistors or FET’s within their circuit design. They draw their power from an external power source and use it to boost or amplify the output signal.

Filter amplification can also be used to either shape or alter the frequency response of the filter circuit by producing a more selective output response, making the output bandwidth of the filter more narrower or even wider. Then the main difference between a “passive filter” and an “active filter” is amplification.

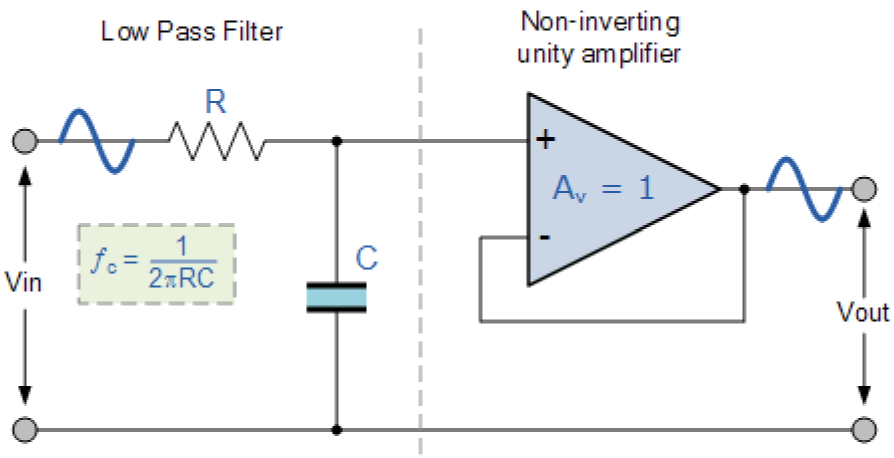
An active filter generally uses an operational amplifier (op-amp) within its design and in the Operational Amplifier tutorial we saw that an Op-amp has a high input impedance, a low output impedance and a voltage gain determined by the resistor network within its feedback loop.

Unlike a passive high pass filter which has in theory an infinite high frequency response, the maximum frequency response of an active filter is limited to the Gain/Bandwidth product (or open loop gain) of the operational amplifier being used. Still, active filters are generally more easier to design than passive filters, they produce good performance characteristics, very good accuracy with a steep roll-off and low noise when used with a good circuit design.

Active Low Pass Filter

The most common and easily understood active filter is the **Active Low Pass Filter**. Its principle of operation and frequency response is exactly the same as those for the previously seen passive filter, the only difference this time is that it uses an op-amp for amplification and gain control. The simplest form of a low pass active filter is to connect an inverting or non-inverting amplifier, the same as those discussed in the Op-amp tutorial, to the basic RC low pass filter circuit as shown.

First Order Active Low Pass Filter

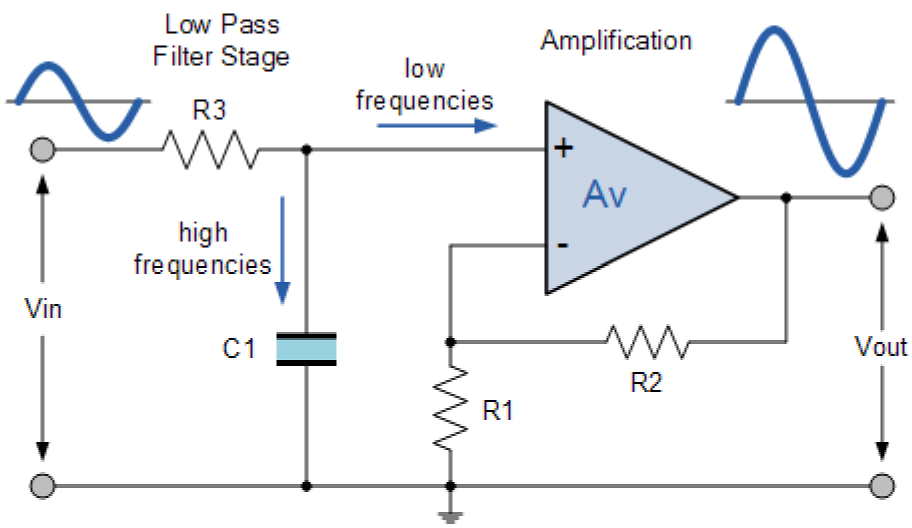


This first-order low pass active filter, consists simply of a passive RC filter stage providing a low frequency path to the input of a non-inverting operational amplifier. The amplifier is configured as a voltage-follower (Buffer) giving it a DC gain of one, Av = +1 or unity gain as opposed to the previous passive RC filter which has a DC gain of less than unity.

The advantage of this configuration is that the op-amps high input impedance prevents excessive loading on the filters output while its low output impedance prevents the filters cut-off frequency point from being affected by changes in the impedance of the load.

While this configuration provides good stability to the filter, its main disadvantage is that it has no voltage gain above one. However, although the voltage gain is unity the power gain is very high as its output impedance is much lower than its input impedance. If a voltage gain greater than one is required we can use the following filter circuit.

Active Low Pass Filter with Amplification



The frequency response of the circuit will be the same as that for the passive RC filter, except that the amplitude of the output is increased by the pass band gain, A_F of the amplifier. For a non-inverting amplifier circuit, the magnitude of the voltage gain for the filter is given as a function of the feedback resistor (R_2) divided by its corresponding input resistor (R_1) value and is given as:

$$\text{DC gain} = \left(1 + \frac{R_2}{R_1}\right)$$

Therefore, the gain of an active low pass filter as a function of frequency will be:

Gain of a first-order low pass filter

$$\text{Voltage Gain, } (Av) = \frac{Vout}{Vin} = \frac{A_F}{\sqrt{1 + \left(\frac{f}{fc}\right)^2}}$$

- Where:
- A_F = the pass band gain of the filter, (1 + R2/R1)
- f = the frequency of the input signal in Hertz, (Hz)
- fc = the cut-off frequency in Hertz, (Hz)

Thus, the operation of a low pass active filter can be verified from the frequency gain equation above as:

- 1. At very low frequencies, $f < fc$ $\frac{Vout}{Vin} \cong A_F$
- 2. At the cut-off frequency, $f = fc$ $\frac{Vout}{Vin} = \frac{A_F}{\sqrt{2}} = 0.707\,A_F$
- 3. At very high frequencies, $f > fc$ $\frac{Vout}{Vin} < A_F$

Thus, the **Active Low Pass Filter** has a constant gain A_F from 0Hz to the high frequency cut-off point, f_C. At f_C the gain is $0.707A_F$, and after f_C it decreases at a constant rate as the frequency increases. That is, when the frequency is increased tenfold (one decade), the voltage gain is divided by 10.

In other words, the gain decreases 20dB (= 20log 10) each time the frequency is increased by 10. When dealing with filter circuits the magnitude of the pass band gain of the circuit is generally expressed in *decibels* or *dB* as a function of the voltage gain, and this is defined as:

Magnitude of Voltage Gain in (dB)

$$Av(dB) = 20\log_{10}\left(\frac{Vout}{Vin}\right)$$

$$\therefore\ -3dB = 20\log_{10}\left(0.707\,\frac{Vout}{Vin}\right)$$

Active Low Pass Filter Example No1

Design a non-inverting active low pass filter circuit that has a gain of ten at low frequencies, a high frequency cut-off or corner frequency of 159Hz and an input impedance of 10KΩ.

The voltage gain of a non-inverting operational amplifier is given as:

$$A_F = 1 + \frac{R_2}{R_1} = 10$$

Assume a value for resistor R1 of 1kΩ rearranging the formula above gives a value for R2 of

$$R_2 = (10 - 1)\ x\ R_1 = 9\ x\ 1k\Omega = 9k\Omega$$

then, for a voltage gain of 10, R1 = 1kΩ and R2 = 9kΩ. However, a 9kΩ resistor does not exist so the next preferred value of 9k1Ω is used instead.

converting this voltage gain to a decibel dB value gives:

$$Gain\ in\ dB = 20\log A = 20\log 10 = 20dB$$

The cut-off or corner frequency (fc) is given as being 159Hz with an input impedance of 10kΩ. This cut-off frequency can be found by using the formula:

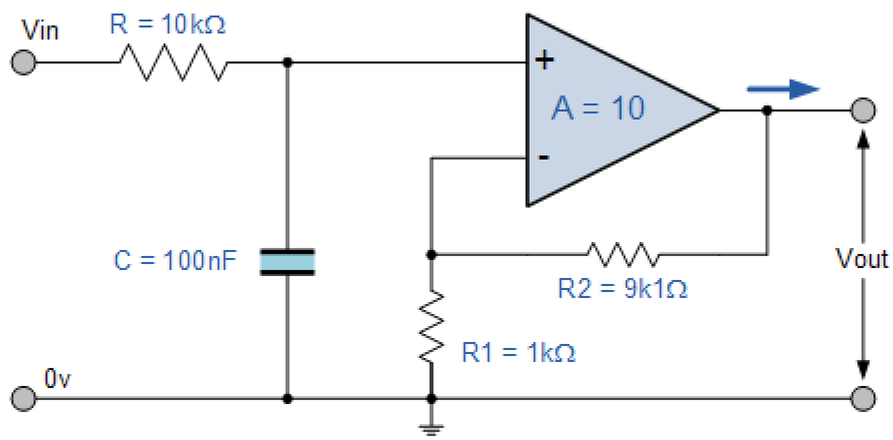
$$fc = \frac{1}{2\pi RC}\ Hz$$ where fc = 159Hz and R = 10kΩ.

then, by rearranging the above formula we can find the value for capacitor C as:

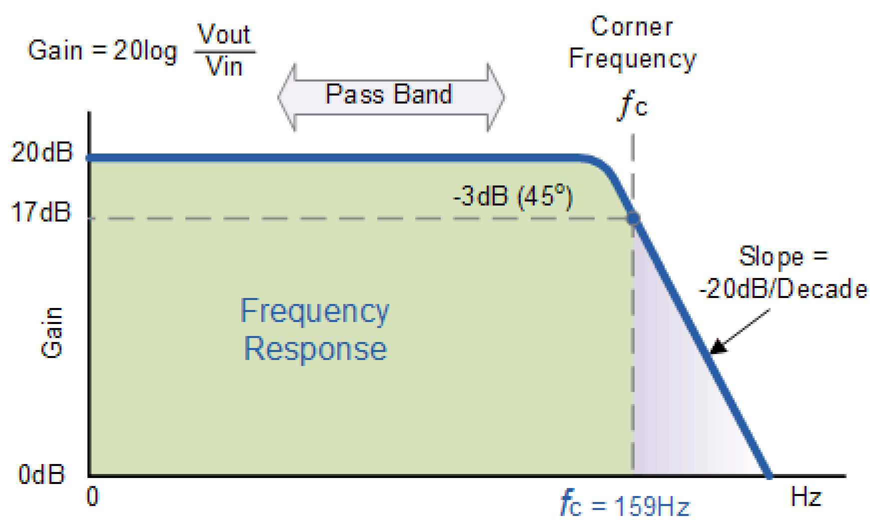
$$C = \frac{1}{2\pi\, fc\, R} = \frac{1}{2\pi\ x\ 159\ x\ 10k\Omega} = 100nF$$

Then the final circuit along with its frequency response is given below as:

Low Pass Filter Circuit.



Frequency Response Curve



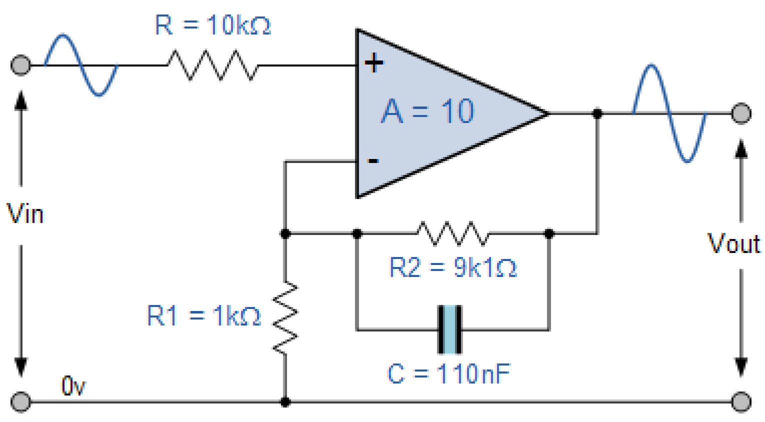
If the external impedance connected to the input of the circuit changes, this change will also affect the corner frequency of the filter (components connected in series or parallel). One way of avoiding this is to place the capacitor in parallel with the feedback resistor R2.

The value of the capacitor will change slightly from being 100nF to 110nF to take account of the 9k1Ω resistor and the formula used to calculate the cut-off corner frequency is the same as that used for the RC passive low pass filter.

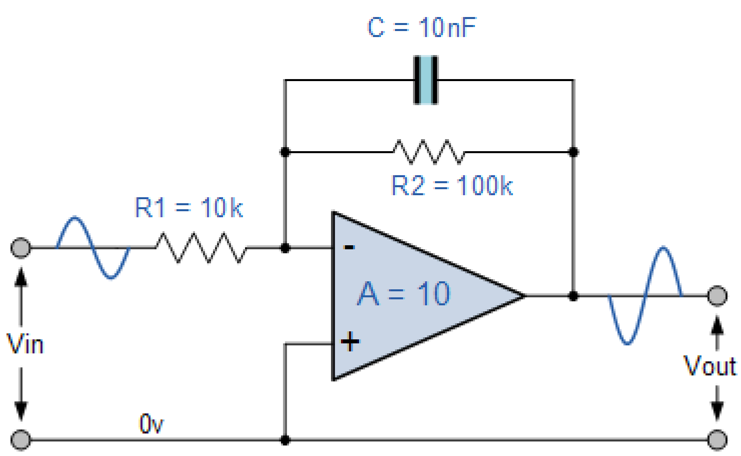
$$fc = \frac{1}{2\pi C R_2} \quad Hertz$$

An example of the new **Active Low Pass Filter** circuit is given as.

Simplified non-inverting amplifier filter circuit



Equivalent inverting amplifier filter circuit

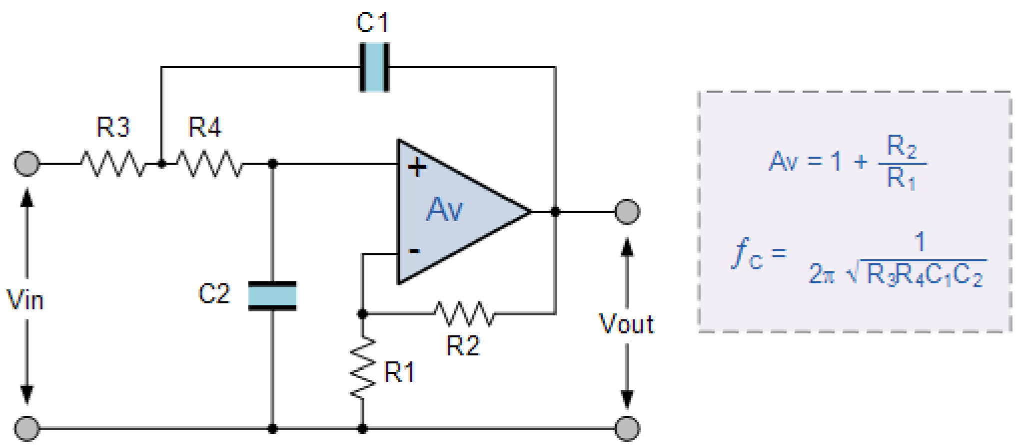


Applications of **Active Low Pass Filters** are in audio amplifiers, equalizers or speaker systems to direct the lower frequency bass signals to the larger bass speakers or to reduce any high frequency noise or "hiss" type distortion. When used like this in audio applications the active low pass filter is sometimes called a "Bass Boost" filter.

Second-order Low Pass Active Filter

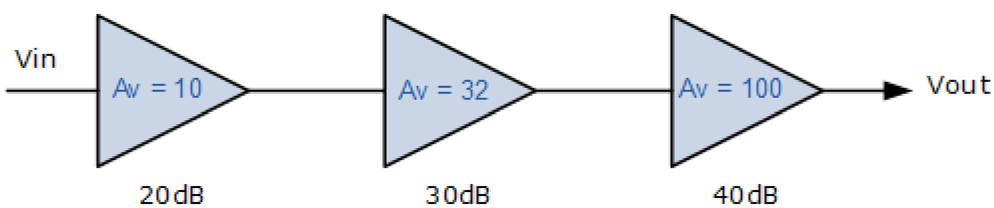
As with the passive filter, a first-order Low Pass Active Filter can be converted into a second-order low pass filter simply by using an additional RC network in the input path. The frequency response of the second-order low pass filter is identical to that of the first-order type except that the stop band roll-off will be twice the first-order filters at 40dB/decade (12dB/octave). Therefore, the design steps required of the second-order active low pass filter are the same.

Second-order Active Low Pass Filter Circuit



When cascading together filter circuits to form higher-order filters, the overall gain of the filter is equal to the product of each stage. For example, the gain of one stage may be 10 and the gain of the second stage may be 32 and the gain of a third stage may be 100. Then the overall gain will be 32,000, (10 x 32 x 100) as shown below.

Cascading Voltage Gain



$$Av = Av_1 \times Av_2 \times Av_3$$

$$Av = 10 \times 32 \times 100 = 32{,}000$$

$$Av(dB) = 20\log_{10}(32{,}000)$$

$$Av(dB) = 90dB$$

$$90dB = 20dB + 30dB + 40dB$$

Second-order (two-pole) active filters are important because higher-order filters can be designed using them. By cascading together first and second-order filters, filters with an order value, either odd or even up to any value can be constructed. In the next tutorial about filters, we will see that Active High Pass Filters, can be constructed by reversing the positions of the resistor and capacitor in the circuit.

Butterworth Filter Design

In the previous filter tutorials we looked at simple first-order type low and high pass filters that contain only one single resistor and a single reactive component (a capacitor) within their RC filter circuit design.

In applications that use filters to shape the frequency spectrum of a signal such as in communications or control systems, the shape or width of the roll-off also called the "transition band", for a simple first-order filter may be too long or wide and so active filters designed with more than one "order" are required. These types of filters are commonly known as "High-order" or "n^{th}-order" filters.

The complexity or Filter Type is defined by the filters "order", and which is dependant upon the number of reactive components such as capacitors or inductors within its design. We also know that the rate of roll-off and therefore the width of the transition band, depends upon the order number of the filter and that for a simple first-order filter it has a standard roll-off rate of 20dB/decade or 6dB/octave.

Then, for a filter that has an n^{th} number order, it will have a subsequent roll-off rate of 20n dB/decade or 6n dB/octave. So a first-order filter has a roll-off rate of 20dB/decade (6dB/octave), a second-order filter has a roll-off rate of 40dB/decade (12dB/octave), and a fourth-order filter has a roll-off rate of 80dB/decade (24dB/octave), etc, etc.

High-order filters, such as third, fourth, and fifth-order are usually formed by cascading together single first-order and second-order filters.

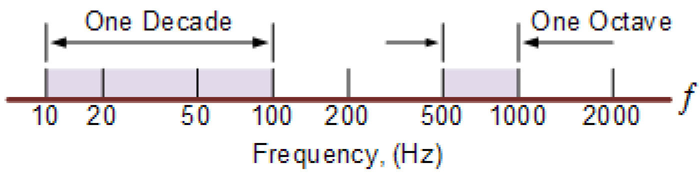
For example, two second-order low pass filters can be cascaded together to produce a fourth-order low pass filter, and so on. Although there is no limit to the order of the filter that can be formed, as the order increases so does its size and cost, also its accuracy declines.

Decades and Octaves

One final comment about *Decades* and *Octaves*. On the frequency scale, a **Decade** is a tenfold increase (multiply by 10) or tenfold decrease (divide by 10). For example, 2 to 20Hz represents one decade, whereas 50 to 5000Hz represents two decades (50 to 500Hz and then 500 to 5000Hz).

An **Octave** is a doubling (multiply by 2) or halving (divide by 2) of the frequency scale. For example, 10 to 20Hz represents one octave, while 2 to 16Hz is three octaves (2 to 4, 4 to 8 and finally 8 to 16Hz) doubling the frequency each time. Either way, *Logarithmic* scales are used extensively in the frequency domain to denote a frequency value when working with amplifiers and filters so it is important to understand them.

Logarithmic Frequency Scale



Since the frequency determining resistors are all equal, and as are the frequency determining capacitors, the cut-off or corner frequency (f_C) for either a first, second, third or even a fourth-order filter must also be equal and is found by using our now old familiar equation:

$$f_C = \frac{1}{2\pi RC} \, Hz$$

As with the first and second-order filters, the third and fourth-order high pass filters are formed by simply interchanging the positions of the frequency determining components (resistors and capacitors) in the equivalent low pass filter. High-order filters can be designed by following the procedures we saw previously in the *Low Pass* and High Pass filter tutorials. However, the overall gain of high-order filters is **fixed** because all the frequency determining components are equal.

Filter Approximations

So far we have looked at a low and high pass first-order filter circuits, their resultant frequency and phase responses. An ideal filter would give us specifications of maximum pass band gain and flatness, minimum stop band attenuation and also a very steep pass band to stop band roll-off (the transition band) and it is therefore apparent that a large number of network responses would satisfy these requirements.

Not surprisingly then that there are a number of “approximation functions” in linear analogue filter design that use a mathematical approach to best approximate the transfer function we require for the filters design.

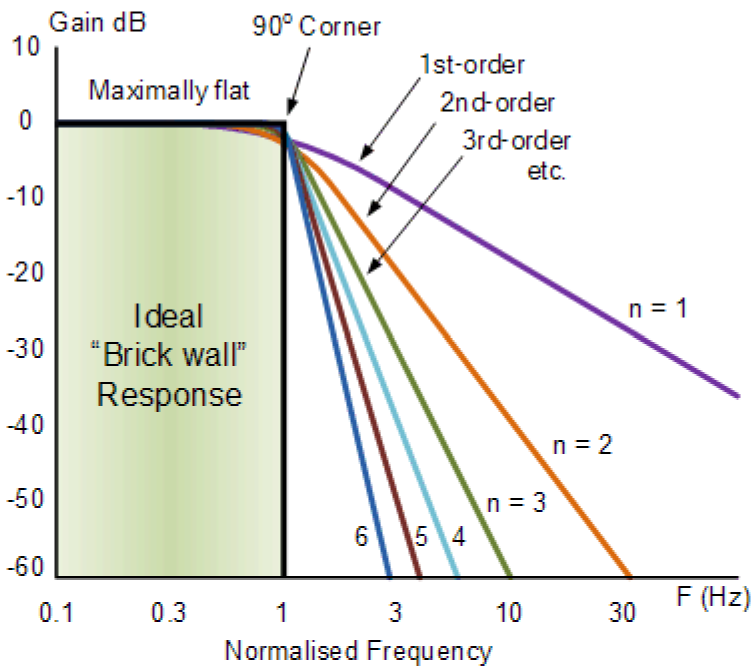
Such designs are known as **Elliptical**, **Butterworth**, **Chebyshev**, **Bessel**, **Cauer** as well as many others. Of these five “classic” linear analogue filter approximation functions only the **Butterworth Filter** and especially the *low pass Butterworth filter* design will be considered here as its the most commonly used function.

Low Pass Butterworth Filter Design

The frequency response of the **Butterworth Filter** approximation function is also often referred to as "maximally flat" (no ripples) response because the pass band is designed to have a frequency response which is as flat as mathematically possible from 0Hz (DC) until the cut-off frequency at -3dB with no ripples. Higher frequencies beyond the cut-off point rolls-off down to zero in the stop band at 20dB/decade or 6dB/octave. This is because it has a "quality factor", "Q" of just 0.707.

However, one main disadvantage of the Butterworth filter is that it achieves this pass band flatness at the expense of a wide transition band as the filter changes from the pass band to the stop band. It also has poor phase characteristics as well. The ideal frequency response, referred to as a "brick wall" filter, and the standard Butterworth approximations, for different filter orders are given below.

Ideal Frequency Response for a Butterworth Filter



Note that the higher the Butterworth filter order, the higher the number of cascaded stages there are within the filter design, and the closer the filter becomes to the ideal "brick wall" response.

In practice however, Butterworth's ideal frequency response is unattainable as it produces excessive passband ripple.

Where the generalised equation representing a "nth" Order Butterworth filter, the frequency response is given as:

$$H_{(j\omega)} = \frac{1}{\sqrt{1+\varepsilon^2\left(\frac{\omega}{\omega_p}\right)^{2n}}}$$

Where: n represents the filter order, Omega ω is equal to $2\pi f$ and Epsilon ε is the maximum pass band gain, (A_{max}). If A_{max} is defined at a frequency equal to the cut-off -3dB corner point (fc), ε will then be equal to one and therefore ε^2 will also be one. However, if you now wish to define A_{max} at a different voltage gain value, for example 1dB, or 1.1220 (1dB = 20logA_{max}) then the new value of epsilon, ε is found by:

$$H_1 = \frac{H_0}{\sqrt{1+\varepsilon^2}}$$

- Where:
- H_0 = the Maximum Pass band Gain, A_{max}.
- H_1 = the Minimum Pass band Gain.

Transpose the equation to give:

$$\frac{H_0}{H_1} = 1.1220 = \sqrt{1 + \varepsilon^2} \quad gives \quad \varepsilon = 0.5088$$

The **Frequency Response** of a filter can be defined mathematically by its **Transfer Function** with the standard Voltage Transfer Function H(jω) written as:

$$H(j\omega) = \left[\frac{Vout(j\omega)}{Vin(j\omega)}\right]$$

- Where:
- Vout = the output signal voltage.
- Vin = the input signal voltage.
- j = to the square root of -1 (√-1)
- ω = the radian frequency ($2\pi f$)

Note: (jω) can also be written as (s) to denote the **S-domain.** and the resultant transfer function for a second-order low pass filter is given as:

$$H(s) = \frac{Vout}{Vin} = \frac{1}{S^2 + S + 1}$$

Normalised Low Pass Butterworth Filter Polynomials

To help in the design of his low pass filters, Butterworth produced standard tables of normalised second-order low pass polynomials given the values of coefficient that correspond to a cut-off corner frequency of 1 radian/sec.

n	**Normalised Denominator Polynomials in Factored Form**
1	$(1+s)$
2	$(1+1.414s+s^2)$
3	$(1+s)(1+s+s^2)$
4	$(1+0.765s+s^2)(1+1.848s+s^2)$
5	$(1+s)(1+0.618s+s^2)(1+1.618s+s^2)$
6	$(1+0.518s+s^2)(1+1.414s+s^2)(1+1.932s+s^2)$
7	$(1+s)(1+0.445s+s^2)(1+1.247s+s^2)(1+1.802s+s^2)$
8	$(1+0.390s+s^2)(1+1.111s+s^2)(1+1.663s+s^2)(1+1.962s+s^2)$
9	$(1+s)(1+0.347s+s^2)(1+s+s^2)(1+1.532s+s^2)(1+1.879s+s^2)$
10	$(1+0.313s+s^2)(1+0.908s+s^2)(1+1.414s+s^2)(1+1.782s+s^2)(1+1.975s+s^2)$

Filter Design – Butterworth Low Pass

Find the order of an active low pass Butterworth filter whose specifications are given as: A_{max} = 0.5dB at a pass band frequency (ωp) of 200 radian/sec (31.8Hz), and A_{min} = -20dB at a stop band frequency (ωs) of 800 radian/sec. Also design a suitable Butterworth filter circuit to match these requirements.

Firstly, the maximum pass band gain A_{max} = 0.5dB which is equal to a gain of **1.0593** (0.5dB = 20log A) at a frequency (ωp) of 200 rads/s, so the value of epsilon ε is found by:

$$1.0593 = \sqrt{1 + \varepsilon^2}$$

$$\therefore \ \varepsilon = 0.3495 \ \ and \ \ \varepsilon^2 = 0.1221$$

Secondly, the minimum stop band gain A_{min} = -20dB which is equal to a gain of **-10** (20dB = 20log A) at a stop band frequency (ωs) of 800 rads/s or 127.3Hz.

Substituting the values into the general equation for a Butterworth filters frequency response gives us the following:

$$H(j\omega) = \frac{H_0}{\sqrt{1 + \varepsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2n}}}$$

$$\frac{1}{10} = \frac{1}{\sqrt{1 \;+\; 0.1221 \left(\frac{800}{200}\right)^{2n}}}$$

$$\therefore\ 100 \;=\; 1 + 0.1221 \;x\; 4^{2n}$$

$$4^{2n} \;=\; \frac{99}{0.1221} \;=\; 810.811$$

$$4^{n} \;=\; \sqrt{810.811} \;=\; 28.475$$

$$\therefore\ n \;=\; \frac{\log 28.475}{\log 4} \;=\; 2.42$$

Since n must always be an integer (whole number) then the next highest value to 2.42 is n = 3, therefore a **“a third-order filter is required”** and to produce a third-order **Butterworth filter**, a second-order filter stage cascaded together with a first-order filter stage is required.

From the normalised low pass Butterworth Polynomials table above, the coefficient for a third-order filter is given as $(1+s)(1+s+s^2)$ and this gives us a gain of 3-A = 1, or A = 2. As A = 1 + (Rf/R1), choosing a value for both the feedback resistor Rf and resistor R1 gives us values of 1kΩ and 1kΩ respectively, (1kΩ/1kΩ + 1 = 2).

We know that the cut-off corner frequency, the -3dB point (ω_o) can be found using the formula 1/CR, but we need to find ω_o from the pass band frequency ω_p then,

$$H(j\omega) = \frac{H_0}{\sqrt{1 + \varepsilon^2 \left(\frac{\omega_O}{\omega_P}\right)^{2n}}}$$

$$3dB \;=\; 1.414 \; at \;\; \omega = \omega_O$$

$$\frac{1}{1.414} = \frac{1}{\sqrt{1 + \varepsilon^2 \left(\frac{\omega_O}{\omega_P}\right)^{2n}}}$$

$$2 \;=\; 1 + \varepsilon^2 \left(\frac{\omega_O}{\omega_P}\right)^{2n}$$

$$\therefore 1 \;=\; \varepsilon \left(\frac{\omega_O}{\omega_P}\right)^{n}$$

$$\omega_O{}^n \;=\; \frac{\omega_P{}^n}{\varepsilon}$$

$$\omega_O{}^3 \;=\; \frac{200^3}{0.3495}$$

$$\omega_O{}^3 \;=\; 22.889 \; x \; 10^6$$

$$\therefore \omega_O \;=\; 283.93 \;\simeq\; 284 \, rads/s$$

So, the cut-off corner frequency is given as 284 rads/s or 45.2Hz, (284/2π) and using the familiar formula 1/CR we can find the values of the resistors and capacitors for our third-order circuit.

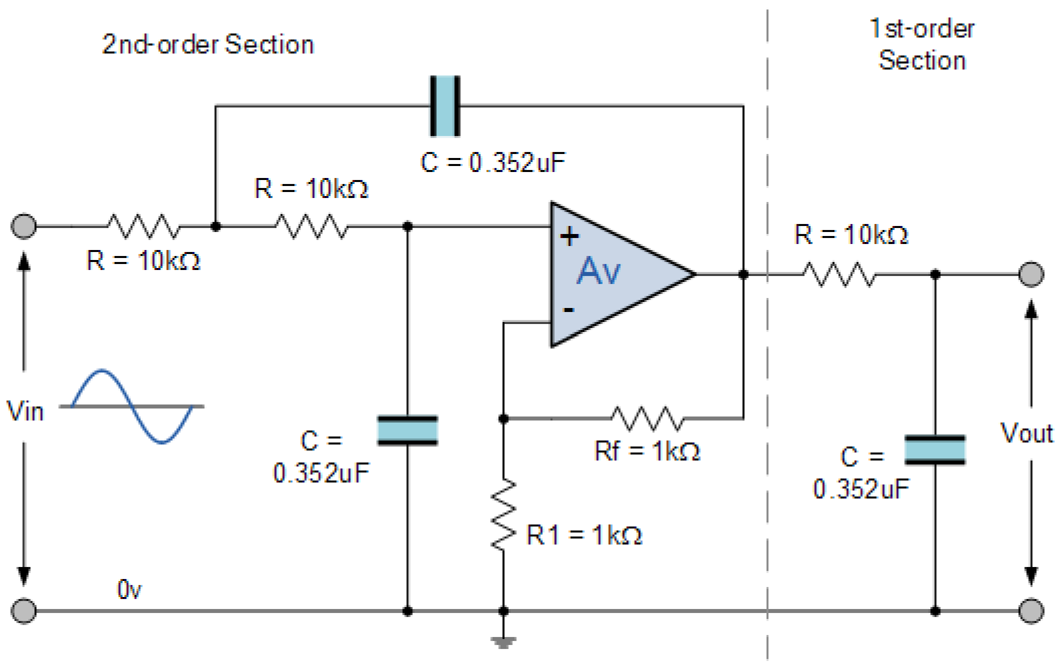
$$284 rads/s = \frac{1}{CR} \quad use\ a\ value\ of\ R = 10k\Omega$$

$$\therefore Capacitor\ C = \frac{1}{284 x 10,000} = 0.352 uF$$

Note that the nearest preferred value to 0.352uF would be 0.36uF, or 360nF.

Third-order Butterworth Low Pass Filter

and finally our circuit of the third-order low pass **Butterworth Filter** with a cut-off corner frequency of 284 rads/s or 45.2Hz, a maximum pass band gain of 0.5dB and a minimum stop band gain of 20dB is constructed as follows.

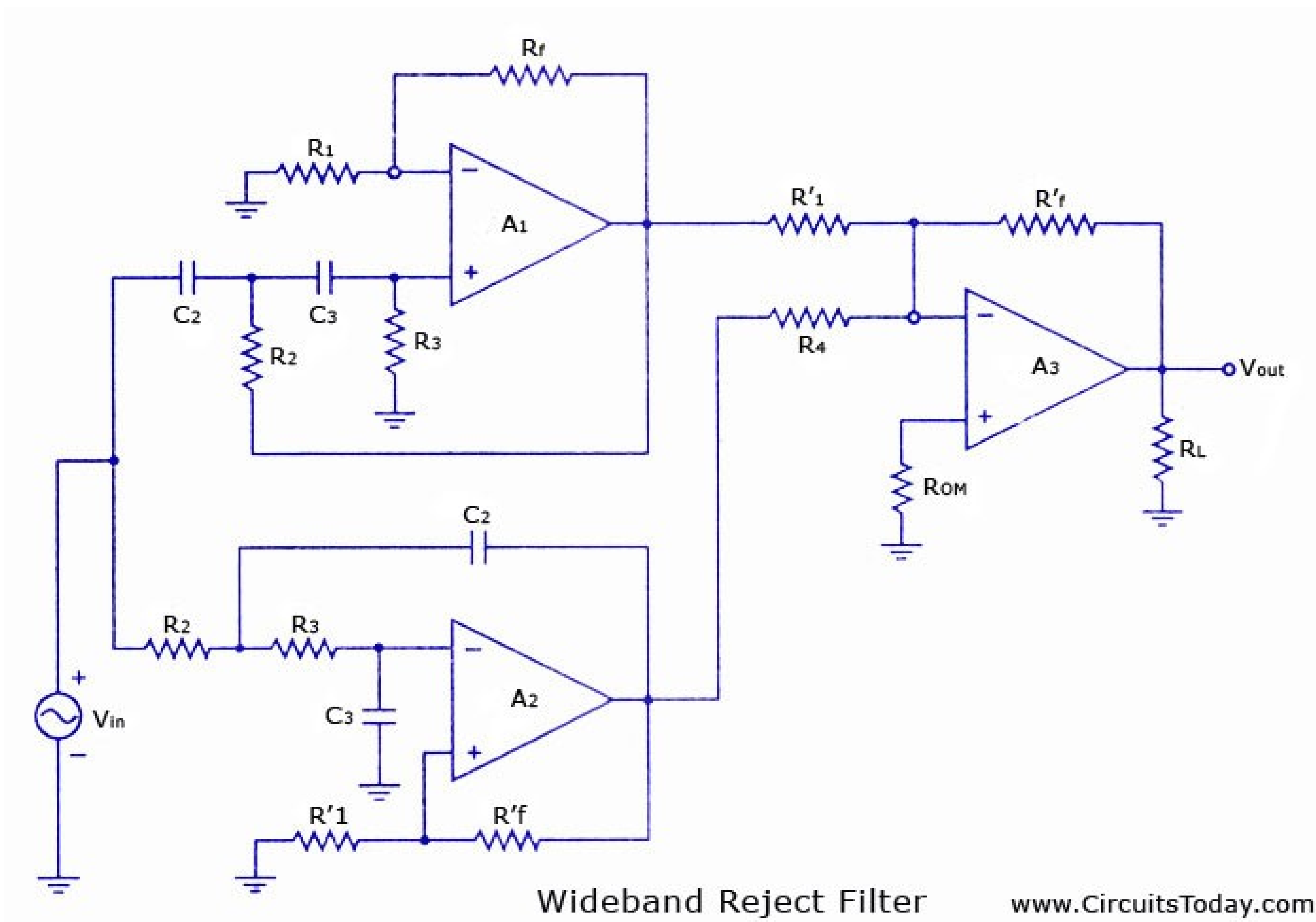


The bandpass filter passes one set of frequencies while rejecting all others. The band-stop filter does just the opposite. It rejects a band of frequencies, while passing all others. This is also called a band-reject or band-elimination filter. Like bandpass filters, band-stop filters

may also be classified as (i) wide-band and (ii) narrow band reject filters.

The narrow band reject filter is also called a notch filter. Because of its higher Q, which exceeds 10, the bandwidth of the narrow band reject filter is much smaller than that of a wide band reject filter.

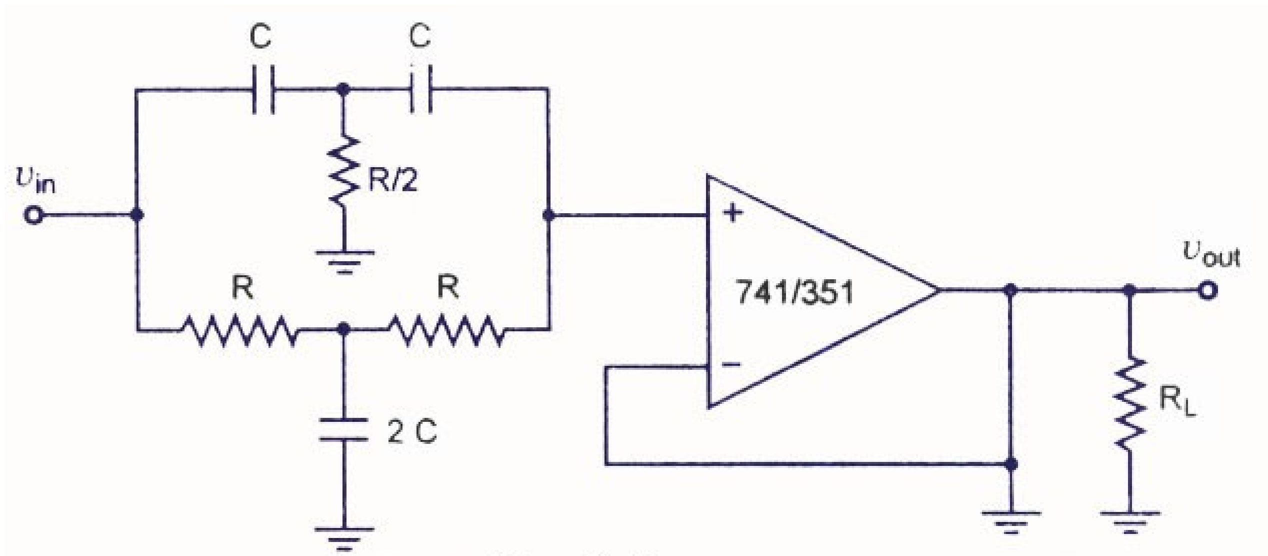
Wide Band-Stop (or Reject) Filter.



Wide band stop filter

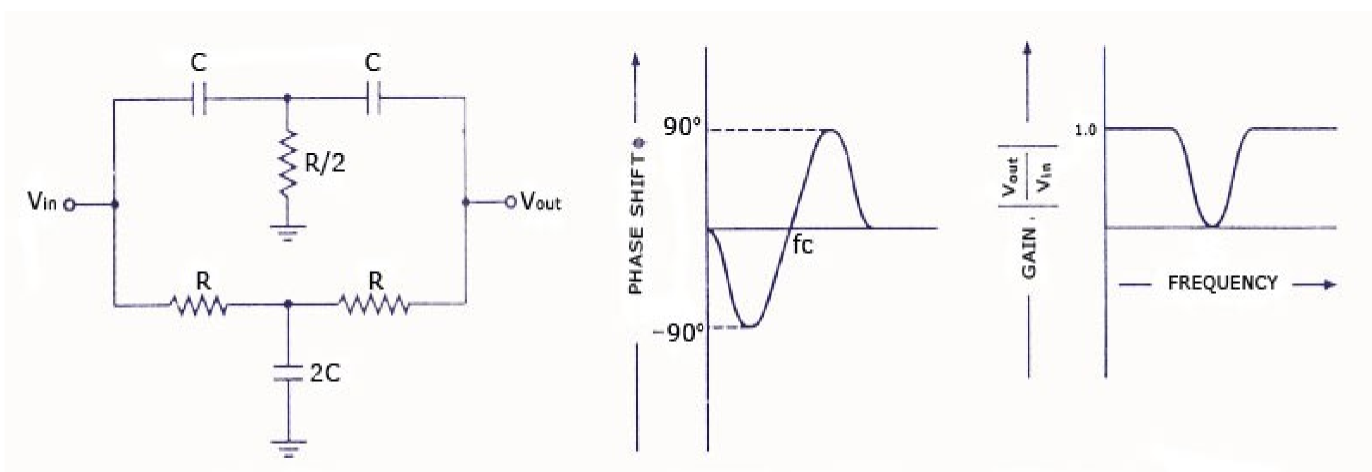
A wide band-stop filter using a low-pass filter, a high-pass filter and a summing amplifier is shown in figure. For a proper band reject response, the low cut-off frequency f_L of high-pass filter must be larger than the high cut-off frequency f_H of the low-pass filter. In addition, the passband gain of both the high-pass and low-pass sections must be equal.

Narrow Band-Stop Filter.



Circuit Diagram

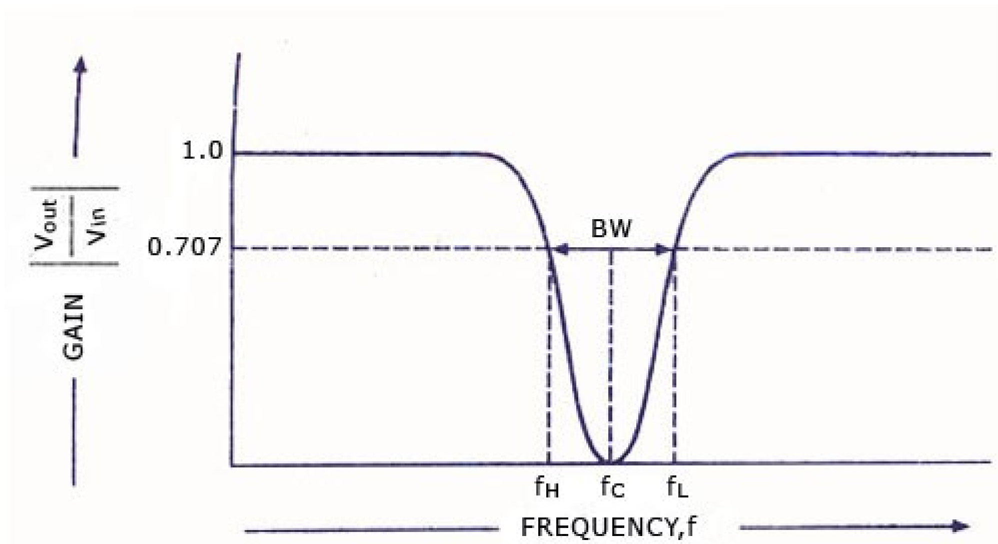
Twin T active notch filter



(a) Circuit Diagram (b) Phase Shift (c) Voltage Gain

Twin T Notch Passive Filter

This is also called a notch filter. It is commonly used for attenuation of a single frequency such as 60 Hz power line frequency hum. The most widely used notch filter is the twin-T network illustrated in fig. (a). This is a passive filter composed of two T-shaped networks. One T-network is made up of two resistors and a capacitor, while the other is made of two capacitors and a resistor.One drawback *of* above notch filter (passive twin-T network) is that it has relatively low figure of merit Q. However, Q of the network can be increased significantly if it is used with the voltage follower, as illustrated in fig. (a). Here the output of the voltage follower is supplied back to the junction of R/2 and 2 C. The frequency response of the active notch filter is shown in fig *(b).*



Frequency Response

Twin T Active Notch Filter

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Notch filters are most commonly used in communications and biomedical instruments for eliminating the undesired frequencies.

A mathematical analysis of this circuit shows that it acts as a lead-lag circuit with a phase angle, shown in fig. (b). Again, there is a frequency f_c at which the phase shift is equal to 0°. In fig. (c), the voltage gain is equal to 1 at low and high frequencies. In between, there is a frequency f_c at which voltage gain drops to zero. Thus such a filter notches out, or blocks frequencies near f_c. The frequency at which maximum attenuation occurs is called the notch-out frequency given by

$f_n = F_c = 2\pi RC$

Notice that two upper capacitors are C while the capacitor in the centre of the network is 2 C. Similarly, the two lower resistors are R but the resistor in the centre of the network is 1/2 R. This relationship must always be maintained.