Sp22-CS-221-01 HW7 Car Written

Jason Alexander Chan

TOTAL POINTS

22 / 16

QUESTION 1

Problem 4: Which car is it? 9 pts

1.1 Part a 5 / 5

✓ **- 0 pts** Correct

- 0.5 pts Using equality when should be proportionality, or doesn't specify

- 1 pts Forgets or doesn't correctly simplify p(c_11)p(c_12)

- 1 pts Minor error in normal term (forgetting norm, using sigma without squaring, etc.)

- 2 pts Somehow assumes e_11 is always the reading for c_11, or similar

- 2 pts Didn't show steps to derive final expression

- 4 pts Significantly incorrect but some reasonable effort

- 5 pts Missing/completely incorrect

1.2 Part b 4 / 4

✓ **- 0 pts** Correct

- 2 pts Fails to explain all permutations results in the same product of priors, or equivalently assignment does not change the probability (e.g. cars are indistinguishable)

- 2 pts Fails to explain there are $$K!$$ possible assignments in total

- 1 pts Fails to mention the same priors

- 1 pts Minor mistake

- 4 pts Blank/incorrect

1.3 Part c 2 / 0

- 0 pts Not attempted/incorrect

✓ **+ 1 pts Correct Treewidth of K**

✓ **+ 1 pts Correcly identified the factors between the cars and c_t and c_{t+1}**

1.4 Part d 4 / 0

+ 0 pts Skipped/Incorrect

✓ **+ 2 pts Factor graph described correctly**

✓ **+ 2 pts Use of factor graph to compute the probabilities described.**

QUESTION 2

Problem 5: Ethics in Advanced Technologies 7 pts

2.1 Part a 4 / 4

✓ **- 0 pts** Correct

- 2 pts No/incorrect answer to the ethics dumping question

- 2 pts No/incorrect answer to whether the tests should be done in Arizona or California

2.2 Part b 3 / 3

✓ **- 0 pts Sufficient answer**

- 3 pts No answer

which one (it is NOT necessarily car 1, it could be any of the cars). On the other hand, D_{t1} is the measured distance of car 1 (we know with certainty that it comes from car 1), the only issue is that we don't observe it directly.

HINT: To reduce notation, you may write, for example, $p(c_{11} \mid e_{11})$ instead of $p(C_{11} = c_{11} \mid E_{11} = e_{11})$.

What we expect: A mathematical expression, with the steps you took to derive that expression, relating (can be proportionality) $p(C_{11} = c_{11}, C_{12} = c_{12} | \mathbf{E_1} = \mathbf{e_1})$ with the PDF of a Gaussian and the priors $p(c_{11})$ and $p(c_{12})$ over car locations.

Your Solution: Since we don't know if e_1 or e_2 belongs to car c_{11} or c_{12}, we need to add the probabilities of two cases.

Case 1: When e_{11} is assigned to c_{11}:

$$\mathrm{p(c_{11})} p(c_{12}) p_{\mathcal{N}}(e_{11}; \|a_1 - c_{11}\|_2, \sigma^2) p_{\mathcal{N}}(e_{12}; \|a_1 - c_{12}\|_2, \sigma^2) \tag{1}$$

Case 2: When e_{11} is assigned to c_{12}:

$$\mathrm{p(c_{12})} p(c_{11}) p_{\mathcal{N}}(e_{11}; \|a_1 - c_{12}\|_2, \sigma^2) p_{\mathcal{N}}(e_{12}; \|a_1 - c_{11}\|_2, \sigma^2) \tag{2}$$

Adding both cases together to find $p(c_{11}, c_{12} \mid e_{11})$ equals:

$$\mathrm{p(c_{11})} p(c_{12}) \Big(p_{\mathcal{N}}(e_{11}; \|a_1 - c_{11}\|_2, \sigma^2) p_{\mathcal{N}}(e_{12}; \|a_1 - c_{12}\|_2, \sigma^2) + p_{\mathcal{N}}(e_{11}; \|a_1 - c_{12}\|_2, \sigma^2) p_{\mathcal{N}}(e_{12}; \|a_1 - c_{11}\|_2, \sigma^2) \Big) \tag{3}$$

1.1 Part a 5 / 5

✓ **- 0 pts** Correct

- 0.5 pts Using equality when should be proportionality, or doesn't specify

- 1 pts Forgets or doesn't correctly simplify p(c_11)p(c_12)

- 1 pts Minor error in normal term (forgetting norm, using sigma without squaring, etc.)

- 2 pts Somehow assumes e_11 is always the reading for c_11, or similar

- 2 pts Didn't show steps to derive final expression

- 4 pts Significantly incorrect but some reasonable effort

- 5 pts Missing/completely incorrect

b. [4 points] Assuming the prior $p(c_{1i})$ of where the cars start out is the same for all i (i.e. for all K cars), show that the number of assignments for all K cars $(c_{11}, \dots, c_{1K})$ that obtain the maximum value of $p(C_{11} = c_{11}, \dots, C_{1K} = c_{1K} \mid \mathbf{E_1} = \mathbf{e_1})$ is at least $K!$ (K factorial). You can also assume that the car locations that maximize the probability above are unique ($C_{1i} \neq c_{1j}$ for all $i \neq j$).

HINT: The priors $p(c_{1i})$ are a probability distribution over the possible starting positions ($t = 1$) of each car i. Note that even if the car positions share the same prior, it doesn't necessarily mean they have the exact same start positions c_{1i} because the start positions are sampled from the prior distribution, which can yield different values each time it is sampled from. However, you should think about what the priors all being the same means intuitively in terms of how we can associate observations with cars.

NOTE: Note: you don't need to produce a complicated proof for this question. It is acceptable to provide a clear explanation based on your intuitive understanding of the scenario.

What we expect: Either a short mathematical argument or concise explanation for why the statement defined in the problem is true.

Your Solution: In the case where the sensor returns $e_{1i} \neq e_{1j}$ for all $i \neq j$ there are K! assignments because there are K options to assign to e_{11}, K-1 choices to assign to e_{12}, .. , and so on until only 1 option to assign to e_{1K}. This means each individual sensor reading is uniquely assigned to a car to maximise the value of $p(C_{11} = c_{11}, \dots, C_{1K} = c_{1K} \mid \mathbf{E_1} = \mathbf{e_1})$

But what about the cases where some $e_{1i} = e_{1j}$ when $i \neq j$? Two possible reasons why this might occur: perhaps a single car is physically occluding all the other cars from view of the sensor. Or if there's no physical occlusion of the cars then perhaps the cars are closely bunched together but sensor's resolution is too low and returns the same sensor reading for some (or all) cars.

In such cases, you could assign the same individual sensor reading to multiple cars e.g. if $e_{11} = e_{12}$ you could assign e_{11} to c_{11} and c_{12} and assign e_{12} to c_{11} and c_{12} and still maximise the value of $p(C_{11} = c_{11}, \dots, C_{1K} = c_{1K} \mid \mathbf{E_1} = \mathbf{e_1})$. In other words, you can have assignments where repeats are allowed, unlike the unique assignments mentioned in the first paragraph. In an extreme scenario imagine if all emissions were the same value then the number of assignments would be K^K.

Hence, there are at least K! assignments.

c. [2 points, extra credit] For general K, what is the treewidth corresponding to the posterior probability over all K car locations at all T time steps conditioned on all the sensor readings:

1.2 Part b 4 / 4

✓ **- 0 pts** **Correct**

- 2 pts Fails to explain all permutations results in the same product of priors, or equivalently assignment does not change the probability (e.g. cars are indistinguishable)

- 2 pts Fails to explain there are $$K!$$ possible assignments in total

- 1 pts Fails to mention the same priors

- 1 pts Minor mistake

- 4 pts Blank/incorrect

$$p(C_{11} = c_{11}, \ldots, C_{tk} = c_{tk}, \ldots, C_{TK} = c_{TK} \mid \mathbf{E_1} = \mathbf{e_1}, \ldots, \mathbf{E_T} = \mathbf{e_T})$$

Briefly justify your answer.For reference, the treewidth of a factor graph is defined as the maximum arity (number of variables that a factor depends on) of any factor created by variable elimination under the best variable elimination ordering. You can find further information, along with an example, that may be relevant to this problem here.

What we expect: The treewidth as a function of K and a justification for why the treewidth is represented by that function.

Your Solution: The tree-width is K. At each timestep t, all variables c_{ti} to c_{tk} are constrained to $\mathbf{E_1}$. There are K variables for one factor $\mathbf{E_1}$. During each time step, there is only one factor per variable for its transition. This pattern repeats for T timesteps. Hence the tree-width is K.

1.3 Part c **2 / 0**

- **- 0 pts** Not attempted/incorrect
- ✓ **+ 1 pts** **Correct Treewidth of K**
- ✓ **+ 1 pts** **Correcly identified the factors between the cars and c_t and c_{t+1}**

d. [4 points, extra credit] Now suppose you change your sensors so that at each time step t, they return the list of exact positions of the K cars, but the list of positions is shifted by a random number of indices (with wrap around). For example, if the true car positions at time step 1 are $c_{11} = (1, 1), c_{12} = (3, 1), c_{13} = (8, 1), c_{14} = (5, 2)$, then $\mathbf{e_1}$ would be $[(1, 1), (3, 1), (8, 1), (5, 2)]$, $[(3, 1), (8, 1), (5, 2), (1, 1)]$, $[(8, 1), (5, 2), (1, 1), (3, 1)]$, or $[(5, 2), (1, 1), (3, 1), (8, 1)]$, each with probability $1/4$. Describe an efficient algorithm for computing $p(c_{ti} \mid \mathbf{e_1}, \dots, \mathbf{e_T})$ for any time step t and car i. Your algorithm should not be exponential in K or T.

What we expect: A description of the factor graph/Bayesian net used to model the problem, including any relevant variables and conditional probabilities. Also, a description of how you would use the factor graph to compute the provided probability. Note that you should try to simplify your expression for the probability as much as possible given the information provided.

Your Solution: An efficient algorithm for computing $p(c_{ti}|e_1, ..., e_T)$ is the forward-backward algorithm based on the smoothing approach. This runs with time complexity $O(K^2T)$ where T is the length of the time sequence and K is the number of cars.

The smoothing approach *asks for the distribution of some hidden variable c_{ti} conditioned on all the evidence, including the future.* We want to *calculate the weighted fraction of paths that pass through c_{ti}.*

The variables of the factor graph/bayesian net are the sets of cars are each timestep: C_1, C_2,... C_T. The conditional probabilities of the emissions are a function of K, since we are given that the probability of a correct assignment of an emission to a car varies with K.

The general forward-backward algorithm is used to compute the probabilities.

$$F_i(c_i) = F_{i-1}(c_{i-1})p(c_i|c_{i-1})\frac{1}{K}p(e_i|c_i) \tag{4}$$

$$B_i(c_i) = F_{i-1}(c_{i+1})p(c_{i+1}|c_i)\frac{1}{K}p(e_{i+1}|c_{i+1}) \tag{5}$$

$$S_i(c_i) = \frac{F_i(c_i)B_i(c_i)}{F_i(c_i)B_i(c_i)} \tag{6}$$

We can use the general forward backward algorithm to find:

$$p(c_t|e_1, ..., e_T) = S_t(c_t) \tag{7}$$

1.4 Part d 4 / 0

+ **0 pts** Skipped/Incorrect

✓ + **2 pts** **Factor graph described correctly**

✓ + **2 pts** **Use of factor graph to compute the probabilities described.**

Problem 5 : Ethics in Advanced Technologies

a. [4 points] You are in charge of public policy for an autonomous vehicle company headquartered in California. Your engineering team is making progress in designing a fully automated vehicle and would like to test it on real roadways. California would require that you apply for autonomous vehicle testing permits and meet regulatory standards designed to protect the safety of other motorists and pedestrians. Given these regulations, testing your vehicles in California secretly without a permit would be illegal. The Governor of Arizona has reached out to offer your company testing in the state without restrictions. Doing so would allow you to test more quickly and without making any changes to your vehicles. On the other hand, testing in Arizona could be considered "ethics dumping," namely "doing research deemed unethical in a scientist's home country in a country or region with laxer ethical rules" and regulations.

Would testing in Arizona be "ethics dumping"? Be sure to explain why you think it is or isn't. Given your answer to this question and other factors you consider to be relevant, should you perform your tests in Arizona or comply with Californian standards? Justify your answer with a reason as to why.

What we expect: In 3-6 sentences, we expect:

i. a yes or no answer to the ethics dumping question
ii. an explanation of why it is or isn't ethics dumping given the definition above
iii. a yes or no answer to whether the tests should be done in Arizona or California
iv. an explanation that justifies why the tests should be done in Arizona or California

Your Solution: Yes it is ethics dumping. Given the definition above, the autonomous vehicle design is immature: *your engineering team is making progress in designing a fully automated vehicle and would like to test it on real roadways.* This appears to be the first time the vehicle will be tested on real roadways. Given the risk of an immature autonomous vehicle, an independent check is necessary to ensure that our design doesn't introduce more risks and harms to motorists and pedestrians who did not voluntarily submit to sharing the roadways with autonomous vehicles. Circumventing this responsibility is unethical.

Yes, the tests should be done in California. The autonomous vehicle design can then be independently checked for conformance to *regulatory standards.* Safety critical systems need independent assurance. The California permit process may reveal design defects that needs to be addressed but the autonomous vehicle design as a whole is better off. If the Arizona option were taken, and an accident occurred, then

2.1 Part a 4 / 4

✓ **- 0 pts** **Correct**

- 2 pts No/incorrect answer to the ethics dumping question

- 2 pts No/incorrect answer to whether the tests should be done in Arizona or California this would completely destroy public confidence in the entire autonomous vehicle industry as well.

b. [3 points] Dual-use technologies are technologies that serve two purposes, typically a military and a civilian purpose. Researchers developing dual-use technologies face a moral dilemma: though they may intend to improve only the peaceful use of the technology, any improvements they make aid others who use the technology in war or in non-state attacks or killings. Tracking of the kind developed in this assignment can be used in self-driving cars or in autonomous weapons systems, such as lethal drones that track people to kill them.

Imagine a researcher who develops a dual-use technology despite knowing about the lethal secondary use of the technology and despite the researcher considering this secondary use unethical. Would the researcher be partially morally responsible for improvements to the lethal use of the technology that result from their discoveries? If not, give one reason why not. If so, explain why they are partially responsible and explain what action(s) the researcher should take to address this (choosing another line of research, building a safeguard, or other)?

What we expect: In 3-6 sentences, we expect:

i. answers yes or no to whether the researcher would be partially morally responsible
ii. provides a reason why they would or would not be partially morally responsible
iii. if yes, describes an action they should take

Your Solution: Yes the researcher would be partially morally responsible. Even though they aren't actively involved in applying their research to military purposes, if their work contributes to improvements to the lethal use of the technology then that assigns some moral responsibility, especially because they know about the lethal secondary use of their technology and considers that use unethical. The researcher has foresight into the consequences of their research and as a result incur some partial responsibility for how it is used. Inaction certainly equates to partial moral responsibility.

The researcher should first raise this ethical concern with their research group. If the research group shares this concern then collectively there is more power to realign the research targets. If the research group doesn't share the same concern, then the researcher could pivot into an adjacent field to eliminate probability that their efforts are directed to military purposes. Alternatively, the researcher could work on developing safeguards to mitigate military use altogether. The final option is to exit that research field.

2.2 Part b 3 / 3

✓ **- 0 pts** **Sufficient answer**

- 3 pts No answer

CS221 Spring 2022: Artificial Intelligence: Principles and Techniques

Homework 7: Car Tracking

SUNet ID:	jchan7
Name:	Jason Chan
Collaborators:	None

By turning in this assignment, I agree by the Stanford honor code and declare that all of this is my own work.

Collaborators: None

By turning in this assignment, I agree by the Stanford honor code and declare that all of this is my own work.

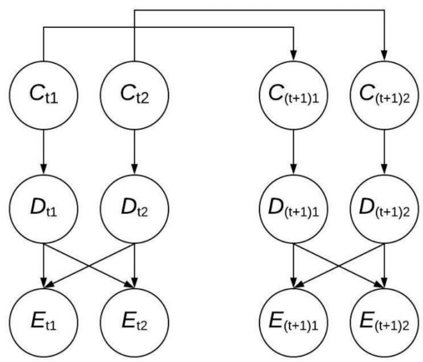
Problem 4 : Which car is it?

So far, we have assumed that we have a distinct noisy distance reading for each car, but in reality, our microphone would just pick up an undistinguished set of these signals, and we wouldn't know which distance reading corresponds to which car. First, let's extend the notation from before: let $C_{ti} \in \mathbb{R}^2$ be the location of the i-th car at the time step t, for $i = 1, \ldots, K$ and $t = 1, \ldots, T$. Recall that all the cars move independently according to the transition dynamics as before.

Let $D_{ti} \in \mathbb{R}$ be the noisy distance measurement of the i-th car at time step t, which is now not directly observed. Instead, we observe the unordered set of distances $\{D_{t1}, \ldots, D_{tK}\}$ as a collective and so cannot attribute any individual measurement in this set to a specific car. (For simplicity, we'll assume that all distances are distinct values.) In other words, you can think of this scenario as the same as observing the list $\mathbf{E_t} = [E_{t1}, \ldots, E_{tK}]$ which is a uniformly random permutation of the (noisy) correctly ordered distances $\mathbf{D_t} = [D_{t1}, \ldots, D_{tK}]$ where index i represents the noisy distance to car i at time t.

For example, suppose $K = 2$ and $T = 2$. Before, we might have gotten distance readings of 8 and 4 for the first car and 5 and 7 for the second car at time steps 1 and 2, respectively. Now, our sensor readings would be permutations of $\{8, 5\}$ (at time step 1) and $\{4, 7\}$ (at time step 2). Thus, even if we knew the second car was distance 5 away at time $t = 1$, we wouldn't know if it moved further away (to distance 7) or closer (to distance 4) at time $t = 2$.

Here is a diagram that shows the flow of information corresponding to the above situation for the case where $K = 2$ and only showing two timesteps, t and $t + 1$. Note that because the observed distances $\mathbf{E_t}$ are a permutation of the true distances $\mathbf{D_t}$, each E_{ti} depends on all of the D_{ti}. Also note that the above diagram is not a Bayes net as E_{t1} and E_{t2} are not conditionally independent given D_{t1} and D_{t2} (however, D_{t1} and D_{t2} are conditionally independent given C_{t1} and C_{t2})



a. [5 points] Suppose we have $K = 2$ cars and one time step $T = 1$. Write an expression for the conditional probability $p(C_{11} = c_{11}, C_{12} = c_{12} \mid \mathbf{E_1} = \mathbf{e_1})$ as a function of the PDF of a Gaussian $p_{\mathcal{N}}(v; \mu, \sigma^2)$ and the prior probability $p(c_{11})$ and $p(c_{12})$ over car locations. Your final answer should not contain variables D_{11}, D_{12}.

   Remember that $p_{\mathcal{N}}(v; \mu, \sigma^2)$ is the probability of a random variable, v, in a Gaussian distribution with mean μ and standard deviation σ.

   HINT: for $K = 1$, the answer would be:

$$p(C_{11} = c_{11} \mid \mathbf{E_1} = \mathbf{e_1}) \propto p(c_{11}) p_{\mathcal{N}}(e_{11}; \|a_1 - c_{11}\|_2, \sigma^2).$$

   where a_t is the position of your car (that you are controlling) at time t. Remember that C_{ti} is the position of the ith observed car at time t. To better inform your use of Bayes' rule, you may find it useful to draw the Bayesian network and think about the distribution of $\mathbf{E_t}$ given $D_{t1}, \ldots, D_{tK}$.

   HINT: Note that the observed variable(s) are the shuffled/randomized distances $\mathbf{E_t} = [E_{t1}, E_{t2}, ..., E_{tK}]$. These are a random permutation of the unobserved noisy distances $\mathbf{D_t} = [D_{t1}, D_{t2}, ..., D_{tK}]$, where D_{t1} is the distance of car 1 at timestep t. Note that E_{t1} is the emission from one of the cars at timestep t, but we aren't sure