MAT 266 Test 2 Review

Approximate Integration, Applications of Integrals, Sequences and Series; sections 6.5, 7.1-7.4, 7.6, 8.1, 8.2 in *Essential Calculus, Early Transcendentals*, 2nd Edition, by James Stewart.

1) Estimate $\int_1^2 \frac{1}{x}dx$ using (write all answers to five decimal places)
   a. The trapezoid rule with $n = 5$.
   b. The midpoint rule with $n = 5$.
   c. Simpson's rule with $n = 4$.

2) Estimate the average temperature, T_{ave}, for one full day with the given data:
   $\left(\text{recall } f_{ave} = \frac{1}{b-a}\int_a^b f(t)dt\right)$

	12:00am	4:00am	8:00am	12:00pm	4:00pm	8:00pm	12:00am
t	0	4	8	12	16	20	24
T	68°F	64°F	69°F	75°F	82°F	76°F	70°F

   a) Using the trapezoid rule.
   b) Using Simpson's rule (round to two decimal places).

3) Find the area of the specified region **exactly** by sketching the region, labeling the intersection points, clearly showing the integral(s) as well as the antiderivatives.
   a. $y = x$ and $y = \frac{1}{9}x^2$
   b. $x = y^2$ and $x = y + 2$
   c. $x = y^2$ and $x = -2y^2 + 3$
   d. $y = \sqrt{x}$, $y = 2x - 6$, and $y = 0$
   e. $y = \sin\left(\frac{\pi x}{2}\right)$ and $y = x^2$

4) Set up the integral and find the volume of the bounded specified region with the **washer/disc** method. Include limits of integration. Include a sketch.
   a. $y = \frac{1}{\sqrt{1+x^2}}$, $x = 0$, and $x = 1$, rotated about the x-axis.
   b. $y = 2\sqrt{x}$, $x = 0$, and $y = 10$, rotated about the x-axis.
   c. Bounded below by $y = 3x^2 + 1$ and above by $y = 4$, rotated about the line $y = 4$.
   d. $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 3$ rotated about the line $y = -1$.

5) Set up the integral to find the volume of the bounded specified region with the **washer/disc** method. Include limits of integration. Include a sketch. **Do not evaluate.**
   a. $y = \sqrt[3]{x}$, $x = 4y$ in Quadrant I, rotated about the y-axis.
   b. $y = e^x$, $y = 10$, and $x = 0$ about the line $y = 12$.

6) Set up the integral and find the volume of the bounded specified region with the **cylindrical shells** method. Include limits of integration. Include a sketch.

  a. $y = \frac{1}{\sqrt{1+x^2}}$, $x = 0$, and $x = 1$, rotated about the y-axis.

  b. $y = x^3$, $y = 8$, and $x = 0$, about the y-axis.

  c. $y = 4 - x^2, y = 3x^2, x = 0$ in Quadrant I, rotated about the line $x = 2$.

7) Set up the integral to find the volume of the bounded specified region with the **cylindrical shells** method. Include limits of integration. Include a sketch. **Do not evaluate.**

  a. $y = \sqrt{x}, y = \frac{x}{2}, x = 0$, rotated about the line $x = -1$.

  b. $y = \sin(x), y = 0, x = \frac{\pi}{2}$, rotated about the line $x = \frac{\pi}{2}$.

8) For each curve, find the arc length on the given interval. Show the integral(s) as well as the antiderivatives.

  a. $y = \frac{4\sqrt{2}\,x^{3/2}}{3} - 1$ from $x = 0$ to $x = 1$.

  b. $y = \frac{x^3}{3} + \frac{1}{4x}$ from $x = 1$ to $x = 3$.

  c. $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$.

  d. $x = \frac{1}{2}(e^y + e^{-y})$ from $y = 0$ to $y = 2$.

9) For each curve, set up the integral to find the arc length on the given interval.

  a. $y = e^{x/3}$ from $x = 0$ to $x = 3$.

  b. $y = \cos(\pi x)$ from $x = 0$ to $x = 2$.

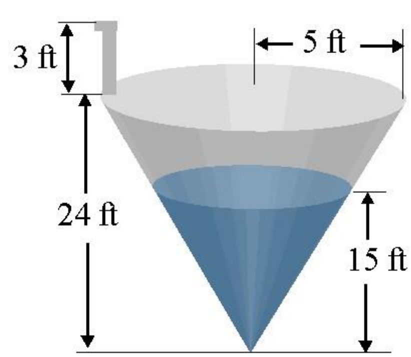
10) A spring has a natural length of 0.2m. If the force required to keep it stretched to a length of 0.6m is 6 Newtons, how much work is required to stretch it from 0.2m to 0.4m? Clearly show the integral(s) as well as the antiderivative.

11) An aquarium is 2m long, 1m wide and 1m deep. It is full of water. Find the work needed to pump out half of the water from the aquarium. [Use the fact that the density of water is 1000kg/m^3]

12) A circular swimming pool has a diameter of 24ft. The sides are 5ft high and the depth of the water is 4ft. How much work is required to pump all of the water over the side. Water weighs 62.5 lbs/ft^3

13) A spherical tank of radius 10 feet is filled with water. Find the work done in pumping all of the water through the top. The weight of the water is 62.5 lbs/ft^3

14) The tank shown contains water to a depth of 15 ft. Find the work required to pump the water out of the spout (the spout is 3 ft high). [Use 62.5 lb/ft^3 as the weight density of the water].

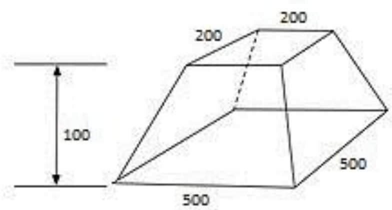


15) Calculate the work required to lift a 10 m chain over the side of a building, assuming that the chain has a density of 8 kg/m.

16) A tank has the shape of a right circular cylinder with height 10 m and radius 4 m. It is filled to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank (the density of water is 1000 kg/m^3).

17) Set up an integral for the volume, in cubic feet, of a frustum of a pyramid with a square base side of 500 feet, square top side of 200 feet, and whose height is 100 feet.

Figure for problems 17 and 18



18) Set up an integral for the work, in foot-pounds, to empty a tank in the shape of a frustum of a pyramid with a square base side of 500 feet, square top side of 200 feet, and whose height is 100 feet, if the tank is initially filled to a depth of 70 feet with water (density ρ = 62.5 lbs/ft^3).

19) The base of a solid S is the parabolic region $\{(x,y)|x^2 \leq y \leq 4\}$. Cross sections perpendicular to the y-axis are squares. Find the volume of S.

20) The base of a solid S is the parabolic region $\{(x,y)|x^2 \leq y \leq 4\}$. Cross sections perpendicular to the x-axis are squares. Find the volume of S.

21) Determine whether the sequence converges or diverges. If it converges, find the limit.

a. $a_n = e^{1/n}$ b. $a_n = \sqrt[n]{3^{2n+1}}$ c. $\left\{\frac{3n}{2n+3}\right\}_{n=1}^{\infty}$

d. $\left\{\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \frac{5}{6}, \dots\right\}$ e. $a_n = \left(1+\frac{1}{n}\right)^n$

22) Determine whether the series converges or diverges. If it converges, find the limit.

a. $\sum_{n=3}^{\infty} \frac{1}{4^n}$ b. $\sum_{k=1}^{\infty} \frac{(-3)^{k+1}}{4^{2k}}$

c. $\sum_{n=1}^{\infty} \frac{2n}{3n+5}$ d. $8 - 6 + \frac{9}{2} - \frac{27}{8} + \cdots$

e. $\sum_{n=1}^{\infty} \cos(n)$ f. $\sum_{k=1}^{\infty} \frac{4^{2k}}{15^k}$

Answers

1) (a) 0.69563 (b) 0.69191 (c) 0.69325

2) (a) 72.5°F (b) 72.22°F

3) (a) $\frac{27}{2}$ (b) $\frac{9}{2}$ (c) 4 (d) $\frac{13}{3}$ (e) $\frac{2}{\pi} - \frac{1}{3}$ (area answers are in square units)

4) (a) $\frac{\pi^2}{4}$ (b) 1250π (c) $\frac{48\pi}{5}$ (d) $2\pi\left(\ln(3) + \frac{1}{3}\right)$ (volume answers are in cubic units)

5) (a) $\pi\int_0^2 (16y^2 - y^6)dy$ (b) $\pi\int_0^{\ln(10)} [(12 - e^x)^2 - 4]dx$

6) (a) $2\pi\left(\sqrt{2} - 1\right)$ cubic units (b) $\frac{96\pi}{5}$ cubic units (c) $\frac{26\pi}{3}$ cubic units

7) (a) $2\pi\int_0^4 (x+1)\left(\sqrt{x} - \frac{x}{2}\right)dx$ (b) $2\pi\int_0^{\pi/2}\left(\frac{\pi}{2} - x\right)\sin(x)\,dx$

8) (a) $\frac{13}{6}$ (b) $\frac{53}{6}$ (c) 12 (d) $\frac{1}{2}(e^2 - e^{-2})$ (length answers are in units)

9) (a) $\int_0^3 \sqrt{1 + \frac{1}{9}e^{2x/3}}\; dx = \frac{1}{3}\int_0^3 \sqrt{9 + e^{2x/3}}\; dx$ (b) $\int_0^2 \sqrt{1 + \pi^2\sin^2(\pi x)}\; dx$

10) 0.3 Newton-meters (Joules)

11) 2450 Joules

12) 108,000π ft-lbs

13) $\frac{2{,}500{,}000\pi}{3}$ ft-lbs

14) $\cong 151001$ ft-lbs

15) 3920 Joules

16) 7,526,400π Joules

17) $\int_0^{100} (-3x + 500)^2\,dx$ or $\int_0^{100} (3x + 200)^2\,dx$

18) $62.5\int_0^{70} (100 - x)(-3x + 500)^2\,dx$ or $62.5\int_{30}^{100} x(3x + 200)^2\,dx$

19) 32 cubic units

20) $\frac{512}{15}$ cubic units

21) (a) 1 (b) 9 (c) $\frac{3}{2}$ (d) does not exist (e) e

22) (a) $\frac{1}{48}$ (b) $\frac{9}{19}$ (c) diverges

(d) $\frac{32}{7}$ (e) diverges (f) diverges