Exercises for Section 4.11

1. Bottles filled by a certain machine are supposed to contain 12 oz of liquid. In fact the fill volume is random with mean 12.01 oz and standard deviation 0.2 oz.

a. What is the probability that the mean volume of a random sample of 144 bottles is less than 12 oz?

b. If the population mean fill volume is increased to 12.03 oz, what is the probability that the mean volume of a sample of size 144 will be less than 12 oz?

2. A 500-page book contains 250 sheets of paper. The thickness of the paper used to manufacture the book has mean 0.08 mm and standard deviation 0.01 mm.

a. What is the probability that a randomly chosen book is more than 20.2 mm thick (not including the covers)?

b. What is the 10th percentile of book thicknesses?

c. Someone wants to know the probability that a randomly chosen page is more than 0.1 mm thick. Is enough information given to compute this probability? If so, compute the probability. If not, explain why not.

3. A commuter encounters four traffic lights each day on her way to work. Let X represent the number of these that are red lights. The probability mass function of X is as follows.

x	0	1	2	3	4
$P(X = x)$	0.1	0.3	0.3	0.2	0.1

What is the probability that in a period of 100 days, the average number of red lights encountered is more than 2 per day?

4. Among all the income-tax forms filed in a certain year, the mean tax paid was \$2000 and the standard deviation was \$500. In addition, for 10% of the forms, the tax paid was greater than \$3000. A random sample of 625 tax forms is drawn.

a. What is the probability that the average tax paid on the sample forms is greater than \$1980?

b. What is the probability that more than 60 of the sampled forms have a tax of greater than \$3000?

5. Bags checked for a certain airline flight have a mean weight of 15 kg with a standard deviation of 5 kg. A random sample of 60 bags is drawn.

a. What is the probability that the sample mean weight is less than 14 kg?

b. Find the 70th percentile of the sample mean weights.

c. How many bags must be sampled so that the probability is 0.01 that the sample mean weight is less than 14 kg?

6. The amount of warpage in a type of wafer used in the manufacture of integrated circuits has mean 1.3 mm and standard deviation 0.1 mm. A random sample of 200 wafers is drawn.

a. What is the probability that the sample mean warpage exceeds 1.305 mm?

b. Find the 25th percentile of the sample mean.

c. How many wafers must be sampled so that the probability is 0.05 that the sample mean exceeds 1.305?

7. The time spent by a customer at a checkout counter has mean 4 minutes and standard deviation 2 minutes.

a. What is the probability that the total time taken by a random sample of 50 customers is less than 180 minutes?

b. Find the 30th percentile of the total time taken by 50 customers.

8. Drums labeled 30 L are filled with a solution from a large vat. The amount of solution put into each drum is random with mean 30.01 L and standard deviation 0.1 L.

a. What is the probability that the total amount of solution contained in 50 drums is more than 1500 L?

b. If the total amount of solution in the vat is 2401 L, what is the probability that 80 drums can be filled without running out?

c. How much solution should the vat contain so that the probability is 0.9 that 80 drums can be filled without running out?

9. The temperature of a solution will be estimated by taking n independent readings and averaging them. Each reading is unbiased, with a standard deviation of $\sigma = 0.5°$C. How many readings must be taken so that the probability is 0.90 that the average is within $\pm 0.1°$C of the actual temperature?

10. Among the adults in a large city, 30% have a college degree. A simple random sample of 100 adults is chosen. What is the probability that more than 35 of them have a college degree?

11. In a process that manufactures bearings, 90% of the bearings meet a thickness specification. A shipment contains 500 bearings. A shipment is acceptable if at least 440 of the 500 bearings meet the specification. Assume that each shipment contains a random sample of bearings.

a. What is the probability that a given shipment is acceptable?

b. What is the probability that more than 285 out of 300 shipments are acceptable?

c. What proportion of bearings must meet the specification in order that 99% of the shipments are acceptable?

12. A machine produces 1000 steel O-rings per day. Each ring has probability 0.9 of meeting a thickness specification.

a. What is the probability that on a given day, fewer than 890 O-rings meet the specification?

b. Find the 60th percentile of the number of O-rings that meet the specification.

c. If the machine operates for five days, what is the probability that fewer than 890 O-rings meet the specification on three or more of those days?

13. Radioactive mass A emits particles at a mean rate of 20 per minute, and radioactive mass B emits particles at a mean rate of 25 per minute.

a. What is the probability that fewer than 200 particles are emitted by both masses together in a five-minute time period?

b. What is the probability that mass B emits more particles than mass A in a two-minute time period?

14. The concentration of particles in a suspension is 30 per mL.

a. What is the probability that a 2 mL sample will contain more than 50 particles?

b. Ten 2 mL samples are drawn. What is the probability that at least 9 of them contain more than 50 particles?

c. One hundred 2 mL samples are drawn. What is the probability that at least 90 of them contain more than 50 particles?

15. The concentration of particles in a suspension is 50 per mL. A 5 mL volume of the suspension is withdrawn.

a. What is the probability that the number of particles withdrawn will be between 235 and 265?

b. What is the probability that the average number of particles per mL in the withdrawn sample is between 48 and 52?

c. If a 10 mL sample is withdrawn, what is the probability that the average number per mL of particles in the withdrawn sample is between 48 and 52?

d. How large a sample must be withdrawn so that the average number of particles per mL in the sample is between 48 and 52 with probability 95%?

16. A battery manufacturer claims that the lifetime of a certain type of battery has a population mean of 40 hours and a standard deviation of 5 hours. Let $\overline{X}$ represent the mean lifetime of the batteries in a simple random sample of size 100.

a. If the claim is true, what is $P(\overline{X} \leq 36.7)$?

b. Based on the answer to part (a), if the claim is true, is a sample mean lifetime of 36.7 hours unusually short?

c. If the sample mean lifetime of the 100 batteries were 36.7 hours, would you find the manufacturer's claim to be plausible? Explain.

d. If the claim is true, what is $P(\overline{X} \leq 39.8)$?

e. Based on the answer to part (d), if the claim is true, is a sample mean lifetime of 39.8 hours unusually short?

f. If the sample mean lifetime of the 100 batteries were 39.8 hours, would you find the manufacturer's claim to be plausible? Explain.

17. A new process has been designed to make ceramic tiles. The goal is to have no more than 5% of the tiles be nonconforming due to surface defects. A random sample of 1000 tiles is inspected. Let X be the number of nonconforming tiles in the sample.

a. If 5% of the tiles produced are nonconforming, what is $P(X \geq 75)$?

b. Based on the answer to part (a), if 5% of the tiles are nonconforming, is 75 nonconforming tiles out of 1000 an unusually large number?

c. If 75 of the sample tiles were nonconforming, would it be plausible that the goal had been reached? Explain.

d. If 5% of the tiles produced are nonconforming, what is $P(X \geq 53)$?
e. Based on the answer to part (d), if 5% of the tiles are nonconforming, is 53 nonconforming tiles out of 1000 an unusually large number?
f. If 53 of the sample tiles were nonconforming, would it be plausible that the goal had been reached? Explain.

18. The manufacture of a certain part requires two different machine operations. The time on machine 1 has mean 0.5 hours and standard deviation 0.4 hours. The time on machine 2 has mean 0.6 hours and standard deviation 0.5 hours. The times needed on the machines are independent. Suppose that 100 parts are manufactured.

a. What is the probability that the total time used by machine 1 is greater than 55 hours?
b. What is the probability that the total time used by machine 2 is less than 55 hours?
c. What is the probability that the total time used by both machines together is greater than 115 hours?
d. What is the probability that the total time used by machine 1 is greater than the total time used by machine 2?

19. Seventy percent of rivets from vendor A meet a certain strength specification, and 80% of rivets from vendor B meet the same specification. If 500 rivets are purchased from each vendor, what is the probability that more than 775 of the rivets meet the specifications?

20. *Radiocarbon dating:* Carbon-14 is a radioactive isotope of carbon that decays by emitting a beta particle. In the earth's atmosphere, approximately one carbon atom in 10^{12} is carbon-14. Living organisms exchange carbon with the atmosphere, so this same ratio holds for living tissue. After an organism dies, it stops exchanging carbon with its environment, and its carbon-14 ratio decreases exponentially with time. The rate at which beta particles are emitted from a given mass of carbon is proportional to the carbon-14 ratio, so this rate decreases exponentially with time as well. By measuring the rate of beta emissions in a sample of tissue, the time since the death of the organism can be estimated. Specifically, it is known that t years after death, the number of beta particle emissions occurring in any given time interval from 1 g of carbon follows a Poisson distribution with rate $\lambda = 15.3e^{-0.0001210t}$ events per minute. The number of years t since the death of an organism can therefore be expressed in terms of λ:

$$t = \frac{\ln 15.3 - \ln \lambda}{0.0001210}$$

An archaeologist finds a small piece of charcoal from an ancient campsite. The charcoal contains 1 g of carbon.

a. Unknown to the archaeologist, the charcoal is 11,000 years old. What is the true value of the emission rate λ?
b. The archaeologist plans to count the number X of emissions in a 25 minute interval. Find the mean and standard deviation of X.
c. The archaeologist then plans to estimate λ with $\widehat{\lambda} = X/25$. What is the mean and standard deviation of $\widehat{\lambda}$?
d. What value for $\widehat{\lambda}$ would result in an age estimate of 10,000 years?
e. What value for $\widehat{\lambda}$ would result in an age estimate of 12,000 years?
f. What is the probability that the age estimate is correct to within ± 1000 years?

4.12 Simulation

When fraternal (nonidentical) twins are born, they may be both boys, both girls, or one of each. Assume that each twin is equally likely to be a boy or a girl, and assume that the sexes of the twins are determined independently. What is the probability that both twins are boys? This probability is easy to compute, using the multiplication rule for