f. (*10 points*) **Inference**

Suppose the Bayesian's manager took a nationwide poll in month t that concluded they had exactly 75 fans. Suppose additionally that in month $t+2$, the Bayesians sell 73 jerseys. What is the probability that in month $t+2$ the Bayesians have 77 fans?

$$\Pr(B_{t+2} = 77 | B_t = 75, J_{t+2} = 73) =$$

we have local dist in the form $P(J_{t+2} | B_{t+2})$, so let's try to massage into that

Bayes rule; $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A) \cdot P(B|A)}{P(B)}$

$$P(A|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{P(B) \cdot P(A \cap C | B)}{P(B) \cdot P(C|B)} = \frac{P(A|B) \cdot P(C|A,B)}{P(C|B)}$$
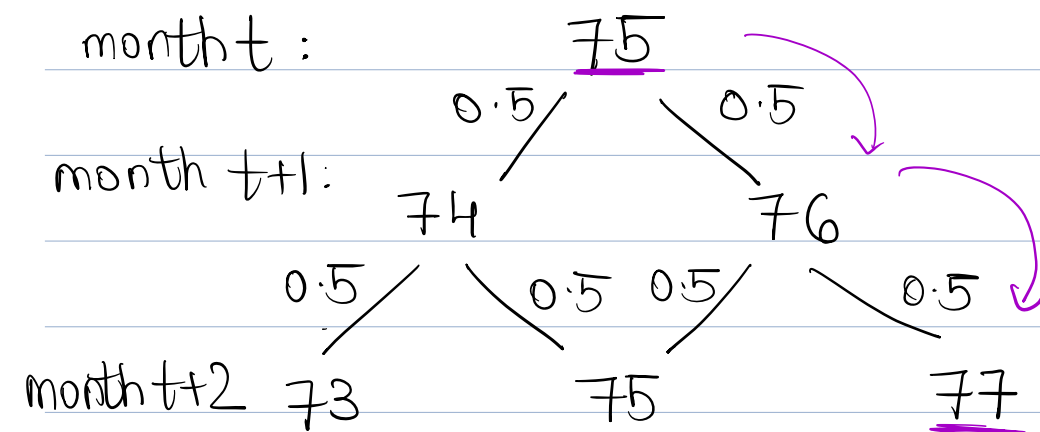
$$\overset{A}{P(B_{t+2}} | \overset{B}{B_t}, \overset{C}{J_{t+2})} = \frac{P(B_{t+2}|B_t) \cdot P(J_{t+2} | B_{t+2}, B_t)}{P(J_{t+2} | B_t)}$$

① $P(J_{t+2} = 73 | B_t = 75, B_{t+2} = 77) \overset{?}{=} P(J_{t+2} = 73 | B_{t+2} = 77)$

b/c B_{t+2} solely determines J_{t+2}. i.e. B_{t+2} is all the info we need to generate J_{t+2}

Look at table $= 0.1$

② $P(B_{t+2}=77|B_t=75)$ don't we need B_{t+1}?
We can do a lil logic, look at our gen story

month t: 75

month $t+1$:   0.5 /   0.5 \
         74   76

month $t+2$ 0.5 /   0.5 \   0.5 /   0.5 \
      73   75   77

$$P(B_{t+2}=77|B_t=75) = 0.5 \times 0.5 = 0.25$$

$P(x) = \sum_y P(x,y) = \sum_y P(y)\cdot P(x|y)$

③ $P(J_{t+2}=73|B_t=75)$ Law of total Prob: $P(x|y) = \sum_z P(x,z|y) = \sum_z \frac{P(x,z,y)}{P(y)}$
$= \sum_z \frac{P(y)\cdot P(z|y)\cdot P(x|z,y)}{P(y)}$

$$= \sum_x P(J_{t+2}=73|B_t=75, B_{t+2}=x)\, P(B_{t+2}=x|B_t=75)$$

Possible x?   73, 75, 77

coz of conditional independence

$= 0.25\, P(J_{t+2}=73|B_{t+2}=73, \cancel{B_t=75}) + 0.5 P(J_{t+2}=73|B_{t+2}=75, \cancel{B_t=75})$
$+ 0.25\, P(J_{t+2}=73|B_{t+2}=77, \cancel{B_t=75})$

$= 0.25 \times 0.3 + (0.25+0.25) \times 0.2 + 0.25 \times 0.1 = 0.2$

$\therefore\ P(B_{t+2}=77|B_t=75, J_{t+2}=73) = \dfrac{0.1 \times 0.25}{0.2} = \boxed{\dfrac{1}{8}}$