MATLAB: Session 2

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Introduction to Dynamic Programming

- Very general and useful method in economics
- Heavily featured in Labour, IO, Macro, Finance, etc
- In programming camp - informal introduction through 3 examples:
  - Cake-eating problem
  - Patent Renewal
  - PCF Championship

Note: No proofs, only methods!

Finite Horizon Problem

Consider the following general problem:

$$V(y_0, 0) = \max_{\{x_t\}_{t=0}^{T-1}} \sum_{t=0}^{T-1} \beta^t u(x_t, y_t)$$

s.t. $y_{t+1} = f(x_t, y_t)$

where y_t is the state variable and x_t is the control variable.

Solution Method: Backward Induction

Backward Induction

$$
\begin{aligned}
V(y_0, 0) &= \max_{\{x_t\}_{t=0}^{T-1}} \sum_{t=0}^{T-1} \beta^t u(x_t, y_t) \\
&= \max_{\{x_t\}_{t=0}^{T-1}} u(x_0, y_0) + \beta \sum_{t=0}^{T-2} \beta^t u(x_{t+1}, y_{t+1}) \\
&= \max_{\{x_t\}_{t=0}^{T-1}} u(x_0, y_0) + \beta V(y_1, 1)
\end{aligned}
$$

There, we can solve the problem, if and only if we knew $V(., 1)$!

BUT, we know that $V(., T) = 0$!! $\quad \rightarrow$ solve for $V(., T-1)$...

Cake-Eating Problem

$$V(y_0) = \max_{\{x_t\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t \log(x_t)$$

subject to

- $y_{t+1} = y_t - x_t$
- $x_t = 0;\ y_{t+1} = 0$
- $y_0 = A$

Backward Induction: Solution Method

Steps

1. Set up V-grid that defines the value function over T states
2. Define state variable grid on which you will evaluate $V(., t)$
3. Set up the loop over backward induction
    - at each t
        - Construct a spline function for $V(., t+1)$
        - Set up grid of potential x_t choices (and satisfies constraints !!)
        - Find optimal choices, saving it as well as the value function

Simulation of AR(1) Process

Consider the following process

$$x_t = \rho_x x_{t-1} + \epsilon_t \quad \text{s.t.} \quad \epsilon_t \sim N(0, \sigma)$$

GOAL: Simulate the Series

- STEP 1: Generate random draws of ϵ_t
- STEP 2: Generate implied evolution of series given the draws

Exercises

1. Evaluate estimated standard deviation of series
2. Construct Figure for evolution of AR(1) process
3. Do the same for AR(2) process