Proof Assignment for Test 2

$$\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$$

Proof:

Let the statement above be $P(n)$.

Base Case: P(0) = ($2^0 = 2^1 - 1$) => ($1 = 1$) is true.

Inductive Step: Suppose $P(n)$ has already been verified for some integer $n \geq 0$

$$\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$$

Assume for n=p, it is true then, ------(1)

$$\sum_{k=0}^{n} 2^k = 2^{p+1} - 1$$

We now show that P(n) is true for n=p+1 and simplify:

$$\sum_{k=0}^{n+1} 2^k = \sum_{k=0}^{n} 2^k + 2^{p+1} = 2^{p+1} - 1 + 2^{p+1}$$

$$= 2^{2p+2} - 1 = 2^{((p+1)+(p+1))} - 1 = R.H.S$$

Therefore we have shown that P(n+1) :

$$\sum_{k=0}^{n+1} 2^k = 2^{((p+1)+(p+1))} - 1$$