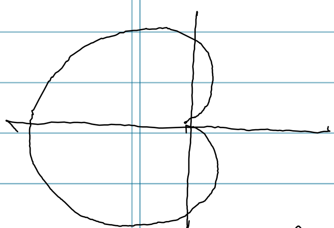
Example: Find the points on the curve
$x = 4(\cos\theta - \cos^2\theta)$, $y = 4(\sin\theta - \sin\theta\cos\theta)$
$0 \le \theta < 2\pi$, where the tangent line is horizontal.

Solution.



$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{4[\cos\theta - (\cos^2\theta - \sin^2\theta)]}{4[-\sin\theta + 2\cos\theta\sin\theta]}$$

$$\frac{dy}{d\theta} = 0; \quad 4[\cos\theta - (\cos^2\theta - \sin^2\theta)] = 0$$

$$-\cos^2\theta + \cos\theta + \sin^2\theta = 0$$

$$(2\cos\theta + 1)(\cos\theta - 1) = 0$$

$$\cos\theta = -\frac{1}{2} \quad \text{or} \quad \cos\theta = 1.$$

$$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, 0.$$

$\frac{dx}{d\theta} \neq 0$ when $\theta = \frac{2\pi}{3}$ & $\frac{4\pi}{3}$.

Tangent is horizontal at $\theta = \frac{2\pi}{3}$ & $\theta = \frac{4\pi}{3}$.

at $\theta = 0$; $\frac{dy}{d\theta} = \frac{dx}{d\theta} = 0$.

$$\lim_{\theta \to 0^+} \frac{dy}{dx} = \lim_{\theta \to 0^+} \frac{4[\cos\theta - (\cos^2\theta - \sin^2\theta)]}{4[-\sin\theta + 2\cos\theta\sin\theta]} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{\theta \to 0^+} \frac{-\sin\theta + 4\cos\theta\sin\theta}{-\cos\theta + 2(\cos^2\theta - \sin^2\theta)}$$

$$= 0$$

Similarly,

$$\lim_{\theta \to 0^-} \frac{dy}{dx} = 0.$$

So, $\frac{dy}{dx} = 0$, at $\theta = 0$.

θ	x	y
0	0	0
$2\pi/3$	-4	$3\sqrt{3}$
$4\pi/3$	-4	$-3\sqrt{3}$