

①

Entropy

$$H(Y) = - \sum_{i=1}^K P(Y=y_i) \log_2 P(Y=y_i)$$

For variable Taste:

$$P(Y = \text{Meh}) = 5/10 = \frac{1}{2}$$

$$P(Y = \text{Yummy}) = 5/10 = \frac{1}{2}$$

$$\rightarrow H(Y) = - \left( \frac{1}{2} \cdot \log_2 \frac{1}{2} + \frac{1}{2} \cdot \log_2 \frac{1}{2} \right) = 1$$

②

Information gain is the decrease in entropy after splitting:

$$IG(X) = H(Y) - H(Y|X)$$

$$\begin{aligned}
 H(Y|X) &= -\frac{3}{10} (1 \log_2 1 + 0 \log_2 0) \quad \text{Visual def} \\
 &\quad - \frac{4}{10} \left( \frac{1}{2} \log_2 \frac{1}{2} + \frac{1}{2} \log_2 \frac{1}{2} \right) \quad \begin{array}{c} \text{Some} \\ \text{none} \\ \text{many} \end{array} \\
 &\quad - \frac{3}{10} (0 \log_2 0 + 1 \log_2 1) \quad \begin{array}{c} Y = \text{Meh}: 3 \\ Y = \text{Yummy}: 0 \end{array} \quad \begin{array}{c} \text{meh}: 2 \\ \text{yummy}: 2 \end{array} \quad \begin{array}{c} \text{meh}: 0 \\ \text{yummy}: 3 \end{array} \\
 &= \frac{4}{10} \quad \begin{array}{l} P(X = \text{some}) = 3/10 \\ P(X = \text{none}) = 4/10 \\ P(X = \text{many}) = 3/10 \end{array}
 \end{aligned}$$

$$\rightarrow IG(x) = 1 - \frac{4}{10} = \frac{3}{5}$$

$$\textcircled{3} \quad H(\text{Taste} \mid \text{VD} = \text{some})$$

$$= - \frac{3}{10} \cdot (1 \log_2 1 + 0 \log_2 0)$$

$$= 0$$

$$H(\text{Taste} \mid \text{VD} = \text{none})$$

$$= - \frac{4}{10} \cdot \left( \frac{1}{2} \log \frac{1}{2} + \frac{1}{2} \log \frac{1}{2} \right)$$

$$= \frac{4}{10}$$