

# Data Stream Analysis

Big Data Management

# Knowledge objectives

1. Explain the difference between generic one-pass algorithms and stream processing
2. Name the two challenges of stream processing
3. Name two solutions to limited processing capacity
4. Name three solutions to limited memory capacity

# Understanding Objectives

1. Decide the probability of keeping a new element or removing an old one from memory to keep equi-probability on load shedding
2. Decide the parameters of the hash function to get a representative result on load shedding
3. Decide the optimum number of hash functions in a Bloom filter
4. Approximate the probability of false positives in a Bloom filter
5. Calculate the weighted average of an attribute considering an exponentially decaying window
6. Decide if heavy hitters will show false positives

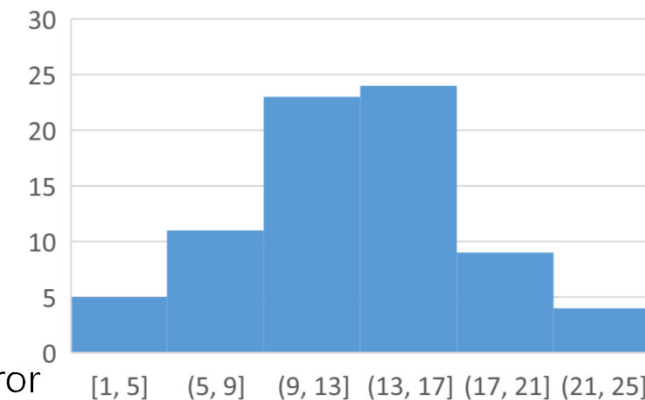
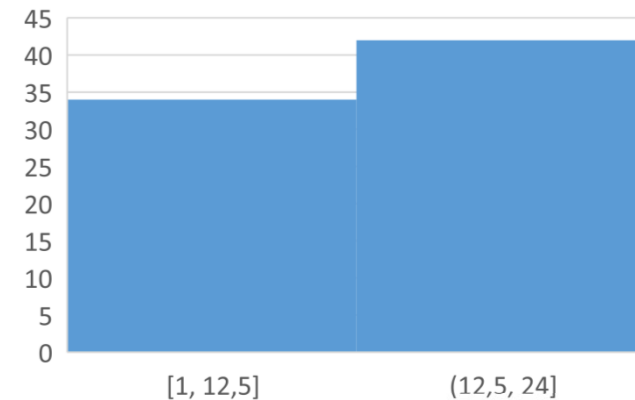
# Challenges and approaches

# Constraints

- Data cannot be stored
  - One-pass algorithms with
    - Bounded processing time
    - Bounded resources (i.e., memory)
      - At most, logarithmic on the size of the stream
    - Answer available at any time
- Processing must be on-line
  - Bounded response time for both
    - a) Summary update
    - b) Response retrieval

# Challenges and approaches

- Limited computation capacity
  - Sampling (i.e., Load shedding)
    - Probabilistically drop stream elements
  - Filtering (i.e., Bloom filters)
- Limited memory capacity
  - Sliding window -> Discard elements
    - Aging (use only most recent data)
  - Exponentially decaying window -> Weight elements
  - Synopsis -> Approximate solutions
    - Examples:
      - Histograms - Works under uniform distribution of values in a bucket
      - Concise sampling - Works under a limited number of distinct values
      - Heavy hitters - Uses logarithmic memory space
      - Sketching - Space needed depends on error and probability of that error

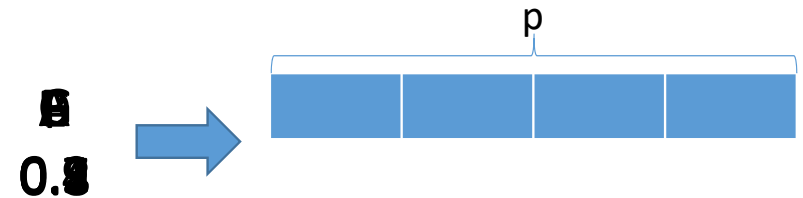


# Load shedding

Sampling data streams

# Load shedding (Keeping equi-probability)

- Mistakes in case of infinite streams:
  - a) Fix the values at the beginning
  - b) Remove old values from memory
- Goal (Uniform Random Sampling):
  - Any subset of elements has the same probability of being in memory at any time
  - Do not want to store any additional information
- Definitions:
  - Memory positions:  $p$
  - Elements seen:  $n$
- Solution (Reservoir sampling):
  - Probability of keeping the new element  $n+1$ 
    - $p/(n+1)$
  - Probability of removing an element from memory
    - $1/p$





# Load shedding (Statement)

"Select a subset of the stream so that answering ad-hoc queries gives a statistically representative result."

Example: Given a stream of tuples [user, query, time], we can store 10% of the tuples. If we randomly keep 1/10 of the tuples, then we would get the wrong answer to "Percentage of duplicate queries for a user"!!!

Definitions:

$s$  = #queries issued once by any user

$d$  = #queries issued twice by any user

No queries issued more than twice

The sample will contain:

$s/10 + 18d/100$  queries issued once

$d/100$  queries issued twice

The answer would be:

$$(d/100)/(s*10/100 + d*18/100 + d/100) = d/(10s + 19d) \neq d/(s + d)$$

Solution:

Keep 1/10 of the users

Before/After	Twice	Once	None
Once	0	$s*1/10$	$s*9/10$
Twice	$d*1/100$	$d*(9/10*1/10 + 1/10*9/10)$	$d*9/10*9/10$
Total	$d*1/100$	$s*10/100 + d*18/100$	...

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# Load shedding (Generalization)

- Queries may need different grouping keys or the key can be compound
  - Use the “group by” set in the hash function
- Memory is limited
  - Take a hash function to a large number of values  $M$  and keep only elements mapping to a value below  $t$ 
    - Dynamically reduce  $t$  as you are running out of memory

$$h(GB) = f(GB) \bmod M < t$$



# Bloom Filters

Filtering data streams

# Bloom filters (Statement)

“Accept those elements in the stream that meet a criterion (based on looking for membership in a set), others are dropped.”

- *Example*

- *Given an e-mail stream of tuples [address,text], we have a list of  $10^9$  allowed addresses (20 bytes each) and only 1GB of memory available.*

- *Solution*

- *Use the memory as an array of bits and map the addresses by means of a hash function ( $h$ : address  $\rightarrow$  bit position)*

- *Note: Some spam will get through the filter*

# Bloom filters (Example with one hash function)

Key values =  $\{IP_1, IP_2\}$

Hash function =  $\{h\}$

Array of bits  $\rightarrow$  0 0 0 0 0 0 0 0 0 0

Build

$$h(IP_1) = 3$$

$$h(IP_2) = 7$$

Probe

$$IP_3 \rightarrow h(IP_3) = 5$$

$$IP_4 \rightarrow h(IP_4) = 3$$

**FALSE POSITIVE!**

# Bloom filters (Example with two hash functions)

Key values =  $\{IP_1, IP_2\}$

Hash functions =  $\{h_1, h_2\}$

Array of bits  $\rightarrow$  0 0 0 0 0 0 0 0 0 0

Build 0 0 0 0 0 0 0 0 0 0

$h_1(IP_1) = 3$        $h_2(IP_1) = 5$

$h_1(IP_2) = 7$        $h_2(IP_2) = 5$

Probe

$IP_3 \rightarrow h_1(IP_3) = 5$        $h_2(IP_3) = 9$

$IP_4 \rightarrow h_1(IP_4) = 3$        $h_2(IP_4) = 7$

**FALSE POSITIVE!**

# Bloom filters (Generalization)

- Elements:
  - A set of  $m$  key values
  - A list of  $k$  hash functions ( $h_i: \text{key} \rightarrow n$ )
  - One array of  $n$  bits ( $n \gg m$ )
- Build:
  - For each element in the probing set, apply all  $k$  hash functions and set to 1 the corresponding bits
- Probing:
  - For each element in the stream, apply all  $k$  hash functions, it will pass only if all corresponding bits are set to 1
- False positives:
  - $(1 - e^{-km/n})^k$
- Optimal
  - $k = (n/m) \cdot \ln 2 \rightarrow (1 - e^{-km/n})^k = (1/2)^k \approx 0.618^{n/m}$

# Bloom filters (Rationale)

- Probability of a bit being set by a hash function at build phase  
 $1/n$
- Probability of a bit NOT being set by a hash function at build phase  
 $1-1/n$
- Probability of a bit NOT being set by a hash function of ANY key at build phase  
 $(1-1/n) \cdot (1-1/n) \cdot \dots \cdot (1-1/n) = (1-1/n)^m = (1-1/n)^{n(m/n)}$ 
  - A good approximation of  $(1-\epsilon)^{1/\epsilon}$  for small  $\epsilon$  is  $1/e$   
 $(1/e)^{m/n} = (e^{-1})^{m/n} = e^{-m/n}$
- Probability of a bit NOT being set by ANY hash function of ANY key at build phase  
 $(e^{-m/n})^k$
- Probability of a bit set by SOME hash function of ANY key at build phase  
 $1-(e^{-m/n})^k = 1-e^{-km/n}$
- Probability of all hash functions finding the bit set in the probing phase  
 $(1-e^{-km/n})^k$



# Exponentially decaying window

# Exponentially decaying window (Statement)

“Do not make a distinction between old and young element, but just weight them.”

- *Example*
  - Find the *currently* most popular movie/topic.
- *Solution:*
  - Keep one weighted *counter per movie/topic*
- *Definitions:*
  - $c$  = small constant (e.g.,  $10^{-6}$  or  $10^{-9}$ )
  - $T$  = current time
  - $f(t) = a_t$  = element at time  $t$  (or 0 if there is no element)
  - $g(T-t) = (1-c)^{(T-t)}$  = weight at time  $T$  of an item obtained at time  $t$
- *Value:*  $\sum f(i) \cdot g(T-i) = \sum_{i=0}^T a_i (1-c)^{T-i}$
- *Process:* Multiply the current counter by  $(1-c)$  and add  $a_t$ 
  - $\text{Counter}(T+1) = \sum_{i=0}^{T+1} a_i (1-c)^{T+1-i} = (\sum_{i=0}^T a_i (1-c)^{T-i}) * (1-c) + a_{T+1} = \text{Counter}(T) * (1-c) + a_{T+1}$
- *Optimizations*
  - a) Being  $X$  the time since the last update, we can multiply by  $(1-c)^X$ , instead
  - b) We might define a threshold to remove from memory the elements when the counter is too small

# Exponentially decaying window (Example)

$c=0.5$

Counter = **0.28125**

Stream

0 1 0 0 1 0 0 ...

# Heavy Hitters (or Frequent Items)

# Heavy hitters (Statement)

“Given a stream, identify the items that occur more than a given percentage ( $\theta$ ) of times.”

- Example:
  - Find the most frequent destinations in a router
  - Find the most frequent queries in a search engine
- Problem:
  - We do not know which will be frequent enough
  - We cannot store all items
    - An exact solution needs to store all items seen
      - $O(n \log(N))$  in the worst case
- Solution – Approximate with false positives
  - Structure:
    - Set of  $1/\theta$  pairs [element, counter]
  - Actions on receiving an element:
    - If the element is in the structure, increase its counter
    - If the element is not in the structure, insert it with counter 1
    - If the set overflows, decrease all counters and remove those with value zero

# Heavy hitters (example)

Required frequency: 33%

Heavy hitters:      a   b   c

a   b   a   b   a   c   c   c   a   a   b   d   e   f   g   h  
↑

Summary (capacity:  $1/0.33 = 3$ )

[a,3]

[b,3]

[c,3]

[d,1]

a:	5/16	=	31.25%
b:	3/16	=	18.75%
c:	3/16	=	18.75%
d:	1/16	=	6.25%
e:	1/16	=	6.25%
f:	1/16	=	6.25%
g:	1/16	=	6.25%
h:	1/16	=	6.25%

# Closing

# Summary

- Stream processing techniques
  - Load shedding
  - Bloom filters
  - Exponentially decaying window
  - Heavy hitters



# References

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