1

- (a) Two propositions are logically equivalent if and only if they have identical truth values for each possible substitutions of propositions for their proposition variables.
- (b) A tautology is a proposition that is always true.
- (c) The negation of "if p then q" is "p and not q".
- (d) "r is a sufficient condition for s" means if r, then s.
- (e) "a is a necessary condition for b" means if b, then a.

2

- (a) If I go to the beach or I go out dancing, then I am done with my work.
- (b) If I don't go to the beach or I don't go out dancing, then I am not done with my work.
- (c) If I am not done with my work, I neither go to the beach nor do I go out dancing.

3

(a) P(a, b) means a likes b, and the universe of discourse for x, y, and z is the set of all people.

$$\exists x \exists y \exists z (P(x,y) \land (x \neq z) \land P(y,z))$$

(b) Universe of discourse: the n in the set of all integers Z

$$\exists c \forall n, c \leq n$$

4

- (a) If you finish your salad, then you can have your ice cream.
- (b) If x is irrational, then $\frac{1}{x}$ is irrational.
- (c) If I get an A in my English class, then I get an A on the final.

5

$$\neg(\forall x \exists y (y > 0 \to (-2 < x \le 6)))$$

$$\exists x \neg (\exists y \big(y > 0 \rightarrow (-2 < x \leq 6) \big))$$

Hieu Pham

$$\exists x \forall y \neg (y > 0 \rightarrow (-2 < x \le 6))$$

$$\exists x \forall y (\neg(y > 0) \rightarrow \neg(-2 < x \le 6))$$

$$\exists x \forall y ((y \le 0) \to (-2 \ge x < 6))$$