

Summary

Let $X \sim \text{Bin}(n_X, p_X)$ and let $Y \sim \text{Bin}(n_Y, p_Y)$. Assume that there are at least 10 successes and 10 failures in each sample, and that X and Y are independent.

To test a null hypothesis of the form $H_0: p_X - p_Y \leq 0$, $H_0: p_X - p_Y \geq 0$, or $H_0: p_X - p_Y = 0$:

- Compute $\hat{p}_X = \frac{X}{n_X}$, $\hat{p}_Y = \frac{Y}{n_Y}$, and $\hat{p} = \frac{X + Y}{n_X + n_Y}$.
- Compute the z -score: $z = \frac{\hat{p}_X - \hat{p}_Y}{\sqrt{\hat{p}(1 - \hat{p})(1/n_X + 1/n_Y)}}$.
- Compute the P -value. The P -value is an area under the normal curve, which depends on the alternate hypothesis as follows:

Alternate Hypothesis

$$H_1: p_X - p_Y > 0$$

$$H_1: p_X - p_Y < 0$$

$$H_1: p_X - p_Y \neq 0$$

P -value

Area to the right of z

Area to the left of z

Sum of the areas in the tails cut off by z and $-z$

Exercises for Section 6.6

- Two extrusion machines that manufacture steel rods are being compared. In a sample of 1000 rods taken from machine 1, 960 met specifications regarding length and diameter. In a sample of 600 rods taken from machine 2, 582 met the specifications. Machine 2 is more expensive to run, so it is decided that machine 1 will be used unless it can be convincingly shown that machine 2 produces a larger proportion of rods meeting specifications.
 - State the appropriate null and alternate hypotheses for making the decision as to which machine to use.
 - Compute the P -value.
 - Which machine should be used?
- Resistors labeled as 100Ω are purchased from two different vendors. The specification for this type of resistor is that its actual resistance be within 5% of its labeled resistance. In a sample of 180 resistors from vendor A, 150 of them met the specification. In a sample of 270 resistors purchased from vendor B, 233 of them met the specification. Vendor A is the current supplier, but if the data demonstrate convincingly that a greater proportion of the resistors from vendor B meet the specification, a change will be made.
 - State the appropriate null and alternate hypotheses.
 - Find the P -value.
 - Should a change be made?
- The article “A Music Key Detection Method Based on Pitch Class Distribution Theory” (J. Sun, H. Li, and L. Ma, *International Journal of Knowledge-based and Intelligent Engineering Systems*, 2011:165–175) describes a method of analyzing digital music files to determine the key in which the music is written. In a sample of 307 pop music selections, the key was identified correctly in 245 of them. In a sample of 347 new-age selections, the key was identified correctly in 304 of them. Can you conclude that the method is more accurate for new-age songs than for pop songs?
- When the light turns yellow, should you stop or go through it? The article “Evaluation of Driver Behavior in Type II Dilemma Zones at High-Speed Signalized Intersections” (D. Hurwitz, M. Knodler, and B. Nyquist, *Journal of Transportation Engineering*, 2011:277–286) defines the “indecision zone” as the period when a vehicle is between 2.5 and 5.5 seconds away from an intersection. At the intersection of Route 7 and North Shrewsbury in Clarendon, Vermont, 154 vehicles were observed to encounter a yellow light in

the indecision zone, and 21 of them ran the red light. At the intersection of Route 62 and Paine Turnpike in Berlin, Vermont, 183 vehicles entered the intersection in the indecision zone, and 20 ran the red light. Can you conclude that the proportion of red-light runners differs between the two intersections?

5. The article “HIV-positive Smokers Considering Quitting: Differences by Race/Ethnicity” (E. Lloyd-Richardson, C. Stanton, et al., *Am J Health Behav*, 2008:3–15) reported that in a group of 230 European-American HIV-positive smokers, 102 of them had used a nicotine patch to try to quit smoking, and in a group of 72 Hispanic-American HIV-positive smokers, 20 had used a nicotine patch. Can you conclude that the proportion of patch users is greater among European-Americans?
6. In August and September 2005, Hurricanes Katrina and Rita caused extraordinary flooding in New Orleans, Louisiana. Many homes were severely damaged or destroyed; of those that survived, many required extensive cleaning. It was thought that cleaning flood-damaged homes might present a health hazard due to the large amounts of mold present in many of the homes. The article “Health Effects of Exposure to Water-Damaged New Orleans Homes Six Months After Hurricanes Katrina and Rita” (K. Cummings, J. Cox-Ganser, et al., *American Journal of Public Health*, 2008:869–875) reports that in a sample of 365 residents of Orleans Parish who had participated in the cleaning of one or more homes, 77 had experienced symptoms of wheezing, and in a sample of 179 residents who had not participated in cleaning, 23 reported wheezing symptoms (numbers read from a graph). Can you conclude that the frequency of wheezing symptoms is greater among those residents who participated in the cleaning of flood-damaged homes?
7. To test the effectiveness of protective packaging, a firm shipped out 1200 orders in regular light packaging and 1500 orders in heavy-duty packaging. Of the orders shipped in light packaging, 20 arrived in damaged condition, while of the orders shipped in heavy-duty packaging, 15 arrived in damaged condition. Can you conclude that heavy-duty packaging reduces the proportion of damaged shipments?
8. Colonoscopy is a medical procedure that is designed to find and remove precancerous lesions in the colon before they become cancerous. The article “Association of Colonoscopy and Death from Colorectal Cancer” (N. Baxter, M. Goldwasser, et al., *Annals of Internal Medicine* 2009:1–8) reports that in a sample of 10,292 people diagnosed with colorectal cancer, 7.0% had previously had a colonoscopy, and in a sample of 51,460 people without colorectal cancer, 9.8% had previously had a colonoscopy. Can you conclude that the percentage of people who have had colonoscopies is greater in those without colorectal cancer?
9. The article “Factors Associated with Exercise Behavior in People with Parkinson Disease” (T. Ellis, J. Cavanaugh, et al., *Physical Therapy*, 2011:1838–1848) reported a survey of patients with Parkinson’s disease. Of 164 patients who said they exercised regularly, 76 reported falling in the previous six months. Of 96 patients who said they did not exercise regularly, 48 reported falling in the previous six months. Can you conclude that the proportion of patients who fall is less for those who exercise than for those who do not?
10. The article “Association Between Exposure to Emissions from the Oil and Gas Industry and Pathology of the Immune, Nervous, and Respiratory Systems, and Skeletal and Cardiac Muscle in Beef Calves” (C. Waldner and E. Clark, *Annals of Environmental and Occupational Health*, 2009:6–27) reports that in a sample of 355 calves less than 4 days old, 106 had lesions on their bronchial lymph nodes, while in a sample of 467 calves more than 4 days old, 147 had such lesions. Can you conclude that the proportion of calves with lesions differs between the two age groups?
11. The article “Long-term Outcomes of Patients Receiving Drug-eluting Stents” (A. Philpott, D. Southern, et al., *Canadian Medical Association Journal*, 2009:167–174) reported that in a sample of 5320 angioplasty patients receiving bare metal stents, 195 died within one year, and in a sample of 1120 patients receiving drug coated stents, 33 died within a year. Can you conclude that the proportions differ between the two groups?
12. In a study conducted by the U.S. Department of Health and Human Services, a sample of 546 boys aged 6–11 was weighed, and it was determined that 87 of them were overweight. A sample of 508 girls aged 6–11 was also weighed, and 74 of them were overweight. Can you conclude that the proportion of boys who are overweight differs from the proportion of girls who are overweight?
13. In order to determine whether to pitch a new advertising campaign more toward men or women, an

advertiser provided each couple in a random sample of 500 married couples with a new type of TV remote control that is supposed to be easier to find when needed. Of the 500 husbands, 62% said that the new remote was easier to find than their old one. Of the 500 wives, only 54% said the new remote was easier to find. Let p_1 be the population proportion of married

men who think that the new remote is easier to find, and let p_2 be the corresponding proportion of married women. Can the statistic $\hat{p}_1 - \hat{p}_2 = 0.62 - 0.54$ be used to test $H_0: p_1 - p_2 = 0$ versus $H_1: p_1 - p_2 \neq 0$? If so, perform the test and compute the P -value. If not, explain why not.

14. The following MINITAB output presents the results of a hypothesis test for the difference $p_1 - p_2$ between two population proportions.

Test and CI for Two Proportions

Sample	X	N	Sample p
1	41	97	0.422680
2	37	61	0.606557

Difference = $p(1) - p(2)$

Estimate for difference: -0.183877

95% CI for difference: $(-0.341016, -0.026738)$

Test for difference = 0 (vs not = 0): $Z = -2.25$ P-Value = 0.024

- Is this a one-tailed or two-tailed test?
- What is the null hypothesis?
- Can H_0 be rejected at the 5% level? How can you tell?

15. The following MINITAB output presents the results of a hypothesis test for the difference $p_1 - p_2$ between two population proportions. Some of the numbers are missing. Fill in the numbers for (a) through (d).

Test and CI for Two Proportions

Sample	X	N	Sample p
1	101	153	(a)
2	(b)	90	0.544444

Difference = $p(1) - p(2)$

Estimate for difference: 0.115686

95% CI for difference: $(-0.0116695, 0.243042)$

Test for difference = 0 (vs not = 0): $Z = (c)$ P-Value = (d)

6.7 Small-Sample Tests for the Difference Between Two Means

The t test can be used in some cases where samples are small, and thus where the Central Limit Theorem does not apply. We present an example.

The article “The Achondroplasia Paternal Age Effect Is Not Explained By an Increase in Mutant Frequency” (I. Tiemann-Boege, W. Navidi, et al., *Proceedings of the National Academy of Sciences*, 2002:14952–14957) describes an experiment in which