CS 310

Assignment 210

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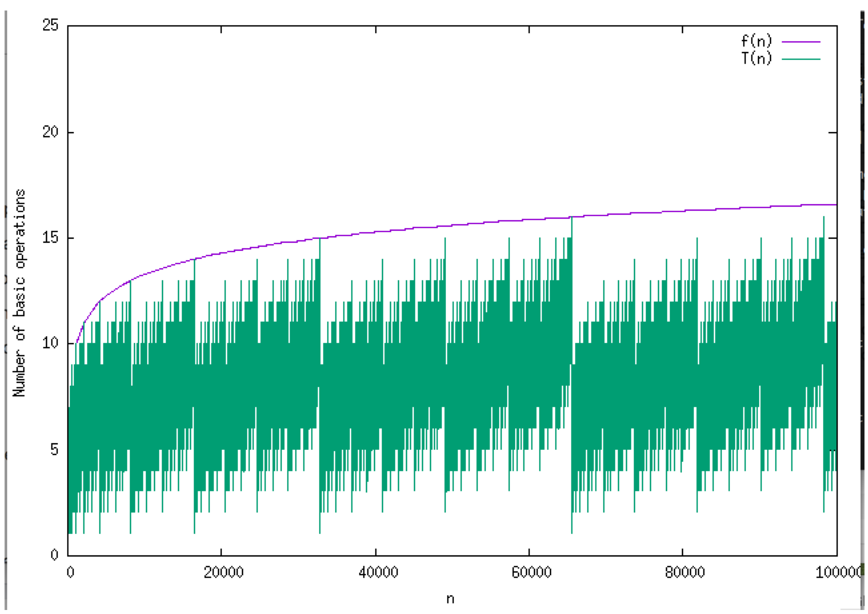
Program 210 implements an algorithm to determine the largest integral power of 2 smaller than or equal to an arbitrary non-negative integer n. It does so by iteratively clearing the least significant set bit in the binary representation of the variable j, which is initially set to n, until j reaches 0. The last positive value of j, which is memoized in another variable i, before the loop termination gives the largest power of 2 not exceeding n. The algorithm will return an error value of 0 if $n = 0$ is input. This is unfortunate since C++ does not offer a positive integer type.

To analyze this algorithm, we choose the bit-manipulating statement on line 26 and 33 as the basic operation as it represents the single most intrinsic, atomic step of this procedure. Furthermore, compared with other statements, it is possibly the most computationally intensive.

The value for n is taken to be value of the formal parameter of the function. It does not represent the size of the input but is the input itself. n is declared as an unsigned integer because inputting a negative number will produce an erroneous result, which should not even exist in the first place since the procedure does not make sense for non-positive integers (including 0).

An empirical analysis of running the algorithm for multiple values of n produces the results shown below.



An analysis of the code itself explains the empirical results when we observe that there is no direct correlation between $T(n)$ and n except for the fact that as n grows larger, $T(n)$ seems to fluctuate more drastically. An examination of the algorithm indicates that the number of basic operations, $T(n)$, is proportional to the number of set bits in the binary representation of n. This supports the earlier observation, since the larger n is, the larger the maximum possible value of $T(n)$ is since the maximum number of set bits in n is higher. Assume that n is strictly positive, $T(n)$ is bounded from below by 1 and from above by the total number of bits in the binary representation of n, $\lceil log_2 n \rceil$ where the ceiling function $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. The lower bound occurs when n is a power of 2 while the upper bound occurs when n is 1 less than a power of 2. This is empirically corroborated by the above graph, in which $T(n)$ lies neatly between the logarithmic curve $f(n) = log_2 n$ and the horizontal line $y = 1$.

Therefore, we conclude that this algorithm is described by

$$T(n) \in \Omega(1) \wedge T(n) \in O(log_2 n)$$