Stock Price Prediction

1. Introduction

- **Motivation:** Through this case study, we aim to explore stock price analysis using standard forecasting methods, addressing the following questions:
  - Can we predict future stock prices by analyzing historical data?
  - Can we develop an inference model that comprehends the relationship between future prices and previous price indicators?
- **Contents:**
  1. We conduct a time-series analysis on the price movements of Bitcoin (BTC) and the S&P 500 index, representing two distinct stocks with unique characteristics.
  2. We construct an AutoRegressive Integrated Moving Average (ARIMA) model, a standard autoregressive model for predicting future trends in a time series.
  3. We build a Bayesian Structural Time Series (BSTS) model, a flexible and modular explanatory model capable of handling the uncertainty associated with price volatility.

2. Background

Data Source and Scope

We utilized the `quantmod` package, designed to assist quantitative traders in exploring and building trading models effortlessly. This package provides daily open, high, low, and close (OHLC) price data for most stocks.

Stocks of Interest

Our analysis focused on the price movements of two representative stocks with distinct characteristics:

1. **Standard & Poor's 500 Index (S&P 500)**

- A market-capitalization-weighted index comprising 500 leading publicly traded companies in the United States.
- Regarded as one of the best indicators of prominent American equities' performance and, by extension, the overall stock market.

2. **Bitcoin (BTC-USD)**

- The world's largest cryptocurrency by market capitalization.
- Known for its high volatility, influenced by many factors such as supply and demand, investor and user sentiments, government regulations, and media hype.

2.1. Dataset

In [ ]:
```
install.packages("quantmod")
install.packages("forecast")
install.packages("highcharter")
install.packages("bsts")
install.packages("reshape2")
install.packages("dplyr")
install.packages("lubridate")
install.packages("ggplot2")
install.packages("gridExtra")
```

```
Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

also installing the dependencies ‘XML’, ‘htmlwidgets’, ‘rlist’, ‘assertthat’, ‘igrap
h’, ‘rjson’


Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

also installing the dependencies ‘BoomSpikeSlab’, ‘Boom’


Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

also installing the dependency ‘plyr’


Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)
```

In [ ]:
```
library(quantmod)
library(pillar)
library(forecast)
library(highcharter)
library(bsts)
library(reshape2)
library(dplyr)
library(lubridate)
library(ggplot2)
library(gridExtra)
```

```
Loading required package: BoomSpikeSlab

Loading required package: Boom


Attaching package: ‘Boom’


The following object is masked from ‘package:stats’:

    rWishart



Attaching package: ‘BoomSpikeSlab’


The following object is masked from ‘package:stats’:

    knots



Attaching package: ‘bsts’


The following object is masked from ‘package:BoomSpikeSlab’:

    SuggestBurn



######################### Warning from 'xts' package ##########################
#                                                                              #
# The dplyr lag() function breaks how base R's lag() function is supposed to  #
# work, which breaks lag(my_xts). Calls to lag(my_xts) that you type or       #
# source() into this session won't work correctly.                            #
#                                                                              #
# Use stats::lag() to make sure you're not using dplyr::lag(), or you can add #
# conflictRules('dplyr', exclude = 'lag') to your .Rprofile to stop           #
# dplyr from breaking base R's lag() function.                                #
#                                                                              #
# Code in packages is not affected. It's protected by R's namespace mechanism #
# Set `options(xts.warn_dplyr_breaks_lag = FALSE)` to suppress this warning.  #
#                                                                              #
################################################################################


Attaching package: ‘dplyr’


The following object is masked from ‘package:pillar’:

    dim_desc

```

```
The following objects are masked from ‘package:xts’:

    first, last


The following objects are masked from ‘package:stats’:

    filter, lag


The following objects are masked from ‘package:base’:

    intersect, setdiff, setequal, union



Attaching package: ‘lubridate’


The following objects are masked from ‘package:base’:

    date, intersect, setdiff, union



Attaching package: ‘gridExtra’


The following object is masked from ‘package:dplyr’:

    combine
```

3. Data Preprocessing

The data obtained consists of daily open, high, low, and close (OHLC) prices. For the purpose of this analysis, we will focus on the closing price, which can be retrieved using the `Cl()` function. The closing price is widely considered a valuable indicator for assessing stock price fluctuations over time.

It's worth noting that some investors prefer to work with adjusted prices rather than closing prices. Adjusted prices take into account corporate actions such as stock splits, dividends, and rights offerings, providing a more accurate representation of a stock's historical performance.

In [ ]:
```
btc_price <- getSymbols('BTC-USD', auto.assign=FALSE, from="2014-01-01")
sp_price <- getSymbols("^GSPC", auto.assign=FALSE, from="2014-01-01")
```

In [ ]:
```
summary(btc_price)
summary(sp_price)
```

```
    Index              BTC-USD.Open      BTC-USD.High      BTC-USD.Low
Min.   :2014-09-17   Min.   :  176.9   Min.   :  211.7   Min.   :  171.5
1st Qu.:2017-02-19   1st Qu.: 1041.7   1st Qu.: 1057.7   1st Qu.: 1021.2
Median :2019-07-26   Median : 8791.7   Median : 8958.5   Median : 8598.4
Mean   :2019-07-26   Mean   :16487.0   Mean   :16869.6   Mean   :16080.0
3rd Qu.:2021-12-29   3rd Qu.:27266.2   3rd Qu.:27788.8   3rd Qu.:26824.2
Max.   :2024-06-03   Max.   :73079.4   Max.   :73750.1   Max.   :71334.1
BTC-USD.Close     BTC-USD.Volume      BTC-USD.Adjusted
Min.   :  178.1   Min.   :5.915e+06   Min.   :  178.1
1st Qu.: 1042.2   1st Qu.:2.341e+08   1st Qu.: 1042.2
Median : 8801.8   Median :1.313e+10   Median : 8801.8
Mean   :16505.1   Mean   :1.725e+10   Mean   :16505.1
3rd Qu.:27282.0   3rd Qu.:2.784e+10   3rd Qu.:27282.0
Max.   :73083.5   Max.   :3.510e+11   Max.   :73083.5
    Index               GSPC.Open      GSPC.High       GSPC.Low
Min.   :2014-01-02   Min.   :1744   Min.   :1756   Min.   :1738
1st Qu.:2016-08-09   1st Qu.:2141   1st Qu.:2150   1st Qu.:2133
Median :2019-03-19   Median :2833   Median :2849   Median :2819
Mean   :2019-03-17   Mean   :3088   Mean   :3104   Mean   :3070
3rd Qu.:2021-10-21   3rd Qu.:3993   3rd Qu.:4018   3rd Qu.:3956
Max.   :2024-05-31   Max.   :5340   Max.   :5342   Max.   :5302
  GSPC.Close    GSPC.Volume        GSPC.Adjusted
Min.   :1742   Min.   :1.297e+09   Min.   :1742
1st Qu.:2140   1st Qu.:3.370e+09   1st Qu.:2140
Median :2837   Median :3.779e+09   Median :2837
Mean   :3089   Mean   :3.952e+09   Mean   :3089
3rd Qu.:3991   3rd Qu.:4.299e+09   3rd Qu.:3991
Max.   :5321   Max.   :9.977e+09   Max.   :5321
```

Data Attributes

The dataset contains the following attributes:

- **Open**: The opening price for that day
- **High**: The highest price paid for the stock that day
- **Low**: The lowest price paid for the stock that day
- **Close**: The closing price for that day
- **Volume**: The volume of stocks traded that day
- **Adjusted**: The adjusted closing price for that day, taking into account corporate actions such as stock splits, dividends, and rights offerings.

Analysis Objective

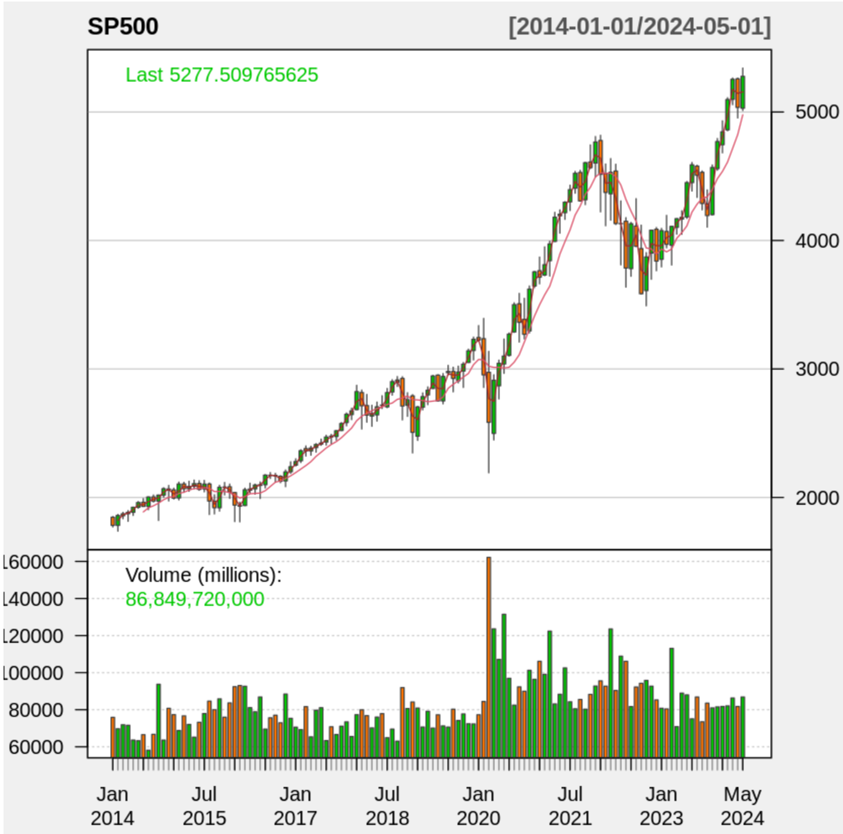
In this project, we will use the **closing price** as the **response variable** for further analysis. The closing price is considered a useful indicator to assess changes in stock prices over time. Therefore, the goal is to predict the future trend of a stock's closing price based on its previous price movements.

In [ ]:
```
chartSeries(to.monthly(btc_price),
            theme = chartTheme("white"),
```

```
            name = "BTC",
            TA = list("addVo()",
                      "addSMA(2)",
                      "addSMA(7, col = 2)"))
```

In [ ]:

```
chartSeries(to.monthly(sp_price),
            theme = chartTheme("white"),
            name = "SP500",
            TA = list("addVo()",
                      "addSMA(2)",
                      "addSMA(7, col = 2)")
                      )
```



Time Series Decomposition

Time series data can exhibit various patterns, and it is often beneficial to decompose a time series into several components, each representing an underlying pattern category.

Additive Decomposition

The additive decomposition model is represented by the following equation:

$$y_t = S_t + T_t + R_t$$

Where:

- y_t is the data
- S_t is the seasonal component
- T_t is the trend-cycle component
- R_t is the remainder component

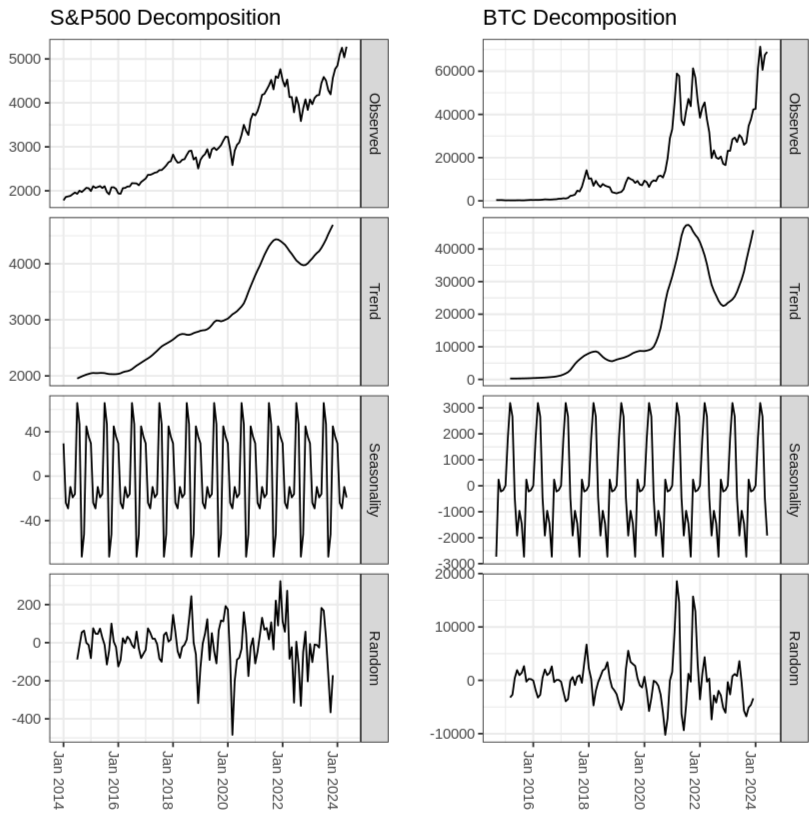
All components are evaluated at period t. The seasonal component changes slowly over time, while the remainder component contains any additional factors present in the time series.

In [ ]:
```
# options(repr.plot.width = 20, repr.plot.height = 10)

sp_close <- Cl(to.monthly(sp_price))
sp_dc <- decompose(as.ts(sp_close))
```

```
btc_close <- Cl(to.monthly(btc_price))
btc_dc <- decompose(as.ts(btc_close))
```

In [ ]:
```
### Extract the components
components <- cbind.data.frame(
  sp_dc$x, sp_dc$trend, sp_dc$seasonal, sp_dc$random, time(to.monthly(sp_price))
)
names(components) <- c("Observed", "Trend", "Seasonality", "Random", "Date")
components <- melt(components, id="Date")
names(components) <- c("Date", "Component", "Value")

### Plot
plot1 <- ggplot(data=components, aes(x=Date, y=Value)) + geom_line() +
  theme_bw() + theme(legend.title = element_blank()) + ylab("") + xlab("") +
  facet_grid(Component ~ ., scales="free") + guides(colour=FALSE) +
  theme(axis.text.x=element_text(angle = -90, hjust = 0)) +
  ggtitle("S&P500 Decomposition")


components <- cbind.data.frame(
  btc_dc$x, btc_dc$trend, btc_dc$seasonal, btc_dc$random, time(to.monthly(btc_price
)
names(components) <- c("Observed", "Trend", "Seasonality", "Random", "Date")
components <- melt(components, id="Date")
names(components) <- c("Date", "Component", "Value")

### Plot
plot2 <- ggplot(data=components, aes(x=Date, y=Value)) + geom_line() +
  theme_bw() + theme(legend.title = element_blank()) + ylab("") + xlab("") +
  facet_grid(Component ~ ., scales="free") + guides(colour=FALSE) +
  theme(axis.text.x=element_text(angle = -90, hjust = 0)) +
  ggtitle("BTC Decomposition")


grid.arrange(plot1, plot2, ncol=2)
```

```
Warning message:
“The `<scale>` argument of `guides()` cannot be `FALSE`. Use "none" instead as
of ggplot2 3.3.4.”
Warning message:
“The `trans` argument of `continuous_scale()` is deprecated as of ggplot2 3.5.0.
ℹ Please use the `transform` argument instead.”
Don't know how to automatically pick scale for object of type <ts>. Defaulting
to continuous.
Warning message:
“Removed 6 rows containing missing values or values outside the scale range
(`geom_line()`).”
Don't know how to automatically pick scale for object of type <ts>. Defaulting
to continuous.
Warning message:
“Removed 6 rows containing missing values or values outside the scale range
(`geom_line()`).”
```



The output displays four plots of our closing price data, which are:

- **Observed**: The original plot of the data.
- **Trend**: The long-term movements in the mean.
- **Seasonal**: The repetitive seasonal fluctuations in the data.
- **Random**: The irregular or random fluctuations not captured by the trend and seasonal components. When the random component is dominant in a data set, accurate forecasting becomes more challenging.

While we observe similar patterns across components in the BTC and S&P 500 prices, there are significant differences in the relative magnitudes of each component.

4. Model

Data Setting

For the following analysis, we will be using weekly stock prices to reduce computational requirements and processing time. Our goal is to predict the stock prices for the next 50 weeks.

AutoRegressive Integrated Moving Average (ARIMA)

Autoregressive Model (AR)

- In a multiple regression model, we forecast the variable of interest using a linear combination of predictors. In an autoregression model, we forecast the variable of interest using a linear combination of past values of the variable itself. The term "autoregression" indicates that it is a regression of the variable against itself.
- An autoregressive model of order p is represented as:

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \varepsilon_t$$

  where ε_t is white noise. This is similar to a multiple regression but with lagged values of y_t as predictors. We refer to this as an $\mathbf{AR}(p)$ model, an autoregressive model of order p.

Moving Average Model (MA)

- A moving average model uses past forecast errors in a regression-like model:

$$y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q}$$

  where ε_t is white noise. We refer to this as an $\mathrm{MA}(q)$ model, a moving average model of order q. However, we do not observe the values of ε_t, so it is not a regression in the usual sense.

ARIMA (AutoRegressive Integrated Moving Average)

- ARIMA combines differencing with autoregression and a moving average model.
- An ARIMA(p, d, q) model is represented as:

$$y'_t = c + \phi_1 y'_{t-1} + \cdots + \phi_p y'_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$

  where y'_t is the differenced series (it may have been differenced more than once).
- p: order of the autoregressive part
- d: degree of first differencing involved
- q: order of the moving average part

In [ ]:
```r
# Data Splitting
btc_price <- to.weekly(btc_price)
sp_price <- to.weekly(sp_price)
n <- 50

btc_train <- head(Cl(btc_price), length(Cl(btc_price))-n)
btc_test  <- tail(Cl(btc_price), n)

sp_train <- head(Cl(sp_price), length(Cl(sp_price))-n)
sp_test  <- tail(Cl(sp_price), n)

sp_len <- length(sp_train)
btc_len <- length(btc_train)
```

In [ ]:
```r
arima_btc <- auto.arima(btc_train, trace=T, allowdrift = FALSE)
```

```
Fitting models using approximations to speed things up...

ARIMA(2,1,2)                    : 8270.241
ARIMA(0,1,0)                    : 8284.204
ARIMA(1,1,0)                    : 8280.503
ARIMA(0,1,1)                    : 8277.139
ARIMA(1,1,2)                    : 8277.422
ARIMA(2,1,1)                    : 8276.118
ARIMA(3,1,2)                    : 8271.845
ARIMA(2,1,3)                    : 8270.843
ARIMA(1,1,1)                    : 8278.106
ARIMA(1,1,3)                    : 8267.085
ARIMA(0,1,3)                    : 8275.087
ARIMA(1,1,4)                    : 8267.561
ARIMA(0,1,2)                    : 8274.721
ARIMA(0,1,4)                    : 8266.286
ARIMA(0,1,5)                    : 8267.89
ARIMA(1,1,5)                    : 8269.226

Now re-fitting the best model(s) without approximations...

ARIMA(0,1,4)                    : 8281.744

Best model: ARIMA(0,1,4)
```

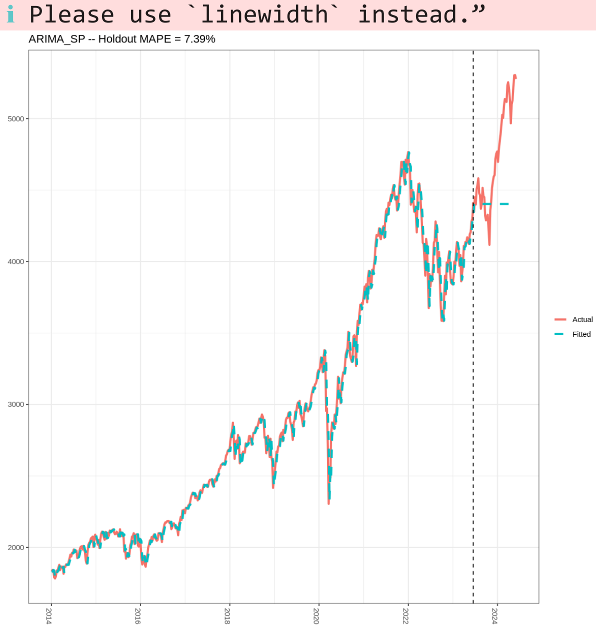
In [ ]:
```
arima_sp <- auto.arima(sp_train, trace=T, allowdrift = FALSE)
```

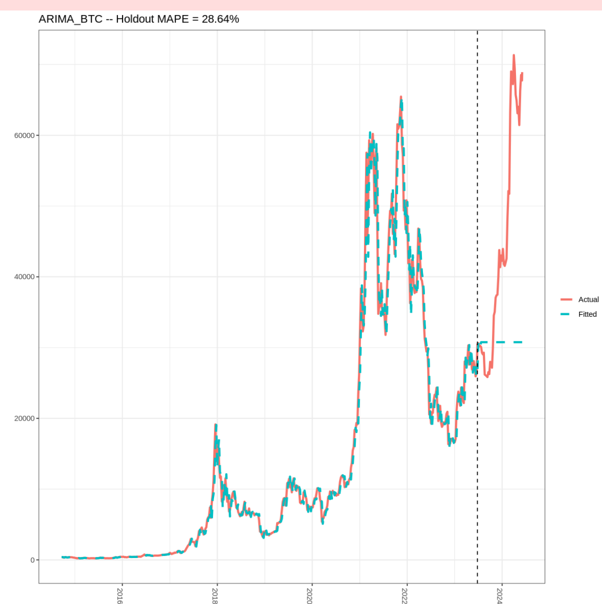
```
Fitting models using approximations to speed things up...

ARIMA(2,1,2)                    : 5630.801
ARIMA(0,1,0)                    : 5628.865
ARIMA(1,1,0)                    : 5629.729
ARIMA(0,1,1)                    : 5628.854
ARIMA(1,1,1)                    : 5631.609
ARIMA(0,1,2)                    : 5630.442
ARIMA(1,1,2)                    : 5633.431

Now re-fitting the best model(s) without approximations...

ARIMA(0,1,1)                    : 5637.451

Best model: ARIMA(0,1,1)
```

In [ ]:
```
d1 <- data.frame(c(fitted(arima_sp), # fitted and predicted
                   predict(arima_sp, n.ahead = n)$pred),
                 Cl(sp_price),
                 time(sp_price)
                )
names(d1) <- c("Fitted", "Actual", "Date")

# MAPE <- filter(d1, year(Date)>2021) %>% summarise(MAPE=mean(abs(Actual-Fitted)/Ac
MAPE <- tail(d1, n) %>% summarise(MAPE=mean(abs(Actual-Fitted)/Actual))

### Plot actual versus predicted
```

```
options(repr.plot.width = 20, repr.plot.height = 10)

plot1 <- ggplot(data=d1, aes(x=Date)) +
  geom_line(aes(y=Actual, colour = "Actual"), size=1.2) +
  geom_line(aes(y=Fitted, colour = "Fitted"), size=1.2, linetype=2) +
  theme_bw() + theme(legend.title = element_blank()) +
  ylab("") + xlab("") +
  geom_vline(xintercept=as.numeric(d1[sp_len,]$Date), linetype=2) +
  ggtitle(paste0("ARIMA_SP -- Holdout MAPE = ", round(100*MAPE,2), "%")) +
  theme(axis.text.x=element_text(angle = -90, hjust = 0))

d1 <- data.frame(c(fitted(arima_btc), # fitted and predicted
                   predict(arima_btc, n.ahead = n)$pred),
                 Cl(btc_price),
                 time(btc_price)
                )
names(d1) <- c("Fitted", "Actual", "Date")

# MAPE <- filter(d1, year(Date)>2021) %>% summarise(MAPE=mean(abs(Actual-Fitted)/Ac
MAPE <- tail(d1, n) %>% summarise(MAPE=mean(abs(Actual-Fitted)/Actual))

### Plot actual versus predicted
options(repr.plot.width = 20, repr.plot.height = 10)

plot2 <- ggplot(data=d1, aes(x=Date)) +
  geom_line(aes(y=Actual, colour = "Actual"), size=1.2) +
  geom_line(aes(y=Fitted, colour = "Fitted"), size=1.2, linetype=2) +
  theme_bw() + theme(legend.title = element_blank()) +
  ylab("") + xlab("") +
  geom_vline(xintercept=as.numeric(d1[btc_len,]$Date), linetype=2) +
  ggtitle(paste0("ARIMA_BTC -- Holdout MAPE = ", round(100*MAPE,2), "%")) +
  theme(axis.text.x=element_text(angle = -90, hjust = 0))

grid.arrange(plot1, plot2, ncol=2)
```

```
Warning message:
“Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
ℹ Please use `linewidth` instead.”
```





Mean Absolute Percentage Error (MAPE)

- MAPE measures the difference of forecast errors and divides it by the actual observation value.
- It is scale-independent, thus can be used to compare a model's performance across different datasets.

Model Performance

We observe that the ARIMA model' s performance on the highly volatile Bitcoin (BTC) and S&P500 is not good, as it completely misses the next uptrend in the price action and instead tries to guide the time series back to its moving average.

This indicates the inability of the ARIMA model to understand the volatility of stock prices. In other words, the model provides no information on the underlying distribution of the time series itself.

4.2. Bayesian Structural Time Series (BSTS)

BSTS is a more transparent approach because its representation does not rely on differencing, lags, and moving averages. You can visually inspect the underlying components of the model. It handles uncertainty in a better way because you can quantify the posterior uncertainty of the individual components, control the variance of the components, and impose prior beliefs on the model. Additionally, any ARIMA model can be recast as a structural model.

Generally, we can write a Bayesian structural model as:

$$Y_t = \mu_t + x_t\beta + S_t + e_t, e_t \sim N\left(0, \sigma_e^2\right)$$
$$\mu_{t+1} = \mu_t + \nu_t, \nu_t \sim N\left(0, \sigma_\nu^2\right)$$

Where:

- x_t: a set of regressors
- S_t: seasonality
- μ_t: the local level term, defines how the latent state evolves over time and is often referred to as the unobserved trend.

Note that the regressor coefficients, seasonality, and trend are estimated simultaneously, which helps avoid strange coefficient estimates due to spurious relationships (similar in spirit to Granger causality). Additionally, due to the Bayesian nature of the model, we can shrink the elements of β to promote sparsity or specify outside priors for the means in case we're not able to get meaningful estimates from the historical data.

In [ ]:
```
btc_price <- to.weekly(btc_price)
sp_price <- to.weekly(sp_price)
```

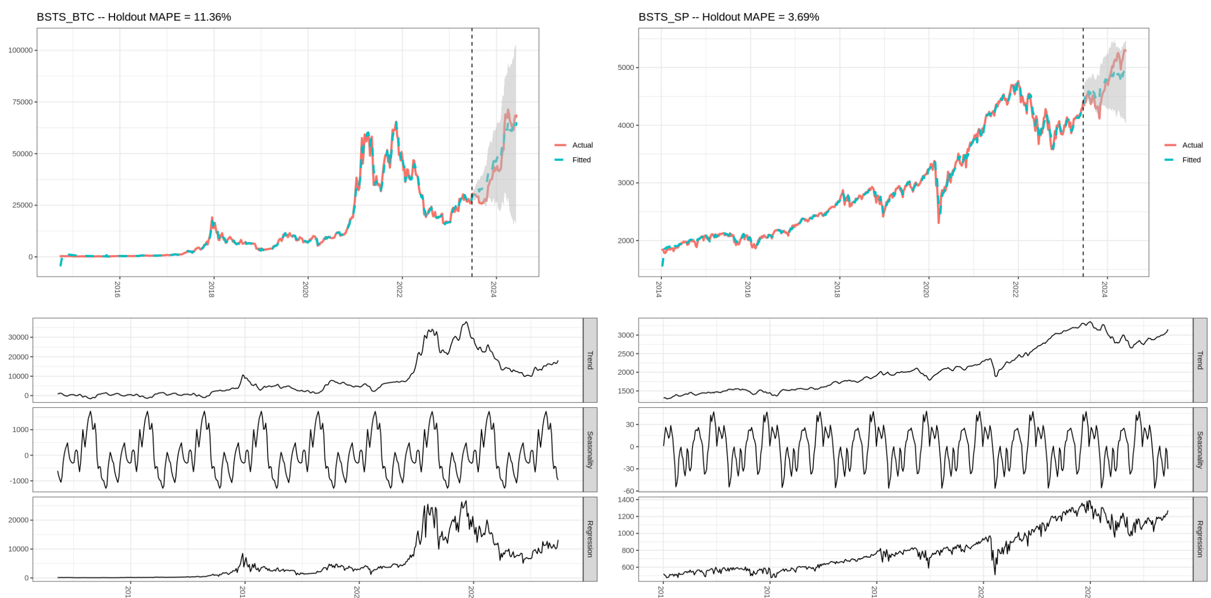
```
n <- 50
num_data_btc <- length(btc_price)/6
num_tr_btc <- num_data_btc - n

# Splitting the data
btc_price <- zoo(btc_price)
btc_train <- btc_price[1:num_tr_btc]
btc_test  <- btc_price[(num_tr_btc+1):num_data_btc]

# Number of period we want to forecast
n <- 50
num_data_sp <- length(sp_price)/6
num_tr_sp <- num_data_sp - n

# Splitting the data
sp_price <- zoo(sp_price)
sp_train <- sp_price[1:num_tr_sp]
sp_test  <- sp_price[(num_tr_sp+1):num_data_sp]
```

4.2.1. Fitting & Predict

In [ ]:
```
ss <- AddLocalLinearTrend(list(), btc_train$btc_price.Close)
ss <- AddSeasonal(ss, btc_train$btc_price.Close, nseasons = 52)
bsts.reg.btc <- bsts(btc_train$btc_price.Close ~ ., state.specification = ss,
                 data = btc_train, niter = 1000, seed=100, expected.model.size=1, p

### Get a suggested number of burn-ins
burn <- SuggestBurn(0.1, bsts.reg.btc)

### Predict
p <- predict.bsts(bsts.reg.btc, newdata=btc_test, burn = burn, quantiles = c(.025,

### Actual versus predicted
d2 <- data.frame(
    c(as.numeric(-colMeans(bsts.reg.btc$one.step.prediction.errors[-(1:burn),])+Cl(
      as.numeric(p$mean)),
    # actual data and dates
    as.numeric(zoo(Cl(btc_price))),
    time(btc_price))
names(d2) <- c("Fitted", "Actual", "Date")

### MAPE (mean absolute percentage error)
MAPE <- tail(d2, n) %>% summarise(MAPE=mean(abs(Actual-Fitted)/Actual))

### 95% forecast credible interval
posterior.interval <- cbind.data.frame(
  as.numeric(p$interval[1,]),
  as.numeric(p$interval[2,]),
  tail(d2, 50)$Date)
names(posterior.interval) <- c("LL", "UL", "Date")

### Join intervals to the forecast
d3 <- left_join(d2, posterior.interval, by="Date")
```

```
### Plot actual versus predicted with credible intervals for the holdout period
options(repr.plot.width = 20, repr.plot.height = 10)
plot1 <- ggplot(data=d3, aes(x=Date)) +
  geom_line(aes(y=Actual, colour = "Actual"), size=1.2) +
  geom_line(aes(y=Fitted, colour = "Fitted"), size=1.2, linetype=2) +
  theme_bw() + theme(legend.title = element_blank()) + ylab("") + xlab("") +
  geom_vline(xintercept=as.numeric(d3[btc_len,]$Date), linetype=2) +
  geom_ribbon(aes(ymin=LL, ymax=UL), fill="grey", alpha=0.5) +
  ggtitle(paste0("BSTS_BTC -- Holdout MAPE = ", round(100*MAPE,2), "%")) +
  theme(axis.text.x=element_text(angle = -90, hjust = 0))

### Get the components
components.withreg <- cbind.data.frame(
  colMeans(bsts.reg.btc$state.contributions[-(1:burn),"trend",]),
  colMeans(bsts.reg.btc$state.contributions[-(1:burn),"seasonal.52.1",]),
  colMeans(bsts.reg.btc$state.contributions[-(1:burn),"regression",]),
  as.Date(time(btc_train)))
names(components.withreg) <- c("Trend", "Seasonality", "Regression", "Date")
components.withreg <- melt(components.withreg, id.vars="Date")
names(components.withreg) <- c("Date", "Component", "Value")

plot3 <- ggplot(data=components.withreg, aes(x=Date, y=Value)) + geom_line() +
  theme_bw() + theme(legend.title = element_blank()) + ylab("") + xlab("") +
  facet_grid(Component ~ ., scales="free") + guides(colour="none") +
  theme(axis.text.x=element_text(angle = -90, hjust = 0))

###
###

ss <- AddLocalLinearTrend(list(), sp_train$sp_price.Close)
ss <- AddSeasonal(ss, sp_train$sp_price.Close, nseasons = 52)
bsts.reg.sp <- bsts(sp_train$sp_price.Close ~ ., state.specification = ss,
                 data = sp_train, niter = 1000, seed=100, expected.model.size=1, pi

### Get a suggested number of burn-ins
burn <- SuggestBurn(0.1, bsts.reg.sp)

### Predict
p <- predict.bsts(bsts.reg.sp, newdata=sp_test, burn = burn, quantiles = c(.025, .9

### Actual versus predicted
d2 <- data.frame(
    c(as.numeric(-colMeans(bsts.reg.sp$one.step.prediction.errors[-(1:burn),])+Cl(s
      as.numeric(p$mean)),
    # actual data and dates
    as.numeric(zoo(Cl(sp_price))),
    time(sp_price))
names(d2) <- c("Fitted", "Actual", "Date")

### MAPE (mean absolute percentage error)
MAPE <- tail(d2, n) %>% summarise(MAPE=mean(abs(Actual-Fitted)/Actual))

### 95% forecast credible interval
posterior.interval <- cbind.data.frame(
  as.numeric(p$interval[1,]),
  as.numeric(p$interval[2,]),
```

```
  tail(d2, 50)$Date)
names(posterior.interval) <- c("LL", "UL", "Date")

### Join intervals to the forecast
d3 <- left_join(d2, posterior.interval, by="Date")

### Plot actual versus predicted with credible intervals for the holdout period
options(repr.plot.width = 20, repr.plot.height = 10)
plot2 <- ggplot(data=d3, aes(x=Date)) +
  geom_line(aes(y=Actual, colour = "Actual"), size=1.2) +
  geom_line(aes(y=Fitted, colour = "Fitted"), size=1.2, linetype=2) +
  theme_bw() + theme(legend.title = element_blank()) + ylab("") + xlab("") +
  geom_vline(xintercept=as.numeric(d3[sp_len,]$Date), linetype=2) +
  geom_ribbon(aes(ymin=LL, ymax=UL), fill="grey", alpha=0.5) +
  ggtitle(paste0("BSTS_SP -- Holdout MAPE = ", round(100*MAPE,2), "%")) +
  theme(axis.text.x=element_text(angle = -90, hjust = 0))

### Get the components
components.withreg <- cbind.data.frame(
  colMeans(bsts.reg.sp$state.contributions[-(1:burn),"trend",]),
  colMeans(bsts.reg.sp$state.contributions[-(1:burn),"seasonal.52.1",]),
  colMeans(bsts.reg.sp$state.contributions[-(1:burn),"regression",]),
  as.Date(time(sp_train)))
names(components.withreg) <- c("Trend", "Seasonality", "Regression", "Date")
components.withreg <- melt(components.withreg, id.vars="Date")
names(components.withreg) <- c("Date", "Component", "Value")

plot4 <- ggplot(data=components.withreg, aes(x=Date, y=Value)) + geom_line() +
  theme_bw() + theme(legend.title = element_blank()) + ylab("") + xlab("") +
  facet_grid(Component ~ ., scales="free") + guides(colour="none") +
  theme(axis.text.x=element_text(angle = -90, hjust = 0))

grid.arrange(plot1, plot2, plot3, plot4, ncol=2, nrow=2)
```



Model Evaluation

- For the S&P 500 index, the BSTS model outperforms the ARIMA model in terms of MAPE score. Additionally, the BSTS model provides a confidence interval for its predictions, generated from the distribution of the MCMC draws. Moreover, while the BSTS model has a significantly higher MAPE score for Bitcoin (BTC), it demonstrates its ability to predict the uptrend followed by a downtrend in future price action. This indicates the model's understanding of the underlying time-series distribution and volatility.
- Furthermore, one of the major advantages of the Bayesian structural model is the ability to visualize the underlying components of the time series using the model's learned distribution.

4.2.2. Bayesian Variable Selection

Spike-and-Slab Prior

- A spike-and-slab prior is a prior distribution that induces sparsity in the posterior. Unlike the Lasso prior (Laplace), which only yields MAP estimates at zero but nonzero posterior, a spike-and-slab prior is a natural way to express a prior belief that most of the regression coefficients are exactly zero by explicitly assigning positive probability of being zero on the spike part of the prior. This directly induces sparsity in the posterior due to conjugacy. Thus, the prediction is computed from the posterior predictive distribution, which is independent of the corresponding variables.
- A spike-and-slab prior can be written as:

$$p\left(\beta, \gamma, \sigma_\epsilon^2\right)=p\left(\beta_\gamma \mid \gamma, \sigma_\epsilon^2\right) p\left(\sigma_\epsilon^2 \mid \gamma\right) p(\gamma)$$

- The last factor is the "spike" part that governs the probability of a given variable being chosen for the model (i.e., having a non-zero coefficient).
- The other factors are modeled to be the "slab" - The part that shrinks the coefficients toward prior expectations (often zero).

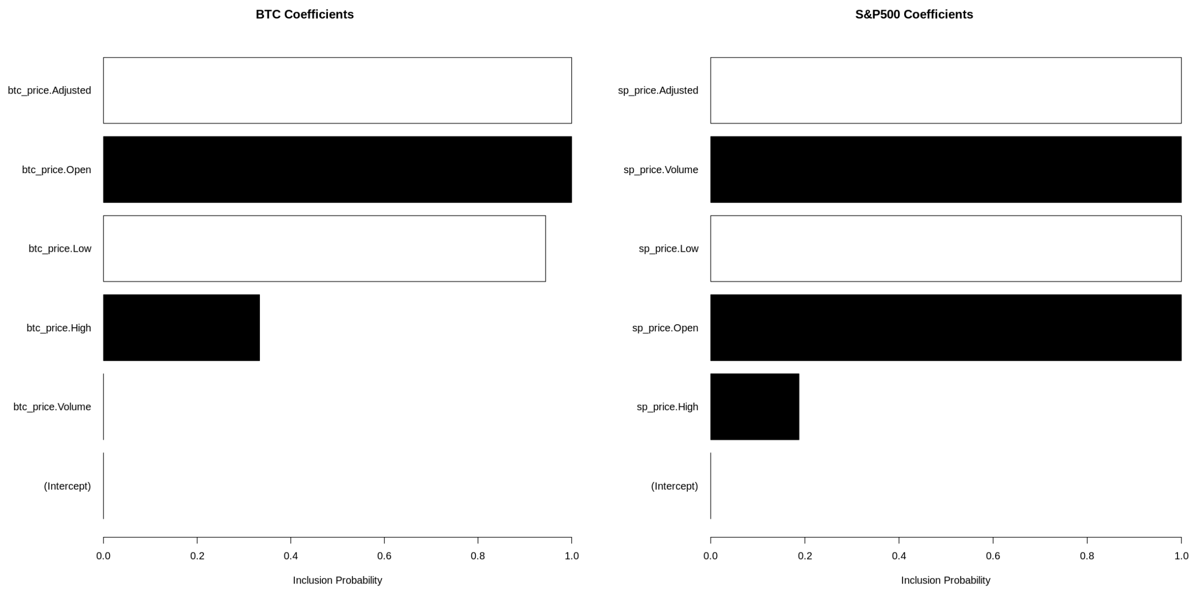
BSTS Model with Spike-and-Slab Priors

- The BSTS model imposes a *spike-and-slab prior* on the regressor coefficients, which is a powerful way of reducing a large set of correlated variables.
- It allows the model to incorporate uncertainties of the coefficient estimates when producing the credible interval for the forecasts.

Prior Parameter Tuning

- We analyze which regression coefficients the previous BSTS models use to predict future prices and try to change the prior parameters (from default values) based on our practical belief to improve performance.

In [ ]:
```
par(mfrow = c(1, 2))
plot(bsts.reg.btc, "coef")
title("BTC Coefficients")
plot(bsts.reg.sp, "coef")
title("S&P500 Coefficients")
```



Through visualizing the inclusion probabilities of all regression coefficients in the previous BSTS models, we observe that both models utilize the Adjusted, High, and Low prices as predictors. However, these variables are highly correlated.

On the other hand, the S&P 500 model also incorporates trading volumes as a predictor. In technical analysis, volume analysis is one of the main indicators representing market strength. Therefore, we may want to include volume as a predictor in the BTC model as well, given the superior performance of the S&P 500 model.

To explicitly impose our prior belief that volume should be included, we use a spike-and-slab prior with the spike parameter set to 1 for the volume coefficient.

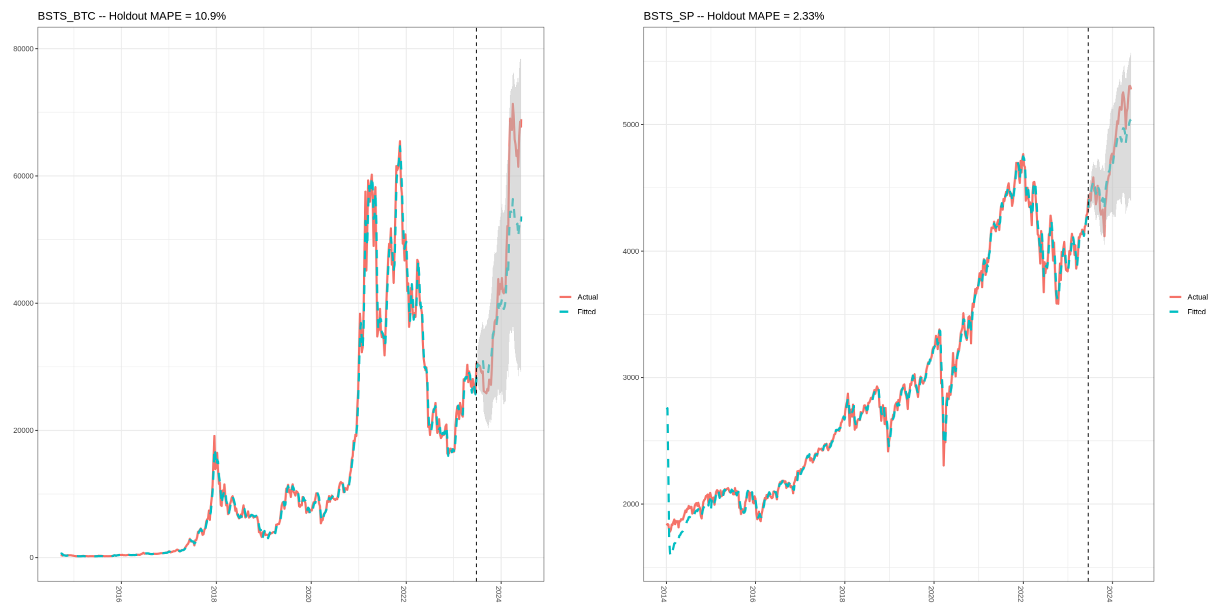
Additionally, to reduce the correlation among price-related coefficients, we aim to use only the Adjusted price as our main predictor while restricting the others. This is achieved by setting the expected model size to 2 (Volume + Adjusted_price).

In [ ]:
```
prior.spikes <- c(0.1,0.1,0.1,0.1,1.0,1.0)
prior.mean   <- c(0  ,0  ,0  ,0  ,0.0,0.5)

### Set up the priors
prior <- SpikeSlabPrior(x=model.matrix(btc_price.Close ~ ., data=btc_train),
                        y=btc_train$btc_price.Close,
                        prior.information.weight = 200,
                        prior.inclusion.probabilities = prior.spikes,
                        optional.coefficient.estimate = prior.mean)

ss <- AddLocalLinearTrend(list(), btc_train$btc_price.Close)
```

```
ss <- AddSeasonal(ss, btc_train$btc_price.Close, nseasons = 52)
bsts.reg.btc <- bsts(btc_train$btc_price.Close ~ ., state.specification = ss, prior
                 data = btc_train, niter = 1000, seed=100, expected.model.size=2, p

### Get a suggested number of burn-ins
burn <- SuggestBurn(0.1, bsts.reg.btc)

### Predict
p <- predict.bsts(bsts.reg.btc, newdata=btc_test, burn = burn, quantiles = c(.025,

### Actual versus predicted
d2 <- data.frame(
    c(as.numeric(-colMeans(bsts.reg.btc$one.step.prediction.errors[-(1:burn),])+Cl(
      as.numeric(p$mean)),
    # actual data and dates
    as.numeric(zoo(Cl(btc_price))),
    time(btc_price))
names(d2) <- c("Fitted", "Actual", "Date")

### MAPE (mean absolute percentage error)
MAPE <- tail(d2, n) %>% summarise(MAPE=mean(abs(Actual-Fitted)/Actual))

### 95% forecast credible interval
posterior.interval <- cbind.data.frame(
  as.numeric(p$interval[1,]),
  as.numeric(p$interval[2,]),
  tail(d2, 50)$Date)
names(posterior.interval) <- c("LL", "UL", "Date")

### Join intervals to the forecast
d3 <- left_join(d2, posterior.interval, by="Date")

### Plot actual versus predicted with credible intervals for the holdout period
options(repr.plot.width = 20, repr.plot.height = 10)
plot1 <- ggplot(data=d3, aes(x=Date)) +
  geom_line(aes(y=Actual, colour = "Actual"), size=1.2) +
  geom_line(aes(y=Fitted, colour = "Fitted"), size=1.2, linetype=2) +
  theme_bw() + theme(legend.title = element_blank()) + ylab("") + xlab("") +
  geom_vline(xintercept=as.numeric(d3[btc_len,]$Date), linetype=2) +
  geom_ribbon(aes(ymin=LL, ymax=UL), fill="grey", alpha=0.5) +
  ggtitle(paste0("BSTS_BTC -- Holdout MAPE = ", round(100*MAPE,2), "%")) +
  theme(axis.text.x=element_text(angle = -90, hjust = 0))

### Get the components
components.withreg <- cbind.data.frame(
  colMeans(bsts.reg.btc$state.contributions[-(1:burn),"trend",]),
  colMeans(bsts.reg.btc$state.contributions[-(1:burn),"seasonal.52.1",]),
  colMeans(bsts.reg.btc$state.contributions[-(1:burn),"regression",]),
  as.Date(time(btc_train)))
names(components.withreg) <- c("Trend", "Seasonality", "Regression", "Date")
components.withreg <- melt(components.withreg, id.vars="Date")
names(components.withreg) <- c("Date", "Component", "Value")

plot3 <- ggplot(data=components.withreg, aes(x=Date, y=Value)) + geom_line() +
  theme_bw() + theme(legend.title = element_blank()) + ylab("") + xlab("") +
  facet_grid(Component ~ ., scales="free") + guides(colour="none") +
```

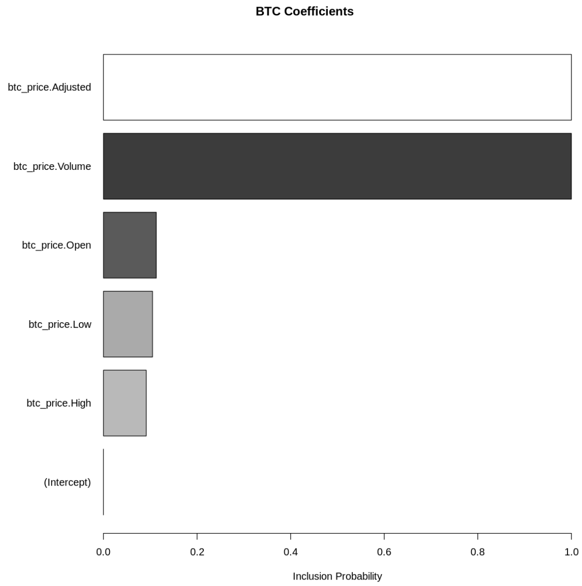
```
  theme(axis.text.x=element_text(angle = -90, hjust = 0))

#####
#####
#####
prior.spikes <- c(0.1,0.1,0.1,0.1,1.0,1.0)
prior.mean   <- c(0  ,0  ,0  ,0  ,0.0,0.5)

### Set up the priors
prior <- SpikeSlabPrior(x=model.matrix(sp_train$sp_price.Close ~ ., data=sp_train),
                        y=sp_train$sp_price.Close,
                        prior.information.weight = 200,
                        prior.inclusion.probabilities = prior.spikes,
                        optional.coefficient.estimate = prior.mean)


ss <- AddLocalLinearTrend(list(), sp_train$sp_price.Close)
ss <- AddSeasonal(ss, sp_train$sp_price.Close, nseasons = 52)
bsts.reg.sp <- bsts(sp_train$sp_price.Close ~ ., state.specification = ss,  prior=p
                 data = sp_train, niter = 1000, seed=100, expected.model.size=2, pi

### Get a suggested number of burn-ins
burn <- SuggestBurn(0.1, bsts.reg.sp)

### Predict
p <- predict.bsts(bsts.reg.sp, newdata=sp_test, burn = burn, quantiles = c(.025, .9

### Actual versus predicted
d2 <- data.frame(
    c(as.numeric(-colMeans(bsts.reg.sp$one.step.prediction.errors[-(1:burn),])+Cl(s
      as.numeric(p$mean)),
    # actual data and dates
    as.numeric(zoo(Cl(sp_price))),
    time(sp_price))
names(d2) <- c("Fitted", "Actual", "Date")

### MAPE (mean absolute percentage error)
MAPE <- tail(d2, n) %>% summarise(MAPE=mean(abs(Actual-Fitted)/Actual))

### 95% forecast credible interval
posterior.interval <- cbind.data.frame(
  as.numeric(p$interval[1,]),
  as.numeric(p$interval[2,]),
  tail(d2, 50)$Date)
names(posterior.interval) <- c("LL", "UL", "Date")

### Join intervals to the forecast
d3 <- left_join(d2, posterior.interval, by="Date")

### Plot actual versus predicted with credible intervals for the holdout period
options(repr.plot.width = 20, repr.plot.height = 10)
plot2 <- ggplot(data=d3, aes(x=Date)) +
  geom_line(aes(y=Actual, colour = "Actual"), size=1.2) +
  geom_line(aes(y=Fitted, colour = "Fitted"), size=1.2, linetype=2) +
  theme_bw() + theme(legend.title = element_blank()) + ylab("") + xlab("") +
  geom_vline(xintercept=as.numeric(d3[sp_len,]$Date), linetype=2) +
```

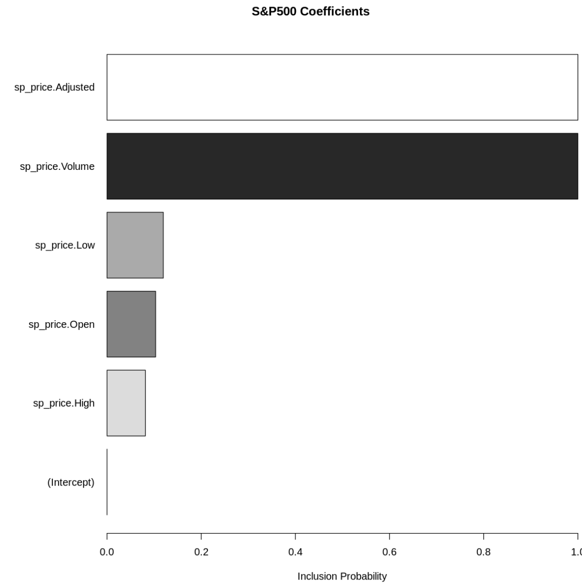
```
  geom_ribbon(aes(ymin=LL, ymax=UL), fill="grey", alpha=0.5) +
  ggtitle(paste0("BSTS_SP -- Holdout MAPE = ", round(100*MAPE,2), "%")) +
  theme(axis.text.x=element_text(angle = -90, hjust = 0))

### Get the components
components.withreg <- cbind.data.frame(
  colMeans(bsts.reg.sp$state.contributions[-(1:burn),"trend",]),
  colMeans(bsts.reg.sp$state.contributions[-(1:burn),"seasonal.52.1",]),
  colMeans(bsts.reg.sp$state.contributions[-(1:burn),"regression",]),
  as.Date(time(sp_train)))
names(components.withreg) <- c("Trend", "Seasonality", "Regression", "Date")
components.withreg <- melt(components.withreg, id.vars="Date")
names(components.withreg) <- c("Date", "Component", "Value")

plot4 <- ggplot(data=components.withreg, aes(x=Date, y=Value)) + geom_line() +
  theme_bw() + theme(legend.title = element_blank()) + ylab("") + xlab("") +
  facet_grid(Component ~ ., scales="free") + guides(colour="none") +
  theme(axis.text.x=element_text(angle = -90, hjust = 0))

grid.arrange(plot1, plot2, ncol=2)
```



The BSTS model with our prior performs slightly better for both BTC and S&P500 with MAPE is 10.9% and 2.33% comparing to 11.26% and 3.69%, respectively

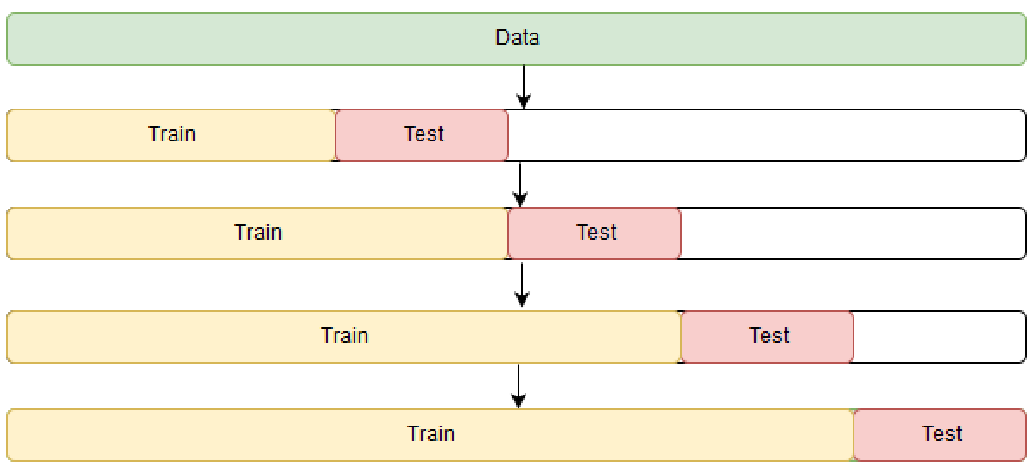
In [ ]:
```
par(mfrow = c(1, 2))
plot(bsts.reg.btc, "coef") #, x="BTC Coefficients")
title("BTC Coefficients")
plot(bsts.reg.sp, "coef")#, xlab="S&P Coefficients")
title("S&P500 Coefficients")
```





4.2.3. Time series cross-validation

- Forward chaining involves cross-validation on a rolling basis.
- The process starts with a small subset of data for training purposes, followed by forecasting for the later data points and then checking the accuracy for the forecasted data points.
- The same forecasted data points are then included as part of the next training dataset, and subsequent data points are forecasted.



In [ ]:
```
bsts.cv.loop <- function(data,
                         number_of_folds,
                         nseasons, model_size,
                         horizon=50,
                         niter=1000,
                         seed=100,
                         verbose=FALSE,
```

```
                          debug=FALSE) {
  mape_v <- c()
  for (fold in 1:number_of_folds) {
    # construct data_train/data_test
    l <- length(data)/6 - fold*horizon
    # if (debug) print(l)
    data <- zoo(data)
    data_train <- data[1:l]
    data_test  <- data[(l+1):(l+horizon)]

    prior.spikes <- c(0.1,0.1,0.1,0.1,1.0,1.0)
    prior.mean   <- c(0  ,0  ,0  ,0  ,0.0,0.5)

    ### Set up the priors
    prior <- SpikeSlabPrior(x=model.matrix(Close ~ ., data=data_train),
                            y=Cl(data_train),
                            prior.information.weight = 200,
                            prior.inclusion.probabilities = prior.spikes,
                            optional.coefficient.estimate = prior.mean)

    ss <- AddLocalLinearTrend(list(), Cl(data_train))
    ss <- AddSeasonal(ss, Cl(data_train), nseasons=nseasons)
    model <- bsts(Close~., data=data_train,
                  state.specification=ss,  expected.model.size=model_size,
                  niter=niter,
                  seed=seed, prior=prior,
                  ping=0)
    burn <- SuggestBurn(0.1, model)

    ### Predict
    pred <- predict.bsts(model, newdata=data_test, burn = burn, quantiles = c(.025,
    # print(length(data_test))
    # print(length(p$mean))

    ### Actual versus predicted
    d2 <- data.frame(
        c(as.numeric(-colMeans(model$one.step.prediction.errors[-(1:burn),])+Cl(dat
          as.numeric(pred$mean)),
        # actual data and dates
        as.numeric(zoo(Cl(data[1:(l+horizon)]))),
        time(data[1:(l+horizon)]))
    names(d2) <- c("Fitted", "Actual", "Date")

    ### MAPE (mean absolute percentage error)
    MAPE <- tail(d2, n) %>% summarise(MAPE=mean(abs(Actual-Fitted)/Actual))

    # evaluation
    mape <- round(MAPE[1,] * 100, 2)
    mape_v <- c(mape_v, mape)
    if (verbose) print(paste0("fold ", fold, ": mape ", mape))
  }
  return(data.frame(mape=mape_v))
}
```

In [ ]:

```
btc_price <- getSymbols('BTC-USD', auto.assign=FALSE, from="2014-01-01")
btc_price <- to.weekly(btc_price)
```

```r
names(btc_price) <- c('Open','High','Low','Close','Volume','Adjusted')

res_btc <- c()
for (ns in c(5, 10, 20)) {
    for (sz in c(1,2,3)) {
        res <- bsts.cv.loop(btc_price, 5, ns, sz)
        res_row <- data.frame(ns=ns,
                              sz=sz,
                              mean_mape=mean(res$mape))
        res_btc <- rbind(res_btc, res_row)
    }
}
print("BTC CV scores:")
print(res_btc)
b <- res_btc[which.min(res_btc$mean_mape),]
cat(paste0("Best model for BTC has ", b$ns, " seasons and ", b$sz, " regression coe

sp_price <- getSymbols('^GSPC', auto.assign=FALSE, from="2014-01-01")
sp_price <- to.weekly(sp_price)
names(sp_price) <- c('Open','High','Low','Close','Volume','Adjusted')

res_sp <- c()
for (ns in c(5, 10, 20)) {
    for (sz in c(1,2,3)) {
        res <- bsts.cv.loop(sp_price, 5, ns, sz)
        res_row <- data.frame(ns=ns,
                              sz=sz,
                              mean_mape=mean(res$mape))
        res_sp <- rbind(res_sp, res_row)
    }
}
cat('\n')
cat("S&P500 CV scores:")
print(res_sp)
b <- res_sp[which.min(res_sp$mean_mape),]
cat(paste0("Best model for S&P500 has ", b$ns, " seasons and ", b$sz, " regression
```

```
Warning message:
“BTC-USD contains missing values. Some functions will not work if objects contain mi
ssing values in the middle of the series. Consider using na.omit(), na.approx(), na.
fill(), etc to remove or replace them.”
Warning message in to.period(x, "weeks", name = name, ...):
“missing values removed from data”
[1] "BTC CV scores:"
  ns sz mean_mape
1  5  1    24.336
2  5  2    24.344
3  5  3    24.050
4 10  1    22.240
5 10  2    22.416
6 10  3    21.794
7 20  1    21.798
8 20  2    22.196
9 20  3    21.714
Best model for BTC has 20 seasons and 3 regression coefficients with MAPE = 21.714
```

5. Discussion

5.1. Impact

The forecasting models employed in this study have demonstrated their capability to provide insightful information regarding future price movements and volatility. This information can serve as a valuable technical indicator, complementing other statistical indicators, to aid traders in identifying optimal entry points for trading stocks. However, it is crucial to recognize that these models, like any other technical indicator, are most effective when combined with sound personal judgment, which remains the pivotal factor influencing the variables in the trading profit optimization model. For optimal performance, the overall model requires sufficient "experience" or training.

Moreover, the explanatory models can facilitate the diagnosis of price time-series characteristics, revealing the strengths and weaknesses of the corresponding stock. By explicitly introducing new variables through the spike-and-slab prior and observing the impact on model performance, we can identify useful indicators of future prices or even the generating distribution of the overall price time-series.

It is essential to emphasize that while these models offer valuable insights, they should be employed in conjunction with other analysis techniques and sound judgment. Successful trading necessitates a holistic approach that considers various factors, including market dynamics, risk management, and a deep understanding of the underlying assets.

5.2. Limitations and Future Direction

The performance of forecasting models is fundamentally correlated with a deep understanding of the corresponding stock's dynamics. Notably, the more volatile the price action, the greater the uncertainty in the model's future predictions. This uncertainty amplifies significantly as the forecast horizon extends further into the future.

There exist numerous technical indicators, either independent or derived from price data, that can be incorporated into the current model to enhance its predictive capabilities.

One such approach is the CausalImpact R package, which implements a method for estimating the causal effect of a designed intervention on a time series. This package builds a Bayesian structural time series model based on multiple comparable control groups (or markets) and utilizes this model to project (or forecast) a series of baseline values for the time period after the event of interest. By incorporating this approach, the model's ability to account for causal factors and measure their impact on the target time series could be augmented, potentially improving its overall performance.

6. Conclusion

Here is a paraphrased and rewritten version:

In this project, we aim to learn and address time-series-related problems through a case study on stock price analysis for Bitcoin (BTC) and the S&P 500 index. First, we conduct a time-series analysis on the price actions of each stock, allowing us to determine their representative characteristics. Subsequently, we construct an ARIMA (Autoregressive Integrated Moving Average) model as a baseline to predict future trends of the stock based on its historical price moving average and lagged values.

We then build an explanatory model for each time series using the Bayesian Structural Time Series (BSTS) method. These models provide an actual understanding of the underlying generative distribution of the time series and effectively handle the uncertainty of price volatility. Finally, we significantly improve the BSTS models through Bayesian variable selection and forward-chaining cross-validation.

The results suggest that the output predictions and confidence intervals from these models can serve as informative indicators of the corresponding stock's future price movements. The BSTS models, augmented by variable selection and cross-validation techniques, offer valuable insights into the underlying dynamics and potential future behavior of the time series, accounting for uncertainties and volatilities inherent in stock price data.

A. References

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- https://www.investopedia.com/
- https://sisifospage.tech/2017-10-30-forecasting-bsts.html
- https://www.rpubs.com/AurelliaChristie/time-series-and-stock-analysis
- https://www.unofficialgoogledatascience.com/2017/07/fitting-bayesian-structural-time-series.html