### Week03 Logically

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# 20) $p \leftrightarrow q$ and (p $\bigwedge$ q) $\bigvee$ (!p $\bigwedge$ !q) are logically equivalent

p	$\mathbf{q}$	$\mathbf{p}\leftrightarrow\mathbf{q}$	$p \wedge q$	!p	$(p \land q) \lor (!p \land)$
0	0	1	0	1	1
0	1	0	0	0	0
1	0	0	0	0	0
1	1	1	1	1	1

#### 21) $!(p \leftrightarrow q)$ and $p \leftrightarrow !q$ are logically equivalent

p	$\mathbf{q}$	!p	!q	$!(\mathbf{p}\leftrightarrow\mathbf{q})$	$\mathbf{p}\leftrightarrow\mathbf{!}\mathbf{q}$
0	0	1	1	0	0
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

22)  $p \rightarrow q$  and  $!q \rightarrow !p$  are logically equivalent

р	q	!p	!q	$\mathbf{p} \to \mathbf{q}$	$\mathbf{!q} \rightarrow \mathbf{!p}$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	0	0
1	1	0	0	1	1

23)  $p \leftrightarrow q$  and  $p \leftrightarrow q$  are logically equivalent

p	$\mathbf{q}$	!p	!q	$\mathbf{!p} \leftrightarrow \mathbf{q}$	$\mathbf{p}\leftrightarrow\mathbf{!}\mathbf{q}$
0	0	1	1	0	0
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

24)  $!(p \oplus q)$  and  $p \leftrightarrow q$  are logically equivalent

 $\oplus = \mathbf{xor}$ 

p	$\mathbf{q}$	!(p + q)	$\mathbf{p} \leftrightarrow \mathbf{q}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	1	1

25) !(p  $\leftrightarrow$  q) and !p  $\leftrightarrow$  q are logically equivalent

p	$\mathbf{q}$	!p	$!(\mathbf{p}\leftrightarrow\mathbf{q})$	$\mathbf{!p} \leftrightarrow \mathbf{q}$
0	0	1	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	0	0

26)  $(p\to q)\bigwedge(p\to r)$  and  $p\to (q\bigwedge r)$  are logically equivalent

p	$\mathbf{q}$	r	$\mathbf{p} \to \mathbf{q}$	$\mathbf{p}  ightarrow \mathbf{r}$	$q \wedge r$	$(\mathrm{p}  ightarrow \mathrm{q}) \ igwedge \ (\mathrm{p}  ightarrow \mathrm{r})$	m p  ightarrow (q  ightharpoonup r)
0	0	1	1	1	0	1	1
0	1	1	1	1	1	1	1
0	0	0	1	1	0	1	1
0	1	0	1	1	0	1	1
1	0	1	0	1	0	0	0
1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0

27) (p  $\rightarrow$  r)  $\bigwedge$  (q  $\rightarrow$  r) and (p  $\bigvee$  q)  $\rightarrow$  r are logically equivalent

p	$\mathbf{q}$	r	$\mathbf{p} \rightarrow \mathbf{r}$	$\mathbf{q} \rightarrow \mathbf{r}$	$p \lor q$	${ m (p  ightarrow r) igwedge { m (q  ightarrow r)}}$	$(p \ \bigvee \ q) \to r$
0	0	1	1	1	0	1	1
0	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	1	0	0	0	1	0	0
1	0	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	0	0	0	0	1	0	0
1	1	0	0	0	1	0	0

28) (p  $\rightarrow$  q)  $\bigvee$  (p  $\rightarrow$  r) and p  $\rightarrow$  (q  $\bigvee$  r) are logically equivalent

p	$\mathbf{q}$	r	$\mathbf{p} \to \mathbf{q}$	$\mathbf{p}  ightarrow \mathbf{r}$	$q \ \lor \ r$	$(\mathrm{p}  ightarrow \mathrm{q}) \ igvee \ (\mathrm{p}  ightarrow \mathrm{r})$	$p \to (q \hspace{0.1cm} \bigvee \hspace{0.1cm} r)$
0	0	1	0	1	1	1	1
0	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	1	0	1	0	1	1	1
1	0	1	0	1	1	1	1
1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	1	0	1	0	1	1	1

29) (p  $\rightarrow$  r)  $\bigvee$  (q  $\rightarrow$  r) and (p  $\bigwedge$  q)  $\rightarrow$  r are logically equivalent

p	$\mathbf{q}$	r	$\mathbf{p} \rightarrow \mathbf{r}$	$\mathbf{q} \rightarrow \mathbf{r}$	$p \wedge q$	$(\mathrm{p}  ightarrow \mathrm{r}) \ igvee \ (\mathrm{q}  ightarrow \mathrm{r})$	$(\mathrm{p} \ igwedge \ \mathrm{q})  ightarrow \mathrm{r}$
0	0	1	1	1	0	1	1
0	1	1	1	1	0	1	1
0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	1	0	0	0	1	0	0

30) !p  $\rightarrow$  (q  $\rightarrow$  r) and q  $\rightarrow$  (p  $\bigvee$  r) are logically equivalent

p	$\mathbf{q}$	r	!p	$\mathbf{q} \rightarrow \mathbf{r}$	$p \ \lor r$	$!\mathrm{p} \to (\mathrm{q} \to \mathrm{r})$	$\mathbf{q} \rightarrow (\mathbf{p} \ \bigvee \ \mathbf{r})$
0	0	1	1	1	1	1	1
0	1	1	1	1	1	1	1
0	0	0	1	1	0	1	1
0	1	0	1	0	0	0	0
1	0	1	0	1	1	1	1
1	1	1	0	1	1	1	1
1	0	0	0	1	1	1	1
1	1	0	0	0	1	1	1

# 31) $p \leftrightarrow q$ and $(p \rightarrow q) \ \bigwedge \ (q \rightarrow p)$ are logically equivalent

p	$\mathbf{q}$	$\mathbf{p} \to \mathbf{q}$	$\mathbf{q} \to \mathbf{p}$	$\mathbf{p}\leftrightarrow\mathbf{q}$	$({f p}  ightarrow {f q}) igwedge ({f q}  ightarrow {f p})$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	1	1	1	1

### 32) p $\leftrightarrow$ q and !p $\leftrightarrow$ !q are logically equivalent

p	$\mathbf{q}$	!p	!q	$\mathbf{p}\leftrightarrow\mathbf{q}$	$\mathbf{!p}\leftrightarrow\mathbf{!q}$
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	1	1