Practice Exam 1 Name: _____

This practice exam is longer than the real exam, but it should give you a good idea of the kind of question I might ask. This exam is open book and note; you may use DyKnow, Moodle, FoCSipedia, and a PDF reader, but not anything else on the internet. Please take some time to check your work. If you need extra space, write on the back. You must show your work to receive any partial credit.

1. (5 points) The `bool` type in ReasonML allows us to do pattern matching on its values. This enables us to write expressions such as

   ```
   switch (test) {
   | true => returnThisIfTrue
   | false => returnThisIfFalse
   }
   ```

   Note that this is equivalent to the following `if` expression:

   ```
   if (test) {
     returnThisIfTrue
   } else {
     returnThisIfFalse
   }
   ```

   One advantage of the pattern-matching expression, however, is that we may match on several values at once. We may also use the wildcard (`_`) to match an arbitrary value:

   ```
   switch (test1, test2) {
   | (false, false) => false
   | (false, true) => true
   | (true, _) => true
   }
   ```

   (a) What logical operation does the above expression compute, in terms of `test1` and `test2`?

   (b) Write a switch statement that corresponds to the implication `test1` $\rightarrow$ `test2`.

2. (5 points) Consider the following ReasonML function:

```
let rec f = (a, nums) => {
  switch (nums) {
  | [] => false
  | [head, ...tail] =>
    if (head < a) {
      f(a, tail)
    } else if (head == a) {
      true
    } else {
      false
    }
  }
}
```

(a) For which of the following will the result be `true` (there may be more than one)?

i. `f(4, [1, 4, 9, 16])`
ii. `f(5, [1, 4, 9, 16])`
iii. `f(25, [1, 4, 9, 16])`
iv. `f(0, [3, 1, 4, 1, 5])`
v. `f(3, [3, 1, 4, 1, 5])`
vi. `f(1, [3, 1, 4, 1, 5])`
vii. `f(5, [3, 1, 4, 1, 5])`

(b) The function call `f(a, nums)` does not always return `true` when the list `nums` contains the value `a`. What additional condition could we place on `nums` so that `f(a, nums)` will correctly tell whether `a` is an element or not?

3. (10 points) Consider the propositional logic expression $(P \vee Q \vee \neg R) \rightarrow (P \wedge Q)$.

   (a) Give a truth table showing the value of this expression for each combination of P, Q, and R.

   (b) Give an equivalent expression using only the operators $\wedge$, $\vee$, and $\neg$.

   (c) Draw a circuit that computes the expression using only AND, OR, and NOT gates.

   (d) Draw another version of the circuit using only two-input NAND gates.

4. (10 points) Suppose we have an integer function T with the following recursive definition:

$$\begin{cases} T(1) = 1 \\ T(N) = 2 \cdot T(N-1) + 1, \qquad (N > 1) \end{cases}$$

(a) Write a ReasonML function that computes T:

(b) Fill in the following table of values. For the last entry find a closed-form expression (that is, expression in terms of N but without any recursive use of T).

$T(1)$	$T(2)$	$T(3)$	$T(4)$	$T(N)$

(c) Prove by induction that your closed-form expression for $T(N)$ is correct for all $N \geq 1$.

5. (10 points) The "absorption law" of Boolean algebra says that $(A \vee (A \wedge B)) \leftrightarrow A$ is a tautology.

   (a) Check that this is true by constructing a truth table:

   (b) Prove that $A \vee (A \wedge B)$ is equivalent to A using the laws of Boolean algebra that we discussed in class.

   (c) Complete this natural deduction proof of the argument $\vdash (A \vee (A \wedge B)) \rightarrow A$:

$\ell_1 : A \vee (A \wedge B) \Rightarrow \{$	
$\quad \ell_2 : A \Rightarrow \{$	
$\quad\quad \ell_3 : A$	ℓ_2
$\quad \}$	
$\quad \ell_4 :$ _____ $\Rightarrow \{$	
$\quad\quad \ell_5 : A$	_____
$\quad \}$	
$\quad \ell_6 : A$	$\vee E,\ \ell_1, \ell_2, \ell_4$
$\}$	
$\ell_7 : (A \vee (A \wedge B)) \rightarrow A$	_____