Summary

Let $X_1, \ldots, X_n$ be a random sample (of any size) from a *normal* population with mean μ. If the standard deviation σ is known, then a level $100(1-\alpha)\%$ confidence interval for μ is

$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \tag{5.12}$$

Occasionally one has a single value that is sampled from a normal population with known standard deviation. In these cases a confidence interval for μ can be derived as a special case of expression (5.12) by setting $n = 1$.

Summary

Let X be a single value sampled from a *normal* population with mean μ. If the standard deviation σ is known, then a level $100(1-\alpha)\%$ confidence interval for μ is

$$X \pm z_{\alpha/2}\sigma \tag{5.13}$$

Exercises for Section 5.3

1. Find the value of $t_{n-1,\alpha/2}$ needed to construct a two-sided confidence interval of the given level with the given sample size:
   a. Level 90%, sample size 12.
   b. Level 95%, sample size 7.
   c. Level 99%, sample size 2.
   d. Level 95%, sample size 29.

2. Find the value of $t_{n-1,\alpha}$ needed to construct an upper or lower confidence bound in each of the situations in Exercise 1.

3. Find the level of a two-sided confidence interval that is based on the given value of $t_{n-1,\alpha/2}$ and the given sample size.
   a. $t = 2.776$, sample size 5.
   b. $t = 2.718$, sample size 12.
   c. $t = 5.841$, sample size 4.
   d. $t = 1.325$, sample size 21.
   e. $t = 1.746$, sample size 17.

4. True or false: The Student's t distribution may be used to construct a confidence interval for the mean of any population, so long as the sample size is small.

5. The article "Wind-Uplift Capacity of Residential Wood Roof-Sheathing Panels Retrofitted with Insulating Foam Adhesive" (P. Datin, D. Prevatt, and W. Pang, *Journal of Architectural Engineering*, 2011:144–154) presents a study of the failure pressures of roof panels. Following are the failure pressures, in kPa, for five panels constructed with 6d smooth shank nails. These data are consistent with means and standard deviations presented in the article.

   3.32 2.53 3.45 2.38 3.01

   Find a 95% confidence interval for the mean failure pressure for this type of roof panel.

6. The following are summary statistics for a data set. Would it be appropriate to use the Student's t distribution to construct a confidence interval from these data? Explain.

N	Mean	Median	StDev
10	8.905	6.105	9.690
Minimum	**Maximum**	**Q1**	**Q3**
0.512	39.920	1.967	8.103

7. The article "An Automatic Visual System for Marble Tile Classification" (L. Carrino, W. Polini, and S. Turchetta, *Journal of Engineering Manufacture*, 2002:1095–1108) describes a measure for the shade of marble tile in which the amount of light reflected by the tile is measured on a scale of 0–255. A perfectly black tile would reflect no light and measure 0, and a perfectly white tile would measure 255. A sample of nine Mezza Perla tiles were measured, with the following results:

204.999	206.149	202.102	207.048	203.496
206.343	203.496	206.676	205.831	

Is it appropriate to use the Student's t statistic to construct a 95% confidence interval for the mean shade of Mezza Perla tile? If so, construct the confidence interval. If not, explain why not.

8. A chemist made eight independent measurements of the melting point of tungsten. She obtained a sample mean of 3410.14 degrees Celsius and a sample standard deviation of 1.018 degrees.

a. Use the Student's t distribution to find a 95% confidence interval for the melting point of tungsten.

b. Use the Student's t distribution to find a 98% confidence interval for the melting point of tungsten.

c. If the eight measurements had been 3409.76, 3409.80, 3412.66, 3409.79, 3409.76, 3409.77, 3409.80, 3409.78, would the confidence intervals above be valid? Explain.

9. Six measurements are taken of the thickness of a piece of 18-gauge sheet metal. The measurements (in mm) are: 1.316, 1.308, 1.321, 1.303, 1.311, and 1.310.

a. Make a dotplot of the six values.

b. Should the t curve be used to find a 99% confidence interval for the thickness? If so, find the confidence interval. If not, explain why not.

c. Six independent measurements are taken of the thickness of another piece of sheet metal. The measurements this time are: 1.317, 1.318, 1.301, 1.307, 1.374, 1.323. Make a dotplot of these values.

d. Should the t curve be used to find a 95% confidence interval for the thickness of this metal? If so, find the confidence interval. If not, explain why not.

10. Fission tracks are trails found in uranium-bearing minerals, left by fragments released during fission events. The article "Yo-yo Tectonics of the Niğde Massif During Wrenching in Central Anatolia" (D. Whitney, P. Umhoefer, et al., *Turkish Journal of Earth Sciences*, 2008:209–217) reports that fifteen tracks on one rock specimen had an average track length of 13 μm with a standard deviation of 2 μm. Assuming this to be a random sample from an approximately normal population, find a 99% confidence interval for the mean track length for this rock specimen.

11. The article "Effect of Granular Subbase Thickness on Airfield Pavement Structural Response" (K. Gopalakrishnan and M. Thompson, *Journal of Materials in Civil Engineering*, 2008:331–342) presents a study of the effect of the subbase thickness on the amount of surface deflection caused by aircraft landing on an airport runway. In six applications of a 160 kN load on a runway with a subbase thickness of 864 mm, the average surface deflection was 2.03 mm with a standard deviation of 0.090 mm. Find a 90% confidence interval for the mean deflection caused by a 160 kN load.

12. The article "Influence of Penetration Rate on Penetrometer Resistance" (J. Oliveira, M. Almeida, et al., *Journal of Geotechnical and Geoenvironmental Engineering*, 2011:695–703) presents measures of penetration resistance for a certain fine-grained soil. Fifteen measurements, expressed as a multiple of a standard quantity, had a mean of 2.64 and a standard deviation of 1.02. Find a 95% confidence interval for the mean penetration resistance for this soil.

13. Ten samples of coal from a Northern Appalachian source had an average mercury content of 0.242 ppm with a standard deviation of 0.031 ppm. Find a 95% confidence for the mean mercury content of coal from this source.

14. The following MINITAB output presents a confidence interval for a population mean.

```
One-Sample T: X

Variable     N       Mean      StDev    SE Mean          95% CI
X           10    6.59635    0.11213    0.03546    (6.51613, 6.67656)
```

a. How many degrees of freedom does the Student's t distribution have?
b. Use the information in the output, along with the t table, to compute a 99% confidence interval.

15. The following MINITAB output presents a confidence interval for a population mean, but some of the numbers got smudged and are now illegible. Fill in the missing numbers for (a), (b), and (c).

```
One-Sample T: X
Variable     N       Mean    StDev    SE Mean       99% CI
X           20    2.39374     (a)     0.52640    ( (b), (c) )
```

16. The concentration of carbon monoxide (CO) in a gas sample is measured by a spectrophotometer and found to be 85 ppm. Through long experience with this instrument, it is believed that its measurements are unbiased and normally distributed, with an uncertainty (standard deviation) of 8 ppm. Find a 95% confidence interval for the concentration of CO in this sample.

17. The article "Filtration Rates of the Zebra Mussel (*Dreissena polymorpha*) on Natural Seston from Saginaw Bay, Lake Huron" (D. Fanslow, T. Nalepa, and G. Lang, *Journal of Great Lakes Research* 1995:489–500) reports measurements of the rates (in mL/mg/h) at which mussels filter seston (particulate matter suspended in seawater).

a. In the year 1992, 5 measurements were made in the Outer Bay; these averaged 21.7 with a standard deviation of 9.4. Find a 95% confidence interval for the mean filtration rate in the Outer Bay.
b. In the year 1992, 7 measurements were made in the Inner Bay; these averaged 8.6 with a standard deviation of 4.5. Should the Student's t distribution be used to find a 95% confidence interval for the mean filtration rate for the Inner Bay? If so, find the confidence interval. If not, explain why not.

5.4 Confidence Intervals for the Difference Between Two Means

We now investigate examples in which we wish to estimate the difference between the means of two populations. The data will consist of two samples, one from each population. The basic idea is quite simple. We will compute the difference of the sample means and the standard deviation of that difference. Then a simple modification of expression (5.1) (in Section 5.1) will provide the confidence interval. The method we describe is based on the results concerning the sum and difference of two independent normal random variables that were presented in Section 4.5. We review these results here:

Let X and Y be independent, with $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$. Then

$$X + Y \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2) \tag{5.14}$$

$$X - Y \sim N(\mu_X - \mu_Y, \sigma_X^2 + \sigma_Y^2) \tag{5.15}$$