Examples with a single quantifier

- Let the universe of discourse be all the ASU students.
- S(x): x is an ASU student who is taking MAT 243.
- M(x): x has been in Mexico.

Write the following sentences in symbolic form using quantifiers and the given predicates.

1. There is a MAT 243 student who has been in Mexico.

$$\exists x(S(x) \wedge M(x))$$

Why not

$$\exists x(S(x) \rightarrow M(x))$$

This statement would be true if there is an ASU student who is not taking MAT 243 even if there is no MAT 243 student who has been in Mexico.

2. Every MAT 243 student has been in Mexico.

$$\forall x(S(x) \rightarrow M(x))$$

Why not

$$\forall x(S(x) \wedge M(x))$$

This statement means that every ASU student is taking MAT 243 and has been in Mexico.

Negation of quantified statements

Write the negation of the following sentences in symbolic form using quantifiers and the given predicates.

3. There is a MAT 243 student who has been in Mexico

$$\exists x(S(x) \land M(x))$$

Negation: There is no MAT 243 student who has been in Mexico.

All MAT 243 students have not been in Mexico.

$$\neg \exists x(S(x) \land M(x)) \equiv \forall x \, \neg(S(x) \land M(x)) \equiv \forall x \, (\neg S(x) \lor \neg M(x)) \equiv$$

$$\boxed{\forall x \, (S(x) \rightarrow \neg M(x))}$$

Write the negation of the following sentences in symbolic form using quantifiers and the given predicates.

4. Every MAT 243 student has been in Mexico

$$\forall x(S(x) \rightarrow M(x))$$

Negation: Not every MAT 243 student has been in Mexico.

There is a MAT 243 students have not been in Mexico.

$$\neg \forall x(S(x) \rightarrow M(x)) \equiv \exists x \neg(S(x) \rightarrow M(x)) \equiv \exists x \neg(\neg S(x) \lor M(x)) \equiv$$

$$\boxed{\exists x\ (S(x) \land \neg M(x))}$$