

# Sp22-CS-221-01 HW1 Foundations Written

Jason Alexander Chan

TOTAL POINTS

26 / 31

QUESTION 1

## Problem 1: Optimization and probability

18 pts

### 1.1 Problem 1a 2 / 2

✓ - 0 pts Correct

- 0.5 pts Missing or incorrect second derivative test / convexity argument in derivation and negative-weights explanation.

- 0.5 pts Missing or incorrect negative-weights explanation

- 1 pts Right idea but derivation error

- 1.5 pts Incorrect or missing minimizer, but correct negative-weights explanation

- 2 pts Answer or shown work is fully incorrect or missing

### 1.2 Problem 1b 3 / 3

✓ - 0 pts Correct, or only minor proof errors

- 1 pts Proof with nontrivial errors but correct intuition

- 2 pts Incorrect proof and nontrivial errors in intuition

- 3 pts Proof and intuition mostly incorrect or missing

### 1.3 Problem 1c 3 / 3

✓ - 0 pts Correct result  $V = \frac{1}{2} \ln \frac{b-a}{2}$

- 0.5 pts minor computation mistake

- 1 pts mistake in probability transition

- 1.5 pts Correct recurrence function with wrong result

- 3 pts no/incorrect answer

### 1.4 Problem 1d 2 / 3

- 0 pts Correct:  $p = 2/3$

- 1 pts Missing/Wrong intuitive interpretation

✓ - 1 pts Missing/Wrong proof of maximum point (why critical point is maximum point)

- 1 pts Wrong p value

- 3 pts Wrong/no answer

### 1.5 Problem 1e 3 / 3

✓ - 0 pts Correct

- 1 pts Insufficient justification for why  $P(A) = P(B)$

- 1 pts Insufficient work after  $P(A) = P(B)$

- 3 pts Completely incorrect/missing answer

### 1.6 Problem 1f 4 / 4

✓ - 0 pts Correct

- 1 pts Mistake in first term

- 1 pts Mistake in second term

- 3 pts Major mistakes

- 1 pts Vector shape error

- 4 pts No solution/progress

QUESTION 2

## Problem 2: Complexity 5 pts

### 2.1 Problem 2a 2 / 2

✓ - 0 pts Correct

- 1 pts Correct answer, no work

- 1 pts Correct handling of  $n^4$  rectangles, but not the 4 components

- 1 pts Correct handling of 4 components but not rectangle options

- 2 pts Completely incorrect

### 2.2 Problem 2b 2 / 3

- 0 pts Correct

- 1 pts No mention of caching

✓ - 1 pts Missing, suboptimal, or incorrect runtime

- 1 pts Incorrect recursive function
- 2 pts Algorithm unclear
- 3 pts Incorrect

#### QUESTION 3

### Problem 3: Ethical Issue Spotting 8 pts

#### 3.1 Problem 3a 1 / 2

- 0 pts Correct
- ✓ - 1 pts Did not copy out and underline the ethical checklist item
- 2 pts Incorrect

#### 3.2 Problem 3b 1 / 2

- 0 pts Correct
- ✓ - 1 pts Did not copy out and underline the ethical checklist item
- 2 pts Incorrect

#### 3.3 Problem 3c 1 / 2

- 0 pts Correct
- ✓ - 1 pts Did not copy out and underline the ethical checklist item
- 2 pts Incorrect

 Please copy out and underline the ethical checklist item to which you are referring as part of your answer.

#### 3.4 Problem 3d 2 / 2

- ✓ - 0 pts Correct
- 2 pts Incorrect

# CS221 Autumn 2021: Artificial Intelligence: Principles and Techniques

## Homework 1: Foundations

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Name: Jason Alexander Chan  
Collaborators: none

*By turning in this assignment, I agree by the Stanford honor code and declare that all of this is my own work.*

Welcome to your first CS221 assignment! The goal of this assignment is to sharpen your math, programming, and ethical analysis skills needed for this class. If you meet the prerequisites, you should find these problems relatively innocuous. Some of these problems will occur again as subproblems of later homeworks, so make sure you know how to do them. If you're unsure about them or need a refresher, we recommend going through our prerequisites module or other resources on the Internet, or coming to office hours.

**Before you get started, please read the Assignments section on the course website thoroughly.**

### Problem 1: Optimization and probability

In this class, we will cast a lot of AI problems as optimization problems, that is, finding the best solution in a rigorous mathematical sense. At the same time, we must be adroit at coping with uncertainty in the world, and for that, we appeal to tools from probability.

- a. Let  $x_1, \dots, x_n$  be real numbers representing positions on a number line. Let  $w_1, \dots, w_n$  be positive real numbers representing the importance of each of these positions. Consider the quadratic function:  $f(\theta) = \sum_{i=1}^n w_i(\theta - x_i)^2$ . Note that  $\theta$  here is a scalar. What value of  $\theta$  minimizes  $f(\theta)$ ? Show that the optimum you find is indeed a minimum. What problematic issues could arise if some of the  $w_i$ 's are negative?

[**NOTE:** You can think about this problem as trying to find the point  $\theta$  that's not too far away from the  $x_i$ 's. Over time, hopefully you'll appreciate how nice quadratic functions are to minimize.]

[**What we expect:** An expression for the value of  $\theta$  that minimizes  $f(\theta)$  and how you got it. A short calculation/argument to show that it is a minimum. 1-2 sentences describing a problem that could arise if some of the  $w_i$ 's are negative.]

**Your Solution:** We solve for the derivative when it equals zero to find the value of  $\theta$  that minimises the expression  $f(\theta)$

$$f(\theta) = \sum_{i=1}^n w_i(\theta - x_i)^2 \tag{1}$$

Differentiating  $f(\theta)$  with respect to  $\theta$  and setting it to zero

$$\frac{\partial f(\theta)}{\partial \theta} = \sum_{i=1}^n 2w_i(\theta - x_i) \quad (2)$$

$$\frac{\partial f(\theta)}{\partial \theta} = 2\theta \sum_{i=1}^n w_i - 2 \sum_{i=1}^n w_i x_i = 0 \quad (3)$$

$$\theta \sum_{i=1}^n w_i - \sum_{i=1}^n w_i x_i = 0 \quad (4)$$

$$\theta = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i} \quad (5)$$

To confirm that this solution is the global minimum we need to ensure that the second derivative is zero or greater (the global minimum of a quadratic expression)

$$\frac{\partial^2 f(\theta)}{\partial \theta^2} = 2 \sum_{i=1}^n w_i \quad (6)$$

From equation 8, if the sum of  $w_i$  is zero or greater then the expression for  $\theta$  in equation 7 is indeed the global minimum. However, if some of the  $w_i$  are negative then there is a possibility that the result of equation 8 is also negative, which means it's not the global minimum.

### 1.1 Problem 1a 2 / 2

✓ - 0 pts Correct

- 0.5 pts Missing or incorrect second derivative test / convexity argument in derivation and negative-weights explanation.

- 0.5 pts Missing or incorrect negative-weights explanation

- 1 pts Right idea but derivation error

- 1.5 pts Incorrect or missing minimizer, but correct negative-weights explanation

- 2 pts Answer or shown work is fully incorrect or missing

- b. In this class, there will be a lot of sums and maxes. Let's see what happens if we switch the order. Let  $f(\mathbf{x}) = \max_{s \in [-1,1]} \sum_{i=1}^d s x_i$  and  $g(\mathbf{x}) = \sum_{i=1}^d \max_{s_i \in [-1,1]} s_i x_i$ , where  $\mathbf{x} = (x_1, \dots, x_d) \in \mathbb{R}^d$  is a real vector and  $[-1, 1]$  means the closed interval from  $-1$  to  $1$ . Which of  $f(\mathbf{x}) \leq g(\mathbf{x})$ ,  $f(\mathbf{x}) = g(\mathbf{x})$ , or  $f(\mathbf{x}) \geq g(\mathbf{x})$  is true for all  $\mathbf{x}$ ? Prove it.

[**HINT:** You may find it helpful to refactor the expressions so that they are maximizing the same quantity over different sized sets.]

[**What we expect:** A short (3-5) line/sentence proof. You should use mathematical notation in your proof, but can also make your argument in words.]

**Your Solution:**

$$f(\mathbf{x}) = \max_{s \in [-1,1]} \sum_{i=1}^d s x_i \quad (7)$$

$$f(\mathbf{x}) = \max_{s \in [-1,1]} s(x_1 + x_2 + \dots + x_d) \quad (8)$$

The max result for  $f(\mathbf{x})$  requires a choice of  $s \in [-1, 1]$  but since  $x \in \mathbb{R}^d$ , any subset of vectors inside the set  $x = x_1, x_2, \dots, x_d$  can be in different 'directions' in a geometric sense. The maximisation will typically be sub-optimal since we can't select the required scalar  $s$  for each vector in this set in the 'same direction'.

Looking at  $g(\mathbf{x})$

$$g(\mathbf{x}) = \sum_{i=1}^d \max_{s \in [-1,1]} s x_i \quad (9)$$

We can see that for each element of the summation  $s x_i$  we may choose a scalar  $s \in [-1, 1]$  to maximise each element in the summation.

$$g(\mathbf{x}) = \max_{s \in [-1,1]} s x_1 + \max_{s \in [-1,1]} s x_2 + \dots + \max_{s \in [-1,1]} s x_d \quad (10)$$

This overcomes the problem mentioned earlier about  $f(\mathbf{x})$  since  $g(\mathbf{x})$  permits each vector  $x$  to be scaled and tuned.

Finally, we can say that  $g(\mathbf{x})$  must at least equal to  $f(\mathbf{x})$  when the same  $s$  is chosen for each summation element in  $g(\mathbf{x})$  as that which is chosen for  $f(\mathbf{x})$ .

Hence

$$g(\mathbf{x}) \geq f(\mathbf{x}) \quad (11)$$

## 1.2 Problem 1b 3 / 3

- ✓ - **0 pts** Correct, or only minor proof errors
- **1 pts** Proof with nontrivial errors but correct intuition
- **2 pts** Incorrect proof and nontrivial errors in intuition
- **3 pts** Proof and intuition mostly incorrect or missing

- c. Suppose you repeatedly roll a fair six-sided die until you roll a 1 or a 2 (and then you stop). Every time you roll a 3, you lose  $a$  points, and every time you roll a 6, you win  $b$  points. You do not win or lose any points if you roll a 4 or a 5. What is the expected number of points (as a function of  $a$  and  $b$ ) you will have when you stop?

[**HINT:** You will find it helpful to define a recurrence. If you define  $V$  as the expected number of points you get from playing the game, what happens if you roll a 3? You lose  $a$  points and then get to play again. What about the other cases? Can you write this as a recurrence?]

[**What we expect:** A recurrence to represent the problem and the resulting expression from solving the recurrence (no more than 1-2 lines).]

**Your Solution:** Let  $E[X]$  be the expected score after  $n$  number of rolls.

$$E[X] = \frac{2}{6}(0) + \frac{2}{6}(E[X]) + \frac{1}{6}(E[X] - a) + \frac{1}{6}(E[X] + b) \quad (12)$$

$$6(E[X]) = 2(E[X]) + (E[X] - a) + (E[X] + b) \quad (13)$$

$$E[X] = \frac{-a + b}{2} \quad (14)$$

### 1.3 Problem 1c 3 / 3

- ✓ - **0 pts** Correct result  $V = \frac{1}{2}$
- **0.5 pts** minor computation mistake
- **1 pts** mistake in probability transition
- **1.5 pts** Correct recurrence function with wrong result
- **3 pts** no/incorrect answer

- d. Suppose the probability of a coin turning up heads is  $p$  (where  $0 < p < 1$ ), and we flip it 6 times and get  $\{T, H, H, H, T, H\}$ . We know the probability (likelihood) of obtaining this sequence is  $L(p) = (1 - p)p^4(1 - p)^2 = p^4(1 - p)^2$ . What value of  $p$  maximizes  $L(p)$ ? Prove/Show that this value of  $p$  maximizes  $L(p)$ . What is an intuitive interpretation of this value of  $p$ ?

[**HINT:** Consider taking the derivative of  $\log L(p)$ . You can also directly take the derivative of  $L(p)$ , but it is cleaner and more natural to differentiate  $\log L(p)$ . You can verify for yourself that the value of  $p$  which maximizes  $\log L(p)$  must also maximize  $L(p)$  (you are not required to prove this in your solution).]

[**What we expect:** The value of  $p$  that maximizes  $L(p)$  and the work/calculation used to solve for it. Note that you must prove/show that it is a maximum. A 1-sentence intuitive interpretation of the value of  $p$ .]

**Your Solution:** Take the log of the expression  $L(p) = p^4(1 - p)^2$

$$\log(L(p)) = \log(p^4(1 - p)^2) \quad (15)$$

$$= \log(p^4) + \log(1 - p)^2 \quad (16)$$

$$= 4\log(p) + 2\log(1 - p) \quad (17)$$

We then take its derivative, set to zero and solve for  $p$

$$0 = 4\frac{1}{p} + 2\frac{-1}{1 - p} \quad (18)$$

$$0 = 4(1 - p) - 2p \quad (19)$$

$$p = \frac{2}{3} \quad (20)$$

Since  $p$  represents the probability of a head in a two sided coin toss,  $p=2/3$  means that the coin biases towards head which explains why it maximises the probability that we get the sequence of  $T, H, H, H, T, H$  in six tosses where we see 4 heads.

#### 1.4 Problem 1d 2 / 3

- **0 pts** Correct:  $p = 2/3$
- **1 pts** Missing/Wrong intuitive interpretation
- ✓ **- 1 pts** Missing/Wrong proof of maximum point (why critical point is maximum point)
- **1 pts** Wrong p value
- **3 pts** Wrong/no answer

- e. Now for a little bit of practice manipulating conditional probabilities. Suppose that  $A$  and  $B$  are two events such that  $P(A|B) = P(B|A)$ . We also know that  $P(A \cup B) = \frac{1}{2}$  and  $P(A \cap B) > 0$ . Prove that  $P(A) > \frac{1}{4}$ .

[**HINT:** Note that  $A$  and  $B$  are not necessarily mutually exclusive. Consider how we can relate  $P(A \cup B)$  and  $P(A \cap B)$ .]

[**What we expect:** A short ( $\sim 5$  line) proof/derivation.]

**Your Solution:** We can relate  $P(A \cup B)$  and  $P(A \cap B)$  by the following

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad (21)$$

We are given that  $P(A \cup B) = \frac{1}{2}$  and that  $P(A \cap B) > 0$

$$\frac{1}{2} = P(A) + P(B) - P(A \cap B) \quad (22)$$

We also know the following property about conditional probabilities:

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A) \quad (23)$$

We are given that  $P(A|B) = P(B|A)$  so combining with the above:

$$P(B) = P(A) \quad (24)$$

Now the expression reduces to

$$\frac{1}{2} = 2P(A) - P(A \cap B) \quad (25)$$

$$\frac{1}{4} = P(A) - \frac{P(A \cap B)}{2} \quad (26)$$

$$P(A) = \frac{1}{4} + \frac{P(A \cap B)}{2} \quad (27)$$

Finally, since we are given  $P(A \cap B) > 0$  it follows that

$$P(A) > \frac{1}{4} \quad (28)$$

### 1.5 Problem 1e 3 / 3

✓ - 0 pts Correct

- 1 pts Insufficient justification for why  $\$P(A)=P(B)\$$

- 1 pts Insufficient work after  $\$P(A)=P(B)\$$

- 3 pts Completely incorrect/missing answer

- f. Let's practice taking gradients, which is a key operation for being able to optimize continuous functions. For  $\mathbf{w} \in \mathbb{R}^d$  (represented as a column vector), and constants  $\mathbf{a}_i, \mathbf{b}_j \in \mathbb{R}^d$  (also represented as column vectors),  $\lambda \in \mathbb{R}$ , and a positive integer  $n$ , define the scalar-valued function

$$f(\mathbf{w}) = \left( \sum_{i=1}^n \sum_{j=1}^n (\mathbf{a}_i^\top \mathbf{w} - \mathbf{b}_j^\top \mathbf{w})^2 \right) + \frac{\lambda}{2} \|\mathbf{w}\|_2^2,$$

where the vector is  $\mathbf{w} = (w_1, \dots, w_d)^\top$  and  $\|\mathbf{w}\|_2 = \sqrt{\sum_{k=1}^d w_k^2} = \sqrt{\mathbf{w}^\top \mathbf{w}}$  is known as the  $L_2$  norm. Compute the gradient  $\nabla f(\mathbf{w})$ .

**[RECALL]:** The gradient is a  $d$ -dimensional vector of the partial derivatives with respect to each  $w_i$ :

$$\nabla f(\mathbf{w}) = \left( \frac{\partial f(\mathbf{w})}{\partial w_1}, \dots, \frac{\partial f(\mathbf{w})}{\partial w_d} \right)^\top.$$

If you're not comfortable with vector calculus, first warm up by working out this problem using scalars in place of vectors and derivatives in place of gradients. Not everything for scalars goes through for vectors, but the two should at least be consistent with each other (when  $d = 1$ ). Do not write out summations over dimensions, because that gets tedious.]

**[What we expect:** An expression for the gradient and the work used to derive it. ( $\sim 5$  lines). No need to expand out terms unnecessarily; try to write the final answer compactly.]

**Your Solution:** We are given that  $\mathbf{w}, \mathbf{a}_i, \mathbf{b}_j \in R^d$  and the below function.

$$f(\mathbf{w}) = \left( \sum_{i=1}^n \sum_{j=1}^n (\mathbf{a}_i^\top \mathbf{w} - \mathbf{b}_j^\top \mathbf{w})^2 \right) + \frac{\lambda}{2} \|\mathbf{w}\|_2^2 \quad (29)$$

Expanding the contents of the summation

$$\sum_{i=1}^n \sum_{j=1}^n (\mathbf{a}_i^\top \mathbf{w} - \mathbf{b}_j^\top \mathbf{w})^2 = \sum_{i=1}^n \sum_{j=1}^n (\mathbf{w}^\top (\mathbf{a}_i \mathbf{a}_i^\top + \mathbf{a}_i \mathbf{b}_j^\top + \mathbf{b}_j \mathbf{a}_i^\top + \mathbf{b}_j \mathbf{b}_j^\top) \mathbf{w}) \quad (30)$$

The gradient of the summation component of  $f(\mathbf{w})$  becomes:

$$\frac{\partial}{\partial \mathbf{w}} \sum_{i=1}^n \sum_{j=1}^n (\mathbf{a}_i^\top \mathbf{w} - \mathbf{b}_j^\top \mathbf{w})^2 = 2\mathbf{w} \sum_{i=1}^n \sum_{j=1}^n (\mathbf{a}_i \mathbf{a}_i^\top + \mathbf{a}_i \mathbf{b}_j^\top + \mathbf{b}_j \mathbf{a}_i^\top + \mathbf{b}_j \mathbf{b}_j^\top) \quad (31)$$

Looking at the L2 norm component of  $f(\mathbf{w})$  and finding its gradient:

$$\frac{\partial}{\partial \mathbf{w}} \frac{\lambda}{2} \|\mathbf{w}\|_2^2 = \frac{\lambda}{2} 2\mathbf{w} = \lambda\mathbf{w} \quad (32)$$

Combining both components we arrive at the gradient of  $f(\mathbf{w})$

$$\nabla f(\mathbf{w}) = 2\mathbf{w} \sum_{i=1}^n \sum_{j=1}^n (\mathbf{a}_i \mathbf{a}_i^\top + \mathbf{a}_i \mathbf{b}_j^\top + \mathbf{b}_j \mathbf{a}_i^\top + \mathbf{b}_j \mathbf{b}_j^\top) + \lambda\mathbf{w} \quad (33)$$

## 1.6 Problem 1f 4 / 4

✓ - 0 pts Correct

- 1 pts Mistake in first term
- 1 pts Mistake in second term
- 3 pts Major mistakes
- 1 pts Vector shape error
- 4 pts No solution/progress

## Problem 2: Complexity

When designing algorithms, it's useful to be able to do quick back-of-the-envelope calculations to see how much time or space an algorithm needs. Hopefully, you'll start to get more intuition for this by being exposed to different types of problems.

- a. Suppose we have an  $n \times n$  grid of points, where we'd like to place 4 arbitrary axis-aligned rectangles (i.e., the sides of the rectangle are parallel to the axes). Each corner of each rectangle must be one of the points in the grid, but otherwise there are no constraints on the location or size of the rectangles. For example, it is possible for all four corners of a single rectangle to be the same point (resulting in a rectangle of size 0) or for all 4 rectangles to be on top of each other. How many possible ways are there to place 4 rectangles on the grid? In general, we only care about asymptotic complexity, so give your answer in the form of  $O(n^c)$  or  $O(c^n)$  for some integer  $c$ .

[**NOTE:** It is unnecessary to consider whether order matters in this problem, since we are asking for asymptotic complexity. You are free to assume either in your solution, as it doesn't change the final answer.]

[**What we expect:** A big-O bound for the number of possible ways to place 4 rectangles and some simple explanation/reasoning for the answer ( $\sim 2$  sentences).]

**Your Solution:** In the first row of the  $n \times n$  grid we can draw  $n$  number of rectangles of increasing length starting in column 1. But we can also draw rectangles that start in column 2, 3...n. The total length shrinks by 1 for each starting column that we shift right. So for the first row we can draw the following number of rectangles

$$n + n - 1 + n - 2 + \dots + 1 = \frac{n(1+n)}{2} \quad (34)$$

We have two axes ( $n \times n$  grid) so the number of possible rectangles becomes

$$\left(\frac{n(1+n)}{2}\right)\left(\frac{n(1+n)}{2}\right) = \left(\frac{n^2(1+n)^2}{2}\right) \quad (35)$$

Whose big O complexity is

$$O(n^4) \quad (36)$$

Since we want to draw 4 rectangles then the big O complexity becomes

$$O(n^{16}) \quad (37)$$

## 2.1 Problem 2a 2 / 2

✓ - 0 pts Correct

- 1 pts Correct answer, no work

- 1 pts Correct handling of  $n^4$  rectangles, but not the 4 components

- 1 pts Correct handling of 4 components but not rectangle options

- 2 pts Completely incorrect

- b. Suppose we have an  $n \times 3n$  grid of points. We start in the upper-left corner (the point at position  $(1, 1)$ ), and we would like to reach the point at the lower-right corner (the point at position  $(n, 3n)$ ) by taking single steps down or to the right. Suppose we are provided with a function  $c(i, j)$  that outputs the cost associated with position  $(i, j)$ , and assume it takes constant time to compute for each position. Note that  $c(i, j)$  can be negative. Define the cost of a path as the sum of  $c(i, j)$  for all points  $(i, j)$  along the path, including both endpoints. Give an algorithm for computing the cost of the minimum-cost path from  $(1, 1)$  to  $(n, 3n)$  in the most efficient way (with the smallest big-O time complexity). What is the runtime (just give the big-O)?

**[What we expect:** A description of the algorithm for computing the cost of the minimum-cost path as efficiently as possible ( $\sim 5$  sentences). The big-O runtime and a short explanation of how it arises from the algorithm.]

**Your Solution:** Define  $p$  as the set of points  $p = (i, j)$  from  $p_1$ , the starting point to  $p_k$ , the end point.

$$p = p_1, p_2, p_3, \dots, p_k \quad (38)$$

The cost of each point is  $c(i, j)$  so the total cost of the path is given by  $J$

$$J = \sum_{i=1}^k c(p_i) \quad (39)$$

Below is my description of Dijkstra's algorithm to find the minimum cost path.

Find lowest cost path

```

create priority queue based on grid points           ▷ Sorted by cost
for each point do
    set cost = inf, set path via = null
end for
set start point's cost = 0                          ▷ Initialise
while end point in priority queue do
    pop priority queue, get its cost and neighbours
    for each neighbour do
        if neighbour in priority queue then
            if neighbour's cost > cost + cost(neighbour) then
                neighbour's cost = current cost + cost(neighbour)
                set path via = current point
            end if
        else
            skip this neighbour
        end if
    end for
end while
Get cost of last popped point                      ▷ The min. cost

```

The best case big-O for Dijkstra using Fibonacci heaps is

$$O(E + V \log V) \quad (40)$$

Where  $V$  is the number of vertices.

$$V = (3n)(n) = 3n^2 \quad (41)$$

And  $E$  is the number of edges. For an  $m \times n$  grid:

$$E = 2mn - m - n = 2(3n)(n) - 3n - n = 6n^2 - 4n \quad (42)$$

Best case big-O using Fibonacci heaps becomes

$$O(6n^2 - 4n + 3n^2 \log 3n^2) \quad (43)$$

$$O(6n^2 - 4n + 3n^2(\log 3 + \log n^2)) \quad (44)$$

$$O(6n^2 - 4n + (3\log 3)n^2 + 6n^3 \log n) \quad (45)$$

Run time is

$$O(n^3 \log n) \quad (46)$$

## 2.2 Problem 2b 2 / 3

- **0 pts** Correct
- **1 pts** No mention of caching
- ✓ **- 1 pts** Missing, suboptimal, or incorrect runtime
- **1 pts** Incorrect recursive function
- **2 pts** Algorithm unclear
- **3 pts** Incorrect

**Your Solution:** The fact that the model “has significantly higher false positive rate for Black and male applicants than for other groups” immediately indicates that bias is encoded for race and gender. A low bias model would have similar false positive rates across race and gender.

3.1 Problem 3a 1 / 2

- 0 pts Correct
- ✓ - 1 pts Did not copy out and underline the ethical checklist item
- 2 pts Incorrect

- b. Stylometry is a way of predicting the author of contested or anonymous text by analyzing the writing patterns in the anonymous text and other texts written by the potential authors. Recently, highly accurate machine learning algorithms have been developed for this task. While these models are typically used to analyze historical documents and literature, they could be used for deanonymizing a wide range of texts, including code [4].

**Your Solution:** Nefarious actors and states may exploit the capability of stylometry to “deanonymizing a wide range of texts, including code” of persecuted groups that need anonymity for their livelihood and economic security such as political opponents, journalists, suppressed ethnicities and whistleblowers. More generally, individuals have the right to remain anonymous on the internet. With a utilitarian lens, the negative social and political outcomes of stylometry, in my opinion, strongly outweighs the academic benefits derived from stylometry.

### 3.2 Problem 3b 1 / 2

- 0 pts Correct
- ✓ - 1 pts Did not copy out and underline the ethical checklist item
- 2 pts Incorrect

- c. A research group scraped millions of faces of celebrities off of Google images to develop facial recognition technology [5]. The celebrities did not give permission for their images to be used in the dataset and many of the images are copyrighted. For copyrighted photos, the dataset provides URL links to the original image along with bounding boxes for the face.

**Your Solution:** On its face, this scenario first seems to be about law and copyrights. Celebrities, as public figures, should reasonably expect their images to be found across the internet. However, it is apparent that the celebrities never consented to being subjects of research tools. Celebrity deep fakes already showcase how images and videos can be abused to “deceive people in ways that cause harm”.

Copyright owners on the other hand, have legal entitlement and rights to their images. If expressed permission or license was not provided by copyright owners then the research group violated copyrights. Providing the URL links to the original image is insufficient.

### 3.3 Problem 3c 1 / 2

- 0 pts Correct
- ✓ - 1 pts Did not copy out and underline the ethical checklist item

- 2 pts Incorrect

 Please copy out and underline the ethical checklist item to which you are referring as part of your answer.

- d. Researchers have recently created a machine learning model that can predict plant species automatically directly from a single photo [6]. The model was trained using photos uploaded to the iNaturalist app by users who consented to use of their photos for research purposes, and the model is only used within the app to help users identify plants they might come across in the wild.

**Your Solution:** There is no obvious ethical dilemma because the risk of unintended harm is not apparent to any stakeholder group now or in the future. The model is expressly used to identify plants and the users supplying the photos consented to the use of their photos for research purposes. There is a question of business ethics if the data set, which was originally collected for research purposes, were to be commercialised.

3.4 Problem 3d 2 / 2

✓ - 0 pts Correct

- 2 pts Incorrect

## Problem 3: Ethical Issue Spotting

One of the goals of this course is to teach you how to tackle real-world problems with tools from AI. But real-world problems have real-world consequences. Along with technical skills, an important skill every practitioner of AI needs to develop is an awareness of the ethical issues associated with AI. The purpose of this exercise is to practice spotting potential ethical concerns in applications of AI - even seemingly innocuous ones.

In this question, you will explore the ethics of four different real-world scenarios using the ethics guidelines produced by a machine learning research venue, the NeurIPS conference. The [NeurIPS Ethical Guidelines](#) list sixteen non-exhaustive concerns under Potential Negative Social Impacts and General Ethical Conduct (the numbered lists). For each scenario, you will write a potential negative impacts statement. To do so, you will first determine if the algorithm / dataset / technique could have a potential negative social impact or violate general ethical conduct (again, the sixteen numbered items taken from the [NeurIPS Ethical Guidelines](#) page). If the scenario does violate ethical conduct or has potential negative social impacts, list one concern it violates and justify why you think that concern applies to the scenario. If you do **not** think the scenario has an ethical concern, explain how you came to that decision. Unlike earlier problems in the homework there are many possible good answers. If you can justify your answer, then you should feel confident that you have answered the question well.

Each of the scenarios is drawn from a real AI research paper. The ethics of AI research closely mirror the potential real-world consequences of deploying AI, and the lessons you'll draw from this exercise will certainly be applicable to deploying AI at scale. As a note, you are **not** required to read the original papers, but we have linked to them in case they might be useful. Furthermore, you are welcome to respond to anything in the linked article that's not mentioned in the written scenario, but the scenarios as described here should provide enough detail to find at least one concern.

[**What we expect:** A 2-5 sentence paragraph for each of the scenarios where you either A. identify at least one ethical concern from the [NeurIPS Ethical Guidelines](#) and justify why you think it applies, or B. state that you don't think a concern exists and justify why that's the case. Chosen scenarios may have anywhere from zero to multiple concerns that match, but you are only required to pick one concern (if it exists) and justify your decision accordingly. Furthermore, copy out and underline the ethical checklist item to which you are referring as part of your answer (i.e.: Severely damage the environment). We have also included a citation in the example solution below, but you are not required to add citations to your response.]

### Example Scenario

You work for a U.S. hospital that has recently implemented a new intervention program that enrolls at-risk patients in programs to help address their chronic medical issues proactively before the patients end up in the hospital. The intervention program automatically identifies at-risk patients by predicting patients' risk scores, which are measured in terms of healthcare costs. However, you notice that for a given risk score tier, the Black patients are considerably sicker when enrolled than white patients, even though their assigned illness risk score is identical. You manually re-assign patients' risk scores based on their current symptoms and notice that the percentage of Black patients who would be enrolled has increased from 17% to over 45% [1].

### Example Solution

This algorithm has likely encoded, contains, or potentially exacerbates bias against people of a certain race or ethnicity since the algorithm predicts healthcare costs. Because access to medical care in the U.S. is unequal, Black patients tend to have lower healthcare costs than their white counterparts [2]. Thus the algorithm will incorrectly predict that they are at lower risk.

- a. An investment firm develops a simple machine learning model to predict whether an individual is likely to default on a loan from a variety of factors, including location, age, credit score, and public record. After looking through their results, you find that the model predicts mainly based on location and that the model mainly accepts loans from urban centers and denies loans from rural applicants [3]. Furthermore, looking at the gender and ethnicity of the applicants, you find that the model has a significantly higher false positive rate for Black and male applicants than for other groups. In a false positive prediction, a model misclassifies someone who does not default as likely to default.

## Problem 4: Programming

In this problem, you will implement a bunch of short functions. The main purpose of this exercise is to familiarize yourself with Python, but as a bonus, the functions that you will implement will come in handy in subsequent homeworks.

**Do not import any outside libraries (e.g. numpy).** Only standard python libraries and/or the libraries imported in the starter code are allowed.

See `submission.py`. No written submission.

## Submission

Submission is done on Gradescope.

**Written:** When submitting the written parts, make sure to select **all** the pages that contain part of your answer for that problem, or else you will not get credit. To double check after submission, you can click on each problem link on the right side and it should show the pages that are selected for that problem.

**Programming:** After you submit, the autograder will take a few minutes to run. Check back after it runs to make sure that your submission succeeded. If your autograder crashes, you will receive a 0 on the programming part of the assignment. Note: the only file to be submitted to Gradescope is `submission.py`.

More details can be found in the Submission section on the course website.

## References

- [1] Obermeyer et al. Dissecting racial bias in an algorithm used to manage the health of populations. 2019.
- [2] Institutue of Medicine of the National Academies. Unequal Treatment: Confronting Racial and Ethnic Disparities in Health Care. 2003.
- [3] Imperial College London. Loan Default Prediction Dataset. 2014.
- [4] Caliskan-Islam et. al. De-anonymizing programmers via code stylometry. 2015.
- [5] Parkhi et al. VGG Face Dataset. 2015.
- [6] iNaturalist. A new vision model. 2020.