1

(a) In an inductive proof verifying the condition for n = 1 (or the lowest possible value) is called the Base Case.

(b) In the induction step first we assume P(n), called the inductive hypothesis for some n ≥ 1,

(c) and then we show that P(n + 1) is true as well.

(d) A recurrence relation is an equation that recursively defines a sequence of elements.

2

Base case:

$$P(1): \left(\sum_{k=1}^{1}(4[1]-3) = 2[1]^2 - [1]\right)$$

$$(1 = 1) \;\; \rightarrow \;\; P(1)\; is\; true\; for\; n = 1.$$

Inductive step:

Let's assume that $\exists n, P(n)$

$$\sum_{k=1}^{n}(4k-3) = 2n^2 - n$$

Then,

$$\sum_{k=1}^{n+1}(4k-3) = \sum_{k=1}^{n}(4k-3) + [4(n+1)-3]$$
$$= \sum_{k=1}^{n}(4k-3) + 4n + 1$$

By inductive hypothesis,

$$= [2n^2 - n] + 4n + 1 = 2n^2 - n + 4n + 1 = 2(n+1)^2 - (n+1)$$

Thus, $\forall n(P(n) \rightarrow P(n+1))$ Q.E.D

3

Base case:

$$P(7)\text{: } (3^7 = 2187, 7! = 5040,\ \ 3^7 < \ 7!)$$

Inductive Step:

Let's assume that $\exists n\{n \in \mathbb{Z} |\ \ 3^n < \ n!\}$

$$P(n+1) = (3^{n+1} < (n+1)!) \ \ \textit{by the inductive hypothesis}$$

$$(3^{n+1} = [3^n \times 3])$$

$$(3^n \times 3) < (n! \times 3) \quad \textit{since } n \geq 7$$

$$< (n+1) \times n! = (n+1)!$$

$$\therefore \forall n\{n \in \mathbb{Z} | n \geq 7, \left(P(n) \rightarrow P(n+1)\right)\}$$

Q.E.D

4

Base case:

$$P(1) = (6|(9^1 - 3^1))$$

Inductive step:

Let's assume that $\exists n, P(n)$, or $(6|(9^n - 3^n))$ for some n

Thus, by definition, $\exists p | 9^n - 3^n = 6p$

Then,

$$9^{n+1} - 3^{n+1} = [(9 \times 9^n) - (3 \times 3^n)]$$

$$= [(6+3) \times 9^n] - [3 \times 3^n] = (6 \times 9^n) + [(3 \times 9^n) - (3 \times 3^n)]$$

$$= (6 \times 9^n) + 3(9^n - 3^n)]$$

$$= (6 \times 9^n) + (3 \times 6p)$$

$$= 6(9^n + 3p)$$

Since n *and* p are integers, $q = 9^n + 3p$ is also an integer.

$$\therefore 9^{n+1} - 3^{n+1} = 6q$$

$\forall n(P(n) \rightarrow P(n+1))$ Q.E.D

5

Part A:

$S = \{\cdots, -8, -4, 0, 4, 8, \cdots\} = \{4n | n \in \mathbb{Z}\}$

Part B:

$1 \in S$

$4n \in S \; if \; n \in S$

Part C:

$1 \in S$

$2 \in S$

$x + 3 \in S \; if \; x \in S$

6

Part A:

S = {1, 00, 01, 10, 11,000, 010, 0000, 0101, 0110, 1010, 1111}

Part B:

I am unclear about proving with Structural Induction for this problem.

7

Part A:

$x^2 = 6x - 9$

$x^2 - 6x + 9 = 0$ (Characteristic equation.)

$(x_1 - 3)(x_2 - 3) = 0$

$x_1 = 3, x_2 = 3$ (Repeated roots.)

$a_n = \alpha_1(x_1)^2 + \alpha_2 n(x_2)^2$

$\quad = \alpha_1(3)^2 + \alpha_2 n(3)^2$ (General form due to repeated roots.)

$a_0 = \alpha_1(3)^0 + \alpha_2 0(3)^0 \rightarrow 4 = \alpha_1 + 0$ (Initial condition.)

$a_1 = \alpha_1(3)^1 + \alpha_2 1(3)^1 \rightarrow 6 = 3\alpha_1 + 3\alpha_2$ (Initial condition.)

$\alpha_1 = 4,\ \alpha_2 = -2$ (Solved based on initial conditions.)

$a_n = 4 \cdot 3^n - 2n \cdot 3^n$ (Closed form representation.)

Part B:

$x^2 = 4x + 5$

$x^2 - 4x - 5 = 0$ (Characteristic equation.)

$(x_1 + 1)(x_2 - 5) = 0$

$x_1 = -1, x_2 = 5$ (Solved for x_1 and x_2 .)

$a_n = \alpha_1(x_1)^2 + \alpha_2(x_2)^2$

$a_n = \alpha_1(-1)^2 + \alpha_2(5)^2$ (General form.)

$a_0 = \alpha_1(-1)^0 + \alpha_2(5)^0 \rightarrow 2 = \alpha_1 + \alpha_2$ (Initial condition.)

$a_1 = \alpha_1(-1)^1 + \alpha_2(5)^1 \rightarrow 8 = -\alpha_1 + 5\alpha_2$ (Initial condition.)

$\alpha_1 = \frac{1}{3}\ ,\ \alpha_2 = \frac{5}{3}$ (Solved based on initial conditions.)

$a_n = \frac{1}{3} \cdot (-1)^n + \frac{5}{3} \cdot 5^n$ (Closed form representation.)