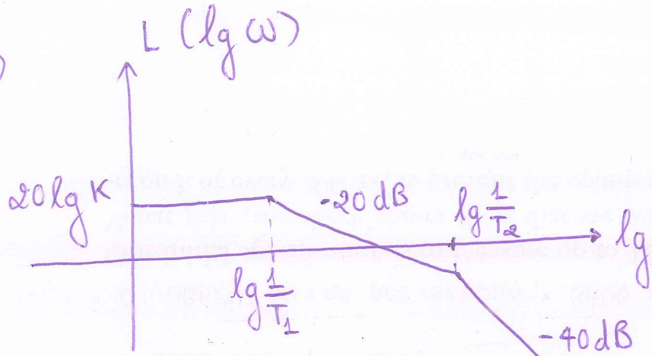
Lý thuyết điều khiển I Đề 1

Bài 1:

1)



$G_2 = G_{21} \cdot G_{22} \cdot G_{23}$

G_{21} hệ số góc 0

$\rightarrow G_{21} = K$

G_{22} hệ số góc -20 từ $\lg\omega = \lg\frac{1}{T_1}$

$\rightarrow G_{22} = \dfrac{1}{T_1 s + 1}$

G_{23} hệ số góc -20 từ $\lg\omega = \frac{1}{T_2}$

$\rightarrow G_{23} = \dfrac{1}{T_2 s + 1}$

Có $\begin{cases} 20\lg K = 20 \\ \lg\dfrac{1}{T_1} = \lg(0{,}1) \\ \lg\dfrac{1}{T_2} = \lg(0{,}25) \end{cases} \rightarrow \begin{cases} K = 10 \\ T_1 = 10 \\ T_2 = 4 \end{cases}$ Vậy $G_2 = \dfrac{10}{(10s+1)(4s+1)}$

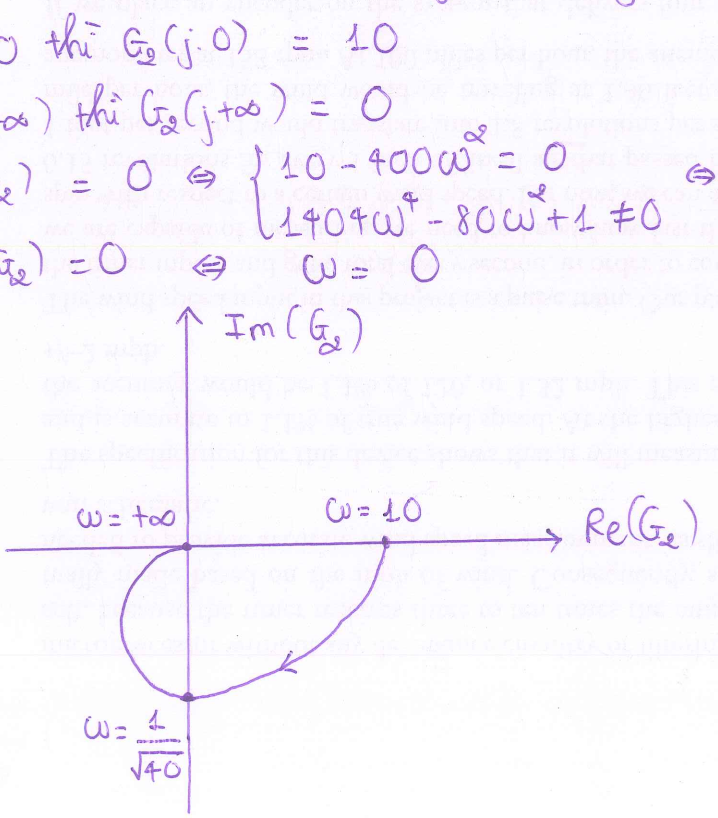
Thay $s = j\omega \rightarrow G_2(j\omega) = \dfrac{10}{(10j\omega+1)(4j\omega+1)} = \dfrac{10 - \cancel{4000\omega^2}\, 400\omega^2}{1404\omega^4 - 80\omega^2 + 1} + j\dfrac{-140\omega}{\cancel{\ldots}\, 1404\omega^4 - 80\omega^2 + 1}$

$\omega = 0$ thì $G_2(j.0) = 10$

$\omega = (+\infty)$ thì $G_2(j+\infty) = 0$

$Re(G_2) = 0 \Leftrightarrow \begin{cases} 10 - 400\omega^2 = 0 \\ 1404\omega^4 - 80\omega^2 + 1 \neq 0 \end{cases} \Leftrightarrow \omega = \dfrac{1}{\sqrt{40}}$

$Im(G_2) = 0 \Leftrightarrow \omega = 0$



2) $G_1 = K_p\left(1+\dfrac{1}{T_I s}\right) = \dfrac{K_p(T_I s+1)}{T_I s}$

$$G_2 = \frac{10}{(10s+1)(4s+1)}$$

$$G_h = G_1 \cdot G_2 \; ; \; G_K = \frac{G_1 G_2}{1+G_1 G_2} = \frac{G_h}{1+G_h}$$

- Nếu chọn $T_I = 10$ thì :

$$G_h = G_1 G_2 = \frac{10K_p}{10s(4s+1)} = \frac{K_p}{s(4s+1)}$$

$$G_K = \frac{K_p}{s(4s+1)+K_p} = \frac{K_p}{4s^2+s+K_p}$$

$$G_K(j\omega) = \frac{K_p}{4(j\omega)^2 + j\omega + K_p} = \frac{K_p}{(K_p - 4\omega^2) + j\cdot\omega}$$

$$|G_K(j\omega)|^2 = \frac{K_p^2}{(K_p-4\omega^2)^2+\omega^2} = \frac{K_p^2}{K_p^2 + (1-8K_p)\omega^2 + 16\omega^4}$$

ω nhỏ $\rightarrow \omega^4 \simeq 0$

$$|G_K(j\omega)| \simeq 1 \Leftrightarrow 1 - 8K_p = 0 \Leftrightarrow K_p = \frac{1}{8}$$

- Nếu chọn $T_I = 4$ thì :

$$G_h = G_1 G_2 = \frac{10K_p}{4s(10s+1)} = \frac{5K_p}{2s(10s+1)}$$

$$G_K = \frac{5K_p}{2s(10s+1)+5K_p} = \frac{5K_p}{20s^2+2s+5K_p}$$

$$G_K(j\omega) = \frac{5K_p}{20(j\omega)^2 + 2(j\omega) + 5K_p} = \frac{5K_p}{(5K_p - 20\omega^2) + j\cdot(2\omega)}$$

$$|G_K(j\omega)|^2 = \frac{(5K_p)^2}{(5K_p)^2 + (4 - 200K_p)\omega^2 + 400\omega^4}$$

ω nhỏ $\rightarrow \omega^4 \simeq 0$

$$|G_K(j\omega)| \simeq 1 \Leftrightarrow 4 - 200K_p = 0 \Leftrightarrow K_p = \frac{1}{50}$$

Vậy có 2 bộ điều khiển $\begin{cases} K_p = 1/8 \; ; \; T_I = 10 \\ K_p = 1/50 \; ; \; T_I = 4 \end{cases}$

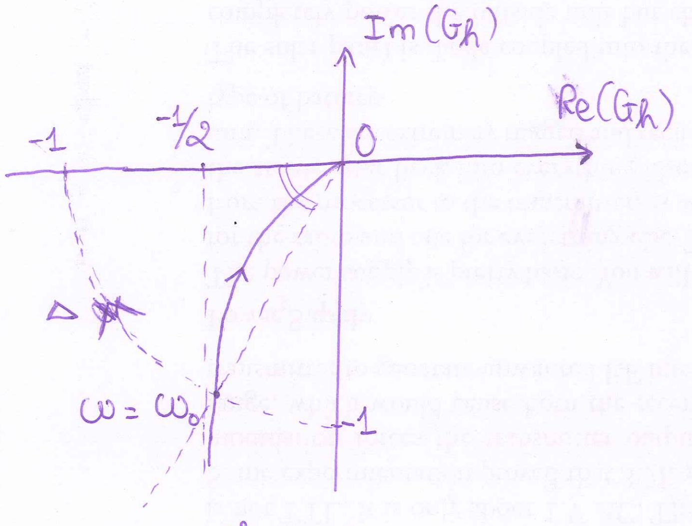
3) $G_h = G_1 G_2 = \dfrac{10k_p(T_I s+1)}{T_I s(10s+1)(4s+1)}$

Nếu $k_p = \frac{10}{8}$, $T_I = 10$ thì: $G_h = \dfrac{1}{8s(4s+1)} = \dfrac{1}{32s^2+8s}$

$$G_h(j\omega) = \frac{1}{32(j\omega)^2+8j\omega} = \frac{-32}{1024\omega^2+64} + j\cdot\frac{-8}{1024\omega^3+64\omega}$$

$\omega = 0 \rightarrow G_h(j.0) = \frac{-1}{2} + j\cdot(-\infty)$

$\omega = +\infty \rightarrow G_h(j.+\infty) = 0$



$$|G_h(j\omega)|^2 = \frac{1}{(32\omega^2)^2+(8\omega)^2} = \frac{1}{1024\omega^4+64\omega^2}$$

$|G_h(j\omega)| = 1 \Leftrightarrow 1024\omega^4 + 64\omega^2 = 1 \Leftrightarrow \omega^2 = 0{,}0129$

$\Leftrightarrow \omega = 0{,}1136 = \omega_0$

Độ dự trữ ổn định:

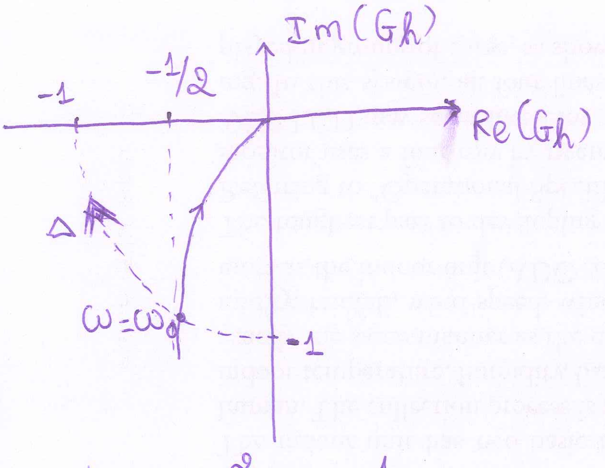
$\Delta = \arctan G_h(j\omega_0) = 1{,}1442$

Nếu $K_p = \frac{1}{50}$, $T_I = 4$ thì: $G_h = \frac{1}{20s(10s+1)} = \frac{1}{200s^2+20s}$

$$G_h(j\omega) = \frac{1}{200(j\omega)^2+20j\omega} = \frac{-1}{200\omega^2+2} + j\cdot\frac{-1}{2000\omega^3+20\omega}$$

$\omega = 0 \rightarrow G_h(j.0) = \frac{-1}{2} + j.(-\infty)$

$\omega = +\infty \rightarrow G_h(j.+\infty) = 0$



$$|G_h(j\omega)|^2 = \frac{1}{(200\omega^2)^2+(20\omega)^2}$$

$|G_h(j\omega)| = 1 \Leftrightarrow 40000\omega^4 + 400\omega^2 = 1 \Leftrightarrow \omega^2 = 0,0021$

$\Leftrightarrow \omega = 0,0458 = \omega_0$

Độ dư trữ ổn định:

$\Delta = \arctan G_h(j\omega_0) = 1,1442$

Bài 2:

1) $\dfrac{dx}{dt} = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} u \rightarrow A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$y = x_1 + a x_2 \rightarrow C = \begin{pmatrix} 1 & a & 0 \end{pmatrix}$

- Kiểm tra tính ổn định:

Có $sI - A = s\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} s-3 & 0 & -1 \\ 0 & s-1 & -1 \\ 0 & -2 & s-2 \end{pmatrix}$

$\det(sI - A) = (s-3)\begin{vmatrix} s-1 & -1 \\ -2 & s-2 \end{vmatrix} - 0.\begin{vmatrix} 0 & -1 \\ -2 & s-2 \end{vmatrix} + 0.\begin{vmatrix} 0 & -1 \\ s-1 & -1 \end{vmatrix}$

$= (s-3)\left[(s-1)(s-2) - (-2).(-1)\right] = s(s-3)^2$

có 1 nghiệm nằm bên phải trục hoành nên không phải là đa thức Hurwitz

vậy nên hệ không ổn định

- Kiểm tra tính quan sát được

Có $CA = \begin{pmatrix} 1 & a & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 3 & a & a+1 \end{pmatrix}$

$CA^2 = CA.A = \begin{pmatrix} 3 & a & a+1 \end{pmatrix}\begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 9 & 3a+2 & 3a+5 \end{pmatrix}$

$\det\begin{pmatrix} C \\ CA \\ CA^2 \end{pmatrix} = \det\begin{pmatrix} 1 & a & 0 \\ 3 & a & a+1 \\ 9 & 3a+2 & 3a+5 \end{pmatrix} = 1.\begin{vmatrix} a & a+1 \\ 3a+2 & 3a+5 \end{vmatrix} - a\begin{vmatrix} 3 & a+1 \\ 9 & 3a+5 \end{vmatrix} + 0$

$= -2 - 6a = -2(3a+1)$

Để hệ quan sát được thì $-2(3a+1) \neq 0 \Leftrightarrow a \neq \dfrac{-1}{3}$

2) cho $a = 1$

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

$$A^2B = A.AB = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ 12 \end{pmatrix}$$

$$N = \begin{pmatrix} B & AB & A^2B \end{pmatrix} = \begin{pmatrix} 0 & 1 & 7 \\ 1 & 2 & 6 \\ 1 & 4 & 12 \end{pmatrix}$$

$$\det(N) = 0 - 1.\begin{vmatrix} 1 & 6 \\ 1 & 12 \end{vmatrix} + 7.\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = 8 \neq 0$$

$\rightarrow$ tồn tại ma trận $S = \begin{pmatrix} \alpha^T \\ \alpha^T A \\ \alpha^T A^2 \end{pmatrix}$

Có $N^{-1} = \begin{pmatrix} 0 & 1 & 7 \\ 1 & 2 & 6 \\ 1 & 4 & 12 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 2 & -1 \\ \frac{-3}{4} & \frac{-7}{8} & \frac{7}{8} \\ \frac{1}{4} & \frac{1}{8} & \frac{-1}{8} \end{pmatrix} \rightarrow \alpha^T = \begin{pmatrix} \frac{1}{4} & \frac{1}{8} & \frac{-1}{8} \end{pmatrix}$

$$\alpha^T A = \begin{pmatrix} \frac{1}{4} & \frac{1}{8} & \frac{-1}{8} \end{pmatrix} \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} \frac{3}{4} & \frac{-1}{8} & \frac{1}{8} \end{pmatrix}$$

$$\alpha^T A^2 = \begin{pmatrix} \frac{3}{4} & \frac{-1}{8} & \frac{1}{8} \end{pmatrix} \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} \frac{9}{4} & \frac{1}{8} & \frac{7}{8} \end{pmatrix}$$

$$\rightarrow S = \begin{pmatrix} \frac{1}{4} & \frac{1}{8} & \frac{-1}{8} \\ \frac{3}{4} & \frac{-1}{8} & \frac{1}{8} \\ \frac{9}{4} & \frac{1}{8} & \frac{7}{8} \end{pmatrix}$$

$\det(sI - A) = s(s-3)^2 = s^3 - 6s^2 + 9s$

$\rightarrow a_0 = 0,\ a_1 = 9,\ a_2 = -6$

Theo đề bài các điểm cực $s_1 = s_2 = s_3 = -3$

$\rightarrow (s+3)^3 = s^3 + 9s^2 + 27s + 27$

$\rightarrow \hat{a}_0 = 27,\ \hat{a}_1 = 27,\ \hat{a}_2 = 9$

$$\rightarrow \begin{cases} r_0' = \hat{a}_0 - a_0 = 27 - 0 = 27 \\ r_1' = \hat{a}_1 - a_1 = 27 - 9 = 18 \\ r_2' = \hat{a}_2 - a_2 = 9 - (-6) = 15 \end{cases}$$

$\rightarrow R' = (27 \quad 18 \quad 15)$

Vậy bộ điều khiển phản hồi âm của hệ là:

$$R = R'S = (27 \quad 18 \quad 15)\begin{pmatrix} \frac{1}{4} & \frac{1}{8} & \frac{-1}{8} \\ \frac{3}{4} & \frac{-1}{8} & \frac{1}{8} \\ \frac{9}{4} & \frac{1}{8} & \frac{7}{8} \end{pmatrix} = (54 \quad 3 \quad 12)$$

$$\begin{cases} \dot{\hat{x}} = (A - BR)\hat{x} + Bu \\ \hat{y} = C\hat{x} + 0.u \end{cases}$$

Bộ quan sát trạng thái:

$$A^T = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix}, \quad C^T = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$A^T C^T \cancel{A^T} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$$

$$(A^T)^2 C^T = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 8 \end{pmatrix}$$

$$N = \cancel{\begin{pmatrix} C^T \\ A^T C^T \\ (A^T)^2 C^T \end{pmatrix}} \cancel{= (1} \; \left(C^T \quad A^T C^T \quad (A^T)^2 C^T \right) = \begin{pmatrix} 1 & 3 & 9 \\ 1 & 1 & 5 \\ 0 & 2 & 8 \end{pmatrix}$$

$$N^{-1} = \begin{pmatrix} \frac{1}{4} & \frac{3}{4} & \frac{-3}{4} \\ 1 & -1 & -1/2 \\ -1/4 & 1/4 & 1/4 \end{pmatrix} \rightarrow s^T = \begin{pmatrix} \frac{-1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$$s^T A^T = \begin{pmatrix} \frac{-1}{4} & \frac{1}{4} & \frac{1}{4} \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{-1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

$$s^T (A^T)^2 = \begin{pmatrix} \frac{-1}{2} & \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix} = \begin{pmatrix} \frac{-1}{2} & \frac{3}{2} & 3 \end{pmatrix}$$

$$T = \begin{pmatrix} s^T \\ s^T A^T \\ s^T (A^T)^2 \end{pmatrix} = \begin{pmatrix} \frac{-1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{-1}{2} & \frac{1}{2} & 1 \\ \frac{-1}{2} & \frac{3}{2} & 3 \end{pmatrix} \rightarrow T^{-1} = \begin{pmatrix} 0 & -3 & 1 \\ 8 & -5 & 1 \\ -4 & 2 & 0 \end{pmatrix}$$

$$T . A^T . T^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -9 & 6 \end{pmatrix} \rightarrow a_0 = 0, \; a_1 = 9, \; a_2 = -6$$

$s_1' = s_2' = s_3' = -4$ là các điểm cực

$\rightarrow (s+4)^3 = s^3 + 12s^2 + 48s + 64 \rightarrow \beta_0 = 64, \; \beta_1 = 48, \; \beta_2 = 12$

$$K = \left(\beta_0 - a_0 \quad \beta_1 - a_1 \quad \beta_2 - a_2 \right) = \left(64 \quad 39 \quad 18 \right)$$

Bộ quan sát $L = (KT)^T = \begin{pmatrix} -44,5 \\ 62,5 \\ 109 \end{pmatrix}$

$$\begin{cases} \dot{\hat{x}} = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & 1 \\ 6 & 2 & 2 \end{pmatrix} \hat{x} + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} u + \begin{pmatrix} -44{,}5 \\ 62{,}5 \\ 109 \end{pmatrix} (y - \hat{y}) \\ \hat{y} = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix} \hat{x} + 0u \end{cases}$$