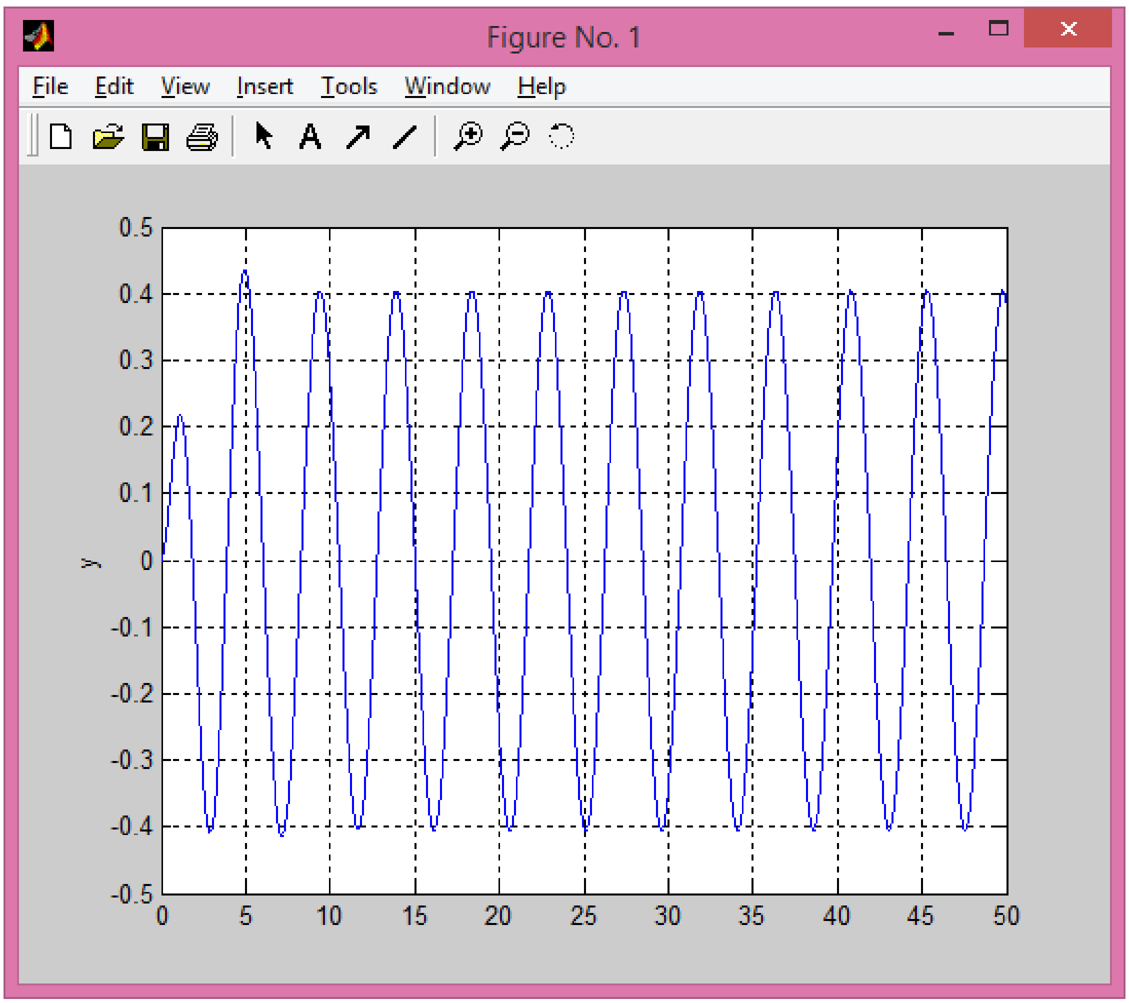
Lab 6

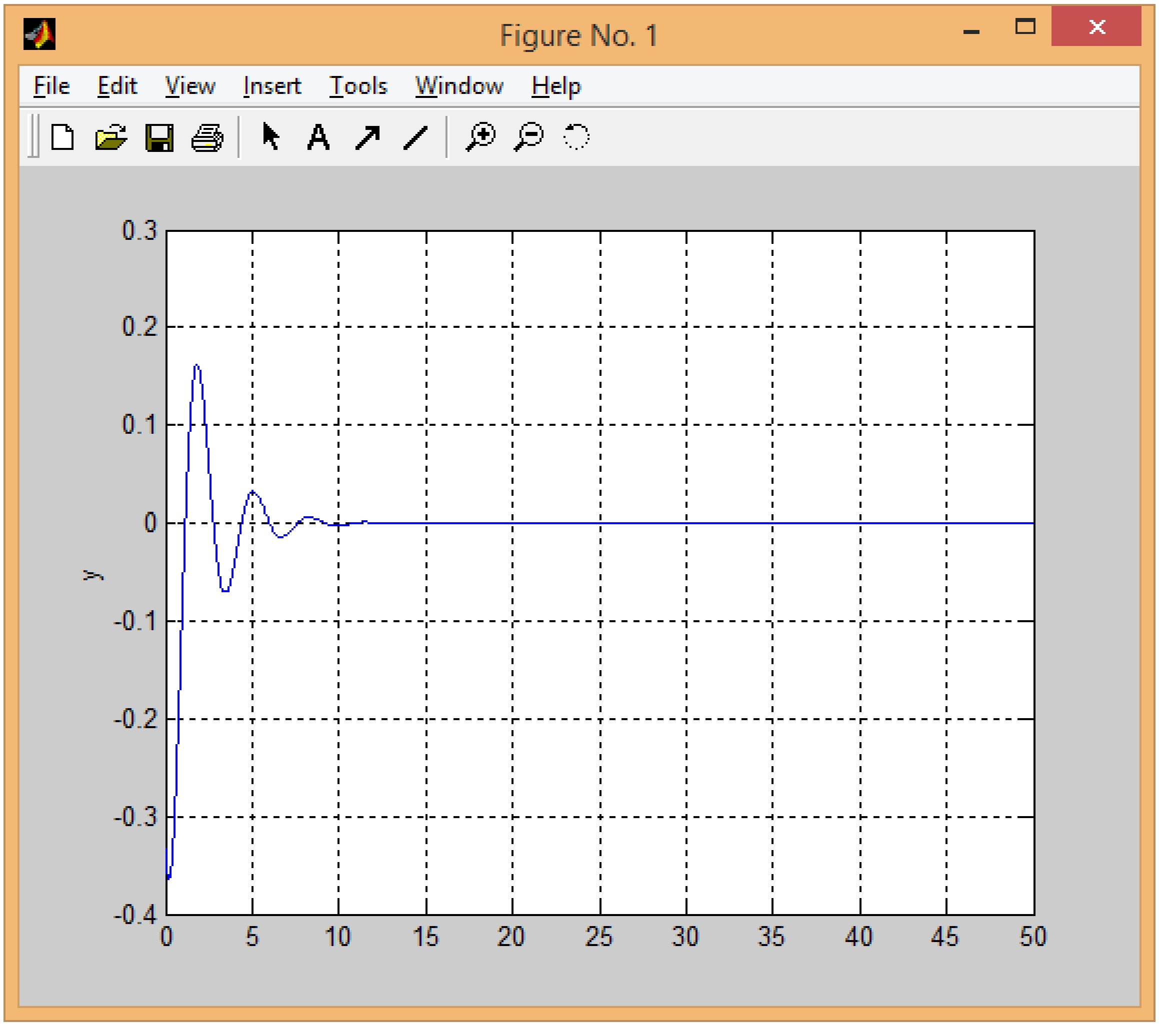
```
function LAB06ex1
omega0 = 2; c = 1; omega = 1.4;
param = [omega0,c,omega];
t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 50;
options = odeset('AbsTol',1e-10,'RelTol',1e-10);
[t,Y] = ode45(@f,[t0,tf],Y0,options,param);
y = Y(:,1); v = Y(:,2);
figure(1)
plot(t,y,'b-'); ylabel('y'); grid on;
t1 = 25; i = find(t>t1);
C = (max(Y(i,1))-min(Y(i,1)))/2;
disp(['computed amplitude of forced oscillation = ' num2str(C)]);
Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c*omega)^2);
disp(['theoretical amplitude = ' num2str(Ctheory)]);
%----------------------------------------------------------------
function dYdt = f(t,Y,param)
y = Y(1); v = Y(2);
omega0 = param(1); c = param(2); omega = param(3);
dYdt = [ v ; cos(omega*t)-omega0^2*y-c*v ];
```



```
% Exercise 1
% part a, the amplitude of forced oscillations

>> alpha=((1*1.4)/((2^2) - (1.4^2)))

alpha =

    0.6863

>> amc=atan(alpha)

amc =

    0.6015
```

The period of oscillation is about 4.49 seconds. Since ω_0 is greater than ω, the first part of the piecewise equation is used. This equations returns: α = 0.6015 radians (I called it *amc*).

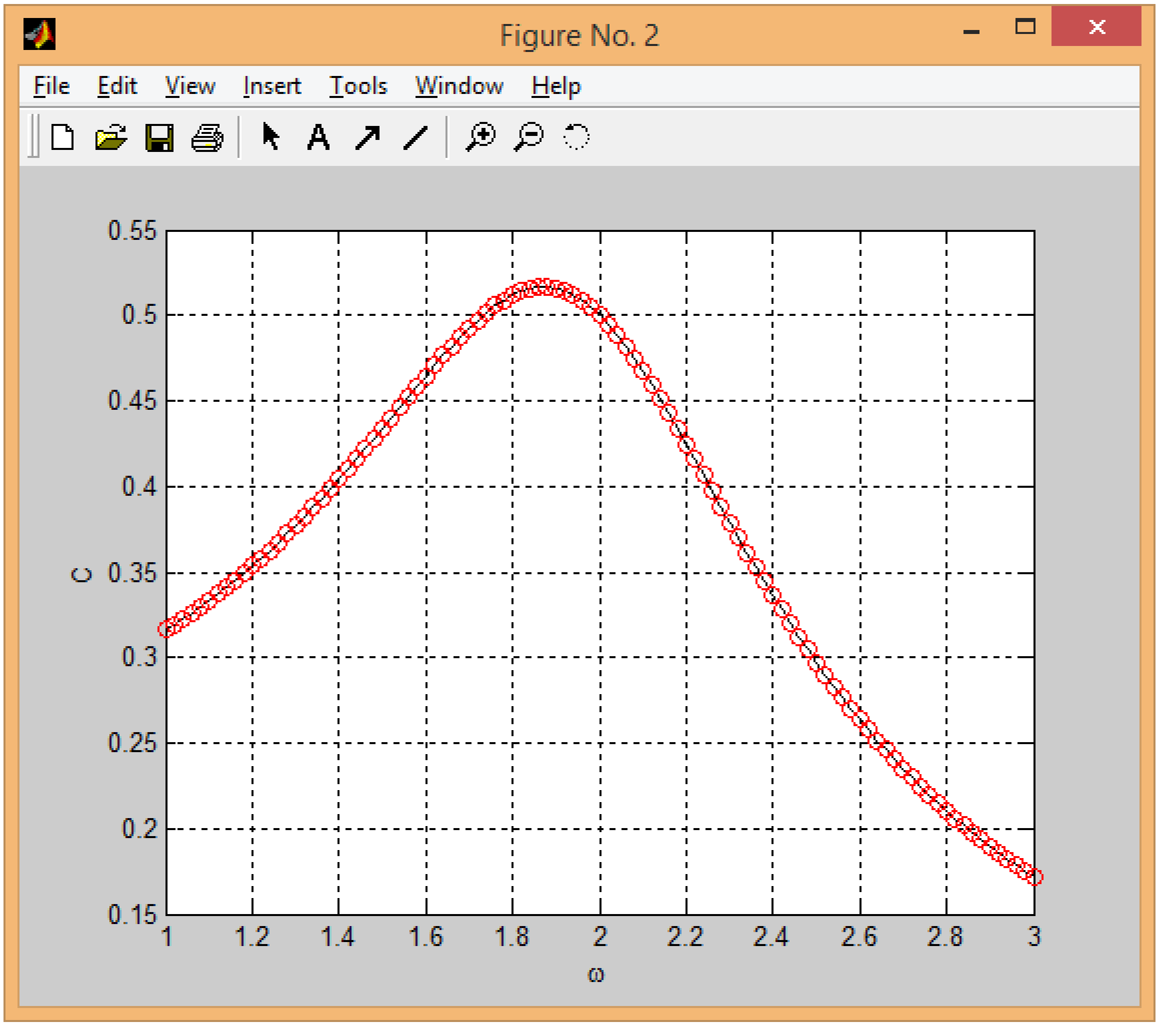
```
% part b

function LAB06ex1
omega0 = 2; c = 1; omega = 1.4;
alpha= atan((c* omega)/((omega0^2) - (omega^2)));
param = [omega0,c,omega];
t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 50;
options = odeset('AbsTol',1e-10,'relTol',1e-10);
[t,Y] = ode45(@f,[t0,tf],Y0,options,param);
y = Y(:,1); v = Y(:,2);
t1 = 25; i = find(t>t1);
C = (max(Y(i,1))-min(Y(i,1)))/2;
y=y-C*(cos(omega*t-alpha))
disp(['computed amplitude of forced oscillation = ' num2str(C)]);
Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c*omega)^2);
disp(['theoretical amplitude = ' num2str(Ctheory)]);
figure(1)
plot(t,y,'b-'); ylabel('y'); grid on;
disp(['alpha = ' num2str(alpha)]);
%-----------------------------------------------------------------
function dYdt = f(t,Y,param)
y = Y(1); v = Y(2);
omega0 = param(1); c = param(2); omega = param(3);
dYdt = [ v ; cos(omega*t)-omega0^2*y-c*v ];



      0.0000
      0.0000

computed amplitude of forced oscillation = 0.40417
theoretical amplitude = 0.40417
alpha = 0.60145
>>
```



The oscillations decayed exponentially because the complementary solution describes the discrepancy between actual oscillations and forced oscillation. Since the oscillations were quickly forced to the specified oscillation equation, the discrepancy diminished quickly.

```
% Exercise 2
% part a

function LAB06ex2
omega0 = 2; c = 1;
OMEGA = 1:0.02:3;
C = zeros(size(OMEGA));
Ctheory = zeros(size(OMEGA));
t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 50; t1 = 25;
for k = 1:length(OMEGA)
    omega = OMEGA(k);
    param = [omega0,c,omega];
    [t,Y] = ode45(@f,[t0,tf],Y0,[],param);
    i = find(t>t1);
    C(k) = (max(Y(i,1))-min(Y(i,1)))/2;
    Ctheory(k) = 1/sqrt((omega0^2-omega^2)^2+(c*omega)^2);% FILL-IN
end
figure(2)
```

```
plot(OMEGA,Ctheory,'k-',OMEGA,Ctheory,'ro'); grid on;% FILL-IN
xlabel('\omega'); ylabel('C');
%-----------------------------------------------------------
function dYdt = f(t,Y,param)
y = Y(1); v = Y(2);
omega0 = param(1); c = param(2); omega = param(3);
dYdt = [ v ; cos(omega*t)-omega0^2*y-c*v ];
```




```
% part b
```

The approximate value of ω that returns the maximum of the graph is ≈1.8733. The corresponding value of C is 0.5167, in which 1.8733 is the practical resonance frequency.

```
% part c

omega0 = 2; c = 1;
OMEGA = 1:0.02:3;
syms W;
f(W)= 1/sqrt(((omega0^2) - (W^2))^2+(c*W)^2);
y=diff(f);
```

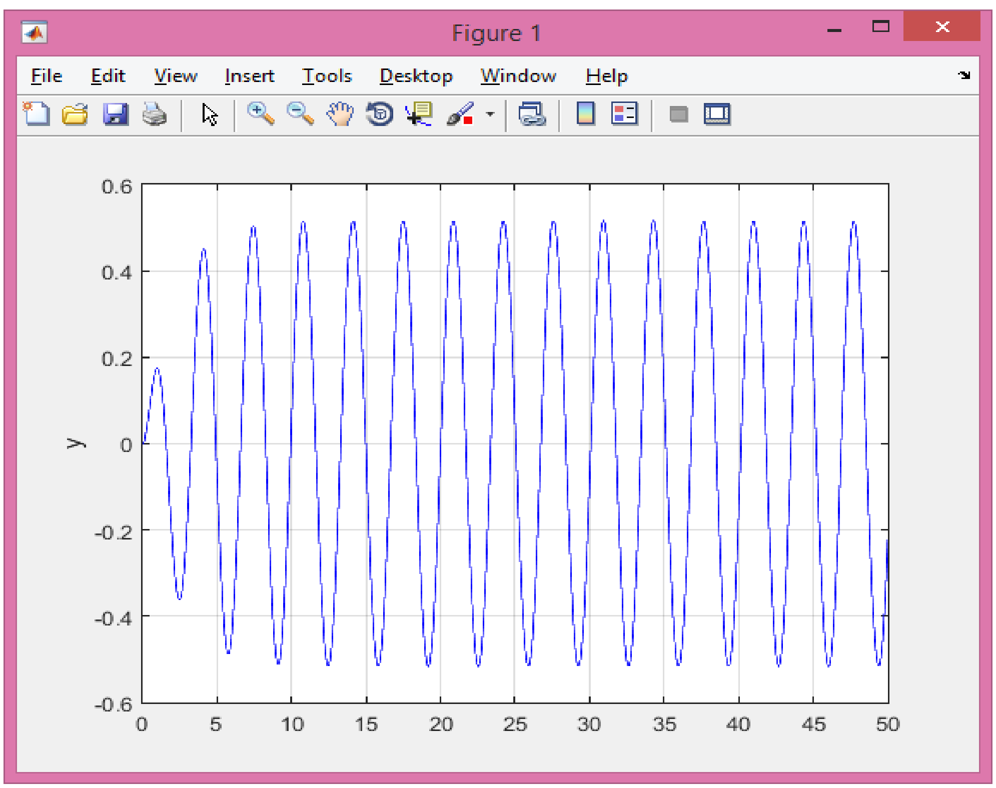
```
z=solve(y)
f(z)

z =

           0
 -14^(1/2)/2
  14^(1/2)/2


ans =

              1/4
 (2*15^(1/2))/15
 (2*15^(1/2))/15
```

As a curiosity, I experimented with MatLab's symbolic function creation mechanism, *syms*, to create a function called *f(ω)*, in addition to using *diff()* function to differentiate a symbolic function, and *solve()* to explore MatLab's solver's capability.

The zero of the function is at ω = 14^(1/2)/2 or 1.871 which returns a value of C of (2*15^(1/2))/15 or 0.5164. This analytical value of ω is just a few ten-thousandths less than the approximation made in part b.

```
% part d
```

The graph from the beginning, with ω = `14^(1/2)/2`



From this graph, the amplitude is about 0.5, which is more than it was in part 1. Theoretically, as ω increases, the amplitude should decrease.

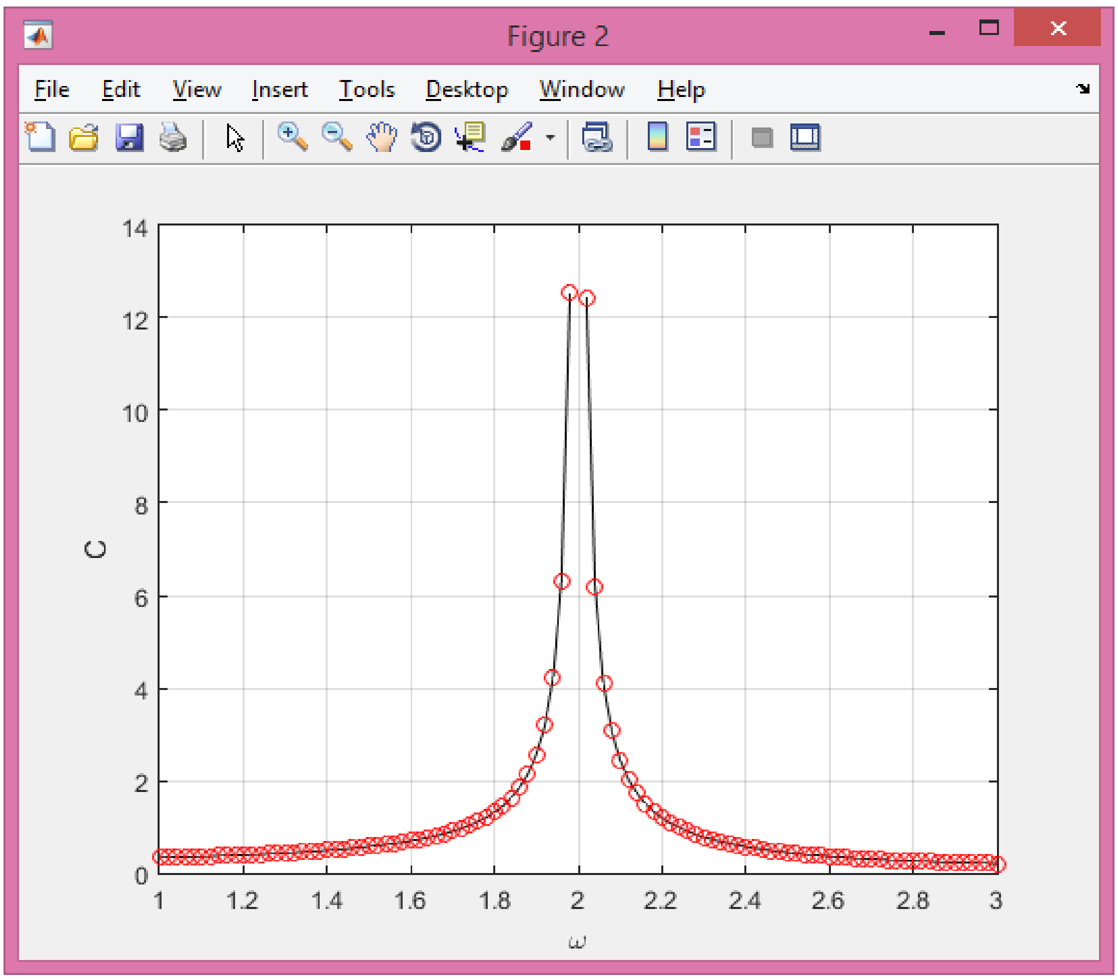
```
% part e
```

As the initial condition c was increased, the amplitude of the graph decreased. As the initial condition ω_0 was increased, the initial amplitude was increased but the amplitude of the forced oscillation was the same.

From the equation, these changes make sense. Because c is in the denominator, so a larger c value would decrease the amplitude. Since ω_0 determines the initial oscillations, it would initially increase but eventually got damped out to the forced oscillation.

```
% Exercise 3

% part a
```

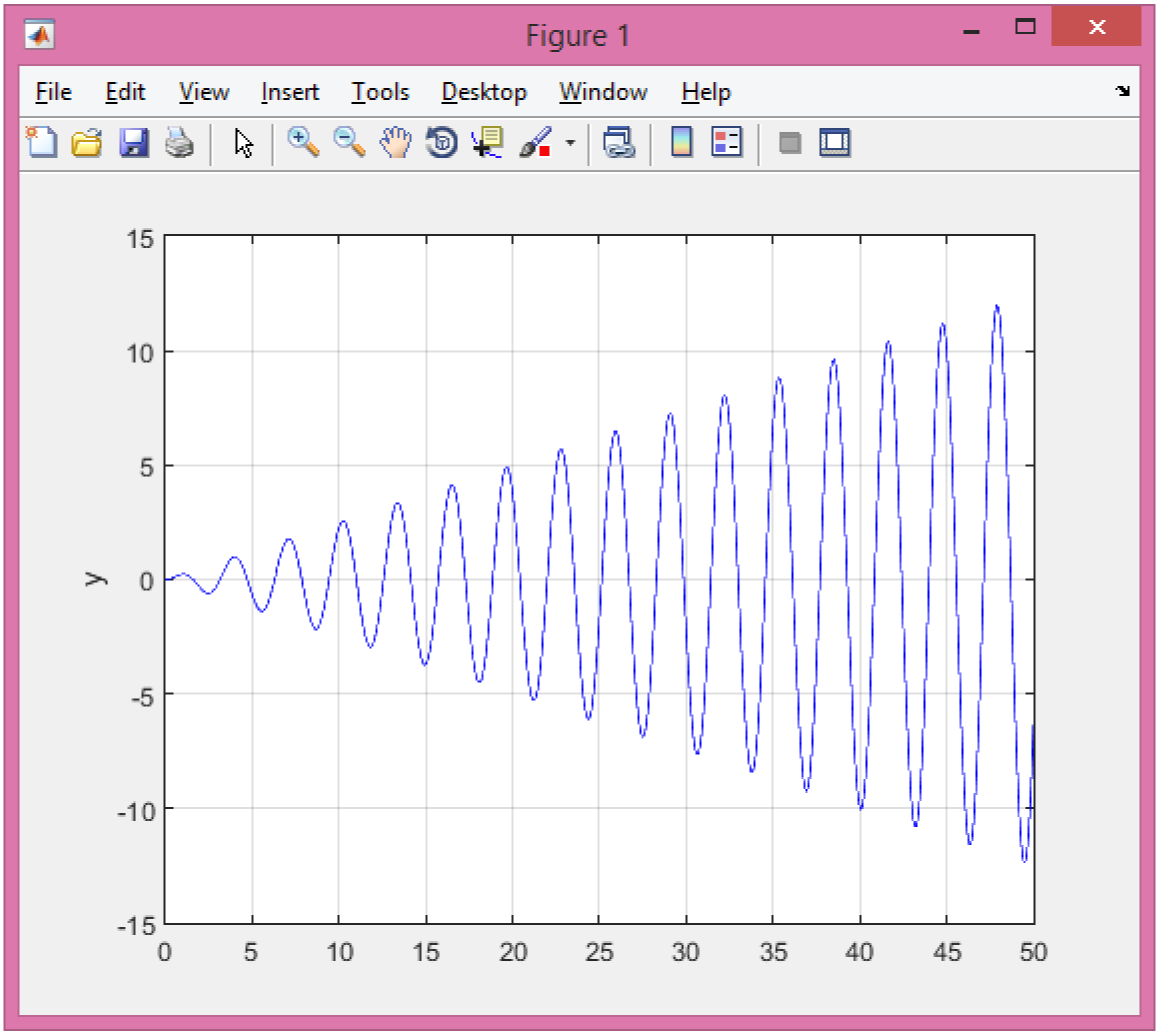
c = 0



maximal amplitude= 12.5
pure resonant frequency= 2
$\omega = \omega_0 = 2$

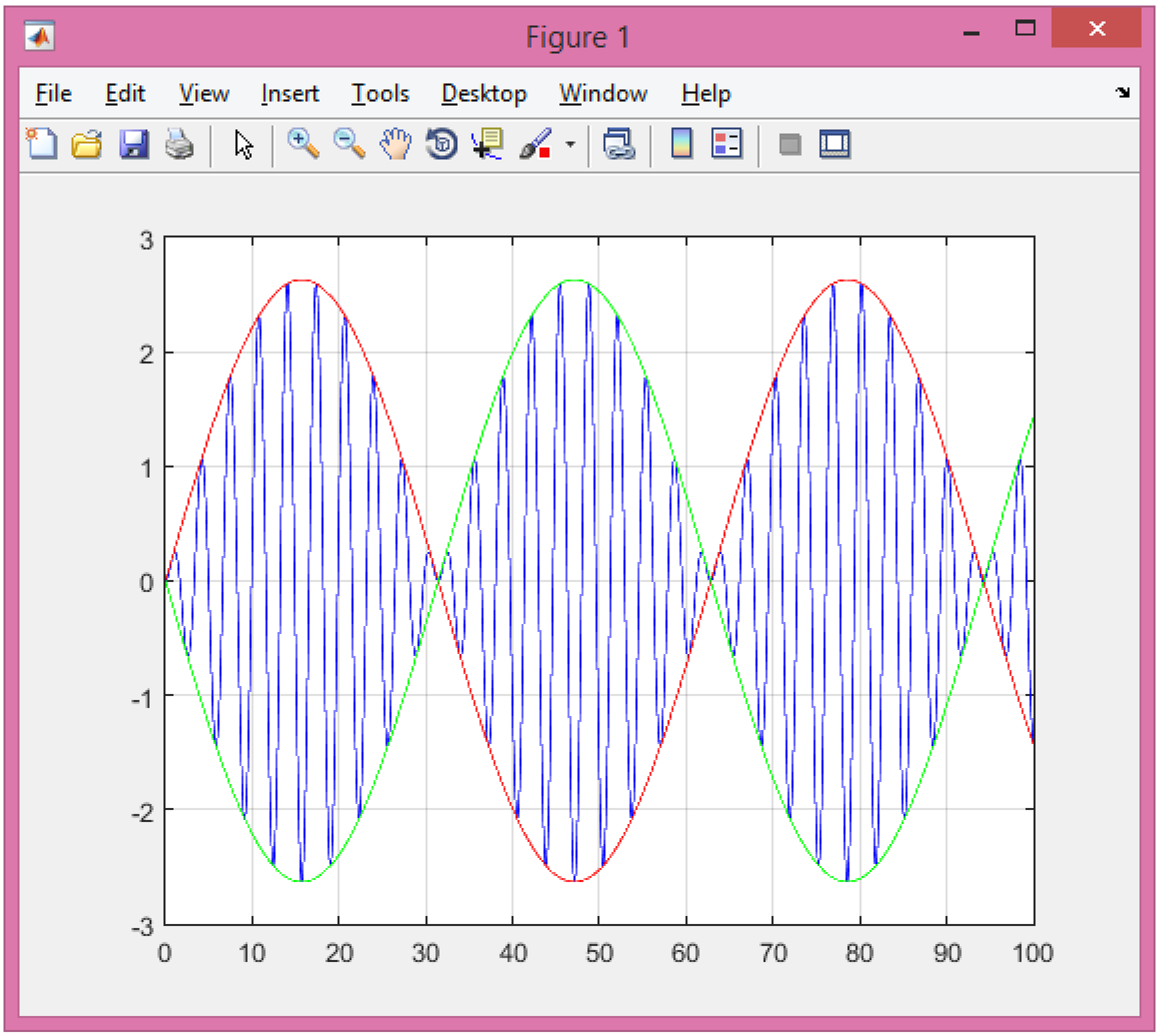
```
% part b
```

$C = 0$; $\omega_0 = 2$



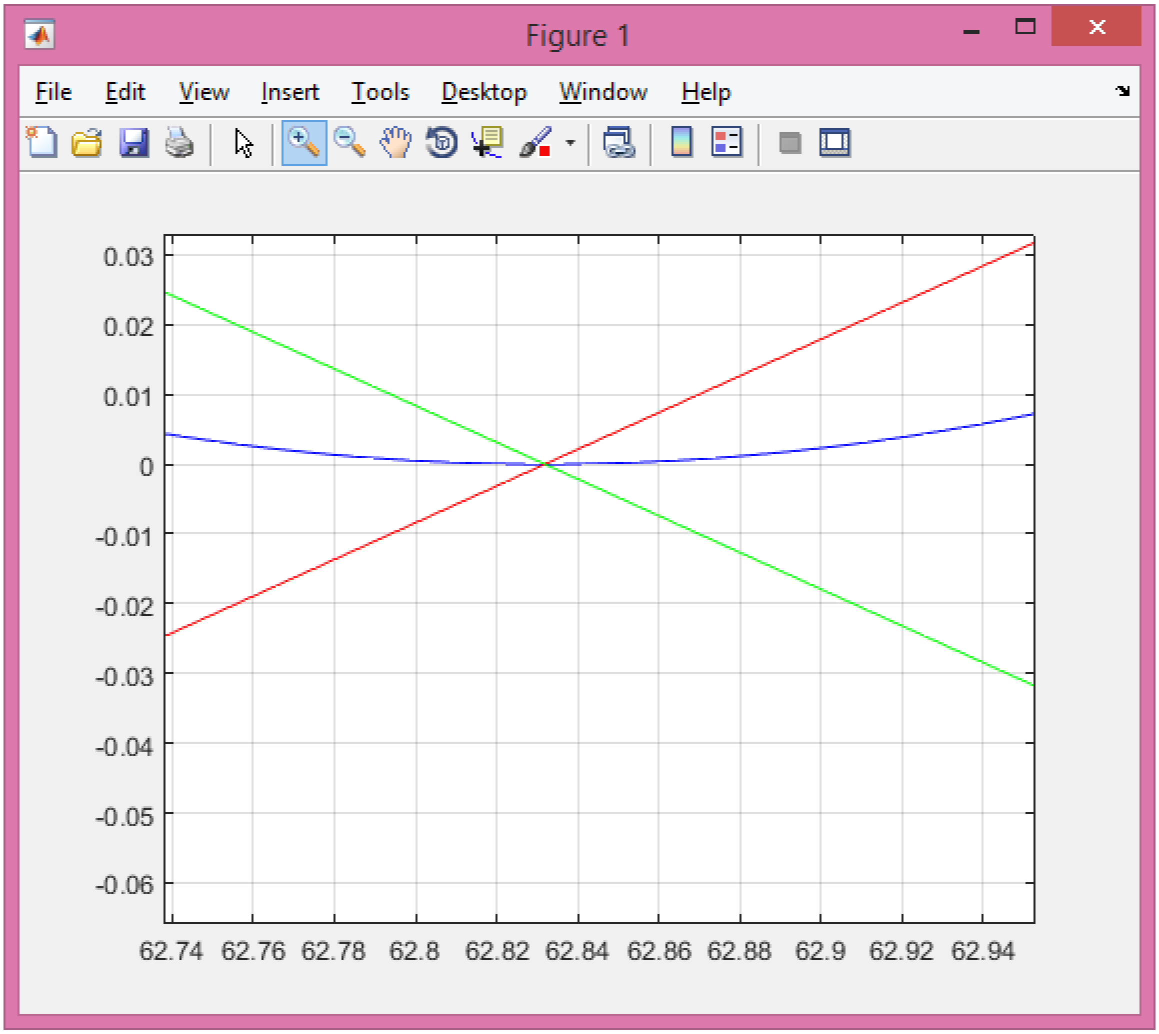

```
computed amplitude of forced oscillation = 12.1743
theoretical amplitude = Inf

% Exercise 4
% part a

function LAB06ex1
omega0 = 2; c = 0; omega = 1.8;
param = [omega0,c,omega];
t0 = 0; y0 = 0; v0 = 0; Y0 = [y0;v0]; tf = 100;
options = odeset('AbsTol',1e-10,'relTol',1e-10);
[t,Y] = ode45(@f,[t0,tf],Y0,options,param);
y = Y(:,1); v = Y(:,2);
C = 1/(omega0^2-omega^2);
A=2*C*sin(.5*(omega0-omega)*t);
figure(1)
plot(t,y,'b-',t,A,'r',t,-A,'g'); grid on;
```

```
t1 = 25; i = find(t>t1);
disp(['computed amplitude of forced oscillation = ' num2str(C)]);
Ctheory = 1/sqrt((omega0^2-omega^2)^2+(c*omega)^2);
disp(['theoretical amplitude = ' num2str(Ctheory)]);

%---------------------------------------------------------------

function dYdt = f(t,Y,param)
y = Y(1); v = Y(2);
omega0 = param(1); c = param(2); omega = param(3);
dYdt = [ v ; cos(omega*t)-omega0^2*y-c*v ];
```




```
computed amplitude of forced oscillation = 1.3158
theoretical amplitude = 1.3158



%part b
```

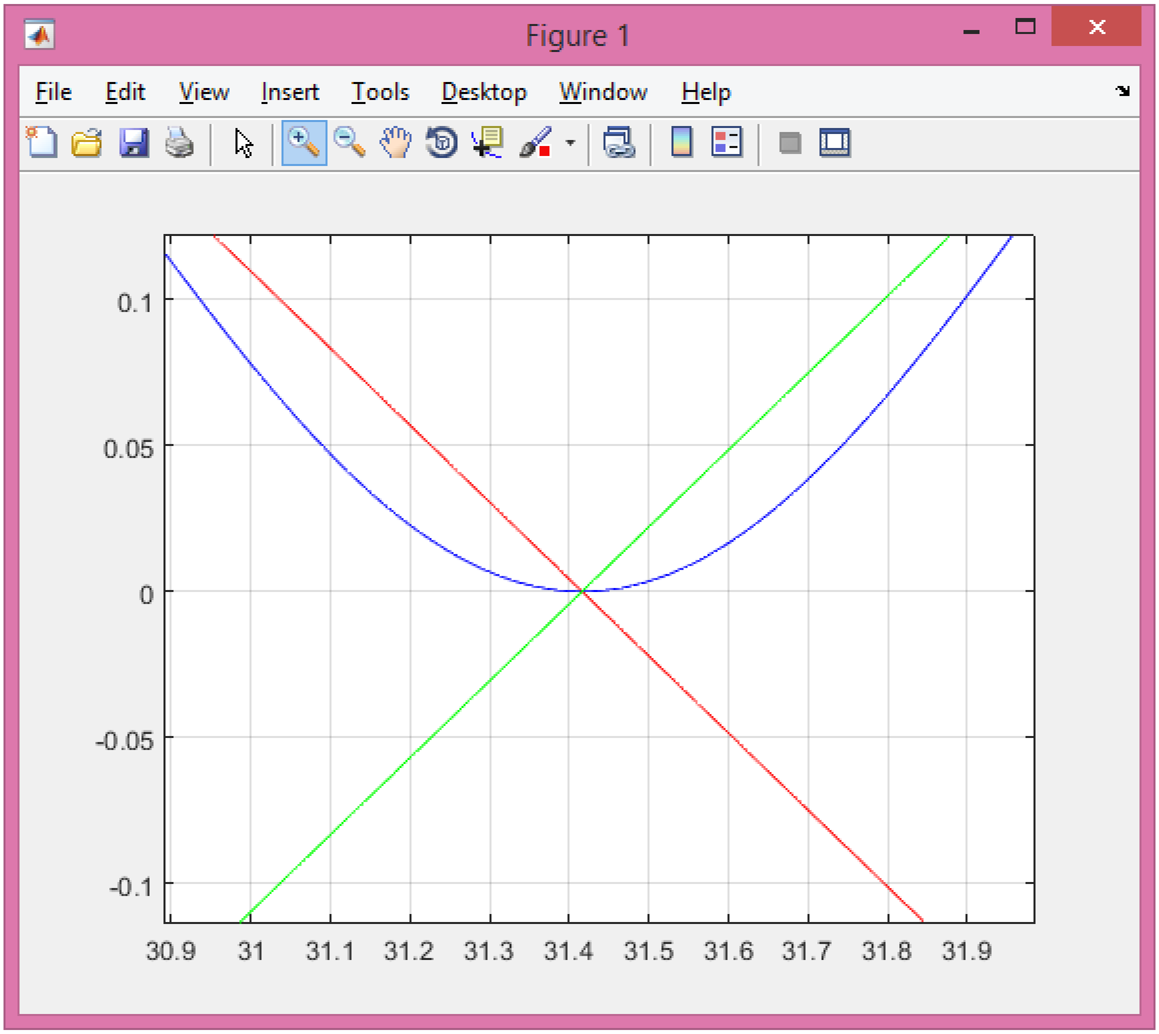
The period of fast oscillation from the equation is 62.83s. This is confirmed by the graph:



It can be observed that the lines pass the zero at the end of one period at about t=62.835s.

```
% part c
```

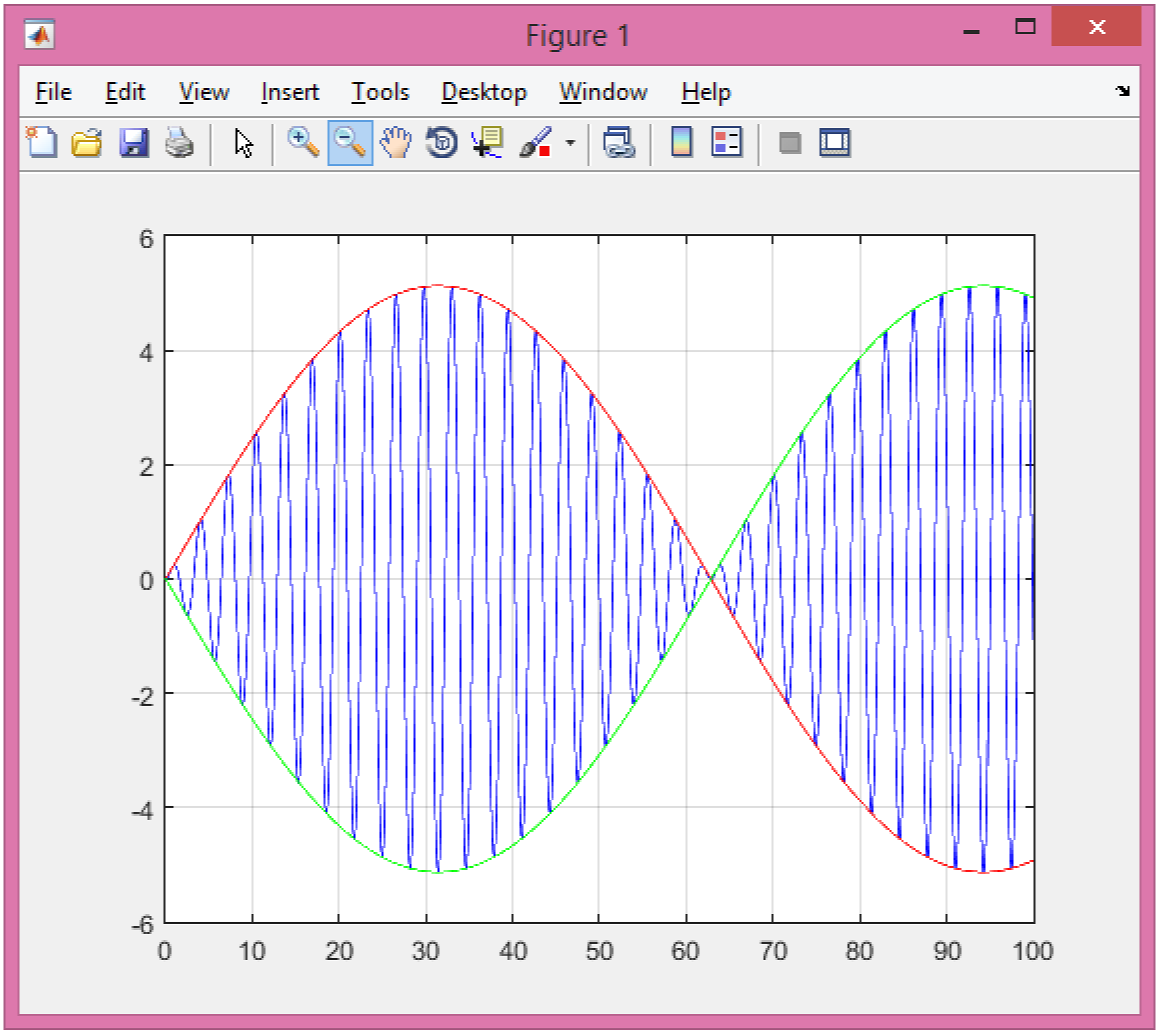
Analytical solution of the length of the graph using the equation $2\pi/(\omega_0 - \omega) = 31.42s$. This value is confirmed by the graph:



As seen from the graph the first intersection after t=0 is at $t \approx 31.42s$.

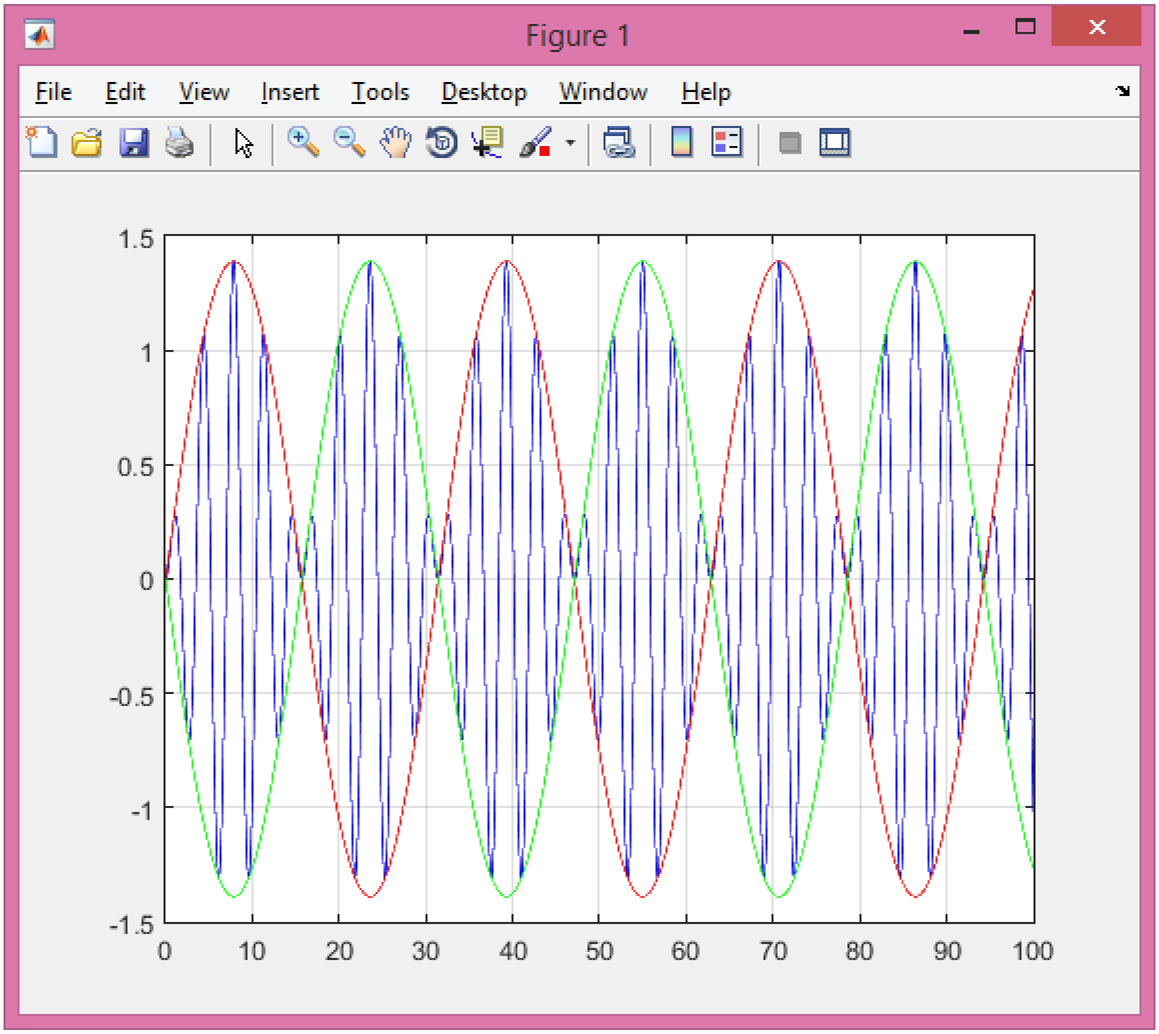
```
% part d
```

$\omega = 1.9$



When omega is changed to 1.9, the period of fast oscillation increases to 125.664s and the length of the beats increases to 62.83s. These values doubled the values from the last graph.

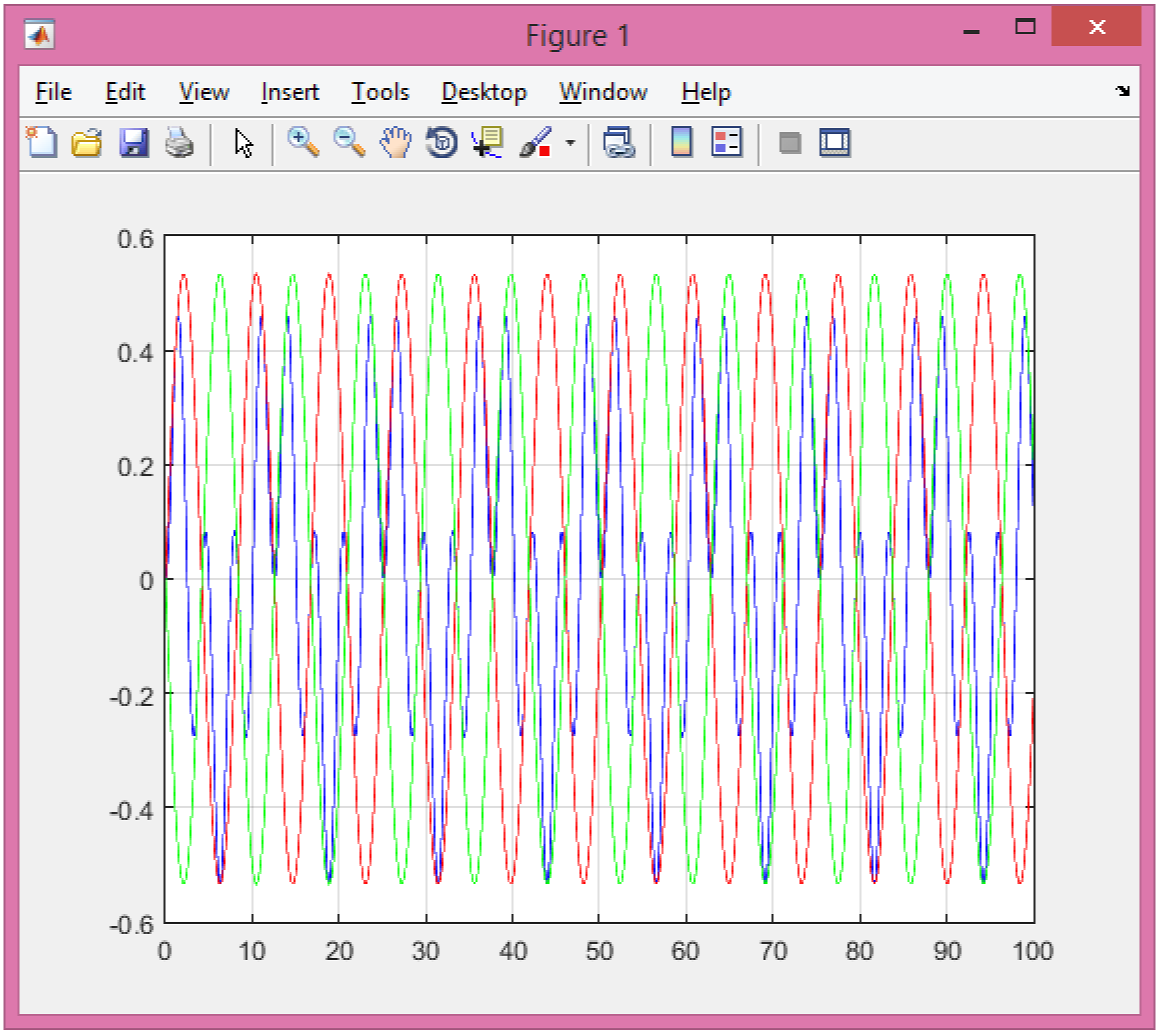
ω = 1.6



When omega is changed to 1.6, the period of fast oscillation decreases to 31.42s and the length of the beats decreases to 15.71s and these values are reduced to half of the original values from the graph where ω = 1.8

```
%part e
```

ω = 0.5



When ω is decreased to 0.5, the beats phenomenon no longer exists and the envelope functions no longer surround the oscillations of the graph. This happens because ω had decreased so much that the period is decreased as well.