# 1

1. An argument is valid iff the conclusion follows logically from the premises, i.e. if the conclusion must be true given that the premises are true.
2. Modus Ponens.
3. Fallacy.
4. A theorem that is an immediate consequence (and perhaps special case) of another theorem.

# 2

### (a)

1. (premise).
2. (premise).
3. (argument).
4. (argument).
5. (Disjunctive syllogism using lines 1, 3).
6. (Modus ponens using lines 4, 5).
7. (Conjunction and commutativity using lines 5, 6) [quod erat demonstrandum].

### (b) Note: The word overweight is taken as “it is not the case that x is fit”.

S(x) = “x is a swimmer”, F(x) = “x is fit”, Universe of discourse = all people.

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Yes, the argument is valid because of the quantifier and it is of the form .

It is Universal Tollens.

# 3

1. . Universe of discourse: x in the set of all real numbers .

This is trivial, as there is a real number x = 1.0, and *P(1.0)* is true.

1. Universe of discourse: x in the set of all integers

Suppose y and x are arbitrary real numbers and y = y. Then x = y2 – 1.

# 4

Suppose *n* is an arbitrary even integer. By definition of even integer, the integer *n* is *even* if there exists an integer *k* such that *n* = 2*k* .

Similarly, Suppose *m* is an arbitrary even integer. By definition of even integer, the integer *m* is *even* if there exists an integer *k* such that *m* = 2*k.*

Then the sum of *m* and *n* is (*m + n*), which is equivalent to *(2k + 2k) = 4k.*

Since *4k = 2(2k)*, which is a multiple of 2, *4k* is also an even number.

Thus, *(m + n) = 2(2k)*, and by definition, *(m + n)* is even.

Therefore, if *m* and *n* are even integers, then their sum is also even. (Q. E. D).

# 5

1. p = “*m* is even”, q = “*n* is even”, Universe of discourse: the set of all integers
2. [premise]
3. [Attempting contraposition]
4. [from line 3]
5. Then,
6. []
7. [Negation to proposition that (*mn*) is even]
8. Thus concludes the proof by contraposition.