CS 310

Assignment 202

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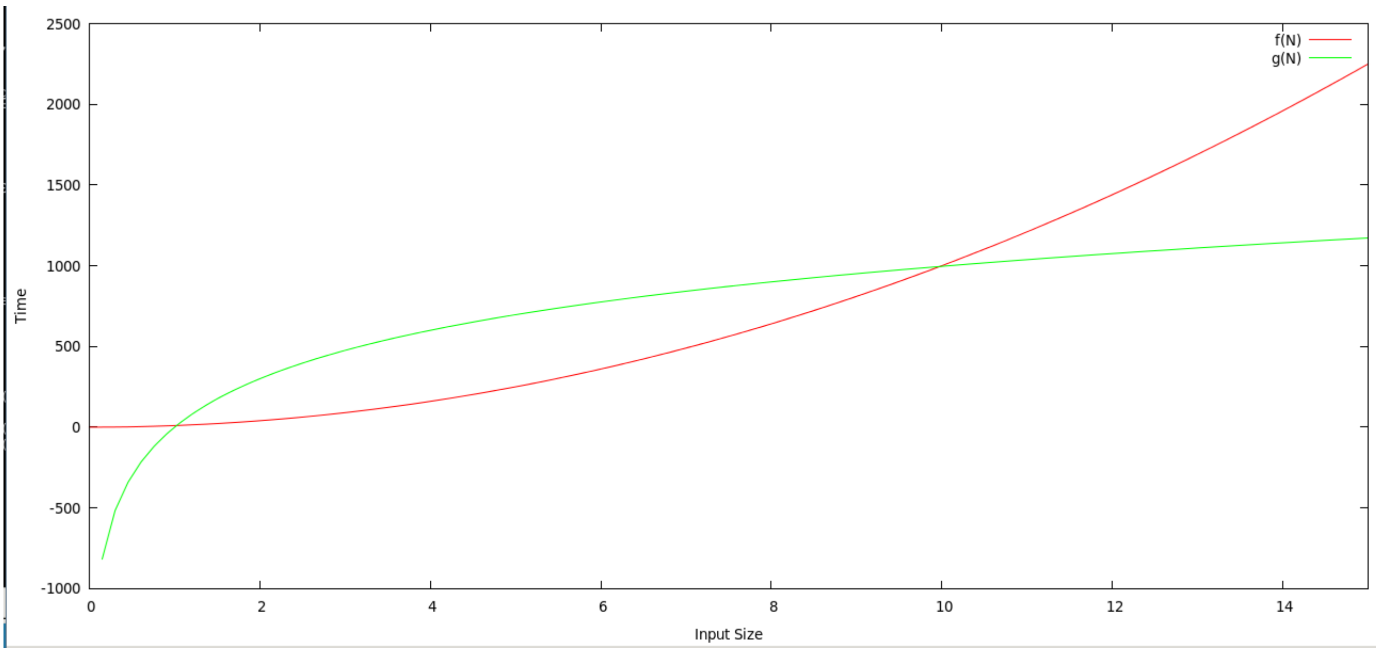
Problem 1. Algorithm A performs $10n^2$ basic operations, and algorithm B performs $300\lg(n)$ basic operations. Which algorithm is better, and at what value of n does the better algorithm start to show its better performance? Illustrate your answer with a graph generated by a program such as gnuplot or wolfram alpha.

Answer: As depicted in the following graph:



Algorithm B is better than algorithm A. Algorithm B begins to show superior performance at $n = 10$.

Problem 2. Using the definitions, show that

$$6n^2 + 20n \in O(n^3)$$

but

$$6n^2 + 20n \notin \Omega(n^3)$$

Answer:
Let $c = 30$ and $n_0 = 1$. Since $6n^2 + 20n \leq cn^3$ for all $n \geq n_0$, $6n^2 + 20n \in O(n^3)$.
Since $6n^2 + 20n$ is of smaller degree than n^3, $6n^2 + 20n$ is asymptotically smaller than n^3. Subsequently, there does not exist any pair of positive numbers c and n_0 such that $6n^2 + 20n \geq cn^3$ for all $n \geq n_0$. Hence, $6n^2 + 20n \notin \Omega(n^3)$.

Problem 3. Given the following algorithm, and assuming that n is an even number, calculate the exact number of times statement `foo` runs, and analyze the algorithm using the the rules (e.g., polynomial).

```
j = 1;
while( j <= n/2 )
{
   i = 1;
   while( i <= j )
   {
      foo;
      i++;
   }
   j++;
}
```

Answer:
The number of times statement `foo` runs is:

$$
\begin{aligned}
T(n) &= \sum_{j=1}^{n/2}(\sum_{i=1}^{j} 1) = \sum_{j=1}^{n/2} j \\
&= 1 + 2 + 3 + ... + n/2 \\
&= (1 + n/2)(n/2)/2 \\
&= (2n + n^2)/8
\end{aligned}
$$

Since $(2n + n^2)/8$ is a polynomial of degree 2, $(2n + n^2)/8 \in \Theta(n^2)$ by the polynomial rule.

Problem 4. Solve the recurrence relation $t_n = 2nt_{n-1}$ where $t_0 = 1$.

Answer: Using the given relation, we have:

$$
\begin{aligned}
t_n &= 2nt_{n-1} \\
&= 2n(2(n-1)t_{n-2}) \\
&= 2^2 n(n-1)t_{n-2} \\
&= 2^{n-(n-2)} \times n!/(n-2)! \times t_{n-2} \\
&= 2^{n-(n-2)} \times n!/(n-2)! \times (2(n-2)t_{n-3}) \\
&= 2^{n-(n-3)} \times n!/(n-3)! \times t_{n-3} \\
&\vdots \\
&= 2^{n-(n-k)} \times n!/(n-k)! \times t_{n-k} \\
&\vdots \\
&= 2^{n-(n-n)} \times n!/(n-n)! \times t_{n-n} \\
&= 2^n n! t_0
\end{aligned}
$$

and therefore $t_n = 2^n n!$ for $n \geq 1$.

Problem 5. Analyze a program whose time complexity is $T(n) = 7T(\frac{n}{4}) + n$.

Answer:
We have: $n \in \Theta(n^1)$.
Applying the Master Theorem, let $a = 7, b = 4, d = 1$.
Since $a > b^d$, $T(n) \in \Theta(n^{log_4 7}) \approx \Theta(n^{1.404})$.